

Labor Markets, Inequality and Hiring Selection

Preliminary

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Motivation

- Labor markets flows involve **selection** at different levels: hiring, poaching, separations.
- Highly productive workers not only earn more, but they also get jobs more often.

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- Highly productive workers not only earn more, but they also get jobs more often.
- Composition of employment is changed through worker flows.
- Selection has an important effect in shaping wage inequality.
- In line with this idea, we document a relation between **worker flows**, **unemployment rate** and **wage inequality**.

What we do

- We develop a framework to match these facts and to understand the impact of hiring selectivity and of the screening process on wage inequality, allowing for on-the-job search.

What we do

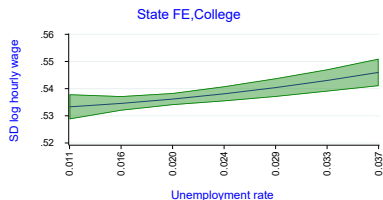
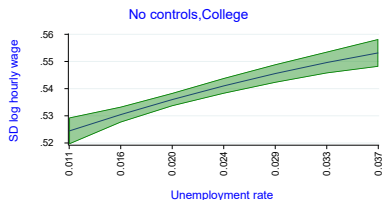
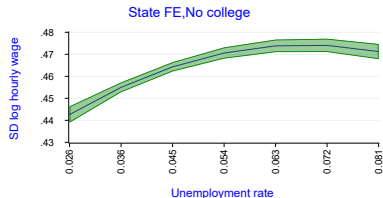
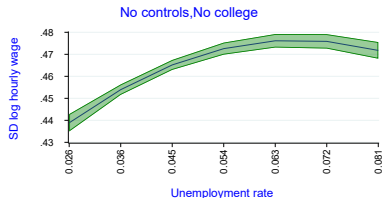
- We develop a framework to match these facts and to understand the impact of hiring selectivity and of the screening process on wage inequality, allowing for on-the-job search.
- We adopt a non-sequential modelling.
- Estimate model, GMM.
- Use model to understand what shocks generate the relation between unemployment & inequality as seen in data.
- Use the model to prescribe optimal policies.

Literature review

- Non-sequential search: Burdett (1977), Blanchard and Diamond (1994), Moen (1999).
 - Non-sequential search in directed search models: Wolthoff (2017), Fernandez-Blanco and Preugschat (2018), Cai et al. (2021).
- Composition of employed-unemployed pool across cycle: Eeckout, Lindenlaub (2019), Engbom (2020), Bradley (2020).

Stylised fact I

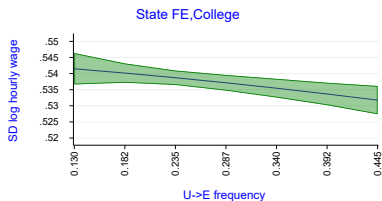
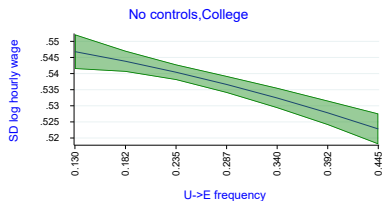
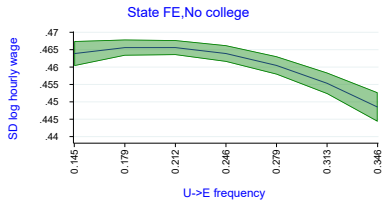
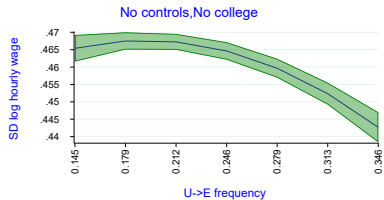
- Wage dispersion vs Unempl. rate.



Sample 1976-2019. US real log hourly wages.

Stylised fact II

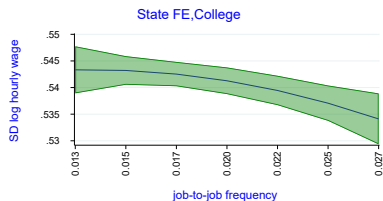
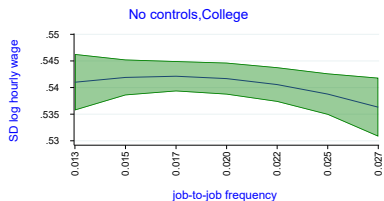
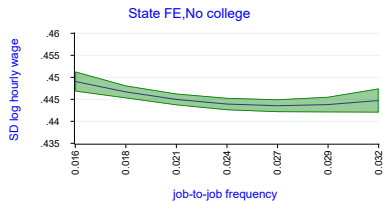
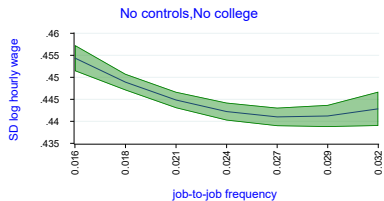
- Wage dispersion vs Job finding rate.



Sample 1976-2019. US real log hourly wages.

Stylised fact III

- Wage dispersion vs Job-to-Job transition rate.



Sample 1994-2019. US real log hourly wages.

Environment

- Time discrete, stationary economy.
- Workers are heterogenous in time-invariant productivity θ and risk-neutral.
- Exogenous distribution of productivities $g(\theta)$.
- Employers are *ex ante* identical and risk-neutral.
- Separation is exogenous with probability η .
- **Non-sequential**, non directed search, general equilibrium.

Environment

- Each vacancy meets a random number of applicants K each period coming from $g_A(\theta)$.
- Large economy $K \sim \text{Poisson}(q = \mathcal{A}/\mathcal{V})$.
- Our conjecture is that employers want to hire the highest θ among interviewees (later verified *numerically*):

$$p(\theta; q) = \sum_{k=1}^{\infty} \underbrace{\frac{q^{k-1} e^{-q}}{(k-1)!}}_{\text{Prob}(K=k)} \underbrace{G_A(\theta)^{k-1}}_{\text{Top of } k-1} = e^{-q(1-G_A(\theta))}$$

- Hence, it is possible to write the probability of being hired as a function of q and the applicant's ranking $x = G_A(\theta)$ such that

$$p(x) = e^{-q(1-x)}$$

Environment

- All the unemployed apply to a randomly picked job.
- A fraction $\lambda \in [0, 1]$ of the employed apply to jobs or received job offers (on-the-job search).
- When the job value is only determined by θ , switching jobs is naturally random.

Environment

- All the unemployed apply to a randomly picked job.
- A fraction $\lambda \in [0, 1]$ of the employed apply to jobs or received job offers (on-the-job search).
- When the job value is only determined by θ , switching jobs is naturally random.
- Applicant pool is a mixture of employed and unemployed:
 $\mathcal{A} \equiv \mathcal{U} + \lambda(1 - \mathcal{U})$.
- Distribution of applicants is a mixture of distributions of unemployed and employed workers.

$$g_A(\theta) = \frac{\mathcal{U}g_U(\theta) + \lambda(1 - \mathcal{U})g_E(\theta)}{\mathcal{U} + \lambda(1 - \mathcal{U})}$$

Distributions

- Plugging the prob of being hired $p(G_A(\theta))$, we get the differential equation

$$g_A(\theta) = \frac{dG_A(\theta)}{d\theta} = \frac{(\eta + \lambda e^{-q(1-G_A(\theta))})g(\theta)}{\mathcal{A}(\eta + e^{-q(1-G_A(\theta))})}$$

Distributions

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- Separable differential equation with closed-form solution.
- Use border conditions such that $G_A(0) = 0$ and $G_A(\infty) = 1$. We get \mathcal{A} to make density to integrate to 1.

Key result: Quantile Mapping

- Mapping between a quantile of the applicants $x = G_A(\theta)$ and a quantile of the population distribution $G(\theta)$:

$$G^{-1}(M(x; q)) = G_A^{-1}(x) = \theta$$

- Key result because we can bypass the unknown distribution of applicants G_A using the primitive G .

Competitive equilibrium

- Lifetime utility of employed:

$$W(\theta) = w(\theta) + \beta \left[\underbrace{(1-\lambda)\eta}_{\eta^*} U + (1 - \underbrace{(1-\lambda)\eta}_{\eta^*}) W(\theta) \right]$$

- Lifetime utility of unemployed:

$$U(\theta) = \rho\theta + \beta \left[\underbrace{p(\theta)}_{\text{JF prob}} W(\theta) + (1 - p(\theta)) U(\theta) \right]$$

- $J(\theta)$ is the firm's value obtained from a worker of type θ :

$$J(\theta) = \theta - w(\theta) + \beta \left[(1-\lambda)\eta V + \lambda p(\theta) V + (1 - \lambda p(\theta) - (1-\lambda)\eta) J(\theta) \right]$$

Competitive equilibrium

- The firm has all the bargaining power:

$$w(\theta) = (1 - \beta)U = \rho\theta$$

- The value of posting a vacancy is

$$V = -\kappa + \beta \mathbb{E}_K[\max\{H(k), V\}] \text{ with}$$
$$H(k) = -\xi k + \int J(\theta)kG_A(\theta)^{k-1}dG_A(\theta)$$

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- Value of posting a vacancy:

$$-\kappa + \beta \left\{ \sum_{k=1}^{\infty} \frac{e^{-q}(q)^k}{k!} \left[-\xi k + \int_0^{\infty} J(v)k(G_A(v))^{k-1}g_A(v)dv \right] + e^{-q}V \right\} = V$$

where $\sum_{k=1}^{\infty} \frac{e^{-q}(q)^k}{k!}$ is the probability of receiving at least one application and e^{-q} is the probability of receiving zero applications.

Competitive equilibrium

- Entry condition: $V = \chi > 0$
- Finally, we get

$$\kappa + \beta\xi q + \chi(1 - \beta e^{-q}) = \beta \int_0^1 J(G^{-1}(M(x; q))) q e^{-q(1-x)} dx$$

Taking the model to the data

- In labor surveys such as monthly CPS since 1994, we directly observe
 - Wage distribution
 - Unemployment rate \mathcal{U}
 - Job finding frequency (UE) \bar{p}_U
 - Separation frequency (EU) η^*
 - Job-to-job frequency (JJ) $\lambda\bar{p}_E$
- We assume the exogenous distribution $G(\theta)$ is log-normal.
- Estimation via GMM using moments given by each percentile of the **wage distribution x , unempl rate, j-t-j and separations.**

Estimation plan

- We map the distribution of employed workers G_E to the population and applicant distributions, G and G_A .
- Quantile mapping recover the population type distribution G if we observed the application distribution G_A .

$$\begin{aligned} \min_{\eta, \lambda, q, \mu, \sigma, \rho} Q = & \left\{ [\mathcal{U}(\lambda, q, \eta) - \tilde{U}]^2 + \varphi_1 [\lambda \bar{p}_E(\lambda, q, \eta) - \tilde{J}]^2 \right. \\ & + \varphi_2 \left[\int_0^1 (\rho G^{-1}(M(x; q, \eta, \lambda)) - \hat{w}(x))^2 dx \right] \\ & \left. + \varphi_3 [\eta(1 - \lambda) - \tilde{E}U]^2 \right\} \end{aligned}$$

where $\hat{w}(x) = \hat{G}_w^{-1} \left(\frac{M(x; \lambda, q, \eta) - \mathcal{A}(\lambda, q, \eta)x}{(1-\lambda)(1-\mathcal{U}(\lambda, q, \eta))} \right)$.

[Details of wage-productivity link](#)

Estimation results

Table: Parameters' estimation

Parameter	Baseline	
	College	Non College
η	0.007	0.017
λ	0.045	0.057
q	2.465	2.867
μ	3.976	3.447
σ	0.533	0.440
ρ	0.330	0.397
Min fun	0.418	0.893

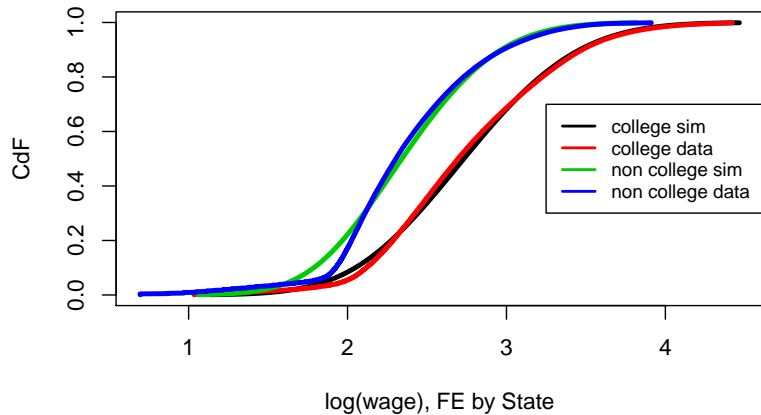
Estimation results: data vs model

Table: Data vs. Model generated moments

Statistic	Baseline	
	College	Non college
Unempl. rate data	0.027	0.065
Unempl. rate model	0.026	0.064
Job-to-job trans. data	0.019	0.023
Job-to-job trans. model	0.019	0.024
Separation rates data	0.006	0.014
Separation rates model	0.007	0.016
Median wage in \$ data	17.267	12.251
Median wage in \$ model	15.361	10.403

Remember that $JJ = \lambda \bar{p}_E$ and $EU = \eta(1 - \lambda)$.

Estimation results: CdF log wages



Closing the model

- Combining with some extra information on β, κ and ξ , we find χ with the free-entry condition:

$$\kappa + \beta\xi\hat{q} + \chi(1 - \beta\hat{q}) = \beta\hat{q} \int_0^1 J \left(G^{-1} \left(M(x; \hat{\eta}, \hat{q}, \hat{\lambda}); \hat{\rho}, \hat{\Gamma}, \chi \right) \right) e^{-\hat{q}(1-x)} dx$$

Closing the model

Vacancy posting and screening costs

- We consider online job posting fees as flow cost: between 200\$ and 375\$ per ad per month → we consider $\kappa = 300$ \$ per month.
- We use the National Employer Survey 1997 (NES97) to compute the average monetary cost that year for recruiting activities (ξ).
- We adapt the idea of Landais et al. (2017) and compute adaption factor ϕ : $\xi_{1997} = \phi \times \text{wage recruiters}_{1997}$

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- Fixed costs χ are of important magnitude (around 200,000 USD and 98,000 USD for college and non college).

What can we learn?

Still work in progress

- Do labor market frictions amplify or reduce *ex ante* inequality?
 - It can be shown that

$$\underbrace{G_E^{-1}(\bar{z}) - G_E^{-1}(z)}_{\approx \text{wage gap}} = \frac{g(G^{-1}(z))}{g(G_E^{-1}(z))} \frac{1 - \mathcal{U}}{\mathcal{E}(x(z, q), q)} \underbrace{(G^{-1}(\bar{z}) - G^{-1}(z))}_{\text{productivity gap}}$$

where \underline{z} and \bar{z} are two quantiles, $x(z, q)$ is the quantile corresponding to the mean value z and $\mathcal{E}(x(z, q), q) = \frac{\rho(x(z, q), q)}{\eta + \rho(x(z, q), q)}$.

- The quantile gap (\underline{z}, \bar{z}) can be amplified or reduced by the economy, and it depends on the unemployment rate level.

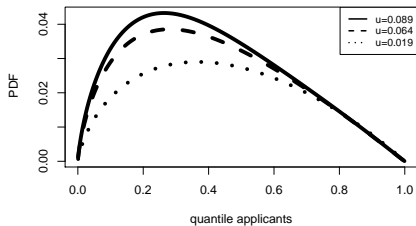
What can we learn?

- We run counterfactuals and get impacts on wage inequality of:
 - ① Changes in the mean of productivity distribution.
 - ② Changes in both mean and spread of productivity distribution.
 - ③ Changes in screening costs ξ .

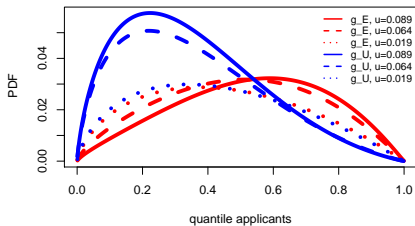
Counterfactual I

Effects of an increase in average productivity (constant spread)

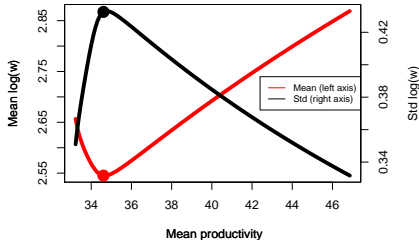
g_A no college



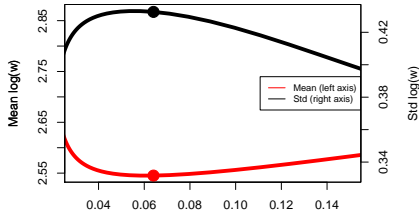
g_E, g_U no college



Wages and mean productivity, no college

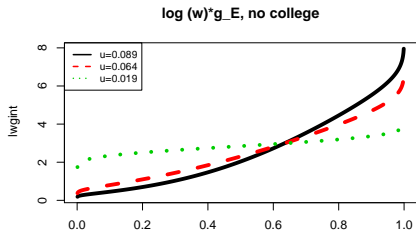
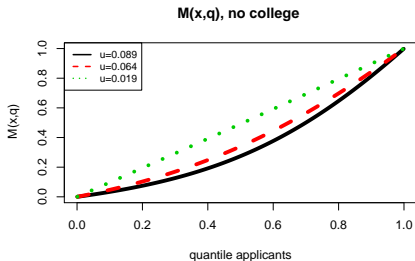
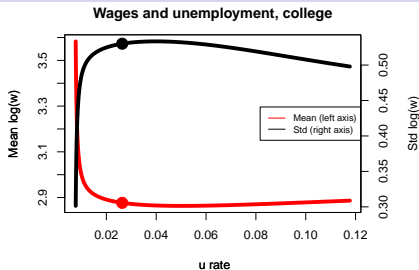
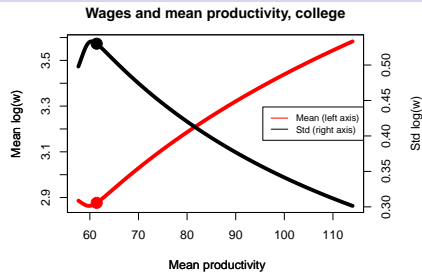


Wages and unemployment, no college



Counterfactual I

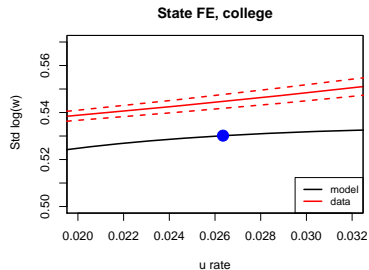
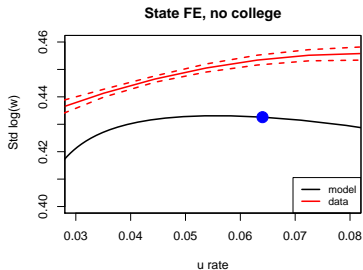
Effects of an increase in average productivity (constant spread)



Counterfactual I

Effects of an increase in average productivity (constant spread)

- Changes in mean, constant spread



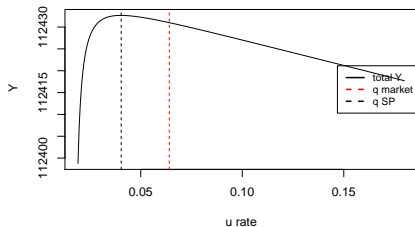
Efficiency analysis

- Standard approach: the Social Planner (SP) is subject to the same frictions as the market economy.

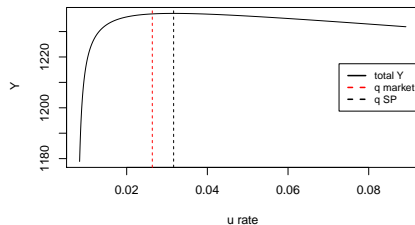
$$\begin{aligned} \max_q Y(q) = & \beta(1 - \mathcal{U}) \int_0^\infty \theta g_E(\theta) d\theta + \beta \mathcal{U} \int_0^\infty \rho \theta g_U(\theta) d\theta + \\ & - \underbrace{[(1 - e^{-q})\beta\xi\mathcal{A}(q) + \chi + \kappa\mathcal{V}]}_{\text{Recruiting costs}} \end{aligned}$$

Efficiency analysis

No College

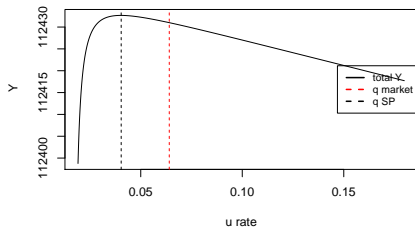


College

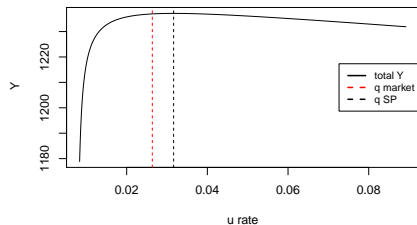


Efficiency analysis

No College



College



- Is there a tax/subsidy $\tau(x; q)$ that restores efficiency?
- Consider a typical tax schedule in public finance:

$$T(y) = y - \tau_0 y^{1-\tau_1}$$

and apply it to the profit function of the firm.

Efficiency analysis

- The net value of a filled job becomes: $(1 - \tilde{t}(\theta))J(\theta)$, where $\tilde{t}(\theta)$ is the average tax rate paid for productivity level θ .
- $\tilde{t}(\theta) = 1 - \tau_0 \theta^{-\tau_1}$
- By applying our usual change of variable:
 $\bar{t}(x) = 1 - \tau_0 (G^{-1}(M(x; q)))^{-\tau_1}$

Efficiency analysis

- The modified free entry condition:

$$\kappa + \beta \xi q(1 - e^{-q}) + \chi(1 - \beta e^{-q}) = \beta \int_0^1 J(G^{-1}(M(x; q))(1 - \bar{t}(x))) e^{-q(1-x)} dx$$

- Additional condition: Balanced Government budget.

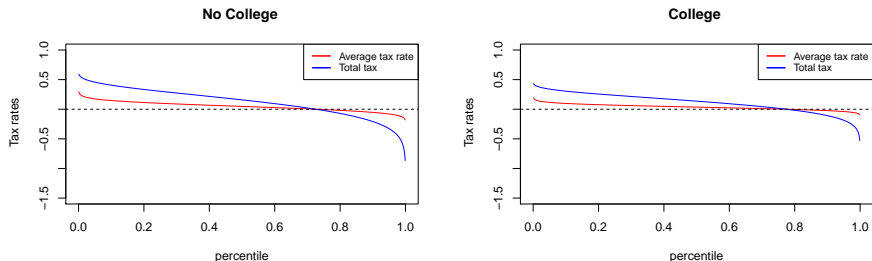
$$(1 - \mathcal{U}(q)) \int_0^{\infty} \tilde{t}(\theta) \theta dG_E(\theta) = 0$$

- Check ex-post that Coincidence Ranking condition is respected:
 $(J(G^{-1}(M(x; q))(1 - \bar{t}(x))))' > 0$

Efficiency analysis

- Optimal values for non college: $\tau_0 = 0.396$ and $\tau_1 = -0.726$, implying a regressive tax and transfer scheme.
- Optimal values for college: $\tau_0 = 0.523$ and $\tau_1 = -0.451$, implying a regressive tax and transfer scheme.

Figure: Tax and transfer schedule



Interpretation of normative issues

- High type workers/matches are the prime reason why employers post vacancies.
- High θ workers/matches generate a positive externality to low types: they spur vacancy posting and decrease selectivity.
- Hence, they are subsidized!

Conclusions

- Labor market agents make screening decisions every day, with impact on inequality.
- We develop a model to study those issues on top of well-known search and matching models.
- Some more work to do:
 - Understand better the mechanisms, analytical measure of relation between *ex ante* and *ex post* inequality.
 - Dive deeper into the differences college/non-college.

Thank you!

Appendix

Linking to the wage distribution

- Given the quantile mapping $\theta = G_A^{-1}(x) = G^{-1}(M(x; q, \eta, \lambda); \Gamma)$, we get

$$G_E(\theta) = \frac{G(\theta) - \mathcal{A}G_A(\theta)}{(1-\lambda)(1-\mathcal{U})}$$

$$G_E(G^{-1}(M(x; q, \eta, \lambda))) = \frac{M(x; q, \eta, \lambda) - \mathcal{A}x}{(1-\lambda)(1-\mathcal{U})}$$

- G_E is not observed, but we have the wage distribution \widehat{G}_w (CPS-ORG data).
- Moreover, wage and productivity rankings are the same (CRE)

$$G_E(G^{-1}(M(x; q, \eta, \lambda))) = \widehat{G}_w(\tilde{w}(x)) = \frac{M(x; q, \eta, \lambda) - \mathcal{A}x}{(1-\lambda)(1-\mathcal{U})}$$

Linking to wage-setting

- We need a wage-setting model to link productivity and wages. In our case, it is simply $\tilde{w} = \rho\theta$.
- Using the quantile mapping, we get

$$\tilde{w}(x) = \rho G^{-1}(M(x; q, \eta, \lambda)) = \hat{G}_w^{-1} \left(\frac{M(x; q, \eta, \lambda) - \mathcal{A}x}{(1 - \lambda)(1 - \mathcal{U})} \right)$$

Back

Key result: Quantile Mapping

- It can be shown that

$$g_U(\theta) = \frac{g(\theta) \frac{\eta}{(\eta+p(\theta))}}{\mathcal{U}}$$

- Therefore, the density of the employed is

$$g_E(\theta) = \frac{g(\theta) \frac{p(\theta)}{(\eta+p(\theta))}}{(1 - \mathcal{U})}$$

- Hence, the distribution of applicants received by firms is

$$g_A(\theta) = \frac{(\eta + \lambda p(\theta))g(\theta)}{\mathcal{A}(\eta + p(\theta))}$$

with $\mathcal{A} \equiv \mathcal{U} + \lambda(1 - \mathcal{U})$.

Key result: Quantile Mapping

- Mapping between a quantile of the applicants $x = G_A(\theta)$ and a quantile of the population distribution $G(\theta)$:

$$G^{-1}(M(x; q)) = G_A^{-1}(x) = \theta$$

$$\text{with } M(x, q) \equiv \frac{m(x, q) - m(0, q)}{m(1, q) - m(0, q)}$$

$$\text{and } m(x, q) \equiv x + \frac{1 - \lambda}{\lambda q} \left(\log \left(\eta + \lambda e^{-q(1-x)} \right) \right)$$

- Key result because we can bypass the unknown distribution of applicants G_A using the primitive G .

Unemployment rate

- Key relation between applicants and unemployed is $\mathcal{A} = \mathcal{U} + \lambda(1 - \mathcal{U})$, the unemployment rate is
- Hence $\mathcal{A} = (m(1, q) - m(0, q))^{-1}$ and $\mathcal{U} = \frac{\mathcal{A} - \lambda}{1 - \lambda}$
- The unemployment rate converges (L'Hôpital rule) to a well-known formula when $\lambda \rightarrow 0$

$$\lim_{\lambda \rightarrow 0} \mathcal{U} = \frac{\eta}{\eta + \frac{1 - e^{-q}}{q}}$$

where $\frac{1 - e^{-q}}{q}$ is the average prob of being hired when there is no on-the-job search.