On-the-Job Search with Hidden Attributes

(PRELIMINARY AND INCOMPLETE)

Tomasz Sulka*

August 17, 2022

Abstract

This paper addresses the issues of optimal search strategy and optimal contract design when some ('hidden') attributes of an alternative are difficult to evaluate *ex ante*, but an individual eventually learns their true value and can conduct 'on-the-job' search. I analyse a model where firms' offers consist of an observable component ('prices', or 'wages') and a hidden component ('quality', or 'benefits') that the individual learns only after accepting a particular offer. Focusing on agents who cannot infer the value of the hidden benefits from the offered wage, but nonetheless take future learning into account, I show first that agents who naively overestimate the option value of searching on the job adopt a search strategy with a lower reservation wage initially. Endogenising the firms' offers, I demonstrate that a pure-strategy equilibrium with inefficiently low benefits is easier to sustain than an equilibrium with socially desirable high benefits, even though the firms make greater profits in the latter case. When different agent types disagree about the option value of searching on the job, there may also exist asymmetric pure-strategy equilibria, in which low wages are paired with high benefits, and vice versa. The logic behind this result is very different from the theory of compensating differentials, however.

JEL: D43, D83, D91, J31, J32, J64

Keywords: consumer search, on-the-job search, hidden attributes, shrouded attributes, naivete

1 Introduction

Important economic decisions typically require individuals to compare complex, multidimensional alternatives and think their way through many possible contingencies. When consumers

*DICE, University of Düsseldorf. Email: sulka@dice.hhu.de

are boundedly rational, i.e. they do not have unlimited information processing capabilities, a better informed party might exploit this by making certain ('headline') attributes of an alternative appear attractive, but shrouding other ('hidden') attributes (Gabaix and Laibson, 2006).[1]

Even though boundedly rational consumers may prefer an option with attractive headline attributes initially, it is natural to imagine that they will eventually learn the total utility they derive from it. Will the consumers stick with the choice they know and understand, or will they search again for a better alternative? How is the possibility to search further 'on the job' reflected in the initial search strategy? How does it affect the incentives of the providers offering complex, multidimensional contracts? These questions appear meaningful across a variety of contexts, but remain largely unexplored in the current literature (see the review below).

In this paper, I analyse a dynamic search problem when alternatives are characterised by a headline as well as a hidden attribute. An individual performs random, sequential search with perfect recall, comparing only the headline attributes of the alternatives she encounters. Once she accepts one of the offers, she additionally learns the hidden attribute of the alternative she selected. Subsequently, the individual has an option to search for a better alternative on the job, or to stick with her current choice. The individual takes this future learning into account when sampling offers initially.

Supposing that search costs increase on the job, I make a distinction between "sophisticated" agents, who anticipate the increase, and "naïve" agents, who do not. This focuses the discussion on the dynamics induced by learning, while holding the initial unobservability of one of the attributes constant across different agent types. Moreover, naiveté about future search cost might arise from time-inconsistent preferences or loss aversion, thus capturing a broad range of prevalent behavioural biases. Performing a partial-equilibrium analysis with a given distribution of offers, I confirm the intuition that an individual who underestimates her future search costs, overestimates the option value of searching on the job, and consequently adopts a lower threshold for accepting an offer initially.

To endogenise offers made by the firms, I analyse a setting in which profit-maximising firms make their offers to a population of sophisticated and naïve individuals. It turns out that a pure-strategy equilibrium with inefficiently poor hidden attributes is easier to sustain than an equilibrium with socially desirable hidden attributes, even though the firms make greater profits in the latter case. The reason behind it is that with the hidden attribute being observable only after the acceptance of an offer, the firms have a greater incentive to deviate and 'surprise' an individual with a poor hidden attribute rather than with a generous one. There may also

[1] Empirical examples of sellers exploiting insufficient attention paid by the consumers to the product's hidden attributes range from printer toner pricing (Gabaix and Laibson, 2006) to mutual fund fee structures (Barber, Odean, and Zheng, 2005), mortgage pricing (Campbell, 2006), and design of retail investment products (Célérier and Vallée, 2017). See Heidhues and Kőszegi (2018) for a recent review of the related literature in behavioural industrial organisation and Wilson (2010) for a model of obfuscation with fully rational consumers.

exist asymmetric pure-strategy equilibria, in which relatively unattractive headline attributes are paired with generous hidden attributes, and vice versa. The logic behind this result is very different from the one underlying the theory of compensating differentials (e.g., Rosen, 1986), which relies on observability and substitutability between the two components of an offer. As a result, although the equilibria predicted by the two theories might be observationally equivalent, they have starkly different efficiency properties.

A particular contractual agreement that I use as an application of on-the-job search with hidden attributes is a multidimensional compensation package, whereby the 'wage' is interpreted as its observable and 'benefits' as its unobservable component. Over the last two decades, the generosity of social security benefits has been substantially reduced in most developed countries, resulting in greater importance of private insurance provisions, often offered at a workplace.[2] Perhaps the most universal example is the rise in importance of private pensions (OECD, 2019). At the same time, there is abundant evidence that a typical household is ill-equipped to evaluate complex financial contracts (e.g., Beshears, Choi, Laibson, and Madrian, 2018; Campbell, 2016; Lusardi and Mitchell, 2014). Thus, the implications of a model that allows job offers to have a hidden component should be relevant for identifying the potential shortcomings of multidimensional compensation packages and the corresponding regulation of labour markets.

Related literature

This paper builds on two, thus far largely independent, strands of the literature. First, I model the employer incentives to offer jobs with either good or bad hidden attributes building on the literature in behavioural industrial organisation studying the design of products with shrouded attributes, such as add-on prices. This literature, starting with Ellison (2005) and Gabaix and Laibson (2006), has analysed a range of topics that arise when firms engage in one-shot competition by choosing their perfectly observable headline price as well as imperfectly observable additional price, see Heidhues and Kőszegi (2018) for a recent review. In settings where consumers can switch providers, even at a cost, the static incentive to exploit naiveté as much as feasible is relaxed by the risk of losing the business of a customer who realises they are being exploited. Arguably, this dynamic consideration is far from negligible in the industries that feature prominently in the literature as relevant applications, that is the credit card, banking, and mobile phone industries. Nonetheless, I am not aware of any previous work accounting for the possibility that an agent eventually learns the true value of the hidden attribute chosen by the

[2]For instance, in 2020, only 69% of total compensation received by an average employee in the US came in the form of wages, with the remaining 31% corresponding to various pecuniary benefits, most importantly health insurance, paid leave, and retirement benefits. The proportion of total compensation received in the form of benefits is higher among high-earners and public sector employees. Compiled from the data of US Bureau of Labor Statistics. Accessed 8.03.2021.

provider and might search for a better alternative 'on the job'. Thus the main contribution of the present paper is to analyse the effects of learning and possible re-contracting on the equilibrium contracts.[3]

In that strand of the literature, the most closely related paper is Gamp and Krähmer (2022) who introduced costly consumer search into the model of deceptive competition, where firms can market either a high- or a low-quality product. There are two types of consumers differing in their expertise. Sophisticated consumers observe price, true product quality, and idiosyncratic match quality when sampling offers, while naïve consumers do not observe match quality and, in addition, underappreciate the quality difference between low- and high-quality products. In this setting, market equilibrium may feature coexistence of 'candid' and 'deceptive' firms offering high and low quality, respectively. While sophisticated consumers search until they encounter a high-quality product with a suitable match, naifs purchase the first product that they sample. In line with previous studies, Gamp and Krähmer (2022) analyse a model that does not account for the effects of consumer learning and subsequent re-contracting.[4]

Second, I model consumer search behaviour in the spirit of the extensive literature on job search (Rogerson, Shimer, and Wright, 2005). By now, there are already several papers that incorporate various considerations from economics and psychology into the search framework and derive policy predictions specific to the assumed biases.[5] Relative to this strand of the literature,

[3]There are several papers that focus on other aspects of dynamic competition in such markets. Johnen (2019) analyses the design of automatic-renewal contracts in a model, where consumers differ in their propensity to passively accept the automatic renewal after learning their utility from consuming for one more period. In contrast to the present paper, this utility is random and does not depend on the provider's choice. Johnen (2020) studies dynamic competition between firms who learn private information about their customers' degree of sophistication by observing their past usage patterns and subsequently target differential offers at consumers they know to be sophisticated and naïve. The consumers, however, remain oblivious to the fact that they are being exploited. Heidhues, Kőszegi, and Murooka (2021) analyse a model where the firms offer an initial price and a switching price. Although consumers have ample opportunities to switch and observe all current and future prices perfectly, and thus there is no role for learning, they may still procrastinate on switching due to naiveté about their time inconsistency.

[4]Chen, Li, and Zhang (forthcoming) study a consumer search problem with horizontally differentiated experience goods, the quality of which becomes known only after purchase. Consumers are short-lived, but those who are active in period 1 leave reviews that allow those who are active in period 2 to distinguish between firms offering low- and high-quality products. Thus, the firms have an incentive to invest in quality and build up reputation. The authors restrict attention to a class of uniform-pricing equilibria, in which quality remains effectively 'hidden' in period 1 even though the consumers are fully rational. Equilibria of this type can only be supported if firms differ in their costs of providing high quality.

[5]DellaVigna and Paserman (2005) derive and test predictions regarding the impact of impatience on exit rates from unemployment, depending on whether the time discounting is exponential or hyperbolic. Paserman (2008) estimates structurally the extent of hyperbolic discounting by setting up a model of job search with endogenous search effort and calibrating it to the data on duration of unemployment spells and accepted wages. Spinnewijn (2015) analyses the optimal design of unemployment insurance when job seekers have biased perceptions of the

the contribution of this paper is to model the benefit component of a job offer as a hidden attribute, which allows to draw novel policy implications when job offers are multidimensional and individuals find some dimensions easier to compare than others.

In most closely related work, Bubb and Warren (2020) develop a theory of employer-sponsored pension plan design, when workers may display two separate behavioural characteristics. First, although the benefit component of a remuneration package is irrelevant in a rational model in absence of any tax advantages, present-biased workers either value its commitment features (if they are sophisticated) or overvalue the employer's matching contributions (if they are naïve). Second, if the default contribution rate is sticky and workers ignore this, the employer has an incentive to design the default contribution rate in a way that minimises worker savings, which reduces the matching contribution. In a dataset covering employer-sponsored pension plans in the US, Bubb and Warren (2020) find that the design of approximately 75% of plans is consistent with their theory of a profit-maximising, rather than paternalistic, employer. In contrast to the present paper, their analysis assumes away any search frictions, learning, or re-contracting.

2 The model

Consider the following model of search and learning about the hidden attribute of an offer. Time is discrete and finite, $t = 1, 2$. In period 1, an individual searches for a job for the first time, performing random, sequential search with perfect recall until she accepts an offer. Each search imposes a fixed cost of $c_1 > 0$. Upon accepting an offer, the individual derives utility from being in employment, thus learning the value of the associated hidden attribute. In period 2, she decides whether or not to continue searching on the job. If the agent is not searching on the job, she derives the same utility from being in employment as in period 1. Otherwise, she searches on the job until she accepts an alternative offer and subsequently derives the associated utility. There is continuum of firms making job offers to individuals, so that the probability of re-sampling the same job in either period is zero.

Suppose that searching on the job imposes a greater cost of $c_2 > c_1$ per search. This is an assumption commonly made in the literature. An increase in search costs while on the job can be due to traditional economic reasons, such as a higher opportunity cost of time (e.g., Burdett, 1978), but it could also reflect psychological factors that lower the propensity to engage in further search while on the job, such as time-inconsistent preferences or loss aversion (see Karle, Schumacher, and Vølund, 2021). To the extent that these alternative mechanisms generate behaviour that is observationally equivalent to an increase in search cost, the model permits a

probability of finding a job. DellaVigna, Lindner, Reizer, and Schmieder (2017) estimate a model of endogenous job search effort with reference-dependent preferences utilising a reform of the unemployment insurance system in Hungary.

broad range of interpretations.

The total utility from being in employment consists of two components - an observable attribute w ('wage') and an unobservable (hidden) attribute θ. An individual who is sampling job offers can perfectly observe and compare their observable attributes. However, she only learns the hidden attribute of a particular offer after accepting it. For an individual i working at firm k, the total utility from being employed in a single period is:

$$u_{ik} = w_k + \theta_{ik}$$

Let $\phi(w)$ denote the density function characterising the distribution of wage offers. Further, suppose that the hidden attribute θ_{ik} is determined jointly by an idiosyncratic match quality ϵ_{ik} and a discretionary component b_k ('benefits') chosen by the employer:

$$\theta_{ik} = b_k + \epsilon_{ik}$$

For simplicity, suppose that ϵ_{ik} is i.i.d. with mean zero. The employer can choose to offer either 'high' or 'low' benefits, i.e. $b_k \in \{\underline{b}, \bar{b}\}$ for some constants $\bar{b} > \underline{b} > 0$.

The agent's objective is to maximise the expected utility from being employed in two periods, net of her search costs. For notational simplicity, we omit time discounting. The value of the outside option (staying out of the labour market) is normalised to 0.

3 Search behaviour

In this section, I analyse search behaviour of an agent facing an exogenously given distribution of job offers consisting of observable and hidden attributes. For the most part, I restrict attention to searching individuals who fail to appreciate the correlation between w and b, even though they have correct beliefs regarding the unconditional distributions of wages and benefits. In other words, in order to model the benefits as a 'hidden' attribute, I assume that the agents display a form of correlation neglect.

Formally, the agents studied here are analogy-based reasoners (Jehiel, 2005), who in response to the complexity of their strategic environment bundle the decision nodes of employers choosing what wages and benefits to offer into two analogy classes, and best-respond to the average behaviour within each analogy class. Although the searching individuals display correlation neglect and are unable to deduce the value of the hidden attribute from the wage offer, they fully internalise the fact that they will learn the value of the hidden attribute upon acceptance of an offer.[6]

[6] Of course, there are alternative ways of capturing correlation neglect in strategic situations. For Bayesian games, Eyster and Rabin (2005) define a concept of a "cursed equilibrium", whereby boundedly rational players

A small but growing literature cited above incorporates various types of psychological biases into models of costly search. Building on these studies, I allow the individuals in period 1 to be either "sophisticated" or "naïve" regarding the future cost of searching on the job. More precisely, while a sophisticated individual correctly perceives the future cost of search to be $c_2 > c_1$, a naïve individual fails to foresee the increase in search costs once the first offer is accepted and expects the future cost of search to be equal to c_1. Thus, the search strategy of a sophisticated agent reflects the impact of hiddenness of benefits on dynamic search, while the search strategy of a naif captures an additional impact of overoptimistic beliefs about the option value of on-the-job search. Distinguishing between naifs and sophisticates defined in this way allows to focus on the dynamics of the model induced by learning and the possibility to search on the job, which constitute the main novelty of this paper.

The purpose of the following partial-equilibrium analysis is to answer the following questions. How do the hidden attributes and naiveté affect the agent's search strategy and ultimately her labour market outcomes? In which economic environments are the effects of naiveté more pronounced? Solving the model backwards, I derive a search strategy adopted by either type. Subsequently, I present some comparative static results relating the wedge between naïve and sophisticated search strategies to the level of search costs and the distribution of headline attributes.

Period 2. Individual i employed at firm k derives (and observes) the total utility of being employed $u_{ik} = w_k + \theta_{ik}$. If the individual searches on the job, her value function of sampling a wage offer w is:

$$v_2(w) = \max \{w + \mathbb{E}[\theta] \ ; \ v_2 - c_2\}$$

where $v_2 = \int_w v_2(w)\phi(w)dw$. The first expression inside the bracket represents the expected utility from accepting the offer. The second expression represents the expected utility from rejecting the offer and searching once more. Notice that $v_2(w)$ is piecewise linear in w, taking a constant value of $v_2 - c_2$ for low w and strictly increasing for higher values of w. Consequently, the optimal search strategy is characterised by a cutoff rule with a reservation wage R_2 which prescribes to continue searching until the first offer with $w \geq R_2$ is sampled (McCall, 1970). Thus the agent's search behaviour is fully characterised by the optimal reservation wage. For

do not fully internalise what other players' actions reveal about their type (or any other private information). Even though players display a form of correlation neglect in their "fully cursed equilibrium", this concept appears conceptually better suited to games with private information, while for the most part this paper analyses a game without any type-uncertainty (i.e., all firms are homogeneous). Nevertheless, the results derived and discussed here are independent of the specific micro-foundation of correlation neglect. See also Eyster (2019) for a taxonomy of errors that people make in strategic environments, summary of the lab and field evidence, and an overview of game-theoretic models incorporating these errors.

all $w < R_2$, $v_2(w) = v_2 - c_2$, while for all $w \geq R_2$, $v_2(w) = w + \mathbb{E}[\theta]$. Combining this with the indifference at the reservation wage, $R_2 + \mathbb{E}[\theta] = v_2 - c_2$, yields:

$$
\begin{aligned}
\int_w v_2(w)\phi(w)dw &= (R_2 + \mathbb{E}[\theta]) \cdot \mathbb{P}[w < R_2] \ + \int_{w \geq R_2} (w + \mathbb{E}[\theta])\phi(w)dw \\
&= R_2 \cdot \mathbb{P}[w < R_2] \ + \int_{w \geq R_2} w\phi(w)dw \ + \ \mathbb{E}[\theta] \\
&= R_2 \ + \ \mathbb{E}[\theta] \ + \ c_2
\end{aligned}
$$

which solves for:

$$
c_2 = \int_{w \geq R_2} \big(w - R_2\big)\phi(w)dw \tag{1}
$$

Intuitively, the optimal reservation wage R_2 equalises the marginal cost of search with the marginal benefit of search. Since the right-hand side of the above equality is strictly decreasing in R_2, the optimal reservation wage is uniquely determined. Note that because the agent perceives the benefits to be independent of the wage offer, her search strategy is a function of the observable component only.

Lemma 1 specifies when an employed individual engages in on-the-job search, depending on her realised utility from being in current employment.

Lemma 1: *In period 2, the worker stays in current employment (searches on the job) when* $u_{ik} = w_k + \theta_{ik} \ \geq (<) \ R_2 + \mathbb{E}[\theta]$.

This observation mirrors the result of McCall (1970), who defines a 'discouraged' worker as one who does not engage in job search at all, because her outside option already provides utility at least corresponding to accepting the optimal reservation wage.

From now on, I will impose for notational convenience that $\mathbb{E}[\theta] = 0$. This expression will then be endogenised in Section 4.

Period 1. The optimal search strategy in period 1 (during the initial job search) depends on the anticipated behaviour once in employment. Even though both types internalise the fact that they will learn the offer's hidden attribute once on the job, the naïve individuals fail to foresee that their search costs will increase to c_2 in period 2. Formally, depending on the type, the beliefs in period 1 are $\hat{c}_2 \in \{c_1, c_2\}$ and the agent expects her future self to adopt a reservation wage $\hat{R}_2$ satisfying:

$$
\hat{c}_2 = \textstyle\int_{w \geq \hat{R}_2} \big(w - \hat{R}_2\big)\phi(w)dw
$$

Since $c_1 < c_2$, $R_2^N > R_2^S = R_2$, with the superscripts N and S referring to naifs and sophisticates, respectively. This yields the following observation.

Lemma 2: *In period 1, a naïve agent overestimates her future propensity to search on the job. Specifically, for realisations $u_{ik} \in [R_2^S, R_2^N)$ she expects to search for a different job, but actually doesn't.*

How is this biased perception reflected in the search strategy in period 1? The value function of sampling an offer with wage w is now:

$$v_1(w) = \max\,\{u_1(w), v_1 - c_1\}$$

where:

$$\begin{aligned} u_1(w) = \big(w + \mathbb{E}[\theta]\big) \;&+\; \mathbb{P}[w + \theta \geq \hat{R}_2] \cdot \big(w + \mathbb{E}[\theta \,|\, \theta \geq \hat{R}_2 - w]\big) \\ &+\; \mathbb{P}[w + \theta < \hat{R}_2] \cdot \big(\hat{v}_2 - \hat{c}_2\big) \end{aligned}$$

$v_1 = \int_w v_1(w)\phi(w)dw$, and the variables $\hat{R}_2, \hat{v}_2, \hat{c}_2$ depend on the beliefs about future search cost. Conditional on accepting the offer, the agent expects to obtain the utility of $w + \mathbb{E}[\theta]$. Furthermore, if the realised utility from being employed turns out to exceed the *perceived* cutoff $\hat{R}_2$, the agent expects to stay in the same employment and not engage in on-the-job search. Otherwise, she expects to search on the job, until an offer paying at least $\hat{R}_2$ is found.

Note that the difference between the sophisticated and naïve agents materialises via their perceived utility from accepting an offer, $u_1(w)$. Three observations are immediate. First, since $R_2^N > R_2^S$, naïve agents underestimate the probability of staying in the same employment or, conversely, they overestimate the probability of searching on the job for any offer sampled in period 1. Second, they overestimate the utility from accepting employment, conditional on staying in the same job, represented by $w + \mathbb{E}[\theta \,|\, \theta \geq \hat{R}_2 - w]$. Third, they overestimate the option value of on-the-job search by overestimating the value from searching as well as not taking into account the increase in search costs. This can be demonstrated as follows:

$$\begin{aligned} \text{For all } w \geq R_2^N, \qquad & v_2^N(w) = v_2^S(w) = w + \mathbb{E}[\theta] \\ \text{For all } w \in [R_2^S, R_2^N), \qquad & v_2^N(w) = v_2^N - c_1 = R_2^N + \mathbb{E}[\theta] > v_2^S(w) = w + \mathbb{E}[\theta] \\ \text{For all } w < R_2^S, \qquad & v_2^N(w) = v_2^N - c_1 > v_2^S(w) = v_2^S - c_2 = R_2^S + \mathbb{E}[\theta] \end{aligned}$$

Overall, as $v_2^N > v_2^S$ and $c_1 < c_2$, $(v_2^N - c_1) > (v_2^S - c_2)$.

Next, I show that the sum of these effects implies that naifs overestimate their utility from accepting any offer w:

$$u_1^N(w) > u_1^S(w) \iff$$

$$\mathbb{P}[w+\theta \geq R_2^N] \cdot \big(w + \mathbb{E}[\theta \,|\, \theta \geq R_2^N - w]\big) \; + \; \mathbb{P}[w+\theta < R_2^N] \cdot \big(v_2^N - c_1\big) >$$
$$> \mathbb{P}[w+\theta \geq R_2^S] \cdot \big(w + \mathbb{E}[\theta \,|\, \theta \geq R_2^S - w]\big) \; + \; \mathbb{P}[w+\theta < R_2^S] \cdot \big(v_2^S - c_2\big)$$

Using the fact that $R_2^N > R_2^S$, expand the right-hand side (RHS) of the inequality:

$$\begin{aligned}
\text{RHS} = \; & \mathbb{P}[w+\theta \geq R_2^N] \cdot \big(w + \mathbb{E}[\theta \,|\, \theta \geq R_2^N - w]\big) \\
& + \mathbb{P}[R_2^N > w+\theta \geq R_2^S] \cdot \big(w + \mathbb{E}[\theta \,|\, R_2^N - w > \theta \geq R_2^S - w]\big) \\
& + \mathbb{P}[w+\theta < R_2^S] \cdot \big(v_2^S - c_2\big)
\end{aligned}$$

Then, $u_1^N(w) > u_1^S(w) \iff$

$$\mathbb{P}[w+\theta < R_2^N] \cdot \big(v_2^N - c_1\big) >$$
$$\mathbb{P}[R_2^N > w+\theta \geq R_2^S] \cdot \big(w + \mathbb{E}[\theta \,|\, R_2^N - w > \theta \geq R_2^S - w]\big) \; + \; \mathbb{P}[w+\theta < R_2^S] \cdot \big(v_2^S - c_2\big)$$

$$\iff \big(v_2^N - c_1\big) > (1-\rho) \underbrace{\big(w + \mathbb{E}[\theta \,|\, R_2^N - w > \theta \geq R_2^S - w]\big)}_{\in [R_2^S, R_2^N)} + \rho\big(v_2^S - c_2\big)$$

where $\rho = \mathbb{P}[w+\theta < R_2^S] \,/\, \mathbb{P}[w+\theta < R_2^N] \in (0,1)$ as long as both probabilities are positive. Since $(v_2^N - c_1) = R_2^N$, the above inequality is indeed strict for any such ρ. The next observation follows.

Lemma 3: *A naïve agent strictly overestimates the utility from accepting any job offer.*

A natural conjecture is that a naïve agent would be too quick to accept a job offer in period 1, given her biased forecast of future search behaviour. As it turns out, that is indeed the case, which I demonstrate in several steps.

First, the optimal search strategy in period 1 is also given by a cutoff rule as long as $d\,u_1(w)/d\,w > 0$.

$$\begin{aligned}
& \mathbb{P}[w+\theta \geq \hat{R}_2] = \int_{\hat{R}_2 - w} \psi(\theta) d\theta \\
& \mathbb{E}[\theta \,|\, \theta \geq \hat{R}_2 - w] = \int_{\hat{R}_2 - w} \theta\psi(\theta) d\theta \,\Big/ \int_{\hat{R}_2 - w} \psi(\theta) d\theta \\
& u_1(w) = \big(w + \mathbb{E}[\theta]\big) \; + \; w \cdot \int_{\hat{R}_2 - w} \psi(\theta) d\theta \; + \int_{\hat{R}_2 - w} \theta\psi(\theta) d\theta \; + \; \big(\hat{v}_2 - \hat{c}_2\big) \cdot \int^{\hat{R}_2 - w} \psi(\theta) d\theta
\end{aligned}$$

Then:

$$
\begin{aligned}
d\,u_1(w)/d\,w = \quad & 1 + \mathbb{P}[w + \theta \geq \hat{R}_2] + w \cdot \big[-\psi(\hat{R}_2 - w)(-1) \big] \\
& + \big[-(\hat{R}_2 - w)\psi(\hat{R}_2 - w)(-1) \big] + \big(\hat{v}_2 - \hat{c}_2 \big) \big[\psi(\hat{R}_2 - w)(-1) \big] \\
= \quad & 1 + \mathbb{P}[w + \theta \geq \hat{R}_2] + \psi(\hat{R}_2 - w) \underbrace{\big[\hat{R}_2 - (\hat{v}_2 - \hat{c}_2) \big]}_{=0} > 0
\end{aligned}
$$

After establishing that $u_1(w)$ is strictly increasing in w, proceeding as in the analysis of the period-2 search rule yields the condition for the optimal reservation wage in period 1, R_1:

$$
c_1 = \int_{w \geq R_1} \big(u_1(w) - u_1(R_1) \big) \phi(w) dw \tag{2}
$$

Second, since $d\,u_1(w)/d\,w = 1 + \mathbb{P}[w + \theta \geq \hat{R}_2]$ and $R_2^N > R_2^S$, we have $\frac{d\,u_1^N(w)}{d\,w} < \frac{d\,u_1^S(w)}{d\,w}$. Thus for any $w > R$:

$$
u_1^N(w) - u_1^N(R) < u_1^S(w) - u_1^S(R)
$$

Third, recall that:

$$
c_1 = \int_{w \geq R_1^N} \big(u_1^N(w) - u_1^N(R_1^N) \big) \phi(w) dw = \int_{w \geq R_1^S} \big(u_1^S(w) - u_1^S(R_1^S) \big) \phi(w) dw
$$

and suppose (by contradiction) that $R_1^N \geq R_1^S$, which would imply that a naïve agent is more picky then her sophisticated counterpart. Then:

$$
\int_{w \geq R_1^S} \big(u_1^S(w) - u_1^S(R_1^S) \big) \phi(w) dw =
$$

$$
= \int_{w \geq R_1^N} \big(u_1^S(w) - u_1^S(R_1^S) \big) \phi(w) dw \; + \; \int_{w = R_1^S}^{R_1^N} \big(u_1^S(w) - u_1^S(R_1^S) \big) \phi(w) dw \geq
$$

$$
\geq \int_{w \geq R_1^N} \big(u_1^S(w) - u_1^S(R_1^N) \big) \phi(w) dw \; + \; \int_{w = R_1^S}^{R_1^N} \big(u_1^S(w) - u_1^S(R_1^S) \big) \phi(w) dw \geq
$$

$$
\geq \int_{w \geq R_1^N} \big(u_1^S(w) - u_1^S(R_1^N) \big) \phi(w) dw >
$$

$$
> \int_{w \geq R_1^N} \big(u_1^N(w) - u_1^N(R_1^N) \big) \phi(w) dw
$$

A contradiction. Thus we must have $R_1^N < R_1^S$ and the final result of this section follows.

Lemma 4: *A naïve agent adopts a strictly lower reservation wage when searching initially, that is $R_1^N < R_1^S$.*

In short, a naïve agent accepts 'too many' inferior job offers, because she overestimates the option value of searching on the job. Since this comparison applies to two types of agents who cannot observe (or infer) the value of the hidden attribute while searching, a natural question arises about whether the wedge between the naif's and sophisticate's search strategies must necessarily lead to an *ex ante* welfare loss. The answer turns out to be no. For example, if lower wages were paired with high benefits in a way that leaves the total utility unaffected, a naïve agent who accepts the first encountered offer would be (coincidentally) maximising her true welfare by minimising the total expected search cost. At the same time, a sophisticated agent could be wastefully searching for higher-paying offers.

More formally, in the appendix I compare the above search strategies to the one adopted by a "fully rational" agent who observes both components of any offer while searching at no extra cost. Lemma A.2 establishes that when faced with a (static) on-the-job search problem in period 2, a boundedly rational agent is never "uniformly more, or less, picky" than her fully rational counterpart in the following sense: She would never reject a wage offer that is accepted by a rational agent even if the associated benefits are low. She would also never accept a wage offer that is rejected by a rational agent even when the associated benefits are high. That is of course due to the fact that a boundedly rational agent takes into account a correct unconditional distribution of benefits when sampling the wage offers. Consequently, if the true correlation between wages and benefits was zero, a (risk neutral) analogy-based reasoner would be *ex ante* as well off as a fully rational agent. Otherwise, correlation neglect leads to a welfare loss, because although a boundedly rational agent attempts to make the right call 'on average', she applies an incorrect joint distribution of wages and benefits.

The dynamic aspect of search requires developing a slightly different intuition. When conducting the search in period 1, a sophisticated agent is also never uniformly more picky than her fully rational counterpart, but she may now be uniformly less picky in the sense that she accepts wage offers that are rejected by a fully rational agent even if the associated benefits are high (Lemma A.3). At first glance, one may conjecture that naifs are thus *ex ante* worse off than sophisticates, because they adopt a strictly lower reservation wage to begin with (Lemma 4). This, however, is not generally true as the notion of 'uniform pickiness' may apply to fictitious compensation packages that are assigned a positive probability under a boundedly rational agent's beliefs, but do not in fact materialise under the true distribution of offers. In what follows, I therefore call the difference $(R_1^S - R_1^N)$ a "naiveté wedge" in search rules, but I do not interpret it as a measure of welfare loss from following a naïve strategy.

Comparative statics

In this section, I examine the impact of the level of search costs and the distribution of wages from which the agent is sampling her offers on the naiveté wedge. This exercise highlights in which environments are the beliefs about future search costs of greater importance.

First, consider how the effects of naiveté vary across populations of otherwise identical agents characterised by different initial search costs, i.e. $c_1 < c_1' < c_2$.[7] Naturally, greater c_1 implies a lower reservation wage for period 1, because the right-hand side of the condition

$$c_1 = \int_{w \geq R_1} \big(u_1(w) - u_1(R_1)\big)\phi(w)dw$$

is strictly decreasing in R_1:

$$\begin{aligned}
\frac{d\,\text{RHS}}{d\,R_1} &= -(u_1(R_1) - u_1(R_1))\phi(R_1) \;+\; \int_{w \geq R_1} -u_1'(R_1)\phi(w)dw \\
&= -u_1'(R_1) \cdot \mathbb{P}[w \geq R_1] < 0
\end{aligned}$$

Recall from above that $d\,u_1(w)/d\,w = 1 + \mathbb{P}[w + \theta \geq \hat{R}_2] > 0$ and therefore

$$\tfrac{d\,\text{RHS}}{d\,R_1} = -\big(1 + \mathbb{P}[R_1 + \theta \geq \hat{R}_2]\big) \cdot \mathbb{P}[w \geq R_1]$$

Given that $R_1^S > R_1^N$ and $R_2^N > R_2^S$, the slope of RHS can be either steeper or flatter for naïve individuals, depending on the underlying distributions of w and θ. In particular, $|\frac{d\,\text{RHS}}{d\,R_1^S}| > |\frac{d\,\text{RHS}}{d\,R_1^N}|$ if and only if the following holds:

$$\big(1 + \mathbb{P}[\theta \geq R_2^S - R_1^S]\big) \;/\; \big(1 + \mathbb{P}[\theta \geq R_2^N - R_1^N]\big) \quad > \quad \mathbb{P}[w \geq R_1^N] \;/\; \mathbb{P}[w \geq R_1^S]$$

where both sides of the inequality are greater than 1.

Note that if the above is satisfied, then for $c_1' > c_1$, $(R_1^{S'} - R_1^{N'}) < (R_1^S - R_1^N)$. On the other hand, if the inequality is reversed, we have $|\frac{d\,\text{RHS}}{d\,R_1^S}| < |\frac{d\,\text{RHS}}{d\,R_1^N}|$ and $(R_1^{S'} - R_1^{N'}) > (R_1^S - R_1^N)$. Finally, R_2^S, R_2^N are invariant to changes in c_1.

These conditions reflect the fact that the impact of naiveté on the sensitivity of the optimal cutoff R_1 to changes in the initial search cost c_1 materialises via two channels. First, overestimating the option value of searching on the job, naifs are more likely to accept any offer in period 1, resulting in a cutoff R_1^N that is relatively insensitive to changes in c_1. Second, anticipating the future increase in search costs sophisticates are more picky in period 1, resulting in a cutoff R_1^S that is relatively insensitive to changes in c_1.

If the first (static) channel dominates, then an increase in the search cost creates a larger wedge between the cutoff rules adopted by sophisticates and naifs, i.e. $(R_1^{S'} - R_1^{N'}) > (R_1^S - R_1^N)$. In contrast, if the second (dynamic) channel dominates, then an increase in the search cost results

[7] The implications of varying period-2 search cost are immediate. The greater c_2, the larger the discrepancy in period-1 search rule between sophisticates and naifs.

in a smaller wedge between the cutoff rules. Thus depending on the underlying distributions for w and θ, populations with higher initial search cost (e.g., inexperienced, not as financially or technically savvy workers) may have either greater or smaller naiveté wedge. These predictions are summarised in the following corollary.

Corollary 1: *If the following holds at the cutoff rules adopted by the sophisticated and naïve agents, respectively:*

$$\big(1+\mathbb{P}[\theta \geq R_2^S - R_1^S]\big) \;/\; \big(1+\mathbb{P}[\theta \geq R_2^N - R_1^N]\big) \quad < (>) \quad \mathbb{P}[w \geq R_1^N] \;/\; \mathbb{P}[w \geq R_1^S]$$

then the naiveté wedge is increasing (decreasing) in the initial search cost c_1*.*

Second, turn to the issue of how the effects of naiveté vary across otherwise identical agents sampling from different wage distributions. Specifically, assume that the wage distribution captured by the density function $\phi^\star(w)$ first-order stochastically dominates the one captured by $\phi(w)$. Naturally, under $\phi^\star(w)$, the reservation wage in period 1 increases, because the right-hand side (RHS) of the optimality condition

$$c_1 = \int_{w \geq R_1} \big(u_1(w) - u_1(R_1)\big)\phi(w)dw$$

is strictly greater under $\phi^\star(w)$ than $\phi(w)$.

Observe that for a fixed R_1, the increase in RHS caused by the shift from $\phi(w)$ to $\phi^\star(w)$ is more pronounced, the steeper $u_1(\cdot)$. It has already been shown that $\frac{du_1^N(w)}{dw} < \frac{du_1^S(w)}{dw}$, because for sophisticated agents the initially accepted wage w is a stronger determinant of the overall utility from accepting an offer. This implies:

$$R_1^{S\star} - R_1^S > R_1^{N\star} - R_1^N \iff \big(R_1^{S\star} - R_1^{N\star}\big) > \big(R_1^S - R_1^N\big)$$

which yields the following result.

Corollary 2: *The naiveté wedge is greater for agents who sample offers from a first-order stochastically dominant distribution of wages.*

4 Optimal offers

Having illustrated the differences in search strategies adopted by naïve and sophisticated agents, in this section I endogenise the offers made by hiring firms by characterising equilibria of the following game. The firms simultaneously choose the wage and benefits they will offer to searching workers, committing the same offer for both periods 1 and 2. The worker's problem is as outlined in the previous section. There is a unit mass of both workers and firms.

Let the equilibrium be defined as a profile of strategies for the workers and the firms, such that all players behave optimally given their beliefs. Every firm holds beliefs consistent with the workers' search strategies and the other firms' offers. Displaying correlation neglect, both sophisticated and naïve workers fail to see a connection between the wage offer and the expected level of benefits, although they have correct beliefs regarding the marginal distributions of offered wages and benefits derived from the firms' strategies. In addition, naifs underestimate their future cost of searching on the job, while sophisticates take into account the correct future search cost when sampling the offers initially. This notion of an equilibrium is similar to a Weak Perfect Bayesian Equilibrium (PBE), except for the systematic biases in the workers' beliefs. Formally, I define the equilibrium in the following way.

Definition 1: *The workers' strategies summarised by* $(R_2, R_2^N, R_1^S, R_1^N)$ *and the firms' strategies summarised by* (w_k, b_k), $k \in [0, 1]$, *constitute a (pure-strategy)* PBE with correlation neglect*, if:*

- *Given the firms' strategies, the workers of both types hold correct beliefs about marginal distributions of wages and benefits, but perceive those components to be independent.*
- *Given their beliefs about the distribution of offers, the workers of both types adopt reservation wage* R_2 *in period 2 which satisfies (1).*
- *Anticipating their period-2 strategy correctly, in period 1 sophisticated workers adopt reservation wage* R_1^S *which satisfies (2). In contrast, naïve workers expect to adopt reservation wage* R_2^N *in period 2 which satisfies (1) parametrised by* $c_1 < c_2$*. Anticipating such period-2 strategy, in period 1 naïve workers adopt reservation wage* R_1^N *which satisfies modified (2).*
- *Given the workers' strategies, the distribution of types, and the strategies adopted by other firms, each firm chooses* (w_k, b_k) *to maximise its expected profits from hiring in periods 1 and 2.*

In addition, the current version of the paper focuses on a class of "sticky" equilibria, where all workers remain in the same job for two periods.

Definition 2: *A PBE with correlation neglect is called* sticky *if no worker searches on the job on the equilibrium path.*

4.1 Deterministic setting

Consider a variant of the model in which $\bar{\epsilon} \to 0$, so that there is no role for match-specific shocks. Offering a compensation package (w, b) costs the firm $w + (1 - \tau)b$ per period, if accepted by a worker. Here, $\tau \in [0, 1)$ represents possible tax advantages or public subsidies to employer benefits.[8] Since the worker's utility aggregates received wages and benefits, it is socially desirable for the firms to offer high benefits $\bar{b}$.

Each firm produces $y > 0$ units of a numeraire good per hired worker using a constant-returns-to-scale technology. Thus the profit from hiring a worker is given by:

$$\pi = y - w - (1 - \tau)b$$

For illustration, consider a problem of a firm offering a compensation package to a population of homogeneous workers, taking the distribution of other firms' offers as given. In period 2, the worker who has accepted the offer initially stays in employment of firm k if:

$$u_{ik} = w_k + b_k \geq R_2 + \mathbb{E}[\theta]$$

where R_2 is given by equation (1). In period 1, the worker accepts the offer if:

$$u_1(w_k) \geq u_1(R_1) \iff w_k \geq R_1$$

where R_1 is given by equation (2). It is now easy to see that for any firm looking to hire a worker in the first place, its wage offer must satisfy $w_k \geq R_1$. Condition $w_k + b_k \geq R_2 + \mathbb{E}[\theta]$ needs to be satisfied only if the firm wishes to keep the worker on for period 2. Thus, under correlation neglect the role of generous benefits is to encourage the worker to stay in employment of the same firm, while the wage determines whether or not the worker accepts the job offer in the first place. This is true for both sophisticated and naïve individuals.

Proceeding further, suppose that fraction $\lambda \in (0, 1)$ of workers are naïve and the rest are sophisticated. What does an equilibrium look like in this case? Focusing on symmetric equilibria in pure strategies, consider a situation when all firms offer some $w = w^*$ and $b = b^*$. Then, since the workers of either type correctly perceive the (degenerate) distributions of offered wages and benefits, there are no differences in behaviour of naïve and sophisticated workers. The worker anticipates (correctly) that she will not search on the job and stay in the same employment for two periods. Thus in period 1, a worker searches exactly once and accepts employment as long as $2(w^* + b^*) - c_1 \geq 0$.

It is easy to see that conditional on b^*, all firms would offer the same wage equal to some R_1. Offering a higher wage would not affect the firm's hiring probability or the worker's decision to stay, while offering a lower wage would result in the worker rejecting the offer. In equilibrium, this uniform wage has to make the workers exactly indifferent between accepting employment

[8]For example, contributions into a workplace pension scheme receive an automatic tax relief in the UK. As of 2020, 15 OECD countries have implemented direct financial incentives to contribute to private pensions, including matching contributions and fixed subsidies from the government (OECD, 2020).

and not entering the labour market, implying $w^* = c_1/2 - b^*$.[9] This is reminiscent of the main result in Diamond (1971).

Under what conditions can a socially desirable outcome with high benefits be sustained in equilibrium? Suppose that $b^* = \bar{b}$ and consider two possible classes of deviations. Since the deviating firm cannot offer a wage lower than w^*, consider first a deviation whereby the firm does not adjust its wage offer and simply combines w^* with low benefits $\underline{b}$. A worker who accepted searches on the job and leaves if $c_2 < \bar{b} - \underline{b}$. However, this still makes the deviator better off as long as $y - w^* - (1-\tau)\underline{b} > 2(y - w^* - (1-\tau)\bar{b})$.[10] On the other hand, if $c_2 \geq \bar{b} - \underline{b}$, the worker does not search on the job and the deviator is strictly better off.

Second, consider a firm that deviates to $\underline{b}$ and simultaneously raises the wage offer to some $\tilde{w}$ in order to prevent the worker from searching on the job. The highest $\tilde{w}$ that the deviator might consider lies along its iso-profit curve and is thus given by $\tilde{w} = c_1/2 - \underline{b} - \tau(\bar{b} - \underline{b})$. Such an offer is accepted by all workers in period 1 as $\tilde{w} > w^*$, but the deviator is worse off if the worker nonetheless leaves, i.e. when $c_2 < \tau(\bar{b} - \underline{b})$. On the other hand, if $c_2 > \tau(\bar{b} - \underline{b})$ there would exist a (strictly) profitable deviation. In sum, there is no profitable deviation from a putative equilibrium with $b^* = \bar{b}$ if $c_2 \leq \tau(\bar{b} - \underline{b})$ and $2(y - w^* - (1-\tau)\bar{b}) \geq y - w^* - (1-\tau)\underline{b}$ are both satisfied. If either one of these conditions fails, there does not exist an equilibrium in which all firms offer generous benefits.

Next, an equilibrium with $b^* = \underline{b}$ always exists (provided that the firms make non-negative profit), since by deviating to $\bar{b}$ a firm would be strictly worse off, irrespective of the parametrisation. This is due to the fact that with benefits being unobservable prior to acceptance of the contract, the deviator still has to offer a wage of $w^* = c_1/2 - \underline{b}$ in order to attract any workers. Thus, switching to generous benefits raises the firm's cost without affecting the worker's decision to stay in current employment. The condition for the firms to make non-negative profit is $y - w^* - (1-\tau)\underline{b} = y + \tau\underline{b} - c_1/2 \geq 0$, which is arguably very weak.

These observations are summarised in the following result.

Proposition 1: *In a deterministic setting, a symmetric pure-strategy equilibrium in which all firms offer low benefits ($b^* = \underline{b}$) exists as long as $y + \tau\underline{b} \geq c_1/2$. A symmetric pure-strategy equilibrium in which all firms offer high benefits ($b^* = \bar{b}$) exists if in addition $c_2 \leq \tau(\bar{b} - \underline{b})$ and $y + \tau\bar{b} \geq (1-\tau)(\bar{b} - \underline{b}) + c_1/2$. In all such equilibria, the distribution of wage offers is degenerate with $w^* = c_1/2 - b^*$.*

When the distribution of offers is degenerate, no worker expects to search while on the job

[9] Suppose instead that the uniform wage offered by all firms strictly exceeds w^*. Then, there would exist a profitable deviation whereby a firm lowers its wage by some $0 < \epsilon < c_1$ and still hires all the workers it encounters, contradicting the equilibrium condition.

[10] Under such unilateral deviation, there are no workers left in the market for the deviating firm to re-hire in period 2.

and the difference between sophisticates and naifs becomes inconsequential. As a result, the conditions above are independent of the distribution of types λ and we can interpret Proposition 1 as capturing the impact of analogy-based reasoning, or correlation neglect, on the incentives of firms to offer either bad or good hidden attributes. Whenever the firms make non-negative profit in the case they offer low benefits, and thus an equilibrium with $b^* = \underline{b}$ exists, an equilibrium with (socially desirable) high benefits $b^* = \bar{b}$ exists only if two additional conditions are met. First, the cost of searching on the job cannot be too high. Otherwise, a firm could deviate to offering low benefits without losing a hired worker. Second, the firms must actually derive a higher profit from employing the worker for both periods rather than letting the worker leave after one period of employment with slightly lower total compensation. In that sense, equilibria with high benefits are more difficult to sustain. Finally, note that when the profits are non-negative in a low-benefit equilibrium and a high-benefit equilibrium exists, the firms are making strictly greater profits in the high-benefit equilibrium due to the associated tax advantages ($\tau > 0$). Thus, imperfect observability of the offered benefits by the workers can be detrimental to the firms.[11]

Even though the firms are homogeneous, one might wonder whether there exist equilibria with more than one wage offer, since naifs and sophisticates adopt different search strategies when the distribution of offers is non-degenerate. Following the logic in Albrecht and Axell (1984) and the discussion above, note that when the distribution of offers is non-degenerate and the two types adopt reservation wages $R_1^S > R_1^N$ in period 1, no firm has an incentive to post a wage offer other than R_1^S or R_1^N. Then, the firms that post $w_k = R_1^S$ hire all workers who contact them, while the firms that post a lower wage $w_k = R_1^N$ can hire naifs only. Because of the trade-off between profits per worker and the number of hires, equilibrium can support differential wage offers as long as total expected profits associated with either offer are equal.

Suppose that fraction $p \in [0, 1]$ of firms offer wage w_1 and the remaining share of firms offer

[11] How would the equilibrium conditions spelled out in Proposition 1 be affected by injection of a small mass of fully rational agents into the population of searching workers? First, there would in principle arise a mutually profitable deviation from the low-benefits equilibrium, whereby the firm offers high benefits and simultaneously reduces its wage dollar-for-dollar. But because such a deviation would attract only the fully rational agents, it cannot increase the deviator's total profit unless their mass is large enough. Second, the deviations from the high-benefits equilibrium discussed above become less profitable when some agents are fully rational. That is because, as argued in the appendix, the fully rational agents perform an essentially static search at a cost of c_1 in the sense that they always stay in originally accepted employment for two periods. Consequently, the deviations that were strictly profitable with analogy-based reasoners who got locked into subpar offers under $c_2 > \tau(\bar{b} - \underline{b})$, may fail to attract any fully rational agents in the first place by the assumption $c_1 < c_2$. Similarly, the deviations that were strictly profitable despite the boundedly rational agent leaving employment after one period are never accepted by the fully rational agents. Qualitatively, these arguments highlight that with a higher share of fully rational agents in the population, the socially desirable equilibria with high benefits become easier to sustain, while the opposite holds for the socially suboptimal equilibria with low benefits.

wage $w_2 > w_1$. Then, naifs accept even the lower wage offer if:

$$u_1^N(w_1) \;\geq\; pu_1^N(w_1) + (1-p)u_1^N(w_2) - c_1 \quad\Longleftrightarrow\quad c_1 \;\geq\; (1-p)(u_1^N(w_2) - u_1^N(w_1)) \tag{3}$$

Sophisticates, on the other hand, search until they encounter the higher wage offer if:

$$c_1 \;<\; (1-p)(u_1^S(w_2) - u_1^S(w_1)) \tag{4}$$

As argued in Section 3, $u_1^N(w_2) - u_1^N(w_1) < u_1^S(w_2) - u_1^S(w_1)$ and thus the above conditions are not contradictory.

In Albrecht and Axell (1984), the difference in worker types boils down to the value of their outside options (i.e., unemployment), and thus the higher of the two wages (w_2) is pinned down by the indifference condition of the type who accepts w_2, but not w_1. This might at first suggest that in the present setting, where it must necessarily be the sophisticates who adopt a higher reservation wage, w_2 is set so as to make them indifferent between accepting employment and leaving the labour market:

$$(1-p)u_1^S(w_2) - c_1 \;=\; 0$$

Notice, however, that this would contradict (4). Thus, a fundamentally different logic applies to the construction of an equilibrium with wage dispersion when the agent types differ in their valuations of the option value of searching on the job, rather than having heterogeneous outside options.

Naifs participate in the labour market as long as:

$$pu_1^N(w_1) + (1-p)u_1^N(w_2) \;\geq\; c_1 \tag{5}$$

while sophisticates participate as long as:

$$(1-p)u_1^S(w_2) \;\geq\; c_1 \tag{6}$$

Conditions (3) and (5) are never contradictory, in that the upper bound they jointly impose on c_1 is always greater than the lower bound. For sophisticates, only condition (4) has bite in the sense that values for c_1 that satisfy (4) also automatically satisfy the participation constraint (6).

When search is random, each firm is contacted by a worker with equal probability. Ignoring for the moment the possibility to re-hire workers in period 2, the expected profit associated with either offer conditional on being contacted by a worker is:

$$\pi(w_2) \;=\; \{1+\mathbb{P}[w_2+b(w_2)\geq R_2+\mathbb{E}[b]]\}\big(y-w_2-(1-\tau)b(w_2)\big)$$

$$\pi(w_1) \;=\; \lambda\{1+\mathbb{P}[w_1+b(w_1)\geq R_2+\mathbb{E}[b]]\}\big(y-w_1-(1-\tau)b(w_1)\big)$$

where $b(w)$ are the benefits associated with wage offer w. In equilibrium, homogeneous firms make differential offers only when they are indifferent, i.e. when $\pi(w_2)=\pi(w_1)$.

What combinations of wages and benefits can arise in such asymmetric pure-strategy equilibria? Consider the following four possibilities.

- $b(w_1)=b(w_2)=\bar{b}$ or $b(w_1)=b(w_2)=\underline{b}$
  Since there is no new information to learn on the job, both sophisticates and naifs expect to stay in initially accepted employment with probability 1, which implies $u_1^S(w)=u_1^N(w)$ for any wage offer w. As a result, all workers initially adopt the same search strategy and thus there does not exist an equilibrium with differential wage offers when firms do not differentiate their benefit offers.

- $b(w_1)=\underline{b}$ and $b(w_2)=\bar{b}$
  Being analogy-based reasoners, the workers expect to encounter an offer of $(w_1,\underline{b})$ with probability p^2, $(w_1,\bar{b})$ with probability $p(1-p)$, $(w_2,\bar{b})$ with probability $(1-p)^2$, and $(w_2,\underline{b})$ with probability $(1-p)p$.

  Recall that for differential wage offers to coexist in equilibrium, $R_1^S>R_1^N$ is required. Since both types of agents acquire the same information during search and display correlation neglect, the only way in which sophisticates and naifs can adopt different reservation wages in period 1 is if they hold different beliefs about the value of on-the-job search for at least some realisations of (w,b). Further, as $c_2>c_1$, for any realisation it is the naifs who perceive on-the-job search to be more beneficial. Let's consider all (perceived) realisations in turn.

  First, independent of the perception of future search costs, no worker expects to search further once she encountered an offer of $(w_2,\bar{b})$. That is because further search can yield no benefit, along any dimension of the offer.

  Second, naifs have to believe that they will engage in on-the-job search for a realisation of $(w_1,\underline{b})$. Otherwise, both worker types would expect (correctly) that the probability of searching on the job is 0. Letting $R_2^N=\max\{\mathbb{E}[w]-c_1;w_2-c_1/(1-p)\}$, notice that a naif does not only overvalue the option to search on the job, but also expects to adopt a different search strategy in that for a wider range of parameters a naif expects to search until a high-wage offer is encountered.

Third, in the case when $b(w_2) = \bar{b}$ in (putative) equilibrium, workers have to actually search on the job for a realisation of $(w_2, \underline{b})$. Otherwise, the high-wage firms could profitably deviate to $\underline{b}$ without losing the worker. If sophisticated workers expect to search on the job for a realisation of $(w_2, \underline{b})$, they also do so for a realisation of $(w_1, \underline{b})$. This already implies that there does not exist a sticky equilibrium with $b(w_1) = \underline{b}$ and $b(w_2) = \bar{b}$. Furthermore, for both realisations with low benefits, naifs also expect to search further. Thus, the only realisation for which naifs can possibly hold wrong beliefs is $(w_1, \bar{b})$.

In the appendix, I consider all parametrisations that capture how the two types expect to search in period 2, if they engage in on-the-job search. Importantly, all of these specifications result in a contradiction, indicating that there cannot exist an equilibrium in which low wages are paired with low benefits, but high wages are paired with high benefits. The reason for this is that it is infeasible to support a decision rule under which naifs (erroneously) expect to search on the job when current employment offers low wage paired with high benefits, but at the same time accept low-paying jobs during the initial search. The realisation $(w_1, \bar{b})$ was nonetheless the only candidate for which naifs could hold wrong beliefs. As one can see, the logic behind non-existence of this type of equilibrium is very different than what would be suggested by the theory of compensating differentials (Rosen, 1986), which relies on observability and substitutability between the two components of an offer.

- $b(w_1) = \bar{b}$ and $b(w_2) = \underline{b}$

  The workers (sophisticated and naïve) expect to encounter an offer of $(w_1, \bar{b})$ with probability p^2, $(w_1, \underline{b})$ with probability $p(1-p)$, $(w_2, \underline{b})$ with probability $(1-p)^2$, and $(w_2, \bar{b})$ with probability $(1-p)p$.

  As before, we have:

$$R_2^S = \max\{\mathbb{E}[w] - c_2; w_2 - c_2/(1-p)\}$$

  and a worker accepts the first sampled offer while searching on the job if $c_2 \geq (1-p)(w_2 - w_1)$. A worker stays in the same employment for period 2 provided that $w + b \geq R_2^S + \mathbb{E}[b]$.

  Regarding specific realisations of wage-benefit pairs, no worker expects to search further once she has received an offer of $(w_2, \bar{b})$ and naifs have to believe they will search on the job if $(w_1, \underline{b})$ realises. Moreover, for $b(w_1) = \bar{b}$ to be part of a sticky equilibrium, a worker has to actually leave a job paying $(w_1, \underline{b})$. Then, we are left with three possible combinations whereby sophisticates' and naifs' beliefs about future on-the-job search disagree. Specifically, naifs might wrongly believe that they will search on the job for a realisation of $(w_1, \bar{b})$, or $(w_2, \underline{b})$, or both.

While all the possible cases are considered in the appendix, here I provide calculations underlying the case that does not result in a contradiction. Specifically, consider parametrisations under which naifs are overoptimistic about searching on the job for a realisation of $(w_2, \underline{b})$ only:

$$R_2^S + \mathbb{E}[b] < R_2^N + \mathbb{E}[b] \leq w_1 + \bar{b}$$
$$R_2^S + \mathbb{E}[b] \leq w_2 + \underline{b} < R_2^N + \mathbb{E}[b]$$

Moreover, when searching on the job, both types expect to accept the first encountered offer:

$$c_2 > c_1 \geq (1-p)(w_2 - w_1)$$

Then $R_2^S = \mathbb{E}[w] - c_2$ and $R_2^N = \mathbb{E}[w] - c_1$, and the above become:

$$R_2^S + \mathbb{E}[b] \leq w_2 + \underline{b} \iff c_2 \geq -p(w_2 - w_1) + p(\bar{b} - \underline{b})$$

$$R_2^N + \mathbb{E}[b] > w_2 + \underline{b} \iff c_1 < -p(w_2 - w_1) + p(\bar{b} - \underline{b})$$

and:

$$R_2^N + \mathbb{E}[b] \leq w_1 + \bar{b} \iff c_1 \geq (1-p)(w_2 - w_1) - (1-p)(\bar{b} - \underline{b})$$

Both types of workers search on the job for a realisation of $(w_1, \underline{b})$ if:

$$R_2^S + \mathbb{E}[b] > w_1 + \underline{b} \iff c_2 < (1-p)(w_2 - w_1) + p(\bar{b} - \underline{b})$$

Then:

$$u_1^S(w_1) = (w_1 + \mathbb{E}[b]) + p(w_1 + \bar{b}) + (1-p)(\mathbb{E}[w + b] - c_2)$$

$$u_1^S(w_2) = 2(w_2 + \mathbb{E}[b])$$

and sophisticates search until they find w_2 in period 1 if the following holds:

$$c_1 < (1-p)(u_1^S(w_2) - u_1^S(w_1)) \iff$$
$$c_1 < (1-p)(w_2 - w_1) + (1-p)(w_2 - pw_1 - (1-p)\mathbb{E}[w]) - (1-p)p(\bar{b} - \mathbb{E}[b]) + (1-p)^2 c_2$$

For naifs:

$$u_1^N(w_1) = (w_1 + \mathbb{E}[b]) + p(w_1 + \bar{b}) + (1-p)(\mathbb{E}[w+b] - c_1)$$

$$u_1^N(w_2) = (w_2 + \mathbb{E}[b]) + p(w_2 + \bar{b}) + (1-p)(\mathbb{E}[w+b] - c_1)$$

and:

$$c_1 \geq (1-p)(u_1^N(w_2) - u_1^N(w_1)) \iff c_1 \geq (1-p^2)(w_2 - w_1)$$

So far, we have not found a contradiction. Moreover, the set of inequalities above reduces to:

(i) $c_1 < (1-p)(w_2 - w_1) + (1-p)(w_2 - pw_1 - (1-p)\mathbb{E}[w]) - (1-p)p(\bar{b} - \mathbb{E}[b]) + (1-p)^2 c_2$

(ii) $c_1 < -p(w_2 - w_1) + p(\bar{b} - \underline{b})$

(iii) $c_1 \geq (1-p^2)(w_2 - w_1)$

(iv) $c_2 < (1-p)(w_2 - w_1) + p(\bar{b} - \underline{b})$

(v) $c_2 \geq -p(w_2 - w_1) + p(\bar{b} - \underline{b})$

Then, in expectation firms earn:

$$\begin{aligned} \pi(w_2) &= 2[y - w_2 - (1-\tau)\underline{b}] \\ \pi(w_1) &= 2\lambda[y - w_1 - (1-\tau)\bar{b}] \end{aligned}$$

per worker that samples their offer. The equal profits condition implies:

$$w_2 = (1-\lambda)y + \lambda w_1 + \lambda(1-\tau)\bar{b} - (1-\tau)\underline{b}$$

Moreover, in equilibrium with differentiated offers conditions, naifs' participation constraint (5) must hold. In this case, this simplifies to:

$$c_1 \leq 2/(2-p)\mathbb{E}[w] + \mathbb{E}[b] + p/(2-p)\bar{b}$$

The equal profits condition above can be used to represent one of the wage variables (e.g. w_2) as a function of the other one (w_1). Then, if there exists such $w_1 \geq 0$ and $p \in (0, 1)$ that satisfy all six inequalities listed above, we have found an equilibrium with a non-degenerate distribution of wages and benefits. Using numerical simulations, it can be shown that the set of exogenous parameters for which the above conditions are met simultaneously is indeed non-empty.[12]

In the appendix, I additionally show that the above is the only possible combination of strategies and beliefs for which equilibria in which low wages are paired with high benefits (and vice versa) may exist. In such equilibria, both sophisticated and naïve agents expect to stay in the same employment if they discover that a job offers high benefits. Nonetheless, while sophisticates anticipate (correctly) that they will remain in a high-wage job with low benefits, naifs expect (erroneously) to search further once low benefits have realised, even if the wage is high. When searching in period 1, sophisticates search until they have sampled a high-wage offer, and then remain in employment for two periods. Overestimating the option value of on-the-job search, naifs accept both low- and high-wage offers in period 1, expecting to search again if low benefits were to realise.

Interestingly, naifs get 'stuck' in high-wage, low-benefits jobs, which are accepted by sophisticates. At the same time, they hold correct beliefs about the propensity to stay in low-wage, high-benefit jobs, which are not considered at all during the initial search by the sophisticates. Being analogy-based reasoners, sophisticates are unwilling to take the risk of accepting a low-paying job and possibly having to search again at a higher cost.

From the firms' perspective, low-wage jobs attract a smaller mass of consumers, but allow the employer to generate greater profits per hired worker, resulting in equivalent total profits. Thus, the employer's cost of providing a compensation package consisting of a low wage and high benefits must be lower than the cost of a compensation package with a high wage and low benefits. However, due to the tax advantages to workplace benefits ($\tau > 0$), the workers accepting low-wage offers are not necessarily worse off than those on high-wage jobs.[13]

[12]For example, for $\lambda = 0.30$, $c_1 = 3$, $c_2 = 7.5$, $y = 200$, $\underline{b} = 5$, $\bar{b} = 24.17$, and $\tau = 0.25$, there exists an equilibrium in which 90% of firms offer a compensation package $(w_1, \bar{b}) = (180.0, 24.17)$, 10% of firms offer a profit-equivalent compensation package $(w_2, \underline{b}) = (195.69, 5)$, and the search behaviour of sophisticated and naïve agents follows the rules described above.

[13]Following from that point, it is in general unclear whether fully rational agents faced with such set of offers would search for a low-wage, high-benefit job, a high-wage, low-benefit job, or perhaps would accept the first sampled offer. For example, under the numerical specification discussed above, the fully rational agents would search until they find a low-wage, high-benefit job, which is markedly different from the search behaviour displayed by either sophisticates or naifs.

The results of the analysis above are summarised in the following.

Proposition 2: *In a deterministic setting, a pure-strategy equilibrium with differential wage offers may only exist when lower wages are paired with high benefits and higher wages with low benefits. There does not exist an equilibrium in which lower wages are paired with lower benefits or in which different wage offers are bundled with the same benefits.*

In contrast to symmetric equilibria with a degenerate distribution of wages and benefits characterised in Proposition 1, the existence of an asymmetric equilibrium relies on the presence of both naïve and sophisticated workers who adopt different strategies when searching for a job initially. Proposition 2 is therefore more useful for understanding how heterogeneous beliefs about future search costs translate into the workers' search strategies and the firms' incentives to offer particular compensation packages, conditional on searching individuals displaying correlation neglect.

Finally, note that the application to multidimensional offers with salient 'wages' and hidden 'benefits' is just one way of framing the predictions of such theoretical framework. The same logic and results apply to settings where firms compete on observable 'price' and unobservable 'quality' of some subscription product.

Generalising the results

Mixed strategies. Allowing the firms to adopt mixed strategies does not affect the conclusions drawn from the analysis above. First, the pure-strategy equilibrium outlined in Proposition 2 is equivalent to a mixed-strategy equilibrium, in which each firm randomises over compensation packages $(w_1, \bar{b})$ and $(w_2, \underline{b})$ with probabilities p and $(1-p)$, respectively. That is because from the firm's perspective, the two offers generate identical expected profits. Second, if there does not exist a profitable, pure-strategy deviation from either equilibrium outlined in Proposition 1, there also does not exist a profitable deviation in mixed strategies.

Furthermore, in a deterministic setting, allowing for mixed strategies cannot be invoked to keep the benefits component 'hidden' from the workers, even in absence of correlation neglect. That is because for a given wage offer w, the firm would never have an incentive to independently randomise over the benefits component b. If the firm prefers the worker to stay for both periods, it would strictly prefer to offer the cheapest benefit level that prevents the worker from searching on the job. If the firm (weakly) prefers the worker to leave, it would strictly prefer to offer the cheapest possible benefit.

Continuous choice of benefits. How would the results be affected if instead of a binary choice of benefits $b \in \{\underline{b}, \bar{b}\}$, the firms could select any benefits level from some interval $b \in [0, B]$? Observing again that for a given wage offer w, the optimal level of benefits to provide is uniquely determined, we can discuss the qualitative robustness of the main results above. First, for any given b, a putative equilibrium with a degenerate distribution of offers necessarily features $w(b)$, such that the worker's participation constraint binds. Then, to verify whether $(w(b), b)$ indeed constitutes an equilibrium, we only need to consider downward deviations from b. Suppose that the firm deviates by lowering the benefits by some $\epsilon > 0$. The deviating firm does not lose the worker, and the deviation is indeed profitable, as long as:

$$w(b) + b - c_2 \;\leq\; w(b) + b - \epsilon \quad \Longleftrightarrow \quad \epsilon \;\leq\; c_2$$

For $c_2 > 0$, we can always find a small enough increment $\epsilon \leq c_2$, such that the deviating firm is strictly better off. Thus, as long as $b > 0$, there must exist such profitable deviation and we obtain a result that is effectively an extreme version of Proposition 1. Namely, when workers cannot observe the benefits component while searching for a job, equilibria with positive benefits are impossible to sustain due to unravelling. Similarly, an equilibrium with differential offers from Proposition 2 no longer exists if the firms can lower their benefits by an arbitrarily small increment.

5 Future work

The above analysis is still incomplete and examining the robustness of the results presented here is currently work in progress. Beyond a class of sticky equilibria, what about situations in which some workers leave employment on the equilibrium path and can be re-hired in period 2? What if the firms are allowed to adjust their offers in period 2?

In the next steps, it may also be informative to introduce uncertainty into the model. One could consider a type of uncertainty that is resolved before the players' choice of strategy, as in models where firms are heterogeneous in their productivity (e.g., Van Den Berg, 2003). Alternatively, uncertainty could be resolved after the players have chosen their strategies, as in models with idiosyncratic match quality (e.g., Gamp and Krähmer, 2022).

A Rational benchmark

Static search

Consider the search behaviour of a *fully rational* agent who observes all components of an offer while searching and contrast it to the strategy adopted by analogy-based reasoners studied in the main body of the paper. Such fully rational agent bears the same search costs when sampling offers from a distribution, but observes both the salient and hidden attributes when searching. Since the utility function is additive in different components of the offer, the agent is concerned with the total compensation $w + b$ when comparing different offers. Her value function from sampling compensation package $w + b$ in period 2 is:

$$\nu(w+b) = \max\{w + b\ ;\ \nu_2 - c_2\}$$

where

$$\nu_2 = \int_{(w+b)} \nu_2(w+b)\psi(w+b)d(w+b)$$

and $\psi(w+b)$ is the probability density function derived from the joint distribution of wage and benefit offers. In the above, we have simplified the expected utility from accepting the offer by assuming that the idiosyncratic match quality has a mean zero, i.e. $\mathbb{E}[\epsilon_{ik}] = 0$. In this case, the optimal strategy is again characterised by a cutoff rule, but with a reservation applied to total compensation.

For all $w + b < \mathcal{R}_2$, we have $\nu_2(w+b) = \nu_2 - c_2$, while for all $w+b \geq \mathcal{R}_2$, $\nu_2(w+b) = w+b$. Combining this with the indifference at the reservation wage, $\mathcal{R}_2 = \nu_2 - c_2$, yields:

$$\begin{aligned}\int_{(w+b)} \nu_2(w+b)\psi(w+b)d(w+b) &= \mathcal{R}_2 \cdot \mathbb{P}[w+b < \mathcal{R}_2] \ + \int_{w+b\geq\mathcal{R}_2} (w+b)\psi(w+b)d(w+b) \\ &= \mathcal{R}_2 \ + \ c_2\end{aligned}$$

which solves for:

$$c_2 = \int_{w+b\geq\mathcal{R}_2} (w+b-\mathcal{R}_2)\psi(w+b)d(w+b)$$

Thus, the optimal reservation compensation $\mathcal{R}_2^*$ equalises the marginal cost of search with the marginal benefit of further search. Since the right-hand side of the above equality is strictly decreasing in $\mathcal{R}_2$, the optimal reservation compensation is unique.

We obtain the following, fully rational version of Lemma 1, which characterises when an employed individual engages in on-the-job search.

Lemma A.1: *In period 2, the rational agent stays in current employment (searches on the job) when* $u_{ik} = w_k + b_k + \epsilon_{ik} \ \geq (<) \ \mathcal{R}_2^*$.

In the main body of the paper, we were able to draw a clear-cut comparison between the perceived levels of reservation wage for period 2, depending on whether the agent is sophisticated or naïve about an increase in her search cost (see Lemma 2). These reservation wages, denoted R_2^S and R_2^N, are an important input into the strategy adopted in period 1. Can a similarly unambiguous comparison be drawn between the cutoffs of agents who either do or do not observe the hidden component when sampling offers? From the optimality conditions for the fully rational and the sophisticated type, we have:

$$\begin{aligned} c_2 \ = \int_{w \geq R_2^S} (w - R_2^S)\phi(w)dw \ &= \int_{w+b \geq \mathcal{R}_2^*} (w + b - \mathcal{R}_2^*)\psi(w+b)d(w+b) \\ &= \ \mathbb{P}[b = \bar{b}] \cdot \int_{w \geq \mathcal{R}_2^* - \bar{b}} (w - (\mathcal{R}_2^* - \bar{b}))\phi(w|\bar{b})dw + \\ &+ \ \mathbb{P}[b = \underline{b}] \cdot \int_{w \geq \mathcal{R}_2^* - \underline{b}} (w - (\mathcal{R}_2^* - \underline{b}))\phi(w|\underline{b})dw \end{aligned} \tag{7}$$

where the third equality follows from the law of total probability. When formulating her cutoff rule in terms of total compensation, the rational agent adopts a smaller reservation wage if she samples $\bar{b}$, but accepts only higher wages if she samples $\underline{b}$. This is the consequence of the rational agent having access to more information when searching.

Because the sophisticated agent takes into account the correct unconditional distribution of wages, it must be the case that:

$$\mathcal{R}_2^* - \bar{b} \ \leq \ R_2^S \ \leq \ \mathcal{R}_2^* - \underline{b} \tag{8}$$

To see this, one can show that either violation of the two inequalities above would lead to a contradiction. It might be helpful to re-write the RHS of equation (7) as follows:

$$
\begin{aligned}
\int_{w+b\geq\mathcal{R}_2^*}(w+b-\mathcal{R}_2^*)\psi(w+b)d(w+b) \;&=\; \mathbb{P}[b=\bar{b}]\cdot\int_{w\geq\mathcal{R}_2^*-\bar{b}}^{\mathcal{R}_2^*-\underline{b}}(w-(\mathcal{R}_2^*-\bar{b}))\phi(w|\bar{b})dw\;+\\
&+\;\mathbb{P}[b=\bar{b}]\cdot\int_{w\geq\mathcal{R}_2^*-\underline{b}}(w-(\mathcal{R}_2^*-\bar{b}))\phi(w|\bar{b})dw\;+\\
&+\;\mathbb{P}[b=\underline{b}]\cdot\int_{w\geq\mathcal{R}_2^*-\underline{b}}(w-(\mathcal{R}_2^*-\underline{b}))\phi(w|\underline{b})dw\\
&\\
&=\;\mathbb{P}[b=\bar{b}]\cdot\int_{w\geq\mathcal{R}_2^*-\bar{b}}^{\mathcal{R}_2^*-\underline{b}}(w-(\mathcal{R}_2^*-\bar{b}))\phi(w|\bar{b})dw\;+\\
&+\;\int_{w\geq\mathcal{R}_2^*-\underline{b}}(w-\mathcal{R}_2^*)\phi(w)dw\;+\\
&+\;\bar{b}\cdot\mathbb{P}[b=\bar{b}]\cdot\mathbb{P}[w\geq\mathcal{R}_2^*-\underline{b}|b=\bar{b}]\;+\\
&+\;\underline{b}\cdot\mathbb{P}[b=\underline{b}]\cdot\mathbb{P}[w\geq\mathcal{R}_2^*-\underline{b}|b=\underline{b}]\\
&\\
&=\;\int_{w\geq\mathcal{R}_2^*-\underline{b}}(w-\mathcal{R}_2^*)\phi(w)dw\;+\\
&+\;\mathbb{P}[b=\bar{b}]\cdot\int_{w\geq\mathcal{R}_2^*-\bar{b}}^{\mathcal{R}_2^*-\underline{b}}(w-\mathcal{R}_2^*)\phi(w|\bar{b})dw\;+\\
&+\;\bar{b}\cdot\mathbb{P}[b=\bar{b}]\cdot\mathbb{P}[w\geq\mathcal{R}_2^*-\bar{b}|b=\bar{b}]\;+\\
&+\;\underline{b}\cdot\mathbb{P}[b=\underline{b}]\cdot\mathbb{P}[w\geq\mathcal{R}_2^*-\underline{b}|b=\underline{b}]
\end{aligned}
$$

First, since all the terms following the last equality are non-negative, having $R_2^S > \mathcal{R}_2^* - \underline{b}$ necessarily leads to a violation of equation (7).

Second, suppose that $R_2^S < \mathcal{R}_2^* - \bar{b}$. Then:

$$
\begin{aligned}
\int_{w\geq R_2^S}\big(w-R_2^S\big)\phi(w)dw \;>\; \int_{w\geq\mathcal{R}_2^*-\bar{b}}\big(w-(\mathcal{R}_2^*-\bar{b})\big)\phi(w)dw \;&=\; \int_{w\geq\mathcal{R}_2^*-\bar{b}}\big(w-\mathcal{R}_2^*)\big)\phi(w)dw\;+\\
&+\;\bar{b}\cdot\mathbb{P}[w\geq\mathcal{R}_2^*-\bar{b}]
\end{aligned}
$$

which from the last line of the above re-arrangements again contradicts (7). That is because:

$$
\begin{aligned}
\int_{w\geq\mathcal{R}_2^*-\bar{b}}\big(w-\mathcal{R}_2^*)\big)\phi(w)dw \;&=\; \int_{w\geq\mathcal{R}_2^*-\underline{b}}\big(w-\mathcal{R}_2^*)\big)\phi(w)dw+\int_{w=\mathcal{R}_2^*-\bar{b}}^{\mathcal{R}_2^*-\underline{b}}\big(w-\mathcal{R}_2^*)\big)\phi(w)dw\;\geq\\
&\geq\int_{w\geq\mathcal{R}_2^*-\underline{b}}(w-\mathcal{R}_2^*)\phi(w)dw\;+\;\mathbb{P}[b=\bar{b}]\cdot\int_{w=\mathcal{R}_2^*-\bar{b}}^{\mathcal{R}_2^*-\underline{b}}(w-\mathcal{R}_2^*)\phi(w|\bar{b})dw
\end{aligned}
$$

and the inequality is strict as long as $\mathbb{P}[b = \underline{b}] \cdot \mathbb{P}[w \in [\mathcal{R}_2^* - \bar{b}, \mathcal{R}_2^* - \underline{b}] \mid b = \underline{b}] > 0$. Secondly:

$$\begin{aligned}\bar{b} \cdot \mathbb{P}[w \geq \mathcal{R}_2^* - \bar{b}] \; &= \; \bar{b} \cdot \mathbb{P}[b = \bar{b}] \cdot \mathbb{P}[w \geq \mathcal{R}_2^* - \bar{b} | b = \bar{b}] + \bar{b} \cdot \mathbb{P}[b = \underline{b}] \cdot \mathbb{P}[w \geq \mathcal{R}_2^* - \bar{b} | b = \underline{b}] \; \geq \\ &\geq \; \bar{b} \cdot \mathbb{P}[b = \bar{b}] \cdot \mathbb{P}[w \geq \mathcal{R}_2^* - \bar{b} | b = \bar{b}] + \underline{b} \cdot \mathbb{P}[b = \underline{b}] \cdot \mathbb{P}[w \geq \mathcal{R}_2^* - \underline{b} | b = \underline{b}]\end{aligned}$$

and the inequality is again strict when $\mathbb{P}[w \in [\mathcal{R}_2^* - \bar{b}, \mathcal{R}_2^* - \underline{b}] \mid b = \underline{b}] > 0$. Thus, we have shown that assuming $R_2^S < \mathcal{R}_2^* - \bar{b}$ also leads to a violation of (7). In sum, we have the following observation.

Lemma A.2: *For a given distribution of wage and benefit offers, (8) holds. That is, when searching in period 2, the reservation wage adopted by an agent who observes the benefits component constitutes an upper (lower) bound for the reservation wage of an agent who does not observe benefits, if the associated benefits are low (high).*

What is the economic interpretation of these regularities? Of course, an agent with an access to superior information calibrates her reservation wage to a particular observed realisation of b. Nevertheless, a boundedly rational agent chooses the right reservation wage 'on average', as she takes the correct (unconditional) distribution of b into account. Put differently, the boundedly rational agent is never uniformly more, or less, picky than her fully rational counterpart, at least when the search problem is static.

Dynamic search

When the agents engage in dynamic search, the comparison between those who are fully rational and boundedly rational becomes markedly different. Recall that the expectation of learning about b while on the job is an important determinant of the period-1 search strategy for a sophisticated agent. In contrast, a fully rational agent already has all the relevant information at her disposal when searching initially, and thus, given the increase in search costs following the acceptance of a particular contract, she only accepts offers to which she is willing to commit for both periods. This implies that in a deterministic setting there is no 'meaningfully dynamic' aspect to the rational agent's search behaviour and the solution derived above could be applied to period-1 search in a straightforward way. Thus, for a fully rational agent we have:

$$\nu_1(w+b) = \max\left\{u_1^R(w+b), \nu_1 - c_1\right\}$$

where $u_1^R(w+b) = 2(w+b)$ and $\nu_1 = \int_{(w+b)} \nu_1(w+b)\psi(w+b)d(w+b)$. Immediately, her optimal reservation compensation solves:

$$c_1 = \int_{w+b \geq \mathcal{R}_1} 2(w+b-\mathcal{R}_1)\psi(w+b)d(w+b)$$

Unsurprisingly, since the agent plans for staying in employment for two periods and $c_1 < c_2$, we have $\mathcal{R}_1 > \mathcal{R}_2$. That is, the fully rational agent adopts a strictly higher acceptance threshold when searching for an offer that binds for an additional period at a lower search cost.

Comparing it to the expected utility that a sophisticated agent derives from accepting a wage w:

$$\begin{aligned} u_1^S(w) \;=\; & \big(w + \mathbb{E}[b]\big) \;+\; \mathbb{P}[w + b \geq R_2^S + \mathbb{E}[b]] \cdot \big(w + \mathbb{E}[b \,|\, b \geq R_2^S + \mathbb{E}[b] - w]\big) \\ & + \; \mathbb{P}[w + b < R_2^S + \mathbb{E}[b]] \cdot \big(v_2^S - c_2\big) \end{aligned}$$

and her optimal cutoff:

$$c_1 \;=\; \textstyle\int_{w \geq R_1} \big(u_1^S(w) - u_1^S(R_1)\big)\phi(w)dw$$

we can notice two key differences between $u_1^R(w+b)$ and $u_1^S(w)$. First, because when a fully rational agent accepts a contract she plans to derive the associated utility exactly twice, the initially accepted wage is a stronger determinant of the overall expected utility from acceptance, i.e. $du_1^R(w+b)/dw \geq du_1^S(w)/dw$. Second, because the sophisticated agent does not observe the hidden component when sampling offers, she may either over- or under-value any given wage offer w, depending on the strategies adopted by the firms.

To see if an analogue of Lemma A.2 obtains, suppose first that $R_1^S > \mathcal{R}_1^* - \underline{b}$, in which case the sophisticated agent would reject at least some offers that are accepted by a fully rational agent who realises that the offered benefits are low. This, however, contradicts the optimality of the two agent type's search rules:

$$\begin{aligned} c_1 \;=\; \int_{w \geq R_1^S} \big(u_1^S(w) - u_1^S(R_1)\big)\phi(w)dw \;&=\; \int_{w+b \geq \mathcal{R}_1^*} 2(w + b - \mathcal{R}_1^*)\psi(w+b)d(w+b) \\ &=\; \mathbb{P}[b = \bar{b}] \cdot \int_{w \geq \mathcal{R}_1^* - \bar{b}} 2(w - (\mathcal{R}_1^* - \bar{b}))\phi(w|\bar{b})dw \,+ \\ &+\; \mathbb{P}[b = \underline{b}] \cdot \int_{w \geq \mathcal{R}_1^* - \underline{b}} 2(w - (\mathcal{R}_1^* - \underline{b}))\phi(w|\underline{b})dw \end{aligned} \tag{9}$$

since we would have:

$$
\begin{aligned}
c_1 &= \int_{w \geq R_1^S} \big(u_1^S(w) - u_1^S(R_1)\big)\phi(w)dw \;< \\
&< \int_{w \geq \mathcal{R}_1^* - \underline{b}} \big(u_1^S(w) - u_1^S(\mathcal{R}_1^* - \underline{b})\big)\phi(w)dw \;= \\
&= \mathbb{P}[b = \bar{b}] \cdot \int_{w \geq \mathcal{R}_1^* - \underline{b}} \big(u_1^S(w) - u_1^S(\mathcal{R}_1^* - \underline{b})\big)\phi(w|\bar{b})dw \;+ \\
&+ \mathbb{P}[b = \underline{b}] \cdot \int_{w \geq \mathcal{R}_1^* - \underline{b}} \big(u_1^S(w) - u_1^S(\mathcal{R}_1^* - \underline{b})\big)\phi(w|\underline{b})dw \;\leq \\
&\leq \mathbb{P}[b = \bar{b}] \cdot \int_{w \geq \mathcal{R}_1^* - \underline{b}} 2(w - (\mathcal{R}_1^* - \underline{b}))\phi(w|\bar{b})dw \;+ \\
&+ \mathbb{P}[b = \underline{b}] \cdot \int_{w \geq \mathcal{R}_1^* - \underline{b}} 2(w - (\mathcal{R}_1^* - \underline{b}))\phi(w|\underline{b})dw \;< \\
&< \mathbb{P}[b = \bar{b}] \cdot \int_{w \geq \mathcal{R}_1^* - \bar{b}} 2(w - (\mathcal{R}_1^* - \bar{b}))\phi(w|\bar{b})dw \;+ \\
&+ \mathbb{P}[b = \underline{b}] \cdot \int_{w \geq \mathcal{R}_1^* - \underline{b}} 2(w - (\mathcal{R}_1^* - \underline{b}))\phi(w|\underline{b})dw \;= \\
&= c_1
\end{aligned}
$$

Thus, it must be the case that $R_1^S \leq \mathcal{R}_1^* - \underline{b}$.

In stark contrast, however, we cannot rule out that $R_1^S < \mathcal{R}_1^* - \bar{b}$, i.e. that the sophisticated agent is uniformly less picky than a fully rational one in the sense that there are at least some offers that are accepted by the sophisticated agent, but are rejected by the fully rational agent even when the associated benefits are high.

To see that this does not lead to a contradiction, consider the following stylised example. Suppose that there are two possible wage levels $\underline{w} < \bar{w}$, which are drawn with probability 1/2. Lower wage $\underline{w}$ is always paired with low benefits $\underline{b}$ and higher wage is always paired with high benefits $\bar{b}$, but being analogy-based reasoners the boundedly rational agents do not realise this and thus expect each benefit level to materialise with the (correct) unconditional probability of 1/2 for every wage offer.

Then, a rational agent searches for the higher-wage offer in period 1 as long as:

$$
c_1 \;\leq\; 1/2 \times 2\big((\bar{w} + \bar{b}) - (\underline{w} + \underline{b})\big) \;=\; (\bar{w} - \underline{w}) + (\bar{b} - \underline{b})
$$

and we have $\mathcal{R}_1^* = \bar{w} + \bar{b}$.

When searching in period 2, a sophisticated agent accepts the first encountered offer as long as:[14]

[14]If the agent was searching until she finds a high-wage offer in period 2, she would also do so in period 1, when the costs are lower and potential benefits of finding a good offer are higher. Thus, for the purposes of this example, we focus on a case when the sophisticated agent terminates her search after the first draw in period 2.

$$c_2 \; > \; 1/2 \times (\bar{w} - \underline{w})$$

Then, $R_2^S = \underline{w}$ and the worker remains in previously accepted employment as long as her realisation satisfies $w + b \geq \underline{w} + \mathbb{E}[b]$. Then, the expected utility from accepting $\bar{w}$ in period 1 is:

$$\begin{aligned} u_1^S(\bar{w}) \; &= \; (\bar{w} + \mathbb{E}[b]) + \\ &+ \; \mathbb{P}[\bar{w} + b \geq \underline{w} + \mathbb{E}[b]] \times (\bar{w} + \mathbb{E}[b \,|\, b \geq \underline{w} - \bar{w} + \mathbb{E}[b]]) + \\ &+ \; \big(1 - \mathbb{P}[\bar{w} + b \geq \underline{w} + \mathbb{E}[b]]\big) \times (\mathbb{E}[w] + \mathbb{E}[b] - c_2) \end{aligned}$$

Consider a parametrisation under which $\bar{w} + \underline{b} < \underline{w} + \mathbb{E}[b]$, so that the sophisticated agent would not stay for two periods in a high-paying job that offers low benefits. Then, the above simplifies to:

$$u_1^S(\bar{w}) \; = \; (\bar{w} + \mathbb{E}[b]) \; + \; 1/2 \times (\bar{w} + \bar{b}) \; + \; 1/2 \times (\mathbb{E}[w] + \mathbb{E}[b] - c_2)$$

while the expected utility from accepting a lower wage offer is:

$$u_1^S(\underline{w}) \; = \; (\underline{w} + \mathbb{E}[b]) \; + \; 1/2 \times (\underline{w} + \bar{b}) \; + \; 1/2 \times (\mathbb{E}[w] + \mathbb{E}[b] - c_2)$$

Then, we have $u_1^S(\bar{w}) - u_1^S(\underline{w}) = 3/2 \times (\bar{w} - \underline{w})$ and the sophisticated agent accepts the first encountered offer in period 1 as long as:

$$c_1 \; > \; 1/2 \times (u_1^S(\bar{w}) - u_1^S(\underline{w})) \; = \; 3/4 \times (\bar{w} - \underline{w})$$

In this example, we have $R_1^S < \mathcal{R}_1^* - \bar{b} \iff \underline{w} < \bar{w}$, which is true by assumption. The decision rules described above are optimal from the perspective of a rational and a sophisticated agent as long as the following set of conditions hold simultaneously:

i) $c_1 \; \leq \; (\bar{w} - \underline{w}) + (\bar{b} - \underline{b})$

ii) $c_2 \; > \; 1/2 \times (\bar{w} - \underline{w})$

iii) $\bar{w} - \underline{w} \; < \; \mathbb{E}[b] - \underline{b} \; = \; 1/2 \times (\bar{b} - \underline{b})$

iv) $c_1 \; > \; 3/4 \times (\bar{w} - \underline{w})$

These conditions are not contradictory with $c_2 > c_1$, and thus the example is valid. For instance, setting $(\bar{w} - \underline{w}) = 0.02$ and $(\bar{b} - \underline{b}) = 0.05$, $c_1 = 0.02$ and $c_2 = 0.04$ satisfy all inequalities above.

Intuitively, the reason why the above example results in $R_1^S < \mathcal{R}_1^* - \bar{b}$ is that a higher wage is always paired with high benefits and thus a rational agent has a strong incentive to search in period 1 until she encounters a high-wage offer. In contrast, from the perspective of a sophisticated boundedly rational agent, accepting any wage offer involves a 'benefits lottery' that is realised after the acceptance. As a result, a sophisticated agent might be "uniformly less picky" in period 1, because accepting even the lower wage leads to the same lottery. This leads to an observation that, although a sophisticated agent is "correct on average" when performing static search, the impact of bounded rationality is fundamentally different in a dynamic setting.

We can summarise the above findings as follows.

Lemma A.3: *When searching in period 1, a boundedly rational agent can be uniformly less picky, but not uniformly more picky, than a fully rational one. That is, while the optimal cutoffs must satisfy* $R_1^S \leq \mathcal{R}_1^* - \underline{b}$*, there exist parametrisations for which* $R_1^S < \mathcal{R}_1^* - \bar{b}$.

B Derivation of Proposition 2

This appendix contains all calculations underlying the claims made in Proposition 2.

$\mathbf{b(w_1) = \underline{b}}$ and $\mathbf{b(w_2) = \bar{b}}$

Recall that in this case, the only realisation for which naifs can possibly hold wrong beliefs is $(w_1, \bar{b})$. This occurs when:

$$R_2^S + \mathbb{E}[b] \leq w_1 + \bar{b} < R_2^N + \mathbb{E}[b]$$

Consider the following cases separately:

a) $c_2 > c_1 \geq (1-p)(w_2 - w_1)$

   This implies that $R_2^S = \mathbb{E}[w] - c_2$ and $R_2^N = \mathbb{E}[w] - c_1$.

   Then:

$$R_2^S + \mathbb{E}[b] \leq w_1 + \bar{b} \iff c_2 \geq (1-p)(w_2 - w_1) - p(\bar{b} - \underline{b})$$

$$R_2^N + \mathbb{E}[b] > w_1 + \bar{b} \iff c_1 < (1-p)(w_2 - w_1) - p(\bar{b} - \underline{b})$$

   which guarantees that naifs hold wrong beliefs regarding the realisation $(w_1, \bar{b})$.

   Note, however, that we cannot simultaneously have $c_1 \geq (1-p)(w_2 - w_1)$ and $c_1 < (1-p)(w_2 - w_1) - p(\bar{b} - \underline{b})$.

b) $c_2 \geq (1-p)(w_2 - w_1) > c_1$

This implies $R_2^S = \mathbb{E}[w] - c_2$ and $R_2^N = w_2 - c_1/(1-p)$.

Then:

$$R_2^S + \mathbb{E}[b] \leq w_1 + \bar{b} \iff c_2 \geq (1-p)(w_2 - w_1) - p(\bar{b} - \underline{b})$$

$$R_2^N + \mathbb{E}[b] > w_1 + \bar{b} \iff c_1 < (1-p)(w_2 - w_1) - p(1-p)(\bar{b} - \underline{b})$$

guarantees that naifs hold wrong beliefs regarding the realisation $(w_1, \bar{b})$.

Sophisticates actually search on the job for a realisation of $(w_2, \underline{b})$ when:

$$R_2^S + \mathbb{E}[b] > w_2 + \underline{b} \iff c_2 < (1-p)(\bar{b} - \underline{b}) - p(w_2 - w_1)$$

this also guarantees that sophisticates (as well as naifs) expect to search for a realisation of $(w_1, \underline{b})$.

Then:

$$u_1^S(w_1) = (w_1 + \mathbb{E}[b]) + (1-p)(w_1 + \bar{b}) + p(\mathbb{E}[w+b] - c_2)$$

$$u_1^S(w_2) = (w_2 + \mathbb{E}[b]) + (1-p)(w_2 + \bar{b}) + p(\mathbb{E}[w+b] - c_2)$$

and sophisticates search until they find w_2 in period 1 if the following holds:

$$c_1 < (1-p)(u_1^S(w_2) - u_1^S(w_1)) \iff c_1 < (2-p)(w_2 - w_1)$$

For naifs, we have:

$$u_1^N(w_1) = (w_1 + \mathbb{E}[b]) + (w_2 + \mathbb{E}[b] - c_1/(1-p))$$

$$u_1^S(w_2) = (w_2 + \mathbb{E}[b]) + (1-p)(w_2 + \bar{b}) + p(w_2 + \mathbb{E}[b] - c_1/(1-p))$$

and:

$$c_1 \geq (1-p)(u_1^N(w_2) - u_1^N(w_1)) \iff c_1 \geq \tfrac{(1-p)}{p}(w_2 - w_1) + (1-p)^2(\bar{b} - \underline{b})$$

However, for $p \in (0,1)$ we cannot simultaneously have the above and $c_1 < (1-p)(w_2 - w_1)$.

c) $(1-p)(w_2 - w_1) > c_2 > c_1$

This implies $R_2^S = w_2 - c_2/(1-p)$ and $R_2^N = w_2 - c_1/(1-p)$.

Then:

$$R_2^S + \mathbb{E}[b] \leq w_1 + \bar{b} \iff c_2 \geq (1-p)(w_2 - w_1) - p(1-p)(\bar{b} - \underline{b})$$

$$R_2^N + \mathbb{E}[b] > w_1 + \bar{b} \iff c_1 < (1-p)(w_2 - w_1) - p(1-p)(\bar{b} - \underline{b})$$

guarantees that naifs hold wrong beliefs regarding the realisation $(w_1, \bar{b})$.

Sophisticates actually search on the job for a realisation of $(w_2, \underline{b})$ when:

$$R_2^S + \mathbb{E}[b] > w_2 + \underline{b} \iff c_2 < (1-p)^2(\bar{b} - \underline{b})$$

this also guarantees that sophisticates (as well as naifs) expect to search for a realisation of $(w_1, \underline{b})$.

Then:

$$u_1^S(w_1) = (w_1 + \mathbb{E}[b]) + (1-p)(w_1 + \bar{b}) + p(w_2 + \mathbb{E}[b] - c_2/(1-p))$$

$$u_1^S(w_2) = (w_2 + \mathbb{E}[b]) + (1-p)(w_2 + \bar{b}) + p(w_2 + \mathbb{E}[b] - c_2/(1-p))$$

and sophisticates search until they find w_2 in period 1 if the following holds:

$$c_1 < (1-p)(u_1^S(w_2) - u_1^S(w_1)) \iff c_1 < (2-p)(w_2 - w_1)$$

For naifs, we have:

$$u_1^N(w_1) = (w_1 + \mathbb{E}[b]) + (w_2 + \mathbb{E}[b] - c_1/(1-p))$$

$$u_1^S(w_2) = (w_2 + \mathbb{E}[b]) + (1-p)(w_2 + \bar{b}) + p(w_2 + \mathbb{E}[b] - c_1/(1-p))$$

and:

$$c_1 \geq (1-p)(u_1^N(w_2) - u_1^N(w_1)) \iff c_1 \geq \tfrac{(1-p)}{p}(w_2 - w_1) + (1-p)^2(\bar{b} - \underline{b})$$

But this condition results in contradiction, just as in case b).

$\mathbf{b(w_1) = \bar{b}}$ and $\mathbf{b(w_2) = \underline{b}}$

In order to limit the number of cases to consider, the calculations below suppose that a worker never searches on a job offering high wage and low benefits. Nevertheless, this does not affect the statement of Proposition 2.

<u>First</u>:

$$R_2^S + \mathbb{E}[b] \leq w_1 + \bar{b} < R_2^N + \mathbb{E}[b]$$
$$R_2^S + \mathbb{E}[b] < R_2^N + \mathbb{E}[b] \leq w_2 + \underline{b}$$

a) $c_2 > c_1 \geq (1-p)(w_2 - w_1)$

$\implies R_2^S = \mathbb{E}[w] - c_2$ and $R_2^N = \mathbb{E}[w] - c_1$.

Then:

$$R_2^S + \mathbb{E}[b] \leq w_1 + \bar{b} \iff c_2 \geq (1-p)(w_2 - w_1) - (1-p)(\bar{b} - \underline{b})$$

$$R_2^N + \mathbb{E}[b] > w_1 + \bar{b} \iff c_1 < (1-p)(w_2 - w_1) - (1-p)(\bar{b} - \underline{b})$$

and:

$$R_2^N + \mathbb{E}[b] \leq w_2 + \underline{b} \iff c_1 \geq -p(w_2 - w_1) + p(\bar{b} - \underline{b})$$

Note, however, that we cannot simultaneously have $c_1 \geq (1-p)(w_2 - w_1)$ and $c_1 < (1-p)(w_2 - w_1) - (1-p)(\bar{b} - \underline{b})$.

b) $c_2 \geq (1-p)(w_2 - w_1) > c_1$

$\implies R_2^S = \mathbb{E}[w] - c_2$ and $R_2^N = w_2 - c_1/(1-p)$.

Then:

$$R_2^S + \mathbb{E}[b] \leq w_1 + \bar{b} \iff c_2 \geq (1-p)(w_2 - w_1) - (1-p)(\bar{b} - \underline{b})$$

$$R_2^N + \mathbb{E}[b] > w_1 + \bar{b} \iff c_1 < (1-p)(w_2 - w_1) - (1-p)^2(\bar{b} - \underline{b})$$

and:

$$R_2^N + \mathbb{E}[b] \leq w_2 + \underline{b} \iff c_1 \geq (1-p)p(\bar{b} - \underline{b})$$

Both types of workers search on the job for a realisation of $(w_1, \underline{b})$ if:

$$R_2^S + \mathbb{E}[b] > w_1 + \underline{b} \iff c_2 < (1-p)(w_2 - w_1) + p(\bar{b} - \underline{b})$$

Then:

$$u_1^S(w_1) = (w_1 + \mathbb{E}[b]) + p(w_1 + \bar{b}) + (1-p)(\mathbb{E}[w+b] - c_2)$$

$$u_1^S(w_2) = 2(w_2 + \mathbb{E}[b])$$

and sophisticates search until they find w_2 in period 1 if the following holds:

$$c_1 < (1-p)(u_1^S(w_2) - u_1^S(w_1)) \iff$$
$$c_1 < (1-p)(w_2 - w_1) + (1-p)(w_2 - pw_1 - (1-p)\mathbb{E}[w]) - p(1-p)(\bar{b} - \mathbb{E}[b]) + (1-p)^2 c_2$$

For naifs:

$$u_1^N(w_1) = (w_1 + \mathbb{E}[b]) + (w_2 + \mathbb{E}[b] - c_1/(1-p))$$

$$u_1^N(w_2) = 2(w_2 + \mathbb{E}[b])$$

and:

$$c_1 \geq (1-p)(u_1^N(w_2) - u_1^N(w_1)) \iff 0 \geq (1-p)(w_2 - w_1)$$

which is a contradiction.

c) $(1-p)(w_2 - w_1) > c_2 > c_1$
$\implies R_2^S = w_2 - c_2/(1-p)$ and $R_2^N = w_2 - c_1/(1-p)$.
Then:

$$R_2^S + \mathbb{E}[b] \leq w_1 + \bar{b} \iff c_2 \geq (1-p)(w_2 - w_1) - (1-p)^2(\bar{b} - \underline{b})$$

$$R_2^N + \mathbb{E}[b] > w_1 + \bar{b} \iff c_1 < (1-p)(w_2 - w_1) - (1-p)^2(\bar{b} - \underline{b})$$

and:

$$R_2^N + \mathbb{E}[b] \leq w_2 + \underline{b} \iff c_1 \geq (1-p)p(\bar{b} - \underline{b})$$

Both types of workers search on the job for a realisation of $(w_1, \underline{b})$ if:

$$R_2^S + \mathbb{E}[b] > w_1 + \underline{b} \iff c_2 < (1-p)(w_2 - w_1) + p(1-p)(\bar{b} - \underline{b})$$

Then:

$$u_1^S(w_1) = (w_1 + \mathbb{E}[b]) + p(w_1 + \bar{b}) + (1-p)(w_2 + \mathbb{E}[b] - c_2/(1-p))$$

$$u_1^S(w_2) = 2(w_2 + \mathbb{E}[b])$$

and sophisticates search until they find w_2 in period 1 if the following holds:

$$\begin{gathered} c_1 < (1-p)(u_1^S(w_2) - u_1^S(w_1)) \iff \\ c_1 < (1-p)(w_2 - w_1) + (1-p)(w_2 - \mathbb{E}[w]) - p(1-p)(\bar{b} - \mathbb{E}[b]) + (1-p)c_2 \end{gathered}$$

For naifs:

$$u_1^N(w_1) = (w_1 + \mathbb{E}[b]) + (w_2 + \mathbb{E}[b] - c_1/(1-p))$$

$$u_1^N(w_2) = 2(w_2 + \mathbb{E}[b])$$

and:

$$c_1 \geq (1-p)(u_1^N(w_2) - u_1^N(w_1)) \iff 0 \geq (1-p)(w_2 - w_1)$$

which yields the same contradiction as in b).

Thus there does not exist an equilibrium in which low wages are paired with high benefits, and naifs are overoptimistic about searching further for a realisation of $(w_1, \bar{b})$ but not $(w_2, \bar{b})$.

<u>Second</u>:

$$\begin{gathered} R_2^S + \mathbb{E}[b] < R_2^N + \mathbb{E}[b] \leq w_1 + \bar{b} \\ R_2^S + \mathbb{E}[b] \leq w_2 + \underline{b} < R_2^N + \mathbb{E}[b] \end{gathered}$$

a) $c_2 > c_1 \geq (1-p)(w_2 - w_1)$

$\implies R_2^S = \mathbb{E}[w] - c_2$ and $R_2^N = \mathbb{E}[w] - c_1$.

Then:

$$R_2^S + \mathbb{E}[b] \leq w_2 + \underline{b} \iff c_2 \geq -p(w_2 - w_1) + p(\bar{b} - \underline{b})$$

$$R_2^N + \mathbb{E}[b] > w_2 + \underline{b} \iff c_1 < -p(w_2 - w_1) + p(\bar{b} - \underline{b})$$

and:

$$R_2^N + \mathbb{E}[b] \leq w_1 + \bar{b} \iff c_1 \geq (1-p)(w_2 - w_1) - (1-p)(\bar{b} - \underline{b})$$

Both types of workers search on the job for a realisation of $(w_1, \underline{b})$ if:

$$R_2^S + \mathbb{E}[b] > w_1 + \underline{b} \iff c_2 < (1-p)(w_2 - w_1) + p(\bar{b} - \underline{b})$$

Then:

$$u_1^S(w_1) = (w_1 + \mathbb{E}[b]) + p(w_1 + \bar{b}) + (1-p)(\mathbb{E}[w+b] - c_2)$$

$$u_1^S(w_2) = 2(w_2 + \mathbb{E}[b])$$

and sophisticates search until they find w_2 in period 1 if the following holds:

$$c_1 < (1-p)(u_1^S(w_2) - u_1^S(w_1)) \iff$$
$$c_1 < (1-p)(w_2 - w_1) + (1-p)(w_2 - pw_1 - (1-p)\mathbb{E}[w]) - (1-p)p(\bar{b} - \mathbb{E}[b]) + (1-p)^2 c_2$$

For naifs:

$$u_1^N(w_1) = (w_1 + \mathbb{E}[b]) + p(w_1 + \bar{b}) + (1-p)(\mathbb{E}[w+b] - c_1)$$

$$u_1^N(w_2) = (w_2 + \mathbb{E}[b]) + p(w_2 + \bar{b}) + (1-p)(\mathbb{E}[w+b] - c_1)$$

and:

$$c_1 \geq (1-p)(u_1^N(w_2) - u_1^N(w_1)) \iff c_1 \geq (1-p^2)(w_2 - w_1)$$

So far, we have not found a contradiction. Moreover, the set of inequalities above reduces to:

(i) $c_1 < (1-p)(w_2-w_1)+(1-p)(w_2-pw_1-(1-p)\mathbb{E}[w])-(1-p)p(\bar{b}-\mathbb{E}[b])+(1-p)^2c_2$

(ii) $c_1 < -p(w_2-w_1)+p(\bar{b}-\underline{b})$

(iii) $c_1 \geq (1-p^2)(w_2-w_1)$

(iv) $c_2 < (1-p)(w_2-w_1)+p(\bar{b}-\underline{b})$

(v) $c_2 \geq -p(w_2-w_1)+p(\bar{b}-\underline{b})$

b) $c_2 \geq (1-p)(w_2-w_1) > c_1$

$\implies R_2^S = \mathbb{E}[w]-c_2$ and $R_2^N = w_2 - c_1/(1-p)$.

Then:

$$R_2^S + \mathbb{E}[b] \leq w_2 + \underline{b} \iff c_2 \geq -p(w_2-w_1)+p(\bar{b}-\underline{b})$$

$$R_2^N + \mathbb{E}[b] > w_2 + \underline{b} \iff c_1 < (1-p)p(\bar{b}-\underline{b})$$

and:

$$R_2^N + \mathbb{E}[b] \leq w_1 + \bar{b} \iff c_1 \geq (1-p)(w_2-w_1)-(1-p)^2(\bar{b}-\underline{b})$$

Both types of workers search on the job for a realisation of $(w_1, \underline{b})$ if:

$$R_2^S + \mathbb{E}[b] > w_1 + \underline{b} \iff c_2 < (1-p)(w_2-w_1)+p(\bar{b}-\underline{b})$$

Then:

$$u_1^S(w_1) = (w_1+\mathbb{E}[b]) + p(w_1+\bar{b}) + (1-p)(\mathbb{E}[w+b]-c_2)$$

$$u_1^S(w_2) = 2(w_2+\mathbb{E}[b])$$

and sophisticates search until they find w_2 in period 1 if the following holds:

$$c_1 < (1-p)(u_1^S(w_2)-u_1^S(w_1)) \iff$$
$$c_1 < (1-p)(w_2-w_1)+(1-p)(w_2-pw_1-(1-p)\mathbb{E}[w])-(1-p)p(\bar{b}-\mathbb{E}[b])+(1-p)^2c_2$$

For naifs:

$$u_1^N(w_1) = (w_1 + \mathbb{E}[b]) + p(w_1 + \bar{b}) + (1-p)(w_2 + \mathbb{E}[b] - c_1/(1-p))$$

$$u_1^N(w_2) = (w_2 + \mathbb{E}[b]) + p(w_2 + \bar{b}) + (1-p)(w_2 + \mathbb{E}[b] - c_1/(1-p))$$

and:

$$c_1 \geq (1-p)(u_1^N(w_2) - u_1^N(w_1)) \iff c_1 \geq (1-p^2)(w_2 - w_1)$$

which contradicts $c_1 < (1-p)(w_2 - w_1)$.

c) $(1-p)(w_2 - w_1) > c_2 > c_1$
$\implies R_2^S = w_2 - c_2/(1-p)$ and $R_2^N = w_2 - c_1/(1-p)$.
Then:

$$R_2^S + \mathbb{E}[b] \leq w_2 + \underline{b} \iff c_2 \geq (1-p)p(\bar{b} - \underline{b})$$

$$R_2^N + \mathbb{E}[b] > w_2 + \underline{b} \iff c_1 < (1-p)p(\bar{b} - \underline{b})$$

and:

$$R_2^N + \mathbb{E}[b] \leq w_1 + \bar{b} \iff c_1 \geq (1-p)(w_2 - w_1) - (1-p)^2(\bar{b} - \underline{b})$$

Both types of workers search on the job for a realisation of $(w_1, \underline{b})$ if:

$$R_2^S + \mathbb{E}[b] > w_1 + \underline{b} \iff c_2 < (1-p)(w_2 - w_1) + p(\bar{b} - \underline{b})$$

Then:

$$u_1^S(w_1) = (w_1 + \mathbb{E}[b]) + p(w_1 + \bar{b}) + (1-p)(w_2 + \mathbb{E}[b] - c_2/(1-p))$$

$$u_1^S(w_2) = 2(w_2 + \mathbb{E}[b])$$

and sophisticates search until they find w_2 in period 1 if the following holds:

$$c_1 < (1-p)(u_1^S(w_2) - u_1^S(w_1)) \iff$$
$$c_1 < (1-p^2)(w_2 - w_1) - (1-p)p(\bar{b} - \mathbb{E}[b]) + (1-p)c_2$$

For naifs:

$$u_1^N(w_1) = (w_1 + \mathbb{E}[b]) + p(w_1 + \bar{b}) + (1-p)(w_2 + \mathbb{E}[b] - c_1/(1-p))$$

$$u_1^N(w_2) = (w_2 + \mathbb{E}[b]) + p(w_2 + \bar{b}) + (1-p)(w_2 + \mathbb{E}[b] - c_1/(1-p))$$

and:

$$c_1 \geq (1-p)(u_1^N(w_2) - u_1^N(w_1)) \iff c_1 \geq (1-p^2)(w_2 - w_1)$$

which again contradicts $c_1 < (1-p)(w_2 - w_1)$.

Thus we cannot rule out that there exists an equilibrium in which low wages are paired with high benefits, and naifs are overoptimistic about searching further for a realisation of $(w_2, \underline{b})$ but not $(w_1, \bar{b})$.

<u>Third</u>:

$$R_2^S + \mathbb{E}[b] \leq w_1 + \bar{b} < R_2^N + \mathbb{E}[b]$$
$$R_2^S + \mathbb{E}[b] \leq w_2 + \underline{b} < R_2^N + \mathbb{E}[b]$$

a) $c_2 > c_1 \geq (1-p)(w_2 - w_1)$

$\implies R_2^S = \mathbb{E}[w] - c_2$ and $R_2^N = \mathbb{E}[w] - c_1$.

Then:

$$R_2^S + \mathbb{E}[b] \leq w_1 + \bar{b} \iff c_2 \geq (1-p)(w_2 - w_1) - (1-p)(\bar{b} - \underline{b})$$

$$R_2^N + \mathbb{E}[b] > w_1 + \bar{b} \iff c_1 < (1-p)(w_2 - w_1) - (1-p)(\bar{b} - \underline{b})$$

$$R_2^S + \mathbb{E}[b] \leq w_2 + \underline{b} \iff c_2 \geq -p(w_2 - w_1) + p(\bar{b} - \underline{b})$$

$$R_2^N + \mathbb{E}[b] > w_2 + \underline{b} \iff c_1 < -p(w_2 - w_1) + p(\bar{b} - \underline{b})$$

Both types of workers search on the job for a realisation of $(w_1, \underline{b})$ if:

$$R_2^S + \mathbb{E}[b] > w_1 + \underline{b} \iff c_2 < (1-p)(w_2 - w_1) + p(\bar{b} - \underline{b})$$

However, $c_1 < (1-p)(w_2 - w_1) - (1-p)(\bar{b} - \underline{b})$ contradicts $c_1 \geq (1-p)(w_2 - w_1)$.

b) $c_2 \geq (1-p)(w_2 - w_1) > c_1$
$\implies R_2^S = \mathbb{E}[w] - c_2$ and $R_2^N = w_2 - c_1/(1-p)$.
Then:

$$R_2^S + \mathbb{E}[b] \leq w_1 + \bar{b} \iff c_2 \geq (1-p)(w_2 - w_1) - (1-p)(\bar{b} - \underline{b})$$

$$R_2^N + \mathbb{E}[b] > w_1 + \bar{b} \iff c_1 < (1-p)(w_2 - w_1) - (1-p)^2(\bar{b} - \underline{b})$$

$$R_2^S + \mathbb{E}[b] \leq w_2 + \underline{b} \iff c_2 \geq -p(w_2 - w_1) + p(\bar{b} - \underline{b})$$

$$R_2^N + \mathbb{E}[b] > w_2 + \underline{b} \iff c_1 < p(1-p)(\bar{b} - \underline{b})$$

Both types of workers search on the job for a realisation of $(w_1, \underline{b})$ if:

$$R_2^S + \mathbb{E}[b] > w_1 + \underline{b} \iff c_2 < (1-p)(w_2 - w_1) + p(\bar{b} - \underline{b})$$

Then:

$$u_1^S(w_1) = (w_1 + \mathbb{E}[b]) + p(w_1 + \bar{b}) + (1-p)(\mathbb{E}[w+b] - c_2)$$

$$u_1^S(w_2) = 2(w_2 + \mathbb{E}[b])$$

and sophisticates search until they find w_2 in period 1 if the following holds:

$$c_1 < (1-p)(u_1^S(w_2) - u_1^S(w_1)) \iff$$
$$c_1 < (1-p)(w_2 - w_1) + (1-p)(w_2 - pw_1 - (1-p)\mathbb{E}[w]) - (1-p)p(\bar{b} - \mathbb{E}[b]) + (1-p)^2 c_2$$

For naifs:

$$u_1^N(w_1) = (w_1 + \mathbb{E}[b]) + (w_2 + \mathbb{E}[b] - c_1/(1-p))$$

$$u_1^N(w_2) = (w_2 + \mathbb{E}[b]) + p(w_2 + \bar{b}) + (1-p)(w_2 + \mathbb{E}[b] - c_1/(1-p))$$

and:

$$c_1 \geq (1-p)(u_1^N(w_2) - u_1^N(w_1)) \iff c_1 \geq (w_2 - w_1) + p(\bar{b} - \mathbb{E}[b])$$

which contradicts $c_1 < (1-p)(w_2 - w_1)$.

c) $(1-p)(w_2 - w_1) > c_2 > c_1$
$\implies R_2^S = w_2 - c_2/(1-p)$ and $R_2^N = w_2 - c_1/(1-p)$.
Then:

$$R_2^S + \mathbb{E}[b] \leq w_1 + \bar{b} \iff c_2 \geq (1-p)(w_2 - w_1) - (1-p)^2(\bar{b} - \underline{b})$$

$$R_2^N + \mathbb{E}[b] > w_1 + \bar{b} \iff c_1 < (1-p)(w_2 - w_1) - (1-p)^2(\bar{b} - \underline{b})$$

$$R_2^S + \mathbb{E}[b] \leq w_2 + \underline{b} \iff c_2 \geq p(1-p)(\bar{b} - \underline{b})$$

$$R_2^N + \mathbb{E}[b] > w_2 + \underline{b} \iff c_1 < p(1-p)(\bar{b} - \underline{b})$$

Both types of workers search on the job for a realisation of $(w_1, \underline{b})$ if:

$$R_2^S + \mathbb{E}[b] > w_1 + \underline{b} \iff c_2 < (1-p)(w_2 - w_1) + p(1-p)(\bar{b} - \underline{b})$$

Then:

$$u_1^S(w_1) = (w_1 + \mathbb{E}[b]) + p(w_1 + \bar{b}) + (1-p)(w_2 + \mathbb{E}[b] - c_2/(1-p))$$

$$u_1^S(w_2) = 2(w_2 + \mathbb{E}[b])$$

and sophisticates search until they find w_2 in period 1 if the following holds:

$$c_1 < (1-p)(u_1^S(w_2) - u_1^S(w_1)) \iff$$
$$c_1 < (1-p^2)(w_2 - w_1) - (1-p)p(\bar{b} - \mathbb{E}[b]) + (1-p)c_2$$

For naifs:

$$u_1^N(w_1) = (w_1 + \mathbb{E}[b]) + (w_2 + \mathbb{E}[b] - c_1/(1-p))$$

$$u_1^N(w_2) = (w_2 + \mathbb{E}[b]) + p(w_2 + \bar{b}) + (1-p)(w_2 + \mathbb{E}[b] - c_1/(1-p))$$

and:

$$c_1 \geq (1-p)(u_1^N(w_2) - u_1^N(w_1)) \iff c_1 \geq (w_2 - w_1) + p(\bar{b} - \mathbb{E}[b])$$

which again contradicts $c_1 < (1-p)(w_2 - w_1)$.

Thus there does not exist an equilibrium in which low wages are paired with high benefits, and naifs are overoptimistic about searching further for a realisation of $(w_1, \bar{b})$ as well as $(w_2, \bar{b})$.

References

Albrecht, J. W. and B. Axell (1984): "An Equilibrium Model of Search Unemployment," *Journal of Political Economy*, 92, 824–840.

Barber, B. M., T. Odean, and L. Zheng (2005): "Out of Sight, Out of Mind: The Effects of Expenses on Mutual Fund Flows," *The Journal of Business*, 78, 2095–2120.

Beshears, J., J. J. Choi, D. Laibson, and B. C. Madrian (2018): "Behavioral Household Finance," in *Handbook of Behavioral Economics: Applications and Foundations 1*, Elsevier, vol. 1, 177–276.

Bubb, R. and P. L. Warren (2020): "An Equilibrium Theory of Retirement Plan Design," *American Economic Journal: Economic Policy*, 12, 22–45.

Burdett, K. (1978): "A Theory of Employee Job Search and Quit Rates," *The American Economic Review*, 68, 212–220.

Campbell, J. Y. (2006): "Household Finance," *The Journal of Finance*, 61, 1553–1604.

——— (2016): "Restoring Rational Choice: The Challenge of Consumer Financial Regulation," *American Economic Review*, 106, 1–30.

Célérier, C. and B. Vallée (2017): "Catering to Investors Through Security Design: Headline Rate and Complexity," *The Quarterly Journal of Economics*, 132, 1469–1508.

Chen, Y., Z. Li, and T. Zhang (forthcoming): "Experience Goods and Consumer Search," *American Economic Journal: Microeconomics*.

DellaVigna, S., A. Lindner, B. Reizer, and J. F. Schmieder (2017): "Reference-Dependent Job Search: Evidence from Hungary," *The Quarterly Journal of Economics*, 132, 1969–2018.

DellaVigna, S. and M. D. Paserman (2005): "Job Search and Impatience," *Journal of Labor Economics*, 23, 527–588.

Diamond, P. A. (1971): "A Model of Price Adjustment," *Journal of Economic Theory*, 3, 156–168.

Ellison, G. (2005): "A Model of Add-On Pricing," *The Quarterly Journal of Economics*, 120, 585–637.

Eyster, E. (2019): "Errors in Strategic Reasoning," in *Handbook of Behavioral Economics: Applications and Foundations 1*, Elsevier, vol. 2, 187–259.

Eyster, E. and M. Rabin (2005): "Cursed Equilibrium," *Econometrica*, 73, 1623–1672.

Gabaix, X. and D. Laibson (2006): "Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets," *The Quarterly Journal of Economics*, 121, 505–540.

Gamp, T. and D. Krähmer (2022): "Competition in search markets with naive consumers," *The RAND Journal of Economics*, 53, 356–385.

Heidhues, P., B. Kőszegi, and T. Murooka (2021): "Procrastination Markets," *Working Paper.*

Heidhues, P. and B. Kőszegi (2018): "Behavioral Industrial Organization," in *Handbook of Behavioral Economics: Applications and Foundations 1*, Elsevier, vol. 1, 517–612.

Jehiel, P. (2005): "Analogy-Based Expectation Equilibrium," *Journal of Economic Theory*, 123, 81–104.

Johnen, J. (2019): "Automatic-renewal contracts with heterogeneous consumer inertia," *Journal of Economics & Management Strategy*, 28, 765–786.

——— (2020): "Dynamic Dompetition in Deceptive Markets," *The RAND Journal of Economics*, 51, 375–401.

Karle, H., H. Schumacher, and R. Vølund (2021): "Consumer Loss Aversion and Scale-Dependent Psychological Switching Costs," *CESifo Working Paper.*

Lusardi, A. and O. S. Mitchell (2014): "The Economic Importance of Financial Literacy: Theory and Evidence," *Journal of Economic Literature*, 52, 5–44.

McCall, J. J. (1970): "Economics of Information and Job Search," *The Quarterly Journal of Economics*, 113–126.

OECD (2019): *Pensions at a Glance 2019*, OECD Publishing, Paris.

——— (2020): *Financial Incentives for Funded Private Pension Plans 2020*, OECD Publishing, Paris.

Paserman, M. D. (2008): "Job Search and Hyperbolic Discounting: Structural Estimation and Policy Evaluation," *The Economic Journal*, 118, 1418–1452.

Rogerson, R., R. Shimer, and R. Wright (2005): "Search-Theoretic Models of the Labor Market: A Survey," *Journal of Economic Literature*, 43, 959–988.

Rosen, S. (1986): "The Theory of Equalizing Differences," *Handbook of Labor Economics*, 1, 641–692.

Spinnewijn, J. (2015): "Unemployed but Optimistic: Optimal Insurance Design with Biased Beliefs," *Journal of the European Economic Association*, 13, 130–167.

Van Den Berg, G. J. (2003): "Multiple Equilibria and Minimum Wages in Labor Markets with Informational Frictions and Heterogeneous Production Technologies," *International Economic Review*, 44, 1337–1357.

Wilson, C. M. (2010): "Ordered Search and Equilibrium Obfuscation," *International Journal of Industrial Organization*, 28, 496–506.