# Competitive Search and the Social Value of Public Information*

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**Abstract**

We study the social value of public information in a competitive search equilibrium with aggregate risk. While perfect information is always optimal, marginal effects of information can be positive, negative or neutral for trade. Equilibria featuring inefficient price dispersion, or the absence of trades in some states of the world, can arise if either some or even all sellers choose a price that implies not selling the good when the demand is low. The salient features of the matching function and aggregate risk matter for how information affects the equilibrium. We also find that entry is in general inefficient.

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## 1 Introduction

What is the right price to set? This is a non-trivial question if sellers do not have perfect information on market conditions before committing to a price. However, some information is publicly available e.g., due to data releases by statistical agencies, media reports, or industry analyses. The precision of such public information can vary, be it because of policy announcements or aggregate uncertainty shocks. In this paper, we study the social value of public information about market conditions in decentralised markets with frictional trade. To this end, we integrate two literatures: on public information, and competitive search with information frictions. Then, we characterize the novel features of equilibria that arise.

In many markets, the exact market conditions may be unknown to market participants. For example, an employer may not know the size of the pool of job applicants that are relevant for its positions or the seller of a house how many people are interested in buying such a house. A restaurateur may not know how much people are willing to pay for a dinner after a lengthy closure of the restaurant and the producer of a novelty product how highly the product is valued. The employer and house owner can get some relevant information from governmental reports on the state of the aggregate labour and financial markets. The restaurateur and producer can learn about the consumers' valuations from industry analyses and media reports.

To the best of our knowledge, our paper is the first to introduce uncertainty about aggregate state into the study of competitive search markets. We show that more precise public information can have negative, positive, or no effect on welfare in competitive search markets with aggregate uncertainty. And, in fact, welfare can be higher in a market with more dispersed prices than a market with less price dispersion. The paper also demonstrates that the effects of aggregate uncertainty in such markets are fundamentally different than the effects of individual-level uncertainty (see, for example, Guerrieri, Shimer, and Wright, 2010, Moen and Rosén, 2011, Delacroix and Shi, 2013, Julien and Roger, 2019, Mayr-Dorn, 2020). First, our model can generate price dispersion even in the absence of individual-level heterogeneity. Second, the model generates market freezes (i.e., situations where, beyond matching frictions, not all trade opportunities are realised) even if gains from trade are positive between all agents on the two sides of the market.

We consider an economy where an indivisible good is traded, there is a fixed population of sellers and two possible states of demand, high or low. Before sellers post a price with commitment, they receive a public signal about the state. The signal outcome is either good or bad, pointing to the high or low demand state respectively. Then buyers direct their search towards sellers if doing so is better than not searching at all. Meetings between buyers and sellers are bilateral. Searching buyers take into account the posted prices and the likelihood of trade. If buyers search, they end up indifferent between meeting with any seller in equilibrium.

We then evaluate the effects of a more informative signal on the expected value of trades and find that the marginal effects of information are ambiguous. We also find that imperfect information on aggregate risk can overturn the standard efficiency properties of equilibria under competitive search. This is because the sellers' incentives are shaped by competition in not one, but two demand states. In particular, there exist prices which are not acceptable to buyers in one of the states of the world. This leads to non-concave and potentially bi-modal expected profit functions. These features of the profit functions complicate the analysis of the model considerably and open the door for mixing over prices.

To begin with, we show that there are two distinct pricing rules when information is imperfect. First, the sellers can try to kill two birds (demand states) with one stone (price), setting a price low enough to be accepted by buyers in each state of demand. We refer to this as *pricing for both states*. Second, they can post a higher price that caters the high-demand state, abandoning any prospects of selling the good when the demand is low. We call this *pricing for the high state only*. Which of the two rules is chosen by the sellers in equilibrium depends on the realisation of the signal and its precision, and the relative appeal of the two demand states for the sellers. The sellers can also be indifferent between the two pricing rules and randomize over prices.

In a competitive search equilibrium, the market prices both the good and the likelihood of a trade taking place. When information is perfect, the optimal price satisfies the Hosios condition - sellers get a fraction of the surplus which is equal to the elasticity of the purchase probability, the Hosios share. Thus, two components determine the optimal price: the likelihood of selling the good and the size of the surplus created by trade. We refer to these as *tightness* and *surplus* risk, respectively. To isolate their effects, we assume that in the high-demand state either only the buyers' population or their valuation of the good is larger than in the low-demand state, while the other characteristics of the market are constant.

We find that under tightness risk, if information is imperfect, all single-price equilibria feature pricing for both states. In particular, no equilibrium exists where all sellers price for the high state only because buyers have the same valuation of the good in the two states. Consider such a hypothetical equilibrium. Although sellers aim to sell only in the high state, the optimal price is also acceptable to buyers in the low state, because the price cannot exceed their valuation.

Under tightness risk, equilibria where sellers mix between pricing for both states and pricing for the high state only also exist. This happens when the buyers' populations are different enough to encourage some sellers to gamble and target the high state only. However, the more sellers follow this strategy, the more profitable it is to target both states. As a result of this stabilising effect, such mixed equilibria exist for a non-degenerate set of signal precisions.

Information can have all three possible effects on trade under tightness risk: positive, nil, or negative. Information is irrelevant if before and after the change in the signal precision the market is in a single-price equilibrium. If sellers set the same price, the selling probabilities are identical and constant across all sellers in each demand state. Information can be either detrimental to or good for trade because equilibria exist where some sellers mix over two prices. Price dispersion that ensues due to sellers' mixing induces different selling probabilities across sellers. This, in turn, is inefficient because the selling probability is concave in the buyer-seller ratio and the good is homogeneous.

Increases in information precision have two effects on the existence of equilibria with price dispersion. First, more precise information can trigger these inefficient equilibria if, by changing the relative profitability of the two states, it makes pricing for the high state only profitable for sellers. We refer to this as the *price competition effect*. Second, better information can undo such equilibria because sellers better align their pricing with the true state of demand when information becomes more precise. Hence, information also has a *revealing effect*.

We show that for large differences in the two states, the expected value of trades monotonically increases in the signal precision. If the states are very different and information becomes precise enough, uninformed sellers mix in equilibrium while imperfectly informed sellers post a single price. In particular, each type of sellers becomes convinced that the true state is the one indicated by the signal outcome and chooses a single price that serves that state better. For intermediate differences between the two states, however, the effects of increasing the signal precision are non-monotone. If, for example, all uninformed sellers are just indifferent between mixing and not mixing, an increase in the signal precision initially leads one type of sellers to mix. A further increase in the signal precision, however, induces them to switch back to pricing for both states.

Under surplus risk, all types of equilibria and effects of information that we have discovered under tightness risk are also present. However, there is also a difference to tightness risk. In particular, if the difference between the surpluses in the two demand states is large, single-price equilibria exist where uninformed sellers price for the high state only and the posted price is above the buyers' low-state valuation of the good. In these equilibria, buyers do not search in the low-demand state at all, leading to a market freeze. When sellers abandon the prospects of selling in the low-demand state, increases in signal precision eventually *increase* welfare, even though they may lead to an increase in price dispersion for intermediate precisions of information. Price dispersion can be, thus, a feature of *more* efficient equilibria, because there is a trade-off between price dispersion and the volume of trade in the low-demand state.

We then extend the model and study the efficiency of equilibrium entry. A benevolent social planner never chooses an equilibrium with price dispersion. Nevertheless, even equilibria with entry and a single price are generically inefficient. This is because a seller benefits from the effect of imperfect information on pricing decisions of the other sellers in the low state: the equilibrium price is higher than under perfect information. But a seller is worse off under imperfect information in the high state, since the equilibrium price is lower than under perfect information. Competition for buyers is fiercer in the low demand state which limits the benefits of imperfect information in the low state. In total, the negative effect dominates the positive effect and entry is inefficiently low.

Finally, we extend the model to allow sellers to post a menu of lotteries, pairs of prices and probabilities of obtaining the good, instead of a single price. Under tightness risk, lotteries do not improve upon the single price. The reason is that buyers in the two states have the same marginal willingness to pay for an increase in the probability of obtaining the good so the probability cannot be used to screen the two states. Under surplus risk, conversely, sellers can in general do better by using lotteries than a single price. The sellers can lower the probability of obtaining the good to below one in the low-state lottery and increase profits. Thus, buyers are with some probability intentionally not served in the low state so that, as with a single price, the equilibria under imperfect information can be inefficient.

**Related Literature.** The paper is related to papers on the interactions of information and search frictions, price setting under imperfect information, and the social value of information.

Other search models where agents face information frictions are the most related. Random search with aggregate risk has been studied by Mauring (2017), Lauermann, Merzyn, and Virág (2018), and Shneyerov and Wong (2020), whereas we study competitive search. Competitive search models (Moen, 1997) in this vein include Guerrieri, Shimer, and Wright (2010), Moen and Rosén (2011), Delacroix and Shi (2013), Julien and Roger (2019), and Mayr-Dorn (2020). These papers study match- or individual-specific uncertainty whereas we study aggregate uncertainty. Our paper generates several novel insights with respect to this literature: the model generates price dispersion in the absence of individual-level heterogeneity (under tightness risk) and rationalises a new market failure, market freezes.

The importance of imperfect information on market conditions for price setting with commitment has been studied in a model of a single monopolist facing changing demand in Keller and Rady (1999) and in monopolistic competition models featuring uncertainty about monetary policy (Mankiw and Reis, 2002, Hellwig and Venkateswaran, 2009, Woodford, 2009). Unlike in those papers, trade is frictional in our model.

Reductions in signal precision in our model can be thought of as uncertainty shocks. In a recent paper, Den Haan, Freund, and Rendahl (2021) show that increases in perceived uncertainty lead to higher unemployment in a labour-market model without free entry. In our environment greater uncertainty (a decrease in the signal precision) is detrimental to the number of matches because of its adverse effect on entry. It can also cause further reduction in the aggregate number of matches if the resulting equilibrium features mixing.

Our model is also related to studies on the role of market transparency in OTC markets and the causes of market freezes. Regarding the former, we provide theoretical support for the empirical results of Schultz (2012) who finds that an increase in transparency reduced price dispersion in the US market for municipal bonds. As for the latter, we show that imperfect information on aggregate risk can lead to collapse of trade in some states of the world, an outcome usually due to adverse selection (Guerrieri et al., 2010, Chiu and Koeppl, 2016).

The result that more precise information can be detrimental has been shown in other settings, including a Burdett-Judd-type search model (Lester, Shourideh, Venkateswaran, and Zetlin-Jones, 2019). In that paper welfare decreases in information if buyers compete fiercely for sellers and adverse selection is moderate; in our model welfare decreases in information if it leads to more dispersed prices or no trading in some states. Morris and Shin (2002) have shown that the provision of public information is detrimental in the presence of strategic complementarity. Lepetyuk and Stoltenberg (2013) demonstrate that releasing public information on aggregate risk in the presence of idiosyncratic risk is detrimental because it dulls the incentives to insure against the latter. In our model, information interacts with search frictions which leads not only to negative, but also positive effects of information.

The structure of the paper is as follows. In section 2 we set up the model. In section 3 we discuss optimal pricing in partial equilibrium. Section 4 contains the results on the equilibrium effects of information on welfare. We consider entry in section 5 and a more general trading mechanism in section 6. The last section concludes.

## 2 Environment

We consider a one-period economy. There is a unit mass of sellers, each has one unit of an indivisible good to sell.

**Aggregate risk.** Sellers do not know the aggregate state $i$ which consists of the size of the population of buyers $\mathcal{B}_i$ and their valuation of the good $v_i$. The aggregate state $i$ is one of two values, pertaining to either high or low demand for the good, $i \in \{H, L\}$ with $\mathcal{B}_H \geq \mathcal{B}_L$, $v_H \geq v_L$, and at least one of these inequalities is strict.[1] Ex ante, both realisations of the state are equally likely. We normalize the utility of sellers from keeping the good and of buyers from not acquiring the good to zero. To purchase the good, buyers produce the seller consumption good on the spot at a linear cost. Sellers' utility over this consumption good is linear as well. Therefore, a trade between a buyer and a seller at a price $p$ yields utility of $p$ to the seller and $v_i - p$ to the buyer.

**Signals and timing.** Firstly, sellers receive a public signal $j \in \{G, B\}$ about $i$. We say that $j = G$ is a *good* signal, and $j = B$ is a *bad* signal. We refer to $G$- and $B$-sellers, for short. The good signal points to the state being high and the bad signal to the state being low with probability $\mu \in [\frac{1}{2}, 1]$. Formally, $\mathbb{P}(j = G|i = H) = \mathbb{P}(j = B|i = L) = \mu$ so that Bayes' law implies $\mathbb{P}(i = H|j = G) = \mathbb{P}(i = L|j = B) = \mu$. Therefore, $\mu = 1/2$ corresponds to no information and $\mu = 1$ to perfect information and we refer to $\mu$ as signal precision. Since signals are public, all sellers receive the same signal. Secondly, given their information set, the sellers post prices with commitment. Thirdly, buyers observe all prices and learn the state $i$. Finally, buyers direct their search towards sellers and buyers who have the chance to buy do so.

**Meetings and submarkets.** Meetings between buyers and sellers are governed by a matching function $M(\mathcal{B}, \mathcal{S})$. Let $x = \mathcal{B}/\mathcal{S}$ be the ratio of buyers to sellers; we refer to $x$ as buyer-seller ratio. A set of sellers posting the same price $p$ and buyers directing their search towards them constitutes a submarket with buyer-seller ratio $x$; submarkets are identifiable up to the $(x, p)$ pair. The probability that a seller meets a buyer is $\lambda(x)$ and the probability that a buyer meets a seller is $\eta(x) = \lambda(x)/x$ with $\lambda'(x) > 0$, $\lambda''(x) < 0$, $\eta'(x) < 0$ and $\eta''(x) > 0$. We also require $\eta(0) = 1$ and $\lambda(0) = 0$. Furthermore, we define $\phi(x) := -x\eta'(x)/\eta(x)$, the elasticity of a buyer's purchasing probability with respect to buyer-seller ratio and call $\phi(x)$ the *Hosios share*. This implies $\phi'(x) \geq 0$ and $x\lambda'(x)/\lambda(x) = 1 - \phi(x)$. Meetings between sellers and buyers are bilateral. That is, a buyer meets one seller or is unmatched, and similarly for a seller.

Because sellers have imperfect information and in anticipation of novel features of some of the model's equilibria, we allow the posting of prices that are not acceptable to buyers in some states of demand. Hence, we permit the buyers not to engage in search and get utility zero as a result. The sellers take this into consideration which gives rise to the following definition.

**Definition 1** (Active and inactive submarkets). *A submarket indexed with $(x, p)$ is* active *when $x > 0$ and* inactive *when $x = 0$.*

[1] Our results generalise to a setting with $N$ distinct states.

**Profit maximisation.** We adopt the market utility approach. The sellers that receive signal $j$ compete against expected utility $V_i^j \geq 0$ that buyers get in state $i$. Given the signal, $j$-sellers post a price to maximise expected profits $\pi^j(p)$:

$$\max_p \pi^j(p) := \left[\mathbb{P}\left(i = H|j\right)\lambda(x_H^j) + \left(1 - \mathbb{P}\left(i = H|j\right)\right)\lambda(x_L^j)\right] p, \tag{1}$$

$$\text{s. t. } x_i^j \left[\eta(x_i^j)\left(v_i - p\right) - V_i^j\right] = 0 \text{ and } x_i^j \geq 0 \text{ for } i \in \{H, L\}, j \in \{G, B\}, \tag{2}$$

$$\text{with } x_L^G = x_H^B = 0 \text{ when } \mu = 1.$$

Sellers choose to either deliver the market utility to the buyers (which then pins down the buyer-seller ratio $x_i^j > 0$), or settle for an inactive submarket. This happens when the price is such that no buyer-seller ratio delivers the market utility for buyers. This is reflected in the multiplication of the market utility constraint by the corresponding buyer-seller ratio in (2).

**Equilibrium.** We focus on symmetric Nash equilibria where each agent chooses their optimal strategy taking as given the strategies of all other agents. As we show in section 3, the solution to the optimisation problem of (1) subject to (2) need not be unique. Hence, we have to consider $j$-sellers mixing over up to $K^j$ prices $p^{j,k}$ with probabilities $\kappa^{j,k}$, for $1 \leq k \leq K^j$. These prices then imply buyer-seller ratios $x_i^{j,k} \geq 0$ in equilibrium.

The expected utility $V_i^{j,k}$ of a buyer in state $i$ who wishes to purchase the good in a submarket indexed with price $p^{j,k}$ and buyer-seller ratio $x_i^{j,k}$ is:

$$V_i^{j,k} = \eta(x_i^{j,k})\left(v_i - p^{j,k}\right). \tag{3}$$

As buyers can abstain from searching, only submarkets with $V_i^{j,k} \geq 0$ can be active (we assume as a tie-breaking rule that buyers search when it brings the same expected utility as not searching). A buyer's strategy is a mapping from the sellers' prices to the probabilities with which to visit each type of seller, taking as given the strategies of other buyers and of sellers. Directed search yields that the market utilities that the sellers take as given satisfy the following conditions in equilibrium:

$$V_i^j = \max_{k \in \mathcal{I}_i^j} V_i^{j,k} = \min_{k \in \mathcal{I}_i^j} V_i^{j,k} \text{ if } \mathcal{I}_i^j \neq \varnothing \text{ and } 0 \text{ otherwise}, \tag{4}$$

where $\mathcal{I}_i^j = \left\{k : x_i^{j,k} > 0\right\}$ is the set of indices of active submarkets in state $i$ when the signal is $j$. In case there are active submarkets, buyers are indifferent among them. If there are no active submarkets in state $i$, then buyers get $V_i^j = 0$. This happens when $\min_{j,k} p^{j,k} > v_i$ in state $i$.

The resulting buyer-seller ratios must be consistent with the total measures of sellers and buyers in each state. The adding-up constraints are:

$$\sum_k^{K^j} \kappa^{j,k} x_i^{j,k} = \mathbb{1}_i^j \mathcal{B}_i, \tag{5}$$

where $\mathbb{1}_i^j = 1$ if $\mathcal{I}_i^j \neq \varnothing$ and $\mathbb{1}_i^j = 0$ otherwise.

As there are two possible realisations of the signal in each state as long as $\mu \neq 1$, condition (5) amounts to up to four constraints. We are now in a position to define the equilibrium of this model.

**Definition 2** (Equilibrium). *A tuple* $\left( \left\{ \kappa^{j,k}, x_i^{j,k}, p^{j,k} \right\}_{k=1}^{K^j}, V_i^j \right)$ *is an* equilibrium *for exogenous parameters* $\Theta = (v_i, \mathcal{B}_i)$, $i \in \{H, L\}$, *and signal precision* $\mu$ *if for each* $j \in \{G, B\}$:

1. *given market utilities* $V_i^j$, *a tuple* $\left\{ x_i^{j,k}, p^{j,k} \right\}$ *solves* (1) *and* (2) *for each* $k$,
2. *market utilities satisfy* (3) *and* (4) *given* $\left\{ x_i^{j,k}, p^{j,k} \right\}_{k=1}^{K^j}$,
3. *buyer-seller ratios and probability weights* $\left\{ x_i^{j,k}, \kappa^{j,k} \right\}_{k=1}^{K^j}$ *are consistent with* (5).

We evaluate the effect of changes in the signal precision $\mu$ via its impact on the ex-ante expected value of surplus generated by trades:

$$W(\Theta, \mu) := \frac{1}{2} \left( \mu \sum_{k=1}^{K^G} \kappa^{G,k} \lambda(x_H^{G,k}) + (1-\mu) \sum_{k=1}^{K^B} \kappa^{B,k} \lambda(x_H^{B,k}) \right) v_H$$

$$+ \frac{1}{2} \left( \mu \sum_{k=1}^{K^B} \kappa^{B,k} \lambda(x_L^{B,k}) + (1-\mu) \sum_{k=1}^{K^G} \kappa^{G,k} \lambda(x_L^{G,k}) \right) v_L = \frac{1}{2} W_H(\Theta, \mu) + \frac{1}{2} W_L(\Theta, \mu).$$

The expected surplus generated by trades is, therefore, an average of surpluses created in each state of demand. Finally, we refer to equilibria where all $j$-sellers post the same price as *pure-strategy equilibria* (PSE) and to equilibria where $j$-sellers post different prices as *mixed-strategy equilibria* (MSE).

# 3 Pricing

As an intermediate step, in this section we investigate the properties of expected profits and the optimal pricing decisions in partial equilibrium. We show that sellers mix over a maximum of two prices (Corollary 1) and derive the partial equilibrium prices both under perfect information (Lemma 1) and imperfect information (Theorems 1 and 2).

In the partial equilibrium, we fix the market utilities of buyers and study the profit maximisation problem. In doing so, we shut down the equilibrium feedback that goes from the composition of the pool of sellers and their pricing decisions to the market utilities of buyers. In equilibrium, the market utilities of buyers would depend on the realisation of the signal, which would affect the pricing decisions of sellers. The prices in turn would then differ across signals, which would affect the market utilities of buyers etc. To isolate the effects of information precision on optimal prices, we must compare the prices posted by the $B$- and $G$-sellers. To make this comparison feasible, we maintain the following simplifying assumption throughout this section.

**Assumption 1** (Partial equilibrium). *Let* $V_i^j = V_i$, *fixed* $\forall \mu \in \left[\frac{1}{2}, 1\right]$ *and* $0 < V_i < v_i$ *for* $i \in \{H, L\}$ *and* $j \in \{G, B\}$.

Let $\bar{p}_i(V_i) = v_i - V_i$ be the upper threshold level for a price in state $i$ to attract buyers. For $p > \bar{p}_i$ buyers don't direct their search towards sellers who post $p$ as doing so yields expected utility strictly lower than $V_i$. As the first step, let's consider a seller who posts a price $p$ with the full knowledge of the underlying state. The expected profits and optimal prices in state $i$ are, respectively:

$$\pi_i(p) = \begin{cases} \lambda\left(x(p)\right) p \text{ and } x(p) \text{ solves } \eta(x(p))(v_i - p) = V_i \text{ for } 0 \leq p \leq \bar{p}_i, \\ 0 \text{ otherwise,} \end{cases}$$

$$p_i^*(V_i) := \arg\max \pi_i(p). \tag{6}$$

Furthermore, we introduce the following assumption on the profit functions.

**Assumption 2** (Regularity condition). *$\pi_i(p)$ is a twice continuously differentiable and a strictly concave function of $p$ on $[0, \bar{p}_i]$.*[2] *Therefore, $p_i^*(V_i)$ are interior on $[0, \bar{p}_i]$.*

Straightforwardly, $\pi_i(p)$ is continuous in $p$ and differentiable everywhere apart from $\bar{p}_i(V_i)$. Observe that the objective functions of $j$-sellers given in (1) are convex combinations of perfect information expected profits in each state with weights determined by the signal they received:[3]

$$\pi^G(p) = \mu\pi_H(p) + (1-\mu)\,\pi_L(p)\,,$$
$$\pi^B(p) = (1-\mu)\,\pi_H(p) + \mu\pi_L(p)\,.$$

These functions are continuous. They are also strictly concave on $[0, \min_i \bar{p}_i(V_i)]$ and separately on $[\min_i \bar{p}_i(V_i), \max_i \bar{p}_i(V_i)]$, but are not strictly concave on $[0, \max_i \bar{p}_i(V_i)]$. Hence, there are at most two local maxima for each $\pi^j(p)$. The two local maxima arise because sellers try to target two states with one price and lead to the following result.

**Corollary 1** (Mixing). *If sellers mix in an equilibrium, they mix over two prices.*

The rationale for this Corollary is that mixing occurs iff the expected profits have two local maxima which yield identical expected profits. This result is independent of Assumption 1.

For what follows, it is useful to introduce the following objects:

$$p^j(V_L, V_H) := \arg\max \pi^j(p) \text{ and } \pi^j(V_L, V_H) := \pi^j(p^j),$$
$$\tilde{p}^j(V_L, V_H) := \arg\max \pi^j(p) \text{ for } p \in \left[0, \min_i \bar{p}_i\right] \text{ and } \tilde{\pi}^j(V_L, V_H) := \pi^j(\tilde{p}^j).$$

For a given signal precision $\mu$ and market utilities $V_H$ and $V_L$, $p^j(V_L, V_H)$ is the optimal price and $\tilde{p}^j(V_L, V_H)$ is the optimal price that leads to active submarkets in both states. Straightforwardly, $p^G(V_L, V_H)$ tends to $p_H(V_H)$ and $p^B(V_L, V_H)$ to $p_L(V_L)$ as $\mu$ tends to 1.

[2] These functions are strictly concave in buyer-seller ratio $x$ as long as the market utility constraint binds. Without our regularity condition, one would have to consider corner solutions; see Wright, Kircher, Julien, and Guerrieri (2019) for a discussion. The regularity condition is satisfied whenever we derive results for a specific matching function (Assumption 5).

[3] A remark on notation: we scrap superscript $j$ whenever signals are perfectly informative. Furthermore, if $\mu = 1/2$, the functions $\pi^j(p)$ are identical and in this case we use superscript $N$ (for no information) instead of $j$.

Our ultimate objects of interest are the global profit maximisers, $p^j(V_L, V_H)$. However, as we show in Theorems 1-2, a necessary step to characterize them is to characterize $\tilde{p}^j(V_L, V_H)$ and $p_i^*(V_i)$ because the globally optimal price is going to be one or the other. For the rest of the derivations in this section, we scrap the dependence of prices on market utilities. Based on the strict concavity and continuity argument, we can provide the following bounds on the profit maximising prices.

**Corollary 2** (Bounds on $p^j$). *Let Assumptions 1 and 2 hold. The profit maximising prices for given market utilities of buyers are bounded by the corresponding perfect information prices:*

$$\min_i p_i^* \leq p^j \leq \max_i p_i^* \quad \forall j \in \{G, B\}\,. \tag{7}$$

This follows from the fact that to the left of $\min_i p_i^*$ the expected profits are strictly increasing, and to the right of $\max_i p_i^*$, strictly decreasing, regardless of the signal realisation. Next, we solve for the perfect information prices.

**Lemma 1** (Perfect information pricing). *Let Assumptions 1 and 2 hold. The solution to the profit maximisation problem with perfectly informative signals under the market utility constraint in* (6) *is:*

$$p_i^* = \phi(x_i^*)v_i, \text{ with the buyer-seller ratio } x_i^* \text{ solving } \eta(x_i^*)\left(1 - \phi(x_i^*)\right)v_i = V_i. \tag{8}$$

The proof is in Appendix A.1. The pricing rule prescribes that a seller gets a share of the surplus $v_i$. The share is equal to the elasticity of the selling probability, evaluated at the buyer-seller ratio consistent with the market utility of buyers, $\phi(x_i^*)$. The tightness that solves the market utility constraint is unique, as $\eta(x_i^*)(1 - \phi(x_i^*))$ is a strictly decreasing function of $x_i^*$, which takes values between zero and one, and $v_i > V_i$.

Next, we characterize the constrained profit maximisers $\tilde{p}^j$. These are the optimal prices that imply active submarkets in both states of demand, unlike prices in the range $[\min_i \bar{p}_i, \max_i \bar{p}_i]$. Hence, we refer to the conditions that define them as *pricing for both states*.

**Lemma 2** (Pricing for both states). *Let Assumptions 1 and 2 hold. The constrained profit maximisation on $[0, \min_i \bar{p}_i]$ has either a corner solution $\tilde{p}^j = \min_i \bar{p}_i$, which happens only when $p^j = \max_i p_i^*$, or an interior solution. For the interior solution, unique buyer-seller ratios $\tilde{x}_i^j > 0$ exist which together with $\tilde{p}^j$ jointly solve:*

$$\eta\left(\tilde{x}_i^j\right)\left(v_i - \tilde{p}^j\right) = V_i, i \in \{H, L\}, j \in \{G, B\},$$

$$\mu\lambda(\tilde{x}_H^G)\left[\frac{\phi(\tilde{x}_H^G)v_H - \tilde{p}^G}{\phi(\tilde{x}_H^G)\left(v_H - \tilde{p}^G\right)}\right] + (1-\mu)\lambda(\tilde{x}_L^G)\left[\frac{\phi(\tilde{x}_L^G)v_L - \tilde{p}^G}{\phi(\tilde{x}_L^G)\left(v_L - \tilde{p}^G\right)}\right] = 0, \tag{9}$$

$$\mu\lambda(\tilde{x}_L^B)\left[\frac{\phi(\tilde{x}_L^B)v_L - \tilde{p}^B}{\phi(\tilde{x}_L^B)\left(v_L - \tilde{p}^B\right)}\right] + (1-\mu)\lambda(\tilde{x}_H^B)\left[\frac{\phi(\tilde{x}_H^B)v_H - \tilde{p}^B}{\phi(\tilde{x}_H^B)\left(v_H - \tilde{p}^B\right)}\right] = 0. \tag{10}$$

The proof is in Appendix A.2. Intuitively, the sellers in this case try to kill two birds (demand states) with one stone (price). The pricing rules (9) and (10) can be thought of as combinations of pricing rules under perfect information. Indeed, equations (9) and (10) imply the following bounds on the price $\tilde{p}^j$:

$$\max_i \phi(\tilde{x}_i^j)v_i \geq \tilde{p}^j \geq \min_i \phi(\tilde{x}_i^j)v_i.$$

For fixed buyer-seller ratios, an increase in $\mu$ pulls the price $\tilde{p}^j$ towards the perfect-information price in the state pointed to by the signal. When $\mu = 1$, the pricing rules (9) and (10) nest the perfect-information case.

Given the general matching function and the arbitrary choice of $V_H$ and $V_L$, to make progress towards the characterisation of the profit maximising prices, we assume that the two demand states $i$ and $-i$ can be ranked strictly in terms of their favourability to the sellers.[4] That is, the profit functions can be ranked on the interval bounded by the perfect information profit maximising prices which, by the virtue of equation (7), contains the imperfect information profit maximising prices.

**Definition 3** (More profitable state)**.** *Let Assumptions 1 and 2 hold. State i is* more profitable *than state $-i$ when the following inequalities hold: $p_i^* \geq p_{-i}^*$ and $\pi_i(p) \geq \pi_{-i}(p)$ on $\left[p_{-i}^*, p_i^*\right]$. State i is* strictly more profitable *than state $-i$ when both inequalities are strict.*

Based on the perfect information pricing (8) and on strict concavity of the expected profit function, we can state a sufficient condition for a state $i$ to be more profitable. This condition requires that in the (strictly) more profitable state, first, the buyers' share of the trade surplus is no greater (strictly smaller) than in the other state. Second, that the ratios of market utilities over the buyers' valuation of the good are ranked accordingly.

**Lemma 3** (Sufficient condition for a state to be more profitable)**.** *Let Assumption 1 hold. State i is (strictly) more profitable if $v_i \geq v_{-i}$ and $V_{-i}/v_{-i} \geq V_i/v_i$ ($V_{-i}/v_{-i} > V_i/v_i$, respectively).*

*Proof.* We provide the proof for the case of state $i$ being strictly more profitable. The two conditions immediately imply, in the light of perfect information pricing (8), that $x_i > x_{-i}^*$ and also $p_i^* > p_{-i}^*$ so that $\pi_i(p_i^*) > \pi_{-i}(p_{-i}^*)$. The market utility constraint in the $i$-state implies that $x_i(p_{-i}^*) > x_i^*$, hence, $\pi_i(p_{-i}^*) > \pi_{-i}(p_{-i}^*)$. The last result and the strict concavity of the profit functions imply that $\pi_i(p) > \pi_{-i}(p)$ on $\left[p_{-i}^*, p_i^*\right]$ as the profit function $\pi_i(p)$ is strictly increasing, while $\pi_{-i}(p)$ is decreasing, on this interval. $\square$

When a state is strictly more profitable, there are two cases to consider, which we characterise in Theorems 1 and 2. In the first case, neither perfect information optimal price exceeds $\min_i \bar{p}_i$, the smaller of the two threshold prices to yield an active submarket, so the expected profit functions are unimodal. In the second case, $\max_i p_i$ exceeds $\min_i \bar{p}_i$ and the profits are bimodal.

To aid building intuition for the working of the model, we assume that the high demand state is strictly more profitable. Because of that, Theorems 1 and 2 have symmetrical counterparts when the ranking of the states is flipped. We later discuss for each Theorem what changes if this happens.

**Assumption 3** (Ranking of demand states)**.** *State H is strictly more profitable.*

[4] The convention that we use is: $a \in \{b, c\} \implies -a = \{b, c\} \setminus a$.

Observe that when $\pi_H(p) > \pi_L(p)$, $\pi^B(p)$ decreases in $\mu$ for a fixed $p$ while the converse is true for $\pi^G(p)$. Because of that, the profit function for the uninformative signal bounds the function $\pi^G(p)$ below and $\pi^B(p)$ above at least on the interval bounded by perfect information profit maximising prices, which also contains the imperfect information profit maximising prices.

**Theorem 1** (Optimal prices, unimodal profits)**.** *Let Assumptions 1, 2 and 3 hold and let also $\min_i \bar{p}_i \geq p_H^*$. Then, $\pi^j(p)$ has a unique global maximum for all $\mu \in \left[\frac{1}{2}, 1\right]$ and each $j$. Hence, mixing over two prices is not optimal and the unique profit maximising prices can be ranked as follows:*

$$p_H^* \geq p^G = \tilde{p}^G \geq p^B = \tilde{p}^B \geq p_L^*. \tag{11}$$

*Furthermore, $p_H^* = p^G$ and $p_L^* = p^B$ iff $\mu = 1$. Also, $p^G = p^B$ iff $\mu = 1/2$.*

The proof is in Appendix A.3. The key take-aways are that under the assumptions of Theorem 1 the profit functions $\pi^j(p)$ are strictly decreasing on $[\min_i \bar{p}_i, \max_i \bar{p}_i]$ and the profit maximising prices of $B$- and $G$-sellers are not greater than the lower price threshold for an active submarket. This means that profit maximizers $p^j$ imply positive buyer-seller ratios for both types of sellers in both states of demand. Next, when $\mu = 1/2$, the profit functions $\pi^j(p)$ coincide and all sellers post the same price. When signals become increasingly informative, $p^G$ tends to $p_H^*$ and $p^B$ to $p_L^*$: each price converges to the perfect-information price of the state indicated by the signal. We illustrate Theorem 1 on the top panel of Figure 1.[5]

The set of profit maximising prices becomes more complicated in the second case, where $p_H > \min_i \bar{p}_i$, which implies that $\min_i \bar{p}_i = \bar{p}_L$. The source of complications is that we must now be concerned with two local maxima of the expected profit functions.

**Theorem 2** (Optimal prices, bimodal profits)**.** *Let Assumptions 1, 2 and 3 hold and let $p_H^* > \bar{p}_L$. Then, $\pi^j(p)$ has a strict local maximum at $p = p_H^*$, for all $\mu \in [\frac{1}{2}, 1)$ and each $j$. Furthermore:*

1. *if $\frac{\pi_H(p_H)}{2} < \pi^N(\tilde{p}^N)$, then $p^B = \tilde{p}^B$ for all $\mu \in [\frac{1}{2}, 1]$ and there is $\bar{\mu}^G$ such that $p^G = \tilde{p}^G$ for $\mu \in [\frac{1}{2}, \bar{\mu}^G)$; when $\mu = \bar{\mu}^G$, $G$-sellers mix between posting $p_H^*$ and $\tilde{p}^G$; and $p^G = p_H^*$ for $\mu \in (\bar{\mu}^G, 1]$;*
2. *if $\frac{\pi_H(p_H)}{2} = \pi^N(\tilde{p}^N)$, then $B$-and $G$-sellers mix between $p_H^*$ and $\tilde{p}^N$ for $\mu = \frac{1}{2}$. When $\mu > \frac{1}{2}$, $G$-sellers post $p^G = p_H^*$ and $B$-sellers post $p^B = \tilde{p}^B$;*
3. *if $\frac{\pi_H(p_H)}{2} > \pi^N(\tilde{p}^N)$, then $p^G = p_H^*$ for all $\mu \in [\frac{1}{2}, 1]$ and there is $\bar{\mu}^B$ such that $p^B = p_H^*$ for $\mu \in [\frac{1}{2}, \bar{\mu}^B)$; when $\mu = \bar{\mu}^B$, $B$-sellers mix between posting $p_H^*$ and $\tilde{p}^B$; and $p^B = \tilde{p}^B$ for $\mu \in (\bar{\mu}^B, 1]$.*

The proof is in Appendix A.4. We illustrate this result on the three bottom panels of Figure 1. The key intuition is that the expected profit function of an uninformed seller is now bimodal, but it converges to the unimodal perfect information expected profit when $\mu$ increases.

On the second panel, the profit maximising price for uninformed sellers targets both states. When $\mu$ increases, this remains the case for $B$-sellers, but there exists a precision level $\bar{\mu}^G$ which makes $G$-sellers indifferent between two prices. Above $\bar{\mu}^G$, $G$-sellers strictly prefer to post $p_H^*$

[5] If the low state is strictly uniformly more profitable, then the ranking of prices in (11) is flipped, the no-information profit is a lower bound for $\pi^B(p)$ and upper bound for $\pi^G(p)$, etc.

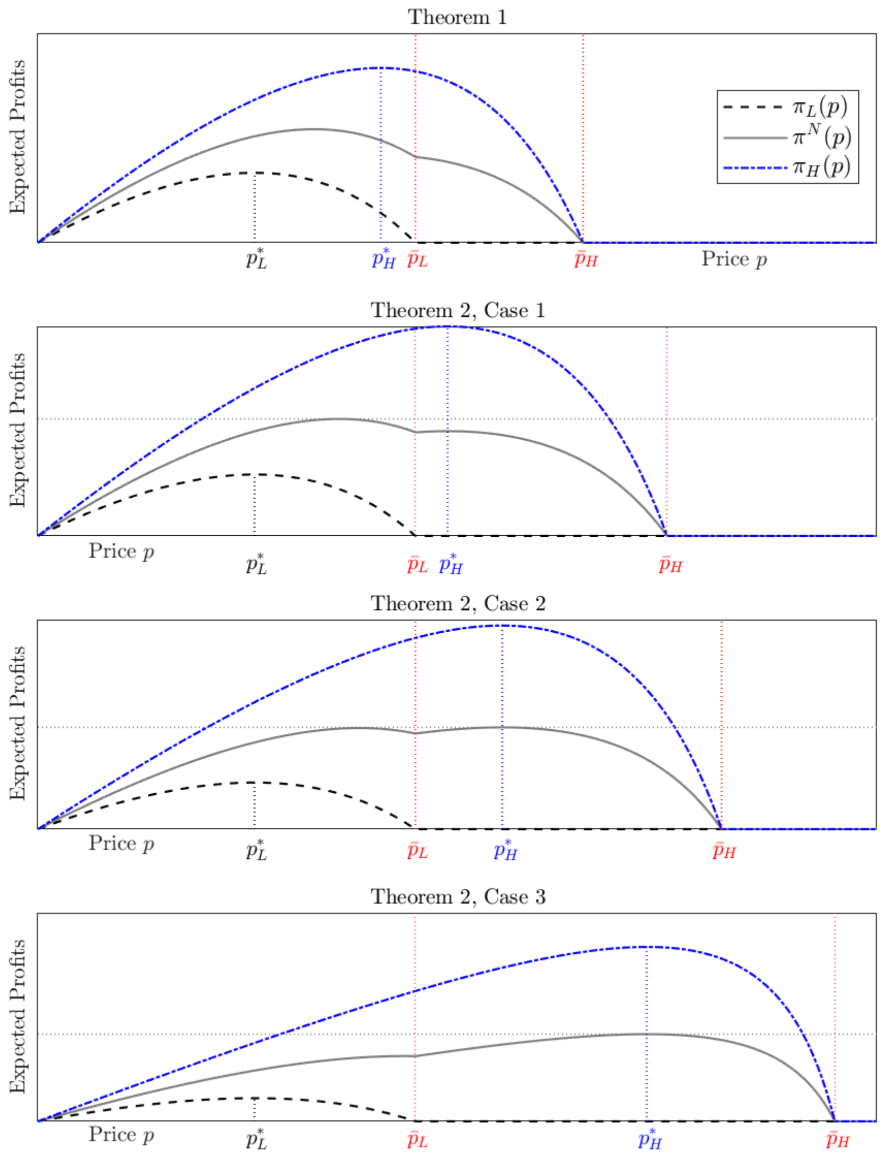


Figure 1: Illustration of Theorems 1 and 2. Expected profits under perfect information (blue - good signal, black - bad signal) and under no information (grey). Same horizontal scale and different vertical scales.

which targets only the high-demand state, leading to an inactive submarket in the low-demand state. On the next panel, the uninformed sellers are indifferent between two prices. When $\mu$ is higher, $G$-sellers post $p_H^*$ and $B$-sellers price for both states. Finally, on the bottom panel, the optimal price for an uninformed seller is $p_H^*$ and it remains optimal to post this price for $G$-sellers when $\mu$ is higher. There exists a signal precision $\bar{\mu}^B$ which makes the $B$-sellers indifferent between two prices and above which they optimally post a price that targets both states.

Summing up, there are two main differences to Theorem 1. First, mixing is possible for some signal precisions. Second, either some or all sellers may settle for posting $p_H^*$ which leads to inactive submarkets in the $L$-state.

As we already hinted at when introducing Assumption 3, Theorem 2 can be derived also for the case when the low-demand state, and not the high-demand state, is more profitable. If this was the case, then, for example, the counterpart to case 1 in Theorem 2 would prescribe that the $B$-sellers always post $p_L^*$ and there exists a threshold $\bar{\mu}^G$ such that for $\mu \in (\bar{\mu}^G, 1]$ the $G$-sellers post $\tilde{p}^G$ etc.

Assuming that market utilities are independent of the signal realisation (i.e., $V_i^j = V_i$) is indispensable if one wants to rank the prices posted by $B$- and $G$-sellers and compare the effects of the signal's precision on prices. In equilibrium, however, the market utilities depend on the signal outcome, which we take into account in the next section.

# 4 Equilibria and the welfare effects of information

In this section, we incorporate the feedback effect that the pricing decisions of sellers have on the market utilities of buyers and investigate the equilibria that arise. We also analyse the welfare effects of more precise information. We start by characterizing the equilibria under the benchmark of perfect information and move on to imperfect information.

## 4.1 Perfect information benchmark

When the sellers know the aggregate state, this is equivalent to them observing the realisation of a perfectly informative signal with precision $\mu = 1$.

**Corollary 3** (Perfect information equilibrium)**.** *The equilibrium under perfect information* $(x_i^*, p_i^*, V_i^*)$*,* $i \in \{L, H\}$*, is:*

$$x_i^* = \mathcal{B}_i,\ p_i^* = \phi(\mathcal{B}_i) v_i \text{ and } V_i^* = \eta(\mathcal{B}_i)\left(1 - \phi(\mathcal{B}_i)\right) v_i.$$

*Thus,* $p_H^* > p_L^*$*. The expected value of trades is* $W(\Theta, 1) = \frac{\lambda(\mathcal{B}_H) v_H + \lambda(\mathcal{B}_L) v_L}{2} =: W^*$*.*

This result is a direct consequence of Lemma 1 and the buyers' adding-up constraints (5). Higher demand benefits sellers via up to three channels. Larger population of buyers makes selling the good more likely. Furthermore, for matching functions that have $\phi'(x) > 0$, the increase in the population of buyers allows the sellers to charge higher prices even if the buyers' valuation of the good $v_i$ is the same in the two states. Finally, the larger the $v_i$, the higher the prices the sellers charge in the perfect-information equilibrium.

## 4.2 Imperfect information

The analysis of imperfect information is impeded by the non-linear nature of the pricing rules given by equations (9)-(10). Therefore, we introduce two alternative simplifying assumptions on the matching function.

**Assumption 4** (Constant Hosios share (CHS) matching function)**.** *Let* $M(\mathcal{B}, \mathcal{S})$ *be such that* $\phi(x) = \phi$*.*

**Assumption 5** (Particular non-CHS matching function)**.** *Let* $M(\mathcal{B},\mathcal{S}) = \frac{\mathcal{B}\mathcal{S}}{\mathcal{B}+\mathcal{S}}$. *Then also* $\lambda(x) = \frac{\mathcal{B}}{\mathcal{S}+\mathcal{B}} = \frac{x}{1+x}$, $\eta(x) = \frac{\mathcal{S}}{\mathcal{S}+\mathcal{B}} = \frac{1}{1+x}$ *and* $\phi(x) = \lambda(x)$.

Assumption 4 is satisfied by the Cobb-Douglas matching function $M(\mathcal{B},\mathcal{S}) = A\mathcal{B}^{\phi}\mathcal{S}^{1-\phi}$ where $A$ is the matching efficiency parameter. The matching function put forth in Assumption 5 can be viewed as a particular case of the matching function $M(\mathcal{B},\mathcal{S}) = \mathcal{B}\mathcal{S}/(\mathcal{B}^{1/\gamma}+\mathcal{S}^{1/\gamma})^{\gamma}$ with $\gamma = 1$ proposed in den Haan, Ramey, and Watson (2000) or the telephone matching function in Stevens (2007). As an illustration of the simplifications these assumptions bring, consider the pricing for both states condition for the $G$-sellers, equation (9), which under Assumption 4 reads:

$$\mu\lambda(\tilde{x}_H^G)\left[\frac{\phi v_H - \tilde{p}^G}{v_H - \tilde{p}^G}\right] + (1-\mu)\lambda(\tilde{x}_L^G)\left[\frac{\phi v_L - \tilde{p}^G}{v_L - \tilde{p}^G}\right] = 0,$$

and which under Assumption 5 reads:

$$\mu\left[\frac{\phi(\tilde{x}_H^G)v_H - \tilde{p}^G}{v_H - \tilde{p}^G}\right] + (1-\mu)\left[\frac{\phi(\tilde{x}_L^G)v_L - \tilde{p}^G}{v_L - \tilde{p}^G}\right] = 0.$$

Next, we also divide the aggregate risk into two polar cases.

**Assumption 6** (Tightness risk)**.** *Let* $\mathcal{B}_H > \mathcal{B}_L$ *and* $v_H = v_L = v$.

**Assumption 7** (Surplus risk)**.** *Let* $\mathcal{B}_H = \mathcal{B}_L = \mathcal{B}$ *and* $v_H > v_L$.

Even though for some applications of our theory Assumptions 6 and 7 need not be met, they bring additional tractability.[6] To see this, note that when trades generate identical surplus in both states of demand (Assumption 6) the pricing for both state equations (9)-(10) become linear in prices as we can remove $v - \tilde{p}^j$ from the denominators. This is not the case, however, under Assumption 7. Hence, we start with tightness risk and build on the results derived for this case to inform our analysis of surplus risk.

### 4.2.1 Tightness risk

In this section, we first demonstrate that, under tightness risk, information is irrelevant for some matching functions (Corollary 4). Then we derive the equilibria (Theorem 3) and demonstrate the diverse welfare effects of information for the matching function of Assumption 5.

Under tightness risk, the perfect information prices differ only up to the Hosios share $\phi(\mathcal{B}_i)$ as per Corollary 3. Because of that, for some matching functions information is irrelevant for the pricing decisions of sellers. For example, Corollary 3 implies that when the matching function features the CHS property, the perfect information equilibrium prices are identical, $p_H^* = p_L^* = \phi v$. In fact, this price also solves the pricing for both states equations regardless of the realisation of the signal and its precision: hence, information does not affect the equilibrium. We formalize this result below.

[6] For example, in a labour-market application, tightness risk corresponds to uncertainty on the participation rate, and surplus risk to uncertainty on aggregate productivity. If workers have information on aggregate productivity and participation in the labour market is costly, higher productivity and greater participation can be correlated.

**Corollary 4** (Irrelevance of information for CHS matching functions, tightness risk)**.** *Let Assumptions 4 and 6 hold. Then, a unique equilibrium exists and is independent of information precision $\mu$. In this equilibrium, all sellers post the same price $p = \phi v$ and welfare is independent of $\mu$, $W(\Theta, \mu) = \frac{1}{2}\left(\lambda(\mathcal{B}_H) + \lambda(\mathcal{B}_L)\right) v = W^*$.*

For the remainder of this section we focus on the particular non-CHS matching function (Assumption 5) which allows us to explicitly derive the relationship between the market utilities of buyers, perfect information prices, prices that target both states, and the corresponding buyer-seller ratios.

**Lemma 4** (Analytical solution, tightness risk)**.** *Let Assumptions 5 and 6 hold. Then, for given market utilities of buyers $V_i^j$, the price set by a seller that is perfectly informed about the underlying state i and the implied buyer-seller ratio are:*

$$p_i^{j*} = v - \sqrt{vV_i^j} \text{ and } x_i^{j*} = \sqrt{\frac{v_i}{V_i^j}} - 1,$$

*while the prices that target both states and the corresponding buyer-seller ratios are:*

$$\tilde{p}^G = v - \sqrt{\left[\mu V_H^G + (1-\mu) V_L^G\right] v}, \quad \tilde{x}_i^G = \frac{\sqrt{\left[\mu V_H^G + (1-\mu) V_L^G\right] v}}{V_i^G} - 1,$$

$$\tilde{p}^B = v - \sqrt{\left[(1-\mu) V_H^B + \mu V_L^B\right] v}, \quad \tilde{x}_i^B = \frac{\sqrt{\left[(1-\mu) V_H^B + \mu V_L^B\right] v}}{V_i^B} - 1.$$

The proof is in Appendix A.5. The prices $p_i^{j*}$ and tightnesses $x_i^{j*}$ derived for demand- and signal-specific market utilities are the equivalent of those in Lemma 1 (where we assumed the market utilities to be only demand-specific).

These results reduce the problem of equilibrium characterisation to a two-step procedure. First, there is a finite number of combinations of price rules used by $G$- and $B$-sellers in equilibrium, as per Theorems 1 and 2.[7] For each of these combinations, solving the buyers' adding-up constraints for market utilities concludes the first step. The second step is to verify that the particular pricing pattern (e.g., both types of sellers pricing for both states) is consistent with profit maximisation given the market utilities of buyers. We use these insights in the proof of the following result.

**Theorem 3** (Equilibria, tightness risk)**.** *Let Assumptions 5 and 6 hold, then:*

1. *the H-state is strictly more profitable,*
2. *there exist thresholds $\tilde{\mathcal{B}}_H^j := \tilde{\mathcal{B}}_H^j(\mathcal{B}_L, \mu)$, $\mathcal{B}_L < \tilde{\mathcal{B}}_H^j < \infty$ such that for $\mathcal{B}_H \in \left(\mathcal{B}_L, \tilde{\mathcal{B}}_H^j\right]$ the equilibrium profit function $\pi^j(p)$ is unimodal. Furthermore, $\partial\tilde{\mathcal{B}}_H^j/\partial\mathcal{B}_L > 0$, $\partial\tilde{\mathcal{B}}_H^G/\partial\mu > 0$, $\partial\tilde{\mathcal{B}}_H^B/\partial\mu < 0$ and $\tilde{\mathcal{B}}_H^G \geq \tilde{\mathcal{B}}_H^B$ with equality only if $\mu = 1/2$,*

[7] Further characterisation of the combinations depends on which state is more profitable.

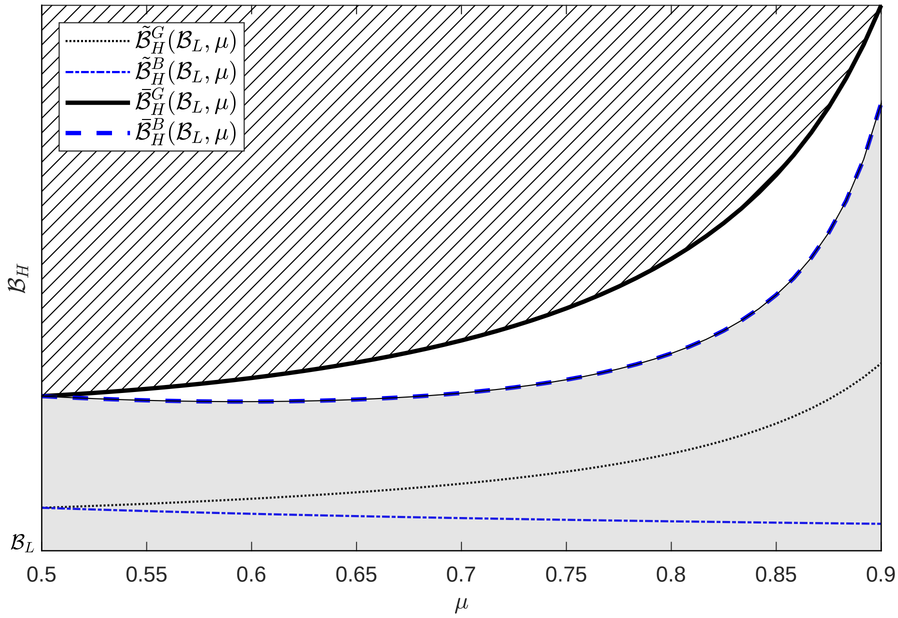


Figure 2: Illustration of Theorem 3 for $v = 1$ and $\mathcal{B}_L = 1/5$. In the shaded area there is a PSE, while in the hatched area there is an MSE, for both signal realisations. In the remaining area, the bad-signal sellers mix and good-signal sellers post a single price.

3. *there exist thresholds $\bar{\mathcal{B}}_H^j := \bar{\mathcal{B}}_H^j(\mathcal{B}_L, \mu)$; $\tilde{\mathcal{B}}_H^j < \bar{\mathcal{B}}_H^j \leq \infty$ such that for $\mathcal{B}_H \in \left(\tilde{\mathcal{B}}_H^j, \bar{\mathcal{B}}_H^j\right)$ the equilibrium profit function $\pi^j(p)$ is bimodal, but pricing for both states maximizes profits. Furthermore, $\lim_{\mu \to 1} \bar{\mathcal{B}}_H^j(\mathcal{B}_L, \mu) = \infty$, and $\bar{\mathcal{B}}_H^j(\mathcal{B}_L, \mu) < \infty$ for $\mathcal{B}_L$ and $\mu$ small enough,*

4. *the thresholds $\bar{\mathcal{B}}_H^j$ have slopes of opposite signs for uninformative signals in the $(\mu, \mathcal{B}_H)$ space:*

$$\lim_{\mu \to \frac{1}{2}} \frac{d}{d\mu} \bar{\mathcal{B}}_H^G = - \lim_{\mu \to \frac{1}{2}} \frac{d}{d\mu} \bar{\mathcal{B}}_H^B.$$

*Therefore, a unique PSE exists iff $\mathcal{B}_H \leq \bar{\mathcal{B}}_H^j(\mathcal{B}_L, \mu)$ and a unique MSE exists iff $\mathcal{B}_H > \bar{\mathcal{B}}_H^j(\mathcal{B}_L, \mu)$ for given signal realisation $j$.*

The proof is in Appendix A.6; we deliver the main intuitions here. The key takeaway is that the model can feature a MSE for a non-trivial set of parameters. We illustrate the thresholds in Theorem 3 for a particular combination of parameters on Figure 2. The two bottom lines correspond to thresholds for unimodal profits when all sellers price for both states, $\tilde{\mathcal{B}}^j$. The two top lines are thresholds for mixing to be an equilibrium, $\bar{\mathcal{B}}^j$. Above each of those lines only a MSE exists for the corresponding signal realisation.[8] Hence, for a given $\mathcal{B}_L$, whenever $\mathcal{B}_H \leq \min_j \min_\mu \bar{\mathcal{B}}_H^j(\mathcal{B}_L, \mu)$, both types of sellers post a single price in equilibrium for all information precisions $\mu$. In the rest of the parameter space, at least one seller type randomizes for $\mu$ sufficiently small.

As illustrated in Figure 2, mixing is an equilibrium outcome for a nondegenerate set of

[8] Unlike the thresholds for unimodal profits, these thresholds cannot be ranked (i.e., the solid blue line can be above the solid black line). We return to this at the end of this section.

parameter values. For mixing to be an equilibrium, a necessary condition is for the profits implied by targeting the high state only to be at least weakly higher than profits implied by pricing for both states. If $\mathcal{B}_H$ grows arbitrarily large, a seller sells almost surely in the high state so $G$-sellers' profits of pricing for the high state only approach $\mu v$, while profits from pricing for both states approach $[\mu + (1-\mu)\phi(\mathcal{B}_L)]^2 v$. For sufficiently low $\mathcal{B}_L$, it is indeed the case that $\mu v > [\mu + (1-\mu)\phi(\mathcal{B}_L)]^2 v$ but this is not so if $\mathcal{B}_L$ is large. The lowest value of $\mathcal{B}_H$ which equates profits from the two pricing strategies, if it exists, is the threshold $\bar{\mathcal{B}}_H^j$ for signal $j$.

To complete the argument that for $\mathcal{B}_H > \max_j \bar{\mathcal{B}}_H^j$, all sellers mix rather than price for the high state only, we rule out the latter as an equilibrium outcome. Suppose that all sellers set the price $p_H^{j*}$. This price is supposed to sell only in the high state, but, in fact, is acceptable to buyers in both demand states as it does not exceed $v$. Therefore, under tightness risk the only alternative to an equilibrium where sellers post a single price targeting both states is an equilibrium where sellers mix.

Whether sellers post a single price or randomize over two prices is important for welfare. If all sellers post a single price (i.e., for parameters in the shaded region in Figure 2), the expected value of trades in state $i$ is $\lambda(\mathcal{B}_i)v$, irrespective of the signal precision. This is also the expected value of trades when the signal is perfectly informative. In this region, thus, welfare is independent of information precision and equal to its highest attainable value $W^*$.

In the rest of the parameter space, at least one seller type randomizes for $\mu$ sufficiently small. When sellers randomize over two prices, welfare decreases in the high state because of price dispersion and in the low state because of an inactive submarket. Assuming that $j$-sellers randomize, choosing the price that targets both states with probability $\kappa$, we have:

$$\begin{aligned} W_H(\Theta, 1) = \lambda(\mathcal{B}_H)v &= \lambda[\kappa\tilde{x}_H^j + (1-\kappa)x_H^j]v \\ &> [\kappa\lambda(\tilde{x}_H^j) + (1-\kappa)\lambda(x_H^j)]v = W_H(\Theta, \mu), \text{ and} \\ W_L(\Theta, 1) = \lambda(\mathcal{B}_L)v &> \lambda(\kappa\tilde{x}_L^j)v > \kappa\lambda(\tilde{x}_L^j)v = W_L(\Theta, \mu). \end{aligned}$$

In this region of the parameter space, there are two possibilities. First, for $\mathcal{B}_H > \bar{\mathcal{B}}_H(\mathcal{B}_L, 1/2)$, the uninformed sellers mix and for sufficiently high information precision neither $G$- nor $B$-sellers do. Hence, there exists a precision of information that leads to welfare increase relative to no information. Second, if $\bar{\mathcal{B}}_H(\mathcal{B}_L, 1/2) > \mathcal{B}_H > \min_j \min_\mu \bar{\mathcal{B}}_H^j(\mathcal{B}_L, \mu)$, the uninformed sellers do not mix, but $B$-sellers mix for some information precision values. When this is the case, welfare decreases below the no-information level for signal precisions which trigger mixing, and then increases back to the highest attainable value.

We demonstrate the three cases on Figure 3 by picking three different values of $\mathcal{B}_H$.[9] To make them comparable, we normalise welfare by the perfect information welfare.[10] The black-dotted line corresponds to the region where all sellers post a single price for all $\mu$. The black-dot-dashed line represents a combination of $(\mathcal{B}_L, \mathcal{B}_H)$ such that if $\mu = 1/2$ both types of sellers mix. Then, for sufficiently high $\mu$, first the $G$-sellers and then the $B$-sellers switch to posting a single price. The blue-dashed line starts at the highest attainable welfare, but increases in

[9] We plot the corresponding equilibrium prices in Online Appendix B.1.
[10] This is necessary because larger population of buyers increases the likelihood of trade.

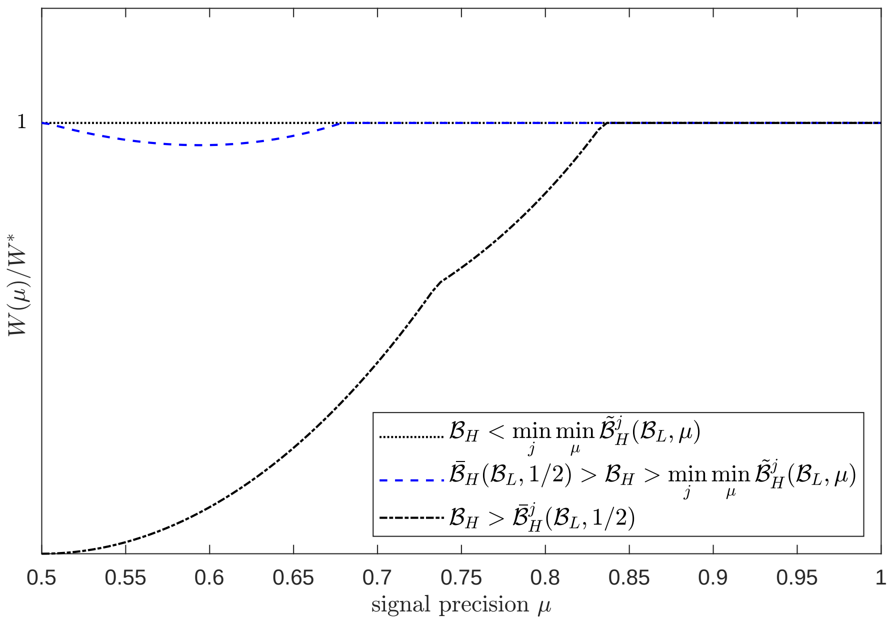


Figure 3: Normalised welfare $W(\mu)/W^*$ under tightness risk as a function of signal precision for three different values of $\mathcal{B}_H$ with $v = 1$ and $\mathcal{B}_L = 1/5$.

signal precision prompt the $B$-sellers to mix over an interval of $\mu$. Altogether, the marginal effects of information on welfare can vary from negative to positive and the key reason for this is the existence of MSE.

In line with intuition, information can have a positive effect on welfare. For $\mu$ high enough, sellers are well-informed about demand and the expected profit functions are unimodal, even after factoring in the equilibrium feedback between pricing and market utilities of buyers. We refer to this effect of information as the *revealing* effect. However, as we have shown, moderate increases in information precision can trigger equilibria with mixing, causing welfare to decrease. This happens due to the interaction of the revealing effect with the *price competition* effect of information. We explain this mechanism below.

Without loss of generality, pick $\mathcal{B}_H = \bar{\mathcal{B}}_H(\mathcal{B}_L, 1/2)$ and an uninformative signal so that all sellers price for both states, the profit function is bimodal and the two local maxima are equal (on Figure 2, the point on the $y$-axis where the two top curves meet). For a switch from PSE to MSE as we increase information precision, it must be the case that eventually a seller can profitably deviate to pricing for the high state only when all other sellers price for both states. When the signal becomes more informative, the PSE price increases (decreases) if the signal is good (bad). This makes the buyers worse off (better off, respectively) which also increases (decreases) perfect information profits in both states. This equilibrium effect of prices on profits is stronger in the low-demand state because the competition for buyers is fiercer there. Therefore, the price competition effect works as follows. When the signal is good, the change in the single price posted in equilibrium shrinks the profitability gap between the two demand states because the high state is uniformly more profitable than the low state. Thus, for $G$-sellers the change favours pricing for both states. For $B$-sellers, conversely, the profitability gap increases and favours pricing for the high state only.

However, an increase in the signal precision also changes the weight assigned to each demand state – the revealing effect – and acts in the opposite direction than the price competition effect. Depending on which of the two effects is stronger, it can be either the *B*- or the *G*-sellers who mix over a larger subset of the parameter space. The net effect depends on the difference between the perfect information profits and on how sensitive the market utilities of buyers are to prices. This in turn depends on the absolute values of $\mathcal{B}_L$ and $\mathcal{B}_H$. For a low $\mathcal{B}_L$ and $\mathcal{B}_H = \bar{\mathcal{B}}_H(\mathcal{B}_L, 1/2)$, the price competition effect is stronger because competition for buyers is fiercer in both states so the expected profits react more to $\mu$. This corresponds to Figure 2 where *B*-sellers randomise for a more precise signal while *G*-sellers do not. A larger $\mathcal{B}_L$ implies a much larger $\bar{\mathcal{B}}_H(\mathcal{B}_L, 1/2)$. For a larger $\mathcal{B}_L$ and $\mathcal{B}_H = \bar{\mathcal{B}}_H(\mathcal{B}_L, 1/2)$, the price competition effect is weaker. As a result, the revealing effect dominates so *G*-sellers mix for a more precise signal, while *B*-sellers do not (illustrated on Figure 5 in Online Appendix B.1.)

#### 4.2.2 Surplus risk

Recall that under surplus risk $\mathcal{B}_H = \mathcal{B}_L = \mathcal{B}$ and $v_H > v_L$. The key source of difficulty in analysing this case is that the pricing equations (9) - (10) are not linear (as for tightness risk), but quadratic. Thus, we focus on the particular matching function proposed in Assumption 5 and uninformative signals to highlight the similarities and differences to tightness risk.[11]

The distinct feature of equilibria under tightness risk is that in all PSE welfare under perfect information, the highest attainable, coincides with that under no information. We show that for surplus risk this is not necessarily the case and welfare in PSE can be strictly lower under no information. The root of equilibria under no information being inefficient is that some, or even all, sellers can opt for posting a price that exceeds $v_L$, leading to a market freeze: an inactive submarket in the low-demand state.

Based on the logic of Theorem 2, three types of equilibria are possible for uninformative signals. First, an equilibrium where sellers price for both states. Such equilibria are qualitatively similar to the equilibria under tightness risk. Second, an equilibrium where sellers mix over two prices. Third, an equilibrium where sellers post a price that exceeds $v_L$. In the third equilibrium sellers post the price that is posted under perfect information in the high-demand state. As our first step, we compare welfare under no and perfect information in the first and third cases.

**Theorem 4** (Equilibria, surplus risk, uninformative signal)**.** *Let Assumptions 5 and 7 hold, and let signals be uninformative, $\mu = 1/2$. Then, there exist threshold values $\bar{v}(v_L, \mathcal{B})$ and $\underline{v}(v_L, \mathcal{B})$ such that:*

1. *when $v_H < \bar{v}(v_L, \mathcal{B})$, the optimal price $p^N$ satisfies $p^N < v_L$ and welfare under no information is equal to welfare under perfect information,*
2. *when $v_H > \underline{v}(v_L, \mathcal{B})$, $p^N > v_L$ and welfare under no information is strictly lower than welfare under perfect information.*

[11] Analogously to tightness risk, we demonstrate the variety of the marginal effects of information on the expected value of trades on Figure 10 in the Online Appendix B.1.2.

*Proof.* 1. When $\mu = 1/2$ and sellers price for both states, the conditions (9) - (10) collapse to a single equation which has a single positive solution, if any:

$$p^N = \frac{(v_H + v_L)(1 + \phi(\mathcal{B})) - \sqrt{(1+\phi(\mathcal{B}))^2(v_H+v_L)^2 - 16\phi(\mathcal{B})v_H v_L}}{4}.$$

The minimal requirement for the price to be consistent with pricing for both states is that $p^N < v_L$ which pins down $\bar{v}(v_L, \mathcal{B})$.
2. When all sellers post $p^N > v_L$, then $p^N = p_H^*$ and the profits are $\pi^N(p_H^*) = \frac{1}{2}\lambda(\mathcal{B})\,\phi(\mathcal{B})\,v_H$. Posting $p^N > v_L$ is optimal iff deviating to posting $\hat{p} = v_L$ leads to lower profits. Any price lower than $v_L$ is an inferior deviation because $p_H^*$ is the profit maximiser in the high-demand state and any single seller deviating to a price weakly lower than $v_L$ sells with certainty.

A single deviating seller doesn't impact the market. Buyers' indifference in the high state requires that there exists a buyer-seller ratio $\hat{x}$ such that:

$$\eta(\hat{x})\,(v_H - v_L) = \eta\,(\mathcal{B})\,(1 - \phi(\mathcal{B}))\,v_H.$$

The profits from this deviation are $\pi^N(v_L) = (1 + \lambda(\hat{x}))\,v_L/2$. Then $\underline{v}(v_L, \mathcal{B})$ is the $v_H$ that solves the following equal-profit condition for given $v_L$ and $\mathcal{B}$:

$$\frac{1}{2}\lambda(\mathcal{B})\,\phi(\mathcal{B})\,v_H = \frac{1}{2}(1 + \lambda(\hat{x}))\,v_L. \qquad \square$$

Thus, when the no-information equilibrium yields no trade in the low-demand state, welfare increases relative to no information for at least some signal precisions $\mu$. We now show that welfare in PSE under surplus risk behaves similarly to tightness risk for all $\mu$ if $v_H$ and $v_L$ are close. The reason is that, if $v_H$ and $v_L$ are close, sellers do not find it optimal to forego selling in the low state and price for both states.

**Lemma 5** (Welfare, surplus risk, similar states)**.** *Let Assumptions 5 and 7 hold. If $v_H \leq v_L + \bar{\varepsilon}$ for some $\bar{\varepsilon} > 0$, both types of sellers price for both states. Then welfare is independent of $\mu$ and equal to $W^*$.*

*Proof.* We argue that both types of sellers price for both states if $v_L$ and $v_H$ are close. Let $v_H > v_L$ and note that pricing for the high state only is more profitable for $G$-sellers than for $B$-sellers. Now assume that $B$-sellers optimally price for both states and $G$-sellers optimally price for the high state only. We rule out the latter possibility for $v_H$ and $v_L$ close enough.

Pricing for the high state only means posting a price $p_H^* = \phi(x_H^G)v_H$ by Lemma 1, where $x_H^G = \mathcal{B}/(\mathcal{B}+1)$. Since $\phi(x) \in [0, 1)$ for all $x \in [0, \frac{\mathcal{B}}{S}]$, we know that for $\phi(x_H^G)$ there exists a $v_L$ such that $\phi(x_H^G)v_H = v_L$, or $v_H = v_L + \varepsilon$ for some $\varepsilon > 0$. Define $\bar{\varepsilon}$ to be the infimum of such $\varepsilon$. Then for all $v_H \leq v_L + \bar{\varepsilon}$, $\phi(x_H^G)v_H < v_L$ for all $\phi(x_H^G)$: even if $G$-sellers intend to price only for the high state, some buyers contact them also in the low state. $\square$

Thus, if $v_H$ and $v_L$ are close, welfare under surplus risk is constant in signal precision in PSE, as was the case for all PSE under tightness risk.

Now we show that welfare in PSE under surplus risk behaves quite differently from tightness risk if $v_H$ and $v_L$ are different enough.

**Lemma 6** (Welfare, surplus risk, different states). *Let Assumptions 5 and 7 hold. If $v_H > \underline{v}(v_L, \mathcal{B})$ and $p^N > v_L$ (i.e., case 2. in Proposition 4), then welfare is monotone increasing in the signal precision for all $\mu \geq \mu_1$.*

*Proof.* Recall that if $v_H > \underline{v}(v_L, \mathcal{B})$, then $p^N > v_L$ or under no information sellers price for the high state only. In a PSE, for any state and price such that buyers buy at the price in the given state, the matching probability is constant $\lambda(\mathcal{B})$. Thus, the welfare comparison in PSE is straightforward. Under perfect information welfare is $W^* = \frac{1}{2}\lambda(\mathcal{B})(v_H + v_L)$ and under no information, $W(\Theta, \frac{1}{2}) = \frac{1}{2}\lambda(\mathcal{B})v_H$.

Now consider $\mu$ high enough such that $B$-sellers want to serve both states with probability one, i.e., post $p^B \leq v_L$ with probability one. Denote the lowest $\mu$ where such behaviour becomes optimal for $B$-sellers by $\mu_1$. We know that such $\mu_1 < 1$ exists because under perfect information $B$-sellers post a price below $v_L$ with probability one. In such an equilibrium, the welfare is

$$W(\Theta, \mu) = \frac{1}{2}\lambda(\mathcal{B})(v_H + \mu v_L).$$

Clearly, $W(\Theta, \mu) \in (W(\Theta, \frac{1}{2}), W^*)$ and increases in $\mu$. $\square$

If $v_H$ and $v_L$ are different enough, then in equilibrium both types of sellers post a price that exceeds $v_L$ if the signal precision is low. This is the new type of inefficiency that can arise in the case of surplus risk, but not tightness risk. If the signal becomes more precise, the $B$-sellers lower the price until it falls below $v_L$. When that happens, welfare is higher than when signals are uninformative.

Intuitively, for intermediate values of $v_H$ a MSE exists. Then, improvements in signal precision can again trigger a switch from a PSE where sellers post a single price that targets both states to a MSE where sellers mix over two prices. When this is the case, welfare can decrease in $\mu$.[12]

## 5 Entry

In this section, we analyse the same model as before, except that sellers can choose whether to enter the market or not. We show that, in general, the free-entry equilibrium is inefficient.

Assume that at the beginning of the period, the signal $j$ arrives and then sellers decide whether to enter or not. To enter, each seller must set up a trading post at a cost $c$. In equilibrium, the expected value of a trading post net of cost $c$ must be equal to zero due to free entry. We formalize the equilibrium concept in this version of the model below.

**Definition 4** (Equilibrium with entry). *A tuple $\left(\left\{\kappa^{j,k}, x_i^{j,k}, p^{j,k}\right\}_{k=1}^{K^j}, V_i^j, \mathcal{S}^j\right)$ is an* equilibrium with entry *for exogenous parameters $\Theta = (v_i, \mathcal{B}_i, c)$, for $i \in \{H, L\}$ and $c > 0$, and signal precision $\mu$ if, for each $j \in \{G, B\}$: conditions 1. and 2. in Definition 2 hold, and*

[12] We illustrate the partition of the parameter space into regions which feature a particular type of equilibrium and provide further details in the Online Appendix B.1.2.

3. *buyer-seller ratios and probability weights* $\left\{x_i^{j,k}, \kappa^{j,k}\right\}_{k=1}^{K^j}$ *are consistent with buyers' adding-up constraints:*

$$\sum_{k}^{K^j} \kappa^{j,k} x_i^{j,k} = \mathbb{1}_i^j \frac{\mathcal{B}_i}{\mathcal{S}^j},$$

*where* $\mathbb{1}_i^j = 1$ *if* $\mathcal{I}_i^j \neq \emptyset$, *and* $\mathbb{1}_i^j = 0$ *if* $\mathcal{I}_i^j = \emptyset$, *the signal realisation is not* $j$ *and* $\mu = 1$, *or* $\mathcal{S}^j = 0$,

4. *there is free entry in setting up a trading post:* $c = \pi^j(p^j)$.

To assess the efficiency of entry, we adjust the definition of welfare which is now the expected surplus generated by trades net of trading post setup costs:

$$\begin{aligned} W(\mathcal{S}^G, \mathcal{S}^B, \Theta, \mu) =& \frac{1}{2}\mathcal{S}^G \left[ \left( \mu \sum_{k=1}^{K^G} \kappa^{G,k} \lambda(x_H^{G,k}) + (1-\mu) \sum_{k=1}^{K^B} \kappa^{B,k} \lambda(x_H^{B,k}) \right) v_H - c \right] \\ &+ \frac{1}{2}\mathcal{S}^B \left[ \left( \mu \sum_{k=1}^{K^B} \kappa^{B,k} \lambda(x_L^{B,k}) + (1-\mu) \sum_{k=1}^{K^G} \kappa^{G,k} \lambda(x_L^{G,k}) \right) v_L - c \right]. \end{aligned}$$

As a benchmark, we characterise the allocations chosen by a planner who only observes the realisation of the signal. The planner decides on the measure of the trading posts and dictates the rule of allocating buyers across sellers. Hence, the planner's choice is not constrained by the free-entry or market-utility constraints. Because of concavity of the trading probabilities, the planner dictates that buyers mix over visiting all sellers with the same probability. Hence, the choice of the measure of trading posts pins down the buyer-seller ratio in each state. This is equivalent to there being only one submarket in each state.

**Definition 5** (Constrained-efficiency)**.** *A pair* $\left(\mathcal{S}_p^G, \mathcal{S}_p^B\right)$ *is a social planner entry for exogenous parameters* $\Theta = (v_i, \mathcal{B}_i, c)$, *for* $i \in \{H, L\}$ *and* $c > 0$, *and signal precision* $\mu$ *if:*

$$\begin{aligned} \left(\mathcal{S}_p^G, \mathcal{S}_p^B\right) \in \arg\max_{\mathcal{S}^G, \mathcal{S}^B} & \frac{1}{2}\mathcal{S}^G \left[\mu\lambda(x_H^G) v_H + (1-\mu)\lambda(x_L^G) v_L - c\right] \\ &+ \frac{1}{2}\mathcal{S}^B \left[(1-\mu)\lambda(x_H^B) v_H + \mu\lambda(x_L^B) v_L - c\right] =: W^p(\mathcal{S}^G, \mathcal{S}^B, \mu). \end{aligned}$$

*with* $x_i^j = \mathcal{B}_i / \mathcal{S}^j$, $j \in \{G, B\}$. *Let the planner's attained level of welfare be* $W^p(\mathcal{S}_p^G, \mathcal{S}_p^B, \Theta, \mu)$. *An equilibrium with entry* $\left(\left\{\kappa^{j,k}, x_i^{j,k}, p^{j,k}\right\}_{k=1}^{K^j}, V_i^j, \mathcal{S}^j\right)$ is constrained-efficient *for* $\Theta$ *and* $\mu$ *if* $\mathcal{S}^j = \mathcal{S}_p^j$, *and* $W(\mathcal{S}^G, \mathcal{S}^B, \Theta, \mu) = W^p(\mathcal{S}_p^G, \mathcal{S}_p^B, \Theta, \mu)$.

The solution to the planner's problem satisfies the first order conditions:

$$\mu\lambda(x_H^G)\phi(x_H^G) v_H + (1-\mu)\lambda(x_L^G)\phi(x_L^G) v_L = c, \tag{12}$$

$$(1-\mu)\lambda(x_H^B)\phi(x_H^B) v_H + \mu\lambda(x_L^B)\phi(x_L^B) v_L = c. \tag{13}$$

These are sufficient as the expected value of trades net of setup costs is strictly concave in levels of entry $(\mathcal{S}^G, \mathcal{S}^B)$. Next, we show in Appendix A.7 that the welfare attained by the planner monotonically increases in signal precision $\mu$ and hence is largest when information is perfect.

The cost of imperfect information is that entry under $G$-signal ($B$-signal) is too low (too high). Finally, when the setup cost $c$ increases, the levels of entry decrease. We discuss constrained efficiency of equilibria with entry next.

**Theorem 5** (Constrained efficiency of equilibria with entry)**.** *Let* $\left(\left\{\kappa^{j,k}, x_i^{j,k}, p^{j,k}\right\}_{k=1}^{K^j}, V_i^j, \mathcal{S}^j\right)$ *be an equilibrium with entry for exogenous parameters* $\Theta = (v_i, \mathcal{B}_i, c)$*, for* $i \in \{H, L\}$ *and* $c > 0$*, and signal precision* $\mu$*. Then, this equilibrium is constrained-efficient iff either Assumptions 4 (CHS matching function) and 6 (Tightness risk) both hold, or the signal is perfectly informative,* $\mu = 1$*.*

*Proof.* First, any equilibrium with entry that features an inactive submarket, be it due to mixing or sellers posting a price that exceeds $v_L$, is not constrained efficient. The planner can choose the same level of entry for each signal, incurs the same costs, and obtains a higher value of trades because buyers mix uniformly over sellers. Hence, we confine attention to combinations of parameters that yield PSE.

Second, when $\mu = 1$, conditions (12) - (13) are identical to the equilibrium free-entry conditions. Indeed, expected profits in an equilibrium with entry then read $\lambda(x_i)\phi(x_i)v_i$ by the virtue of Corollary 3. For the case of $\mu < 1$, when Assumptions 4 and 6 hold, the planner's optimality conditions (12) - (13) are equivalent to the equilibrium free-entry conditions as then $\phi(x_i^j)v_i = \phi v$. This completes the proof of the if- part of Theorem 5 as each of the planner conditions has a unique solution for the level of entry.

For the only if- part, we describe the key intuition focussing, without loss of generality, on the case of an uninformative signal. To do this, we investigate the consequences of parting with Assumption 4. Hence, we assume that Assumptions 5 (Particular non-CHS matching function) and 6 hold simultaneously.[13] When $\mu = 1/2$ there is just one level of equilibrium entry, $\mathcal{S}^G = \mathcal{S}^B = \mathcal{S}$, and one level chosen by the planner, $\mathcal{S}_p$. The equilibrium free-entry condition reads:

$$\frac{1}{4}\left[\lambda(x_H) + \lambda(x_L)\right]\left[\phi(x_H) + \phi(x_L)\right] v = c, \text{ with } x_i = \frac{\mathcal{B}_i}{\mathcal{S}}.$$

Now, assuming $\mathcal{S}_p = \mathcal{S}$ leads to a contradiction, as:

$$\underbrace{c = \frac{\lambda(x_H) + \lambda(x_L)}{2}\frac{\phi(x_H) + \phi(x_L)}{2}v}_{\text{free entry}} < \underbrace{\frac{\left[\lambda(x_H)\phi(x_H) + \lambda(x_L)\phi(x_L)\right]}{2}v = c}_{\text{planner's optimality condition}}.$$

Therefore, for the same cost of a trading post setup, the planner chooses a level of entry that is strictly larger than that in equilibrium. □

The inefficiency of entry is due to the effect of imperfect information on pricing. The price that the imperfectly informed sellers set exceeds the perfect information price in the low state and is below the perfect information price in the high state. Hence, an atomistic seller benefits from the effect of imperfect information on the pricing decisions of other sellers in the low state and is worse off in the high state. The competition for buyers being fiercer in the low-demand

[13] Alternatively, one can part with Assumption 6, keeping Assumption 4 in place etc.

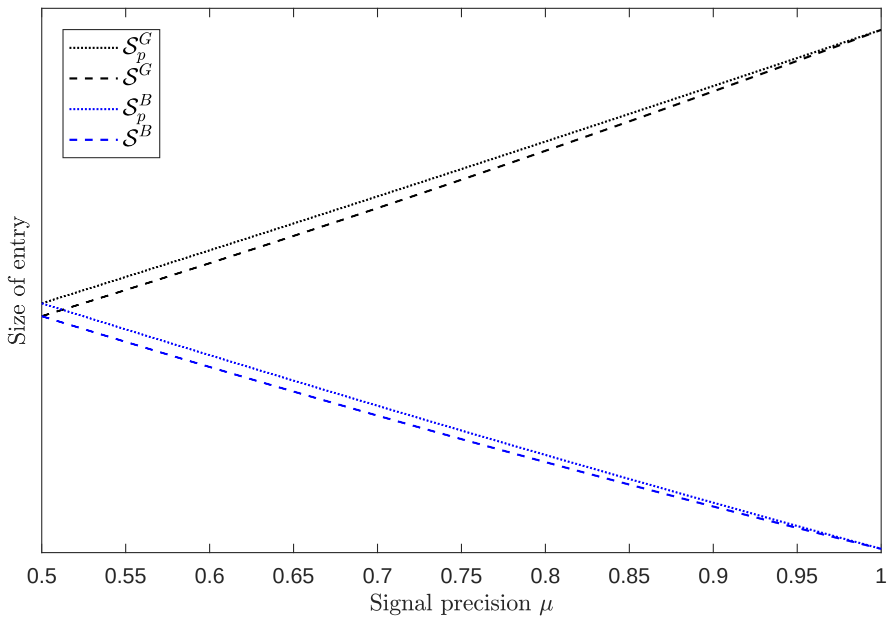


Figure 4: Illustration of Theorem 5 for $v = 1$, $\mathcal{B}_L = 1$ and $\mathcal{B}_H = 3/2$.

state limits the gains there and eventually the latter effect dominates the former, leading to an inefficiently low entry. We illustrate this on Figure 4.

Put differently, the ability of the planner to ignore competition between sellers and its interaction with buyers' indifference is akin to perfect-information pricing in each state for a given signal (the $\phi(x_i^j)v_i$ terms in (12) - (13)) conditional on the level of entry. Prices set in an equilibrium assuming planner entry, however, are not equal to the perfect-information prices so the free-entry and planner's optimal entry conditions in general do not hold simultaneously.

## 6 Lotteries

Throughout the paper we have assumed that the sellers can post at most one price. Riley and Zeckhauser (1983) showed that posting a single price is the best that an uninformed seller can do under individual-level uncertainty and bilateral trade so it is instructive to learn whether this is still the case under aggregate uncertainty.[14] We show that this is not always the case.

Allowing sellers to post trading mechanisms that depend on the state of demand $i$ is akin to assuming away the effects of imperfect information on aggregate state. To see this, note that under such superior technology, posting two aggregate-state-dependent prices would always lead to full information equilibria. Hence, we consider more realistic mechanisms that depend solely on individual-level meetings.

As a seller can meet at most one buyer, the most general trading mechanism is a menu of lotteries $\left(p_i^j, \theta_i^j\right)_i$ that prescribe the price $p_i^j$ a buyer must pay upon meeting with a seller and the probability $\theta_i^j$ to obtain the good. Such a menu can screen buyers, as long as the following

[14] Delacroix and Shi (2013) show that when the sellers are (privately) informed about the quality of the good they offer, a two-part pricing scheme can improve efficiency upon posting a single price. In our framework signaling considerations are absent and a two-part pricing scheme collapses to a single price.

incentive compatibility (IC) constraints hold:

$$\theta_i^j v_i - p_i^j \geq \theta_{-i}^j v_i - p_{-i}^j, i \in \{H, L\}, j \in \{G, B\}. \tag{14}$$

This mechanism requires the sellers to be able to commit to withholding the good even though doing so brings no additional benefits to them. The price posting assumption we have worked with throughout the paper is a special case of this more general mechanism with $\theta_i^j = 1$ and $p_i^j = p^j$.

Our first result pertaining to these lotteries is that they bring no additional benefit under tightness risk.

**Lemma 7** (Irrelevance of lotteries, tightness risk)**.** *Let Assumption 6 hold. Then $\theta_i^j = 1$ and $p_i^j = p^j$.*

*Proof.* Under Assumption 6, the IC constraints (14) become:

$$\theta_L^j v - p_L^j \geq \theta_H^j v - p_H^j \text{ and } \theta_H^j v - p_H^j \geq \theta_L^j v - p_L^j \implies \theta_L^j v - p_L^j = \theta_H^j v - p_H^j.$$

The buyers must be indifferent between the two lotteries. The sellers' profits are unaffected by $\theta_i^j$ and differentiating the probabilities between types does not bring any benefits. Hence it's optimal to set $\theta_i^j = 1$ and correspondingly $p_L^j = p_H^j = p^j$. □

Our second result is that when the sellers can post menus of lotteries, there is always trade under surplus risk because the sellers can increase their profits relative to the equilibrium with no trading in the low state under single price posting. Analogously to other results in section 4.2.2, we show this for uninformative signals.

**Lemma 8** (Lotteries dominate single price, surplus risk)**.** *Let Assumption 7 hold and let also $v_H \geq \underline{v}(v_L, \mathcal{B})$ and $\mu = 1/2$ as in Proposition 4. Then, posting a single price is not optimal.*

*Proof.* When assumptions of Lemma 8 are met, sellers post $p^N = \phi(\mathcal{B})v_H$ which leads to lack of trade in the low demand state. However, when sellers can post lotteries, they can increase their profits. They can choose the same price $p_H^j = \phi(\mathcal{B})v_H$ and guarantee the purchase in the high demand state, $\theta_H^j = 1$, but select a different incentive-compatible offer for buyers in the low-demand state. The conditions to do so are:

$$\theta_L^j v_L - p_L^j \geq v_L - \phi(\mathcal{B})v_H,$$
$$v_H - \phi(\mathcal{B})v_H \geq \theta_L^j v_H - p_L^j.$$

The first constraint can be tightened to $\theta_L^j v_L - p_L^j \geq 0$, as buyers in the low demand state do not want to purchase the good at the price designed for the high state by assumption. Offering exactly zero to buyers in the low demand state is compatible with their market utility being zero (due to no search) which implies $p_L^j = \theta_L^j v_L$. Then, the highest probability of acquiring the good when demand is low that screens the states is the one that leaves the buyers in the high-demand state indifferent:

$$\theta_L^j = \frac{(1 - \phi(\mathcal{B}))\, v_H}{v_H - v_L}.$$ □

Note, when such a lottery is posted, it is still the case that welfare under perfect information is higher than under no information, which was the distinct feature of certain equilibria under surplus risk. However, when sellers post lotteries instead of a single price under surplus risk it is still the case that some buyer-seller meetings will result in no trade. Thus, while lotteries help avoid market freezes, the frequency of trades can still be significantly lower (just simply not equal to zero) than under full information.

## 7 Conclusions

We extend the standard model of competitive search with uncertainty about market conditions and analyse the effects of increases in the precision of public information on trade. We obtain several results which point to the importance of interactions between the properties of the matching function and the type of underlying aggregate risk on the one hand and information on the other.

Our results are readily applied to studying other environments with competitive search and either exogenous or endogenous public information on aggregate risk. An example of the former is a model in which sellers publicly learn about past trades of other sellers. As for the latter, the signal precision can be a, potentially costly, choice variable of a policy maker. We hope to spur future research on interactions of imperfect information, aggregate risk and competitive search. One such promising avenue is to consider an endogenous private information choice problem and another is making the model dynamic.

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# A Proofs and derivations

## A.1 Proof of Lemma 1

We differentiate the expected profits with respect to price and then implicitly differentiate the market utility constraint with respect to price as well to arrive at the following first order condition:

$$\lambda'(x_i^*)p_i^* = -\frac{\lambda(x_i^*)}{\eta(x_i^*)}\eta'(x_i^*)(v_i - p_i^*).$$

Multiplying both sides by $x_i^*$ and using the definition of $\phi(x)$, we obtain:

$$(1-\phi(x_i^*))\,p_i^* = \phi(x_i^*)\,(v_i - p_i^*) \implies p_i^* = \phi(x_i^*)v_i.$$

## A.2 Proof of Lemma 2

Following analogous steps as in the previous case, we obtain:

$$\mu\lambda(\tilde{x}_H^G) + (1-\mu)\lambda(\tilde{x}_L^G) = \left[-\mu\frac{\lambda'(\tilde{x}_H^G)\eta(\tilde{x}_H^G)}{\eta'(\tilde{x}_H^G)(v_H - \tilde{p}^G)} - (1-\mu)\frac{\lambda'(\tilde{x}_L^G)\eta(\tilde{x}_L^G)}{\eta'(\tilde{x}_L^G)(v_L - \tilde{p}^G)}\right]\tilde{p}^G.$$

Multiplying the components of the sum on the right hand side by $\frac{x_i^G\lambda(x_i^G)}{x_i^G\lambda(x_i^G)}$ and rearranging yields the formula in the Lemma.

## A.3 Proof of Theorem 1

By the definition of perfect information maximisers, $p_i^* < p < \bar{p}_i \implies \frac{d\pi_i(p)}{dp} < 0$ so that $\pi^j(p)$ is strictly decreasing on $(p_H^*, \bar{p}_H]$ and also strictly decreasing on $[\min_i \bar{p}_i, \max_i \bar{p}_i]$ for $j = G, B$. Thus, there is no local maximum on this interval.

We also have that $\pi^j(p)$ are strictly increasing on $(0, p_L^*)$. Hence the (by strict concavity, unique) maximisers are interior for imperfectly informative signals: $p^j \in (p_L, p_H)$ for $\mu < 1$. By Assumption 3, $\frac{d\pi_H}{dp} > \frac{d\pi_L}{dp}$ on $[p_L^*, p_H^*]$. Also:

$$\frac{d\pi^G(p)}{dp} = \mu\frac{d\pi_H}{dp} + (1-\mu)\frac{d\pi_L}{dp} \text{ and } \frac{d\pi^B(p)}{dp} = (1-\mu)\frac{d\pi_H}{dp} + \mu\frac{d\pi_L}{dp}.$$

This implies $\frac{d\pi^G(p)}{dp} \geq \frac{d\pi^B(p)}{dp}$ at any given $p$ in this interval with equality only if $\mu = 1/2$. Therefore, $\tilde{p}^G \geq \tilde{p}^B$ with equality only if $\mu = 1/2$.

## A.4 Proof of Theorem 2

For this proof we continue with a slight abuse of notation, treating $\mu$ as the second argument of expected profits, next to the price $p$. In this vein, observe that by the virtue of Assumption 3:

$$p \in [p_L^*, \bar{p}_H] \implies \frac{\partial\pi^G(p)}{\partial\mu} = -\frac{\partial\pi^B(p)}{\partial\mu} = \pi_H(p) - \pi_L(p) > 0.$$

Next, as $p_H^* > \bar{p}_L > p_L^*$, there trivially is a local maximum at $p_H^*$ for both $\pi^j(p)$. For the remainder of the proof we rely on differences between maximisers of $\pi^j(p)$ on the two intervals $[0, \bar{p}_L]$ and $[\bar{p}_L, \bar{p}_H]$ as functions of $\mu$. We define:

$$\Delta^j(\mu) := \pi^j(p_H^*) - \pi^j\left(\tilde{p}^j\right).$$

Whenever $\Delta^j(\mu) > 0$, the global maximiser of $\pi^j(p)$ is $p_H^*$, when $\Delta^j(\mu) = 0$, the $j$-seller is indifferent between posting $p_H^*$ and $\tilde{p}^j$ and when $\Delta^j(\mu) < 0$, then the global maximiser of $\pi^j(p)$ is $\tilde{p}^j$. This implies the following:

$$\frac{d\Delta^G(\mu)}{d\mu} = \pi_H(p_H^*) - \left[\pi_H\left(\tilde{p}^G\right) - \pi_L\left(\tilde{p}^G\right)\right] - \frac{d\pi^G\left(\tilde{p}^G\right)}{d\tilde{p}^G}\frac{d\tilde{p}^G}{d\mu},$$

$$\frac{d\Delta^B(\mu)}{d\mu} = -\pi_H(p_H^*) + \left[\pi_H\left(\tilde{p}^B\right) - \pi_L\left(\tilde{p}^B\right)\right] - \frac{d\pi^B\left(\tilde{p}^B\right)}{d\tilde{p}^B}\frac{d\tilde{p}^B}{d\mu}.$$

There are two cases to consider here. Either $\tilde{p}^j$ is an interior solution and then by envelope theorem $d\pi^j\left(\tilde{p}^j\right)/d\tilde{p}^j = 0$, or it is a corner solution (which can only happen for $j = G$) and then $d\tilde{p}^j/d\mu = 0$. Furthermore, by Assumption 3 and $p_H^*$ being the maximiser:

$$\pi_H(p_H^*) > \pi_H\left(\tilde{p}^G\right) > \pi_L\left(\tilde{p}^G\right) \text{ and } \pi_H(p_H^*) > \pi_H\left(\tilde{p}^B\right) > \pi_L\left(\tilde{p}^B\right).$$

Hence, we conclude that $\frac{d\Delta^G(\mu)}{d\mu} > 0$ and $\frac{d\Delta^B(\mu)}{d\mu} < 0$ for $\mu \in \left(\frac{1}{2}, 1\right)$. Note also that $\Delta^G(1) = \pi_H(p_H^*) > 0$ and $\Delta^B(1) = -\pi_H(p_H^*) < 0$. We now go over the cases listed in Theorem 2.

1. In this case we have $\Delta^G\left(\frac{1}{2}\right) < 0$ and $\Delta^B\left(\frac{1}{2}\right) < 0$ and so the $B$-sellers always stick to posting $\tilde{p}^B$ and by Darboux theorem there exists a unique $\bar{\mu}^G$ such that $\Delta^G\left(\bar{\mu}^G\right) = 0$ which then implies the optimal price posting behaviour described in the Theorem.

3. Here we have $\Delta^G\left(\frac{1}{2}\right) > 0$ and $\Delta^B\left(\frac{1}{2}\right) > 0$. Therefore, $p_H^*$ remains the global profit maximising price for $G$-sellers, but by Darboux theorem there exists a unique $\bar{\mu}^B$ such that $\Delta^B\left(\bar{\mu}^B\right) = 0$ which then implies the optimal price posting behaviour described in the Theorem.

2. Finally, we have $\Delta^G\left(\frac{1}{2}\right) = \Delta^B\left(\frac{1}{2}\right) = 0$ and then both sellers are indifferent between $p_H^*$ and $\tilde{p}^N$ for $\mu = \frac{1}{2}$. Pricing behaviour for $\mu > \frac{1}{2}$ follows similar logic to the other two cases.

### A.5 Proof of Lemma 4

Let Assumption 5 hold. We have $\eta(x)\left(1 - \phi(x)\right) = \frac{1}{(1+x)^2}$. Hence, (8) implies:

$$x_i^{j*} = \sqrt{\frac{v_i}{V_i^j}} - 1 \implies \phi(x_i^{j*}) = 1 - \sqrt{\frac{V_i^j}{v_i}} \implies p_i^{j*} = v_i - \sqrt{v_i}\sqrt{V_i^j}.$$

If $p \leq v - V_i^j$, the buyers' market utility constraint becomes:

$$x(p)\left(\frac{v-p}{1+x(p)} - V_i^j\right) = 0 \implies x(p) = \frac{v-p-V_i^j}{V_i^j},$$

and the perfect information profits are:

$$\pi_i(p) = \lambda(x(p))p = \frac{v-p-V_i^j}{v-p}p, \tag{15}$$

If $p > v - V_i^j$, and $x(p) = \pi_i(p) = 0$. We proceed to derive the pricing-for-both-states prices, focussing on the $G$-sellers. The derivations for $B$-sellers are analogous. Based on the perfect information profits, the $G$-sellers' profit function for prices that don't exceed the lower price threshold $\bar{p}_i$ is:

$$\pi^G(p) = \mu\frac{v-p-V_H^G}{v-p}p + (1-\mu)\frac{v-p-V_L^G}{v-p}p \text{ for } p \leq \min_i \bar{p}_i = \min_i v - V_i^j.$$

The FOC for profit maximisation implies:

$$\left(v - \tilde{p}^G\right)^2 = \mu v V_H^G + (1-\mu)vV_L^G,$$

which is solved by the price stated in the Lemma. The buyer-seller ratios $\tilde{x}_i^G$ then follow from evaluating the market utility constraint at this price.

### A.6 Proof of Theorem 3

**The $H$ state is strictly more profitable.** First, let's assume all sellers post the same price $p^j < v$. Then, $V_H^j = \eta(\mathcal{B}_H)(v - p^j) < \eta(\mathcal{B}_L)(v - p^j) = V_L^j$. Observe that this inequality also allows us to rule out $V_H^j = V_L^j$ as then the sellers would indeed optimally post a single price. Finally, let's consider $V_L^j < V_H^j$ and the sellers randomising over two prices, $p_{j,1}^* < p_{j,2}^*$, with weights $\kappa$ and $1-\kappa$, respectively. For this to be optimal, it must be that $p_{j,2}^*$ targets the strictly more profitable low state only. $p_{j,2}^*$ implies the buyer-seller ratio according to the perfect information condition while $p_{j,1}^*$ is a price that targets both states. The buyers' adding-up constraints imply the following:

$$\kappa x_H^{j,1} = \mathcal{B}_H > \mathcal{B}_L = \kappa x_L^{j,1} + (1-\kappa)x_L^{j,2} \iff \kappa(x_H^{j,1} - x_L^{j,1}) > (1-\kappa)x_L^{j,2}$$

Without loss of generality, consider $G$-sellers. The optimal pricing conditions then imply:

$$x_H^{G,1} = \frac{\sqrt{\left[\mu V_H^G + (1-\mu)V_L^G\right]v}}{V_H^G} - 1 < \frac{\sqrt{\left[\mu V_H^G + (1-\mu)V_L^G\right]v}}{V_L^G} - 1 = x_L^{G,1}.$$

Therefore, we arrive at a contradiction: $V_H^G > V_L^G \implies x_L^{G,2} < 0$ (this goes through analogously for $B$-sellers). Hence, the high demand state is strictly more profitable and we can discard the possibility that a price targets the low demand state only.

**Thresholds $\tilde{\mathcal{B}}_H^j$ for unimodal profits.** When the public signal is good and sellers set the price that targets both states, the buyer's adding up constraints read:

$$\frac{\sqrt{\left[\mu V_H^G + (1-\mu)V_L^G\right] v}}{V_H^G} - 1 = \mathcal{B}_H \text{ and } \frac{\sqrt{\left[\mu V_H^G + (1-\mu)V_L^G\right] v}}{V_L^G} - 1 = \mathcal{B}_L.$$

Therefore, $V_L^G(\mathcal{B}_L + 1) = V_H^G(\mathcal{B}_H + 1)$ which leads to:

$$V_H^G = \frac{1}{\mathcal{B}_H + 1}\left(\frac{\mu}{\mathcal{B}_H + 1} + \frac{1-\mu}{\mathcal{B}_L + 1}\right) v \text{ and } V_L^G = \frac{\mathcal{B}_H + 1}{\mathcal{B}_L + 1} V_H^G. \tag{16}$$

Now we must consider two cases. The first case is that of the unimodal profit function, so that $v - \sqrt{vV_H^G} = p_H^G \leq \bar{p}_L^G = v - V_L^G \iff V_L^G \leq \sqrt{vV_H^G}$. When this condition is met, the optimal solution is to price for both states. Now

$$V_L^G \leq \sqrt{vV_H^G} \iff (1-\mu)\mathcal{B}_H \leq (\mathcal{B}_L + 1)^3 - \mu\mathcal{B}_L - 1.$$

Hence, we have $\tilde{\mathcal{B}}_H^G(\mathcal{B}_L, \mu) = [(\mathcal{B}_L + 1)^3 - \mu\mathcal{B}_L - 1]/(1-\mu)$. Straightforwardly, $\partial\tilde{\mathcal{B}}_H^G/\partial\mathcal{B}_L > 0$ and $\partial\tilde{\mathcal{B}}_H^G/\partial\mu > 0$. Analogous steps for the bad signal give:

$$V_H^B = \frac{1}{\mathcal{B}_H + 1}\left(\frac{1-\mu}{\mathcal{B}_H + 1} + \frac{\mu}{\mathcal{B}_L + 1}\right) v \text{ and } V_L^B = \frac{\mathcal{B}_H + 1}{\mathcal{B}_L + 1} V_H^B, \tag{17}$$

etc. Therefore, we find $\tilde{\mathcal{B}}_H^B(\mathcal{B}_L, \mu) = \left[(\mathcal{B}_L + 1)^3 - (1-\mu)\mathcal{B}_L - 1\right]/\mu$ and so $\partial\tilde{\mathcal{B}}_H^B/\partial\mathcal{B}_L > 0$ but $\partial\tilde{\mathcal{B}}_H^B/\partial\mu < 0$. Finally, note that $\tilde{\mathcal{B}}_H^j(\mathcal{B}_L, \mu) > \mathcal{B}_L$ and $\mathcal{B}_H^G(\mathcal{B}_L, 1/2) = \mathcal{B}_H^B(\mathcal{B}_L, 1/2)$.

**Thresholds $\bar{\mathcal{B}}_H^j$ for pricing for both states; bimodal profits.** Next, it can be the case that the profit function is bimodal, and yet it's optimal to price for both states. Hence, for parameter values which yield $(1-\mu)\mathcal{B}_H > (\mathcal{B}_L + 1)^3 - \mu\mathcal{B}_L - 1$ we must compare profits of both pricing rules. When a $G$-seller targets the high state only, its profit is:

$$\pi^G(p_H^G) = \mu\left(v - 2\sqrt{vV_H^G} + V_H^G\right),$$

while its profit when targeting both states is:

$$\pi^G(\tilde{p}^G) = v - 2\sqrt{\left(\mu V_H^G + (1-\mu)V_L^G\right) v} + \mu V_H^G + (1-\mu)V_L^G.$$

Letting $\mathcal{B}_H \to \infty$ gives:

$$\lim_{\mathcal{B}_H \to \infty} \pi^G(\tilde{p}^G) - \pi^G(p_H^G) = v\left[(1-\mu)\left(1 + \frac{1-\mu}{(\mathcal{B}_L + 1)^2}\right) - 2\frac{1-\mu}{\mathcal{B}_L + 1}\right].$$

Observe that $\lim_{\mathcal{B}_L \to 0}\lim_{\mathcal{B}_H \to \infty} \pi^G(\tilde{p}^G) - \pi^G(p_H^G) < 0$ while $\lim_{\mathcal{B}_L \to \infty}\lim_{\mathcal{B}_H \to \infty} \pi^G(\tilde{p}^G) - \pi^G(p_H^G) > 0$. Hence, for $\mathcal{B}_L$ sufficiently small, $\mathcal{B}_H$ exists such that the profit function is bimodal and the two maxima are equal.

**Existence and uniqueness of PSE.** The derivations for both thresholds assume that sellers post prices that target the high state only and hence provide a constructive proof of a PSE with the market utilities of buyers given by (16) - (17). By construction, a PSE does not exist when $\mathcal{B}_H > \bar{\mathcal{B}}^j$.

**Existence and uniqueness of MSE.** Again, we focus on the $G$-sellers without loss of generality. For mixing to be optimal, it must be the case that the sellers are indifferent between two prices: $\pi^G(\tilde{p}^G) - \pi^G(p_H^G) = 0$, or

$$(1-\mu)\left(v + V_L^G\right) + 2\left[\mu\sqrt{V_H^G v} - \sqrt{\left(\mu V_H^G + (1-\mu)V_L^G\right)v}\right] = 0.$$

The price that targets the high state only must exceed what the buyers are willing to pay in the low state $v - \sqrt{vV_H^G} > v - V_L^G \iff V_L^G > \sqrt{vV_H^G}$.

We can not rely on the derivations for $V_i^j$ from the earlier parts of the proof, as the buyers' adding up constraints now are (this follows from the analytical solution for prices and buyer-seller ratios):

$$\kappa\frac{\sqrt{\left[\mu V_H^G + (1-\mu)V_L^G\right]v}}{V_H^G} + (1-\kappa)\left(\sqrt{\frac{v}{V_H^G}}\right) = \mathcal{B}_H + 1,$$

$$\frac{\sqrt{\left[\mu V_H^G + (1-\mu)V_L^G\right]v}}{V_L^G} = \frac{\mathcal{B}_L}{\kappa} + 1. \tag{18}$$

These can be combined to give:

$$\sqrt{vV_H^G} = \frac{V_H^G\left(\mathcal{B}_H + 1\right) - V_L^G\left(\mathcal{B}_L + \kappa\right)}{1-\kappa}. \tag{19}$$

Therefore, the equal-profit condition now becomes:

$$\mu V_H^G = \frac{1-\kappa}{2\left(\mathcal{B}_H + 1\right)}\left[V_L^G\left(2\frac{\mathcal{B}_L}{\kappa} + 1 + \mu + 2\frac{\mu\left(\mathcal{B}_L + \kappa\right)}{1-\kappa}\right) - (1-\mu)v\right]. \tag{20}$$

The existence and uniqueness of MSE follows from some algebra involving equations (18), (19) and (20). After combining the equations, we arrive at

$$\mathcal{B}_H = \frac{\left[\mu\kappa(1+\mathcal{B}_L) - \sqrt{\mu}(1-\kappa)\mathcal{B}_L\right]\left[2\mathcal{B}_L + \kappa(1-\mu) + 2\sqrt{\mu}\mathcal{B}_L\right]}{(1-\mu)(\mathcal{B}_L - \kappa\sqrt{\mu})^2} - 1,$$

which defines a relationship between $\mathcal{B}_H$ and $\kappa$: for any given $\mathcal{B}_H$, the $\kappa$ that solves this equation (if any) gives the MSE mixing probability. We can show that this $\mathcal{B}_H$ decreases in $\kappa$ so a unique MSE exists, if any. Note that the limit $RHS_{\kappa=1} = \frac{\mu(1+\mathcal{B}_L)[2\mathcal{B}_L + (1-\mu) + 2\sqrt{\mu}\mathcal{B}_L]}{(1-\mu)(\mathcal{B}_L - \sqrt{\mu})^2} - 1$ defines $\bar{\mathcal{B}}_H$: a unique MSE exists for all $\mathcal{B}_H > \bar{\mathcal{B}}_H$. But $RHS_{\kappa=0} = \frac{-2\sqrt{\mu}(1+\sqrt{\mu})}{(1-\mu)} - 1 < 0$ so for any $\mathcal{B}_H > \bar{\mathcal{B}}_H$, the equilibrium mixing probability is some $\kappa > \bar{\kappa}$. The $RHS \to \infty$ if $\kappa \to \frac{\mathcal{B}_L}{\sqrt{\mu}} =: \bar{\kappa}$. Therefore, a MSE only exists for $\mathcal{B}_H > \bar{\mathcal{B}}^j$ and is unique.

**Slopes of thresholds for MSE at $\mu = 1/2$** The thresholds that delimit the region of MSE existence can be further characterised at the limiting case of an uninformative signal. We show below that the thresholds $\bar{\mathcal{B}}^j_H(\mu, \mathcal{B}_L)$ have opposite slopes at $\mu = 1/2$ unless their slope is equal to zero there.[15]

**Lemma 9.** *Let Assumptions 5 and 6 hold. Assume that all sellers price for both states. Then, the level curves of difference in profits from pricing for both states and pricing for the high state only have slopes of opposite signs at $\mu = \frac{1}{2}$ for the B and G signals in the $(\mu, \mathcal{B}_H)$ space for fixed $\mathcal{B}_L$ and $v$.*

*Proof.* The $G$-threshold for the two pricing strategies yielding equal profits is:

$$0 = (1-\mu)(v + V_L^G) + 2\sqrt{vV_H^G}\left[\mu - \sqrt{\mu + (1-\mu)\frac{\mathcal{B}_H + 1}{\mathcal{B}_L + 1}}\right],$$

as in the main text. For the $B$-sellers we obtain:

$$0 = \mu(v + V_L^B) + 2\sqrt{vV_H^B}\left[(1-\mu) - \sqrt{(1-\mu) + \mu\frac{\mathcal{B}_H + 1}{\mathcal{B}_L + 1}}\right]$$

Let's first investigate the behaviour of the profit function of $G$-sellers. Eventually, we are interested in $\frac{d\mathcal{B}_H}{d\mu}$ when the two pricing strategies yield exactly equal profits (which is how the curves $\bar{\mathcal{B}}^j_H(\mu, \mathcal{B}_L)$ are defined), but our derivations carry over for any level curve of difference in profits (from pricing for both states and pricing for high state only *when all other sellers price for both states*). Without loss of generality, let's normalize $v = 1$. The impact of $\mu$ on the maximum for the price targeting the high state only is:

$$\frac{d}{d\mu}\pi^G(p_H^{G*}) = \underbrace{\left[1 - 2\sqrt{V_H^G} + V_H^G\right]}_{>0,\text{ direct effect}} + \underbrace{\mu\frac{\partial V_H^G}{\partial \mu}\left(1 - \frac{1}{\sqrt{V_H^G}}\right)}_{>0,\text{ indirect (equilibrium) effect}}$$

When $\mu$ increases, it increases the profits attained by a deviation to pricing for high state only (when all other sellers continue to price for both states). The value of this maximum is $\mu\pi_H(p_H^{G*})$. Hence, the direct effect is $\pi_H(p_H^{G*})$ and the indirect effect captures the shift in the profit function $\pi_H$ due to increase in $\mu$ (and the shift of the corresponding locally optimal price) which operates via general equilibrium effect of higher prices which makes the buyers worse off. Formally, the profit function can also be written as $\pi^G(\mu, V_H^G, V_L^G)$ and the effect of $\mu$ is:

$$\frac{d\pi^G}{d\mu} = \frac{\partial \pi^G}{\partial \mu} + \frac{\partial \pi^G}{\partial V_H^G}\frac{\partial V_H^G}{\partial \mu} + \frac{\partial \pi^G}{\partial V_L^G}\frac{\partial V_L^G}{\partial \mu}$$

with the latter effect (via $V_L^G$) being zero at $p_H^{G*}$, but not at $\tilde{p}^G$:

[15] This happens for a single value of $\mathcal{B}_L$.

$$\frac{d}{d\mu}\pi^G(\tilde{p}^G) = \underbrace{\left(V_H^G - V_L^G\right)\left(1 - \frac{1}{\sqrt{\mu V_H^G + (1-\mu)V_L^G}}\right)}_{>0\text{, direct effect}}$$
$$+ \underbrace{\left(\mu\frac{\partial V_H^G}{\partial \mu} + (1-\mu)\frac{\partial V_L^G}{\partial \mu}\right)\left(1 - \frac{1}{\sqrt{\mu V_H^G + (1-\mu)V_L^G}}\right)}_{>0\text{, indirect (equilibrium) effect}}.$$

The signs of responses of market utilities follow from the explicit expressions for $V_i^G$ and imply that $\frac{\partial V_i^G}{\partial \mu} < 0$ (the economic intuition for this is that increases in $\mu$ lead to weakly higher prices in equilibrium when the signal is good). At $\tilde{p}^G$, the direct effect is $\pi_H(\tilde{p}^G) - \pi_L(\tilde{p}^G) < \pi_H(p_H^{G*})$. Summing up, the direct effect of increase in $\mu$ is to increase expected profits of $G$-sellers and this effect is stronger at the pricing for high state only maximum. The indirect effect (of $\mu$ via $V_i^G$) is stronger for $V_L^G$:

$$\frac{\partial V_H^G}{\partial \mu} = \frac{1}{\mathcal{B}_H + 1}\left(\frac{1}{\mathcal{B}_H + 1} - \frac{1}{\mathcal{B}_L + 1}\right),$$
$$\frac{\partial V_L^G}{\partial \mu} = \frac{1}{\mathcal{B}_L + 1}\left(\frac{1}{\mathcal{B}_H + 1} - \frac{1}{\mathcal{B}_L + 1}\right) = \frac{\mathcal{B}_H + 1}{\mathcal{B}_L + 1}\frac{\partial V_H^G}{\partial \mu}.$$

Let's now redo this exercise for $\mathcal{B}_H$:

$$\frac{d}{d\mathcal{B}_H}\pi^G(p_H^{G*}) = \mu\frac{\partial V_H^G}{\partial \mathcal{B}_H}\left(1 - \frac{1}{\sqrt{V_H^G}}\right),$$
$$\frac{d}{d\mathcal{B}_H}\pi^G(\tilde{p}^G) = \left(\mu\frac{\partial V_H^G}{\partial \mathcal{B}_H} + (1-\mu)\frac{\partial V_L^G}{\partial \mathcal{B}_H}\right)\left(1 - \frac{1}{\sqrt{\mu V_H^G + (1-\mu)V_L^G}}\right),$$

with the partial derivatives of market utilities with respect to $\mathcal{B}_H$ being:

$$\frac{\partial V_H^G}{\partial \mathcal{B}_H} = -\frac{1}{(\mathcal{B}_H + 1)^2}\left(\frac{2\mu}{\mathcal{B}_H + 1} + \frac{1-\mu}{\mathcal{B}_L + 1}\right) < 0,$$
$$\frac{\partial V_L^G}{\partial \mathcal{B}_H} = -\frac{\mu}{(\mathcal{B}_H + 1)^2(\mathcal{B}_L + 1)} < 0.$$

We now redo the same calculations for $B$-signals:

$$\frac{d}{d\mu}\pi^B(p_H^{B*}) = -\left[1 - 2\sqrt{V_H^B} + V_H^B\right] + (1-\mu)\frac{\partial V_H^B}{\partial \mu}\left(1 - \frac{1}{\sqrt{V_H^B}}\right),$$

$$\frac{d}{d\mu}\pi^B(\tilde{p}^B) = \left(V_L^B - V_H^B\right)\left(1 - \frac{1}{\sqrt{(1-\mu)V_H^B + \mu V_L^B}}\right)$$
$$+ \left((1-\mu)\frac{\partial V_H^B}{\partial \mu} + \mu\frac{\partial V_L^B}{\partial \mu}\right)\left(1 - \frac{1}{\sqrt{(1-\mu)V_H^B + \mu V_L^B}}\right),$$

$$\frac{d}{d\mathcal{B}_H}\pi^B(p_H^{B*}) = (1-\mu)\frac{\partial V_H^B}{\partial \mathcal{B}_H}\left(1 - \frac{1}{\sqrt{V_H^B}}\right),$$

$$\frac{d}{d\mathcal{B}_H}\pi^B(\tilde{p}^B) = \left((1-\mu)\frac{\partial V_H^B}{\partial \mathcal{B}_H} + \mu\frac{\partial V_L^B}{\partial \mathcal{B}_H}\right)\left(1 - \frac{1}{\sqrt{(1-\mu)V_H^B + \mu V_L^B}}\right).$$

The responses of market utilities to $\mu$ and $\mathcal{B}_H$ for the $B$ signal are:

$$\begin{aligned}
\frac{\partial V_H^B}{\partial \mu} &= \frac{1}{\mathcal{B}_H+1}\left(\frac{1}{\mathcal{B}_L+1} - \frac{1}{\mathcal{B}_H+1}\right) \\
\frac{\partial V_L^B}{\partial \mu} &= \frac{1}{\mathcal{B}_L+1}\left(\frac{1}{\mathcal{B}_L+1} - \frac{1}{\mathcal{B}_H+1}\right) = \frac{\mathcal{B}_H+1}{\mathcal{B}_L+1}\frac{\partial V_H^B}{\partial \mu} \\
\frac{\partial V_H^B}{\partial \mathcal{B}_H} &= -\frac{1}{(\mathcal{B}_H+1)^2}\left(\frac{\mu}{\mathcal{B}_L+1} + \frac{2(1-\mu)}{\mathcal{B}_H+1}\right) < 0 \\
\frac{\partial V_L^B}{\partial \mathcal{B}_H} &= -\frac{(1-\mu)}{(\mathcal{B}_H+1)^2(\mathcal{B}_L+1)} < 0
\end{aligned}$$

Let's take stock. First, the effects of $\mu$ on the market utilities of buyers differ only up to a sign with respect to signal realisation. That is, $\partial V_i^j/\partial\mu = -\partial V_i^{-j}/\partial\mu$. Next, let's consider the effect of increase in $\mu$ at $\mu = \frac{1}{2}$, so that $V_H^B = V_H^G = V_H$ and $V_L^B = V_L^G = V_L$. This implies the following:

$$\lim_{\mu\to\frac{1}{2}}\frac{d}{d\mu}\left(\pi^G(p_H^{G*}) - \pi^G(\tilde{p}^G)\right) = -\lim_{\mu\to\frac{1}{2}}\frac{d}{d\mu}\left(\pi^B(p_H^{B*}) - \pi^B(\tilde{p}^B)\right) \text{ and}$$

$$\lim_{\mu\to\frac{1}{2}}\frac{d}{d\mathcal{B}_H}\left(\pi^G(p_H^{G*}) - \pi^G(\tilde{p}^G)\right) = \lim_{\mu\to\frac{1}{2}}\frac{d}{d\mathcal{B}_H}\left(\pi^B(p_H^{B*}) - \pi^B(\tilde{p}^B)\right). \qquad \square$$

### A.7 Properties of planner entry

The FOCs read:

$$\mu\lambda(x_i^j)v_i + (1-\mu)\lambda(x_{-i}^j)v_{-i} - c - \mathcal{S}^j\left[\mu\lambda'(x_i^j)\frac{x_i^j}{\mathcal{S}^j}v_i + (1-\mu)\lambda'(x_{-i}^j)\frac{x_{-i}^j}{\mathcal{S}^j}v_{-i}\right] = 0,$$

for $(j=G, i=H)$ and $(j=B, i=L)$, which follows from the definition of $x_i^j$. Next, we remove the $\mathcal{S}^j$, divide by the corresponding $\lambda(x_i^j)$, and use $1 - \frac{\lambda'(x_i^j)x_i^j}{\lambda(x_i^j)} = 1 - (1-\phi(x_i^j))$. This leads to the conditions (12)- (13) presented in the main text (strict concavity of the planner's objective function follows directly from SOCs). We then differentiate these conditions with respect to $\mu$:

$$\frac{d\mathcal{S}_p^j}{d\mu} = \frac{\lambda(x_i^j)\phi(x_i^j)v_i - \lambda(x_{-i}^j)\phi(x_{-i}^j)v_{-i}}{\mu\left[\lambda'(x_i^j)\phi(x_i^j) + \lambda(x_i^j)\phi'(x_i^j)\right]\frac{v_i x_i^j}{\mathcal{S}^j} + (1-\mu)\left[\lambda'(x_{-i}^j)\phi(x_{-i}^j) + \lambda(x_{-i}^j)\phi'(x_{-i}^j)\right]\frac{v_{-i}x_{-i}^j}{\mathcal{S}^j}},$$

for $(j = G, i = H)$ and $(j = B, i = L)$. We find that the entry when the signal is good (bad) is increasing (decreasing) in the signal precision.

Next, we differentiate the social welfare function with respect to $\mu$ at the optimal levels of entry $\left(\mathcal{S}_p^G, \mathcal{S}_p^G\right)$ which, by the envelop theorem, reads:

$$\frac{dW(\mathcal{S}_p^G, \mathcal{S}_p^B, \Theta, \mu)}{d\mu} = \frac{1}{2}\left[\mathcal{S}_p^G \left(\lambda(x_H^G)v_H - \lambda(x_L^G)v_L\right) + \mathcal{S}_p^B \left(\lambda(x_L^B)v_L - \lambda(x_H^B)v_H\right)\right].$$

At $\mu = 1/2$, $dW(\mathcal{S}_p^G, \mathcal{S}_p^B, \Theta, \mu)/d\mu = 0$ because signal index disappears and everything cancels out. The second derivative is proportional to:

$$\left[\lambda(x_H^G)\phi(x_H^G)v_H - \lambda(x_L^G)\phi(x_L^G)v_L\right]\frac{d\mathcal{S}_p^G}{d\mu} + \left[\lambda(x_L^B)\phi(x_L^B)v_L - \lambda(x_H^B)\phi(x_H^B)v_H\right]\frac{d\mathcal{S}_p^B}{d\mu} \geq 0,$$

with equality only for $\mu = 1/2$. By continuity of the welfare function and compactness of the set $[1/2, 1]$, the highest welfare is at $\mu = 1$.

# B Online Appendix

## B.1 Figures

### B.1.1 Tightness risk

We illustrate the switch of the ranking of the $\bar{\mathcal{B}}_H^j(\mathcal{B}_L, \mu)$ thresholds on Figure 5. The difference between this case and Figure 2 is that $\mathcal{B}_L$ has now been increased. While we had $\bar{\mathcal{B}}_H^G(\mathcal{B}_L, \mu) \geq \bar{\mathcal{B}}_H^B(\mathcal{B}_L, \mu)$ on Figure 2, this ranking is flipped on Figure 5. The price competition effect now dominates the revealing effect not for the bad, but for the good signal when $\mu$ is sufficiently close to $1/2$.

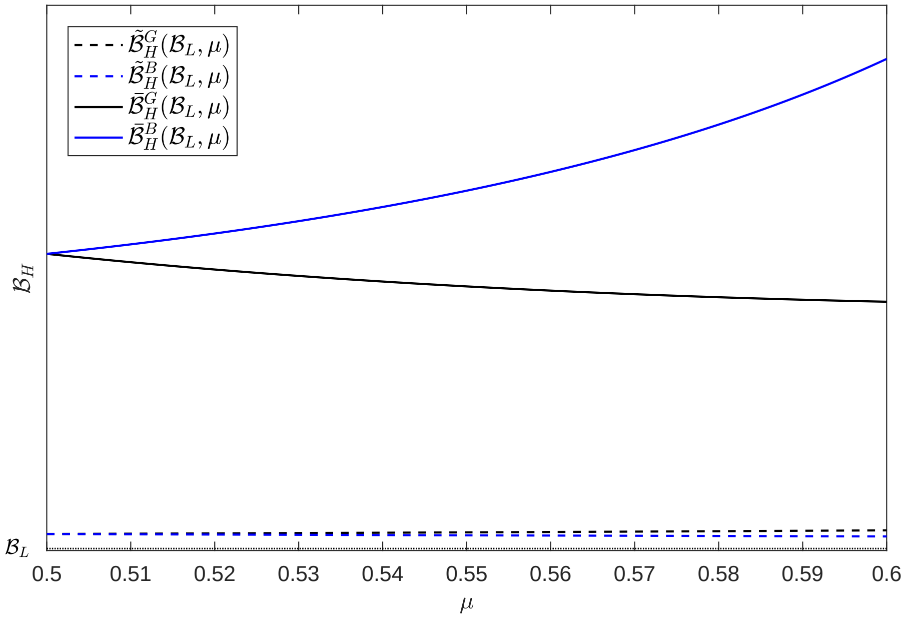


Figure 5: Illustration of Theorem 3 for $v = 1$ and $\mathcal{B}_L = 1/2$.

Next, we plot equilibrium prices that correspond to the three effects of information demonstrated on Figure 3. Prices on Figure 6 correspond to the black-dotted case on Figure 3 (pure strategy equilibrium for all signal precision values, no effect of information on the expected value of trades). Then, Figure 7 corresponds to the blue-dashed case on Figure 3 (increase in $\mu$ initially trigger an inefficient MSE). Finally, prices on Figure 8 correspond to the black-dashed case on Figure 3 (a MSE for uninformative signals, positive effect of information).

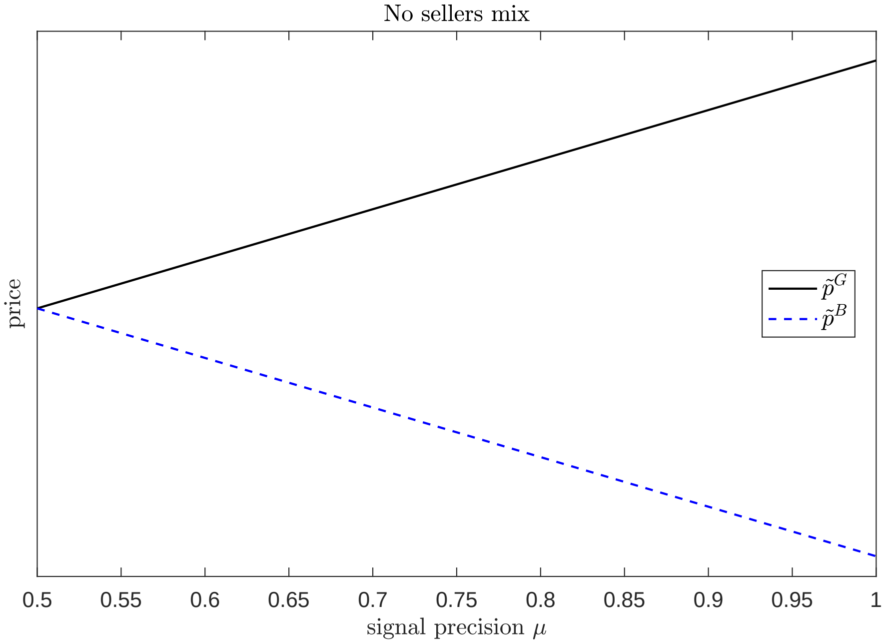


Figure 6: Equilibrium prices in the region with PSE (Figure 3).

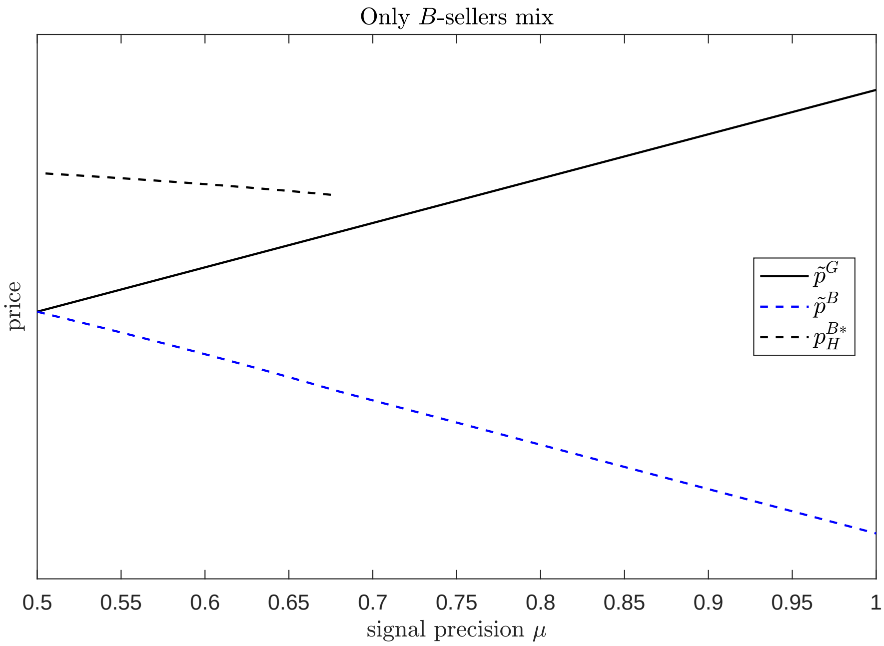


Figure 7: Equilibrium prices in the region with PSE for $G$-sellers and MSE for $B$-sellers (Figure 3).

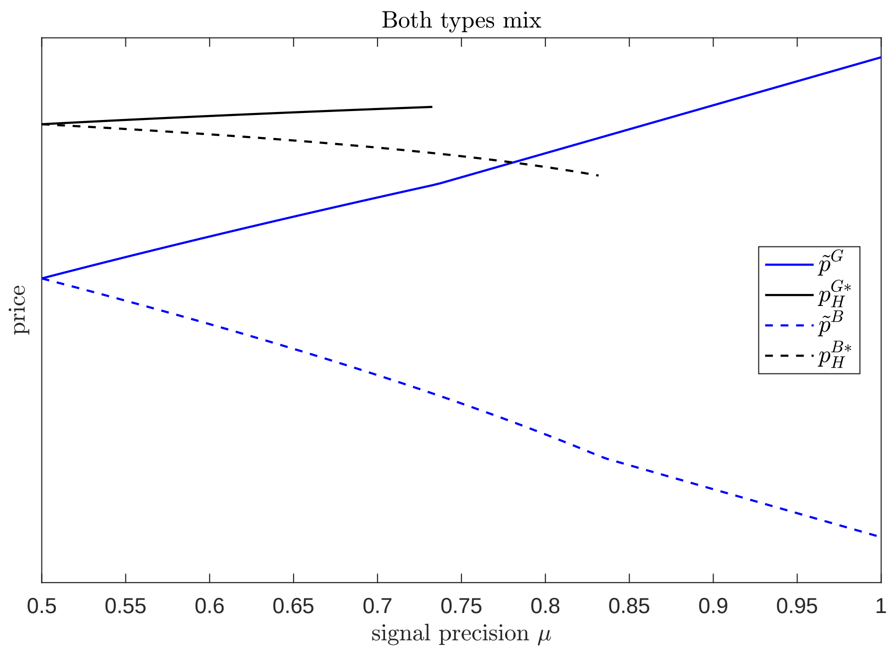


Figure 8: Equilibrium prices in the region with MSE for both types of sellers (Figure 3).

### B.1.2 Surplus risk

We numerically implement the procedure of finding thresholds that separate various type of equilibria regions of the $(v_H, \mu)$ parameter space for a given $v_L$ value. Fully analogously to the tightness risk case, we find $\tilde{v}^j_H(v_L, \mu)$ such that when all sellers price for both states, when $v_H < \tilde{v}^j_H(v_L, \mu)$ the equilibrium profits are unimodal. Next, we find $\bar{v}^j_H(v_L, \mu)$ such that for $v_H > \bar{v}^j_H(v_L, \mu)$ pricing for both states is no longer an equilibrium outcome. However, there is an additional type of threshold under surplus risk, $\hat{v}^j_H(v_L, \mu)$. When $v_H \geq \hat{v}^j_H(v_L, \mu)$ then pricing for the high state only is the unique equilibrium. Thus, under surplus risk MSE exists when $\hat{v}^j_H(v_L, \mu) > v_H > \bar{v}^j_H(v_L, \mu)$. These thresholds are illustrated on Figure 9.

Next, we again fix $v_H$ at three distinct values and investigate the effects of increasing $\mu$ on the (normalised by perfect information value) expected value of trades. We demonstrate the results of this exercise on Figure 10. Firstly, there exists a region with PSE for all signal precisions. There information is again irrelevant for welfare (the blue-dotted line). Next, we pick $v_H$ such that increases in information precision trigger a MSE and for further increases in$\mu$, also the pricing-for-high-state-only equilibrium. Information in this case has a negative effect on the expected value of trades over an interval of signal precision values (the blue-dashed line). Finally, we pick $v_H$ such that we start in the pricing-for-high-state-only equilibrium and information increases welfare (the black-dashed line).

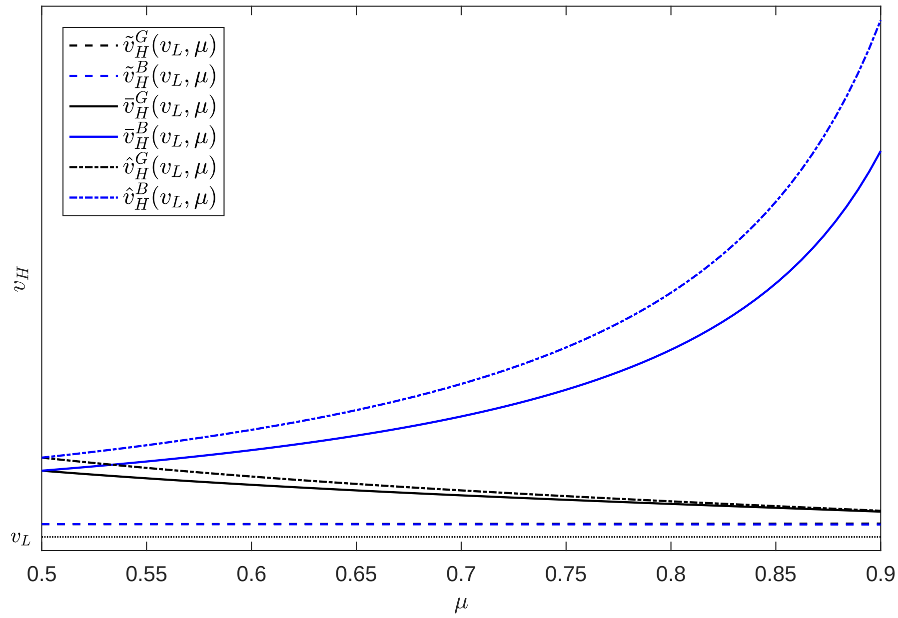


Figure 9: Thresholds for characterisation of equilibria under surplus risk

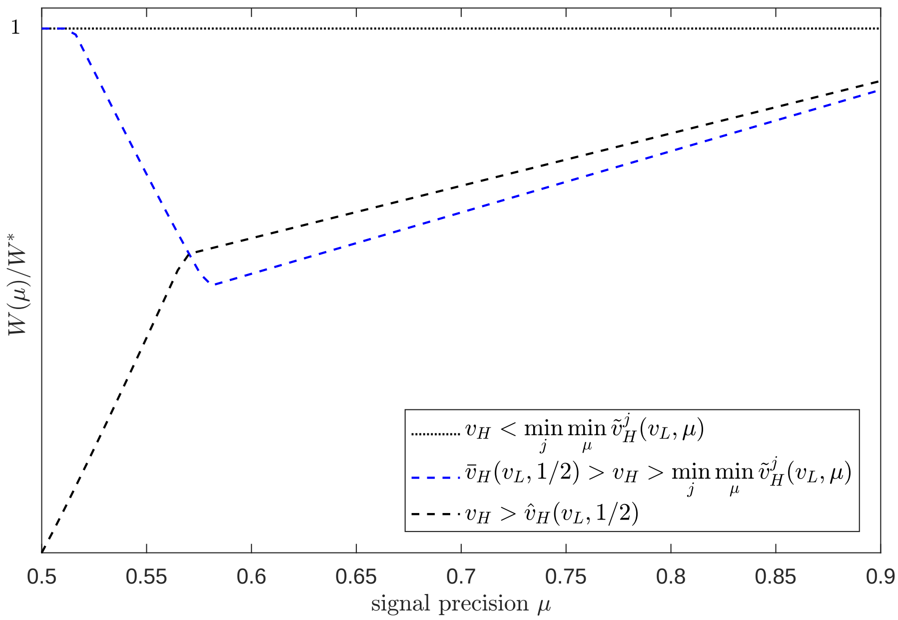


Figure 10: Normalised welfare $W(\mu)/W^*$ under surplus risk as a function of signal precision for three different values of $v_H$ with $v_L = 1$ and $\mathcal{B} = 1$.