Decomposing Identification Gains and Evaluating IVs Identification Power for Partially Identified Average Treatment Effects

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Introduction

- Causal analysis is widely used in empirical studies.
- Conventional IV estimators are often used when treatment is endogenous.
- In homogenous treatment effect models, using any valid IV can lead to point identification and correct estimation of the average treatment effect (ATE).
- However in heterogeneous TE models, different IV estimates different local ATEs (Imbens and Angrist 1994), and classical IV estimand is no longer ATE but may be a quantity with no interpretable meaning (Heckman, Urzua and Vytlacil 2006).

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- However in heterogeneous TE models, different IV estimates different local ATEs (Imbens and Angrist 1994), and classical IV estimand is no longer ATE but may be a quantity with no interpretable meaning (Heckman, Urzua and Vytlacil 2006).
- In empirical studies, it is not uncommon to arrive at very different ATE estimates when using alternative IVs, suggesting evidence against homogeneous TE.
- Once heterogeneous TE is allowed, ATE is often partially identified, so the identified sets for ATE offer a more appropriate way for estimating ATE than the conventional IV models.

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- The role of IVs in partially identified models has been discussed in the econometric literature (Manski 1990; Heckman and Vytlacil 2001; Chesher 2005, 2010; Chiburis 2010; Li, Poskitt and Zhao 2019).
- Kitagawa (2009) uses the size of the identified set to measure "identification power".

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- Kitagawa (2009) uses the size of the identified set to measure "identification power".
- However the partially identified models or the mechanism through which the IV strength translates to identification gains has not been well appreciated by empirical practitioners, despite of econometric developments (Freedman and Sekhon 2010; Mourifie and Meango 2014; Han and Vytlacil 2017).

Objectives

- This paper aims to summarise and synthesise the existing econometric results on IV and ATE bounds, and illustrate the complex role of IVs in ATE identification.
- We focus on models with binary outcome and binary endogenous treatment.
- We use the reduction in the size of the identification set as a measure for identification gain.
- We study the roles of IVs and their interplays with other factors in achieving identification gains.

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- This paper aims to summarise and synthesise the existing econometric results on IV and ATE bounds, and illustrate the complex role of IVs in ATE identification.
- We focus on models with binary outcome and binary endogenous treatment.
- We use the reduction in the size of the identification set as a measure for identification gain.
- We study the roles of IVs and their interplays with other factors in achieving identification gains.
- We use the example of Shaikh and Vytlacil (2011) bounds (SV bounds), and study its identification gain against the benchmark of Manski (1990) bounds without IVs.
- We construct a novel decomposition of the identification gains to measure contributions from instrument validity and relevance, instrument strength, and the impact of exogenous covariates.

Objectives

- We propose an index for Instrument Identification Power (or IIP) to measure the IV contribution to identification gains.
- We provide graphical illustration of bound reduction, and numerical illustration and simulation for finite sample performance of the decomposition.
- Our simulation also shed light on ranking/selecting alternative instrument sets and detection of IV relevancy in finite sample.
- Such information on IV identification power can be useful for future treatment/instrument allocation.
- Finally we present an empirical application to the study of women's childbearing and LFP (Angrist, Evans 1998), and decompose the identification gains achieved by the two IVs and the use of covariates in this example.

SV Bounds and ATE Identification

Counterfactual framework: $Y = DY_1 + (1 - D)Y_0$.

$$ATE(x) = E[Y_1 - Y_0|X = x].$$

Joint threshold crossing model Shaikh & Vytlacil (2011, Econometrica):

$$Y = 1[\nu_1(D, X) > \varepsilon_1],$$
$$D = 1[\nu_2(X, Z) > \varepsilon_2].$$

- Y outcome, D endogenous treatment, X covariates, Z IVs;

Assumption 1. Shaikh and Vytlacil [2011]

(a) $(\varepsilon_1, \varepsilon_2)'$ a strictly positive density on $\mathbb{R}^2$ with unknown $F_{\varepsilon_1,\varepsilon_2}$ CDF.

(b) (Exogeneity) $(X, Z) \perp (\varepsilon_1, \varepsilon_2)$.

(c) (Relevance) $\nu_2(X, Z)|X$ is non-degenerate.

(d) Support of (X, Z) is compact.

(e) ν_1, ν_2 are unknown functions, continuous in both arguments.

SV Bounds and ATE Identification

Conditional propensity score: $P(X,Z) = \Pr[D=1|X,Z]$.
SV bounds:

$$L^{SV}(x) = \sup_{p\in\Omega_{P|x}} \left\{\Pr[Y=1,D=1|x,p] + \sup_{x'\in\mathbf{X}_{1+}(x)} \Pr[Y=1,D=0|x',p]\right\}$$
$$- \inf_{p\in\Omega_{P|x}} \left\{\Pr[Y=1,D=0|x,p] + p \inf_{x'\in\mathbf{X}_{0+}(x)} \Pr[Y=1|x',p,1]\right\};$$

$$U^{SV}(x) = \inf_{p\in\Omega_{P|x}} \left\{\Pr[Y=1,D=1|x,p] + (1-p) \inf_{x'\in\mathbf{X}_{1-}(x)} \Pr[Y=1|x',p,0]\right\}$$
$$- \sup_{p\in\Omega_{P|x}} \left\{\Pr[Y=1,D=0|x,p] + \sup_{x'\in\mathbf{X}_{0-}(x)} \Pr[Y=1,D=1|x',p]\right\}.$$

Intersection over x' "similar" to x: $\Pr(D=1|x,z) = \Pr(D=1|x',z')$;

SV bounds: $\mathrm{ATE}(x) \in \left[L^{SV}(x), U^{SV}(x)\right]$;

SV bounds width: $\omega^{SV}(x) = U^{SV}(x) - L^{SV}(x)$.

Determinants of ATE Bounds: CPS

Conditional propensity score (CPS): $P(X,Z) = \Pr[D=1|X,Z]$.

Proposition 1 (Shaikh & Vytlacil 2011; CPS - with support condition)

Let $(\underline{p}, \overline{p}) := (\inf\{p \in \Omega_P\}, \sup\{p \in \Omega_P\})$. Under Ass. 1, if $\Omega_{X,P} = \Omega_X \times \Omega_P$, the SV bounds are sharp. In addition, for any $x \in \Omega_X$

(i) $L^{SV}(x)$ is weakly increasing as $\underline{p}$ decreases or as $\overline{p}$ increases;

(ii) $U^{SV}(x)$ is weakly decreasing as $\underline{p}$ decreases or as $\overline{p}$ increases;

(iii) $\omega^{SV}(x)$ is weakly decreasing as $\underline{p}$ decreases or as $\overline{p}$ increases.

Determinants of ATE Bounds: CPS

Proposition 2 (CPS - without support condition)

Let $(\underline{p}(x), \overline{p}(x)) := (\inf\{p \in \Omega_{P|x}\}, \sup\{p \in \Omega_{P|x}\})$. Under Ass 1, for any $x \in \Omega_X$ there exists an outer set such that $[L^{SV}(x), U^{SV}] \subseteq [\underline{L}^{SV}(x), \overline{U}^{SV}]$. Moreover,

(i) $\underline{L}^{SV}(x)$ is weakly increasing as $\underline{p}(x)$ decreases or as $\overline{p}(x)$ increases;

(ii) $\overline{U}^{SV}(x)$ is weakly decreasing as $\underline{p}(x)$ decreases or as $\overline{p}(x)$ increases;

(iii) $\overline{\omega}(x) = \overline{U}^{SV}(x) - \underline{L}^{SV}(x)$ is weakly decreasing as $\underline{p}(x)$ decreases or as $\overline{p}(x)$ increases.

- **So**: the extremes of the CPS capture IV *strength*.
- "Identification at infinity" is one special case.

Determinants of ATE Bounds (Endogeneity Degree)

Denote a single parameter copula $C(\cdot,\cdot;\rho)$ satisfying

- ρ: dependence parameter;
- **concordant ordering**: $C(F_{\varepsilon_1},F_{\varepsilon_2};\rho_1) \leq C(F_{\varepsilon_1},F_{\varepsilon_2};\rho_2)$, for any $\rho_1 < \rho_2$;
- **Unknown** $F_{\varepsilon_1}, F_{\varepsilon_2}$ and $C(\cdot,\cdot;\rho)$.

Assumption 2

$F_{\varepsilon_1,\varepsilon_2} = C(F_{\varepsilon_1},F_{\varepsilon_2};\rho)$.

Proposition 3 (Endogeneity Degree)

Under Assumptions 1 and 2,

(i) if $\text{ATE}(x) \geq 0$, then $\overline{\omega}(x)$ weakly $\uparrow$ in ρ;

(ii) if $\text{ATE}(x) < 0$, then $\overline{\omega}(x)$ weakly $\downarrow$ in ρ.

- **So**: sign and degree of endogeneity asymmetrically affect IV identification;
- IV strength and degree of endogeneity jointly determine the IV identification power.

Determinants of ATE Bounds (Covariates)

Proposition 4 (Covariates)

Under Assumption 1,

- (Chiburis 2010) for any $x \in \Omega_X$, if there is no $(x', z'), (x, z) \in \Omega_{X,Z}$ such that $P(x, z) = P(x', z')$, then $[L^{SV}(x), U^{SV}(x)] = [\underline{L}^{SV}(x), \overline{U}^{SV}(x)]$;
- if random variable $\nu_1(D, X)|D$ is degenerate, then $[L^{SV}(x), U^{SV}(x)] = [\underline{L}^{SV}(x), \overline{U}^{SV}(x)]$.

- SV bounds = outer set, if no matching or X has no impacts on Y;
- In general, **covariates support and variability** will help to improve identification.

Decomposition of Identification Gains

Benchmark: ATE bounds of Manski [1990] without IVs: "worst case scenario"

$$L^M(x) = -\Pr(Y = 1, D = 0|x) - \Pr(Y = 0, D = 1|x),$$
$$U^M(x) = \Pr(Y = 1, D = 1|x) + \Pr(Y = 0, D = 0|x),$$

- Always includes zero $\Rightarrow$ fails to identify the sign of ATE;
- Always with width one $\Rightarrow$ widest bounds width for binary outcome;
- Does not use information of IVs or covariates to gain identification.

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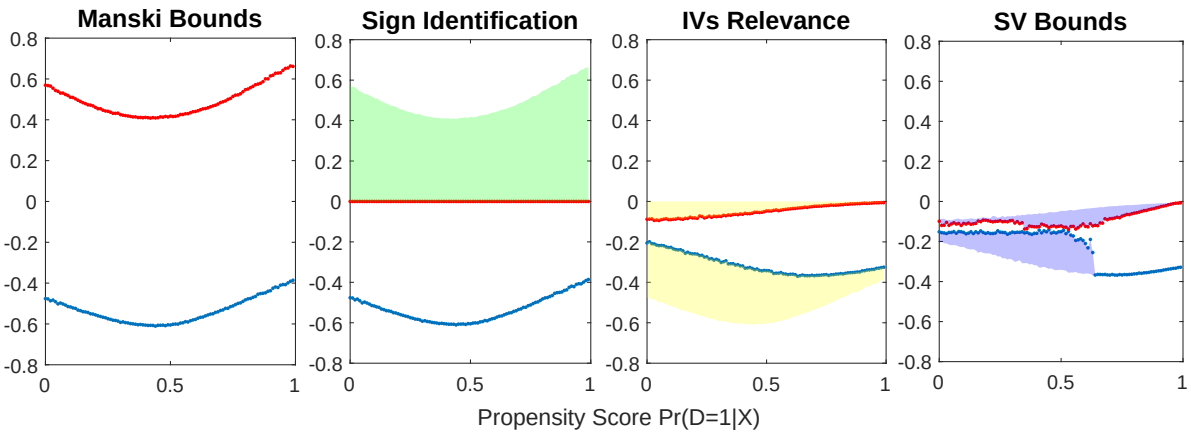
Decomposition:

Figure: LFP (Angrist, Evans (1998))



- $C_1(x)$: IV validity and relevance;
- $C_2(x)$: IV strength;
- $C_3(x)$: Covariates;
- $C_4(x)$: Unexplained;

Measure of IV Identification Power (IIP)

For $\forall x \in \Omega_X$, define the IVs identification power (IIP) as

$$IIP(x) := \begin{cases} 1 - \overline{\omega}(x) = C_1(x) + C_2(x), & \text{if } Z \text{ relevant} \\ 0, & \text{if } Z \text{ irrelevant,} \end{cases}$$

Proposition 4 (IIP)

(i) Standardized: $IIP(x) \in [0, 1]$;

(ii) $IIP(x) = 0 \Leftrightarrow Z$ is irrelevant (redundant);
$IIP(x) = 0 \Rightarrow$ SV bounds reduces to Manski bounds;

(iii) $IIP(x) = 1 \Leftarrow Z$ "identification at infinity";
$IIP(x) = 1 \Rightarrow \text{ATE}(x)$ point identified.

1. **Interpretable** at limit points, and within $[0, 1]$;
2. **Comparable** across different sets of IVs: e.g. $IIP(x) = 0.1$ v.s. $IIP(x) = 0.5$;
3. Percentage reduction of bound width from Manski bound width due to IVs alone.

Numerical Illustration: Relationship with γ,ρ,β

$$Y = 1[\alpha D + \beta X + \varepsilon_1 > 0], \quad D = 1[\gamma Z + \pi X + \varepsilon_2 > 0].$$

γ: IV strength, ρ: endogeneity, β: covariates

Figure: Determinants of ATE Bounds

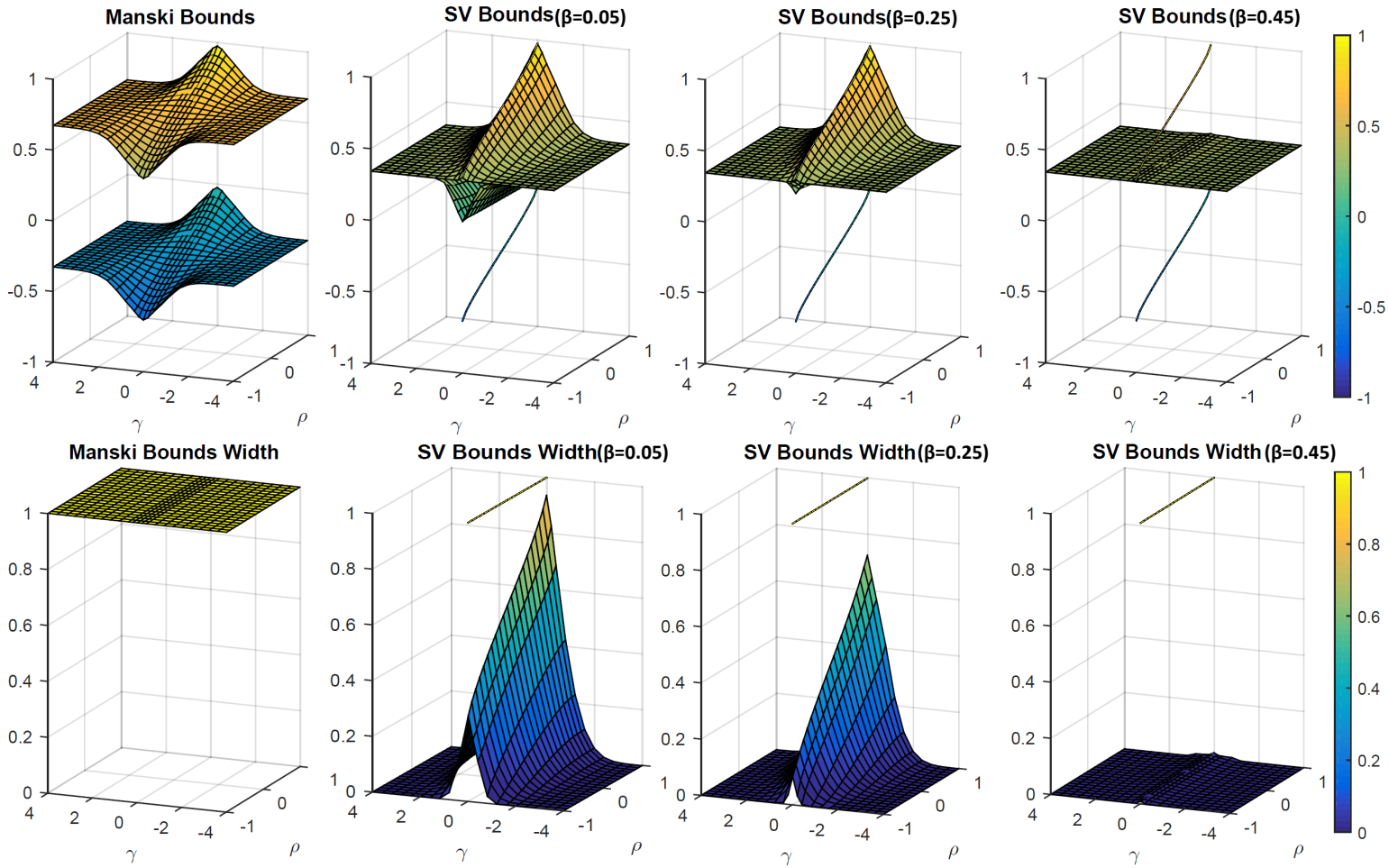
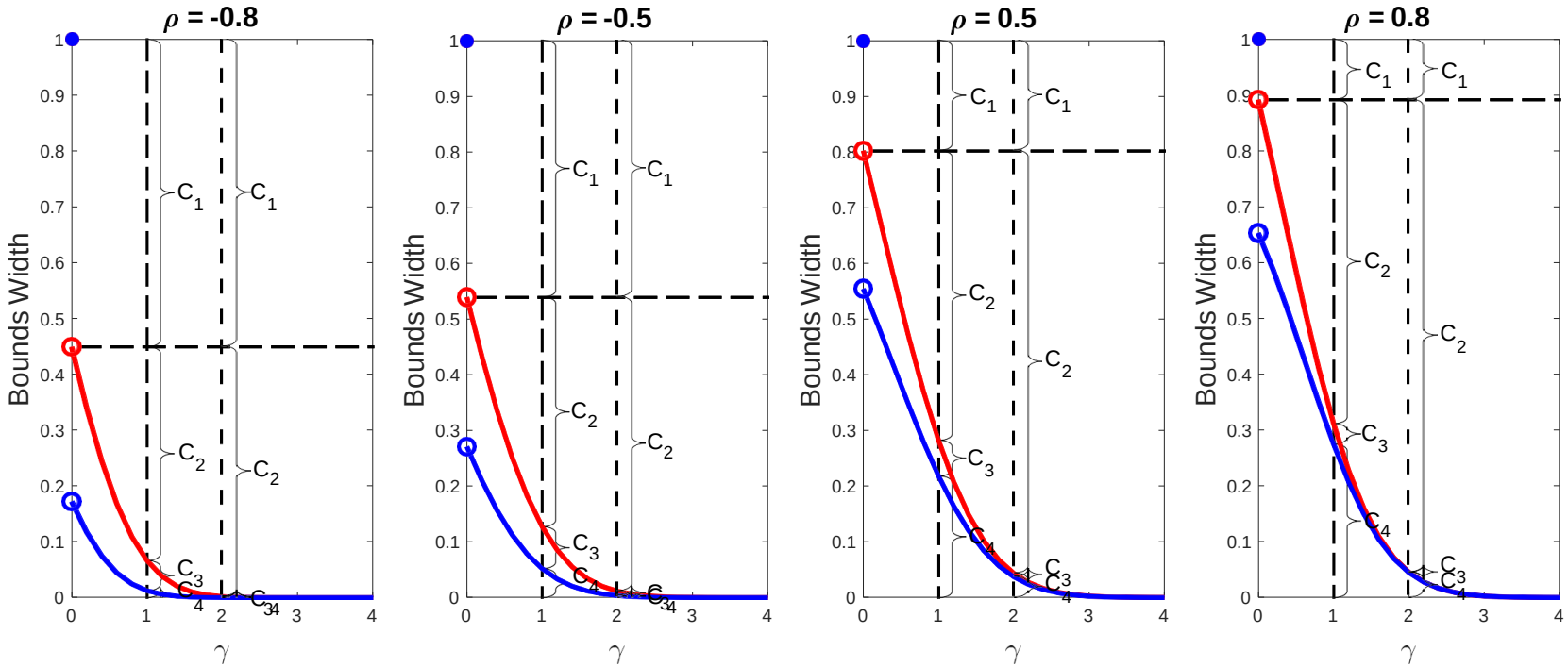


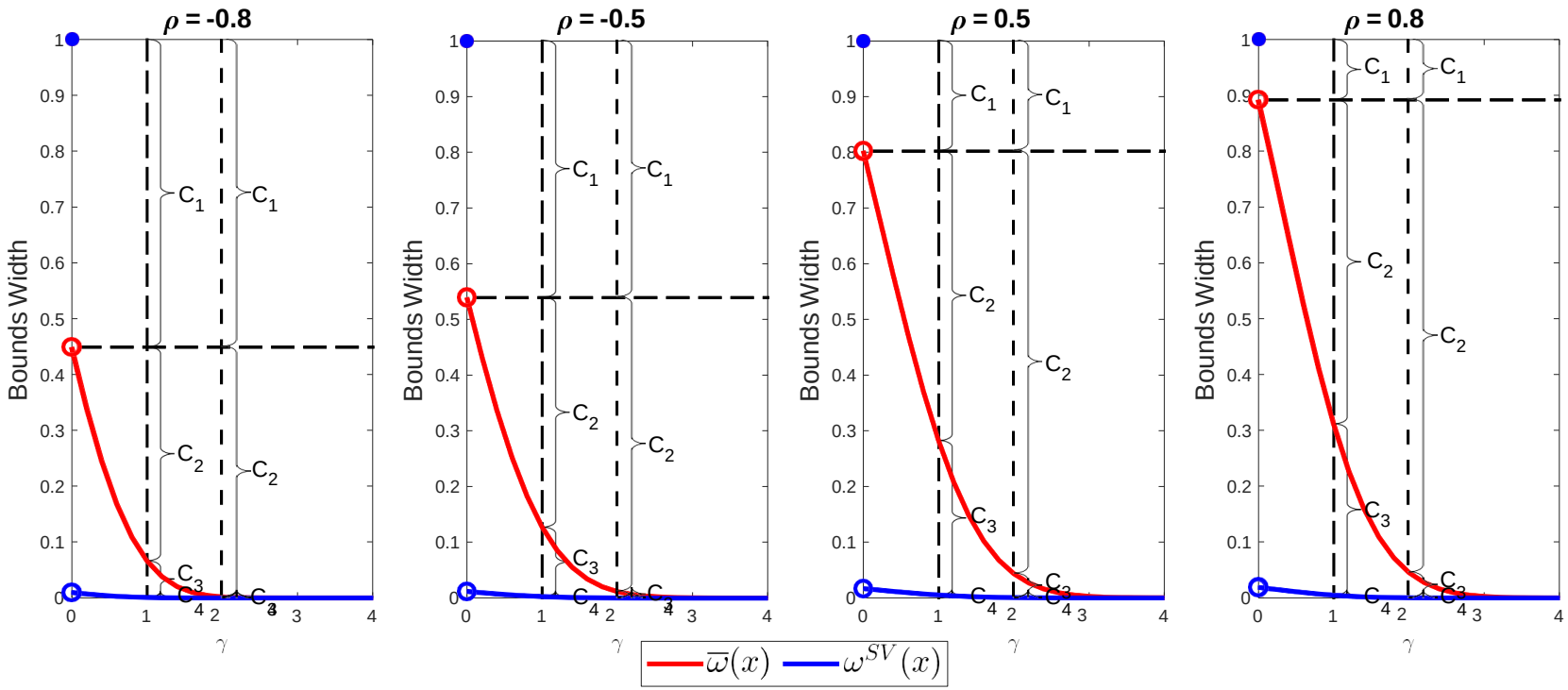
Figure: Decomposition of Identification Gains ($\beta = 0.25$)



x-axis: CPS expanding

Red: $\overline{\omega}(x)$; **Blue**: $\omega^{SV}(x)$.

Figure: Decomposition of Identification Gains ($\beta = 0.45$)



x-axis: CPS expanding

Red: $\overline{\omega}(x)$; **Blue**: $\omega^{SV}(x)$.

Finite Sample Evaluation IV Strength and Relevance

$$\text{DGP:} \quad \begin{aligned} Y &= 1[\alpha D + \beta X + \varepsilon_1 > 0], \\ D &= 1[\pi X + \gamma_1 Z_1 + \gamma_2 Z_2 + \varepsilon_2 > 0], \end{aligned}$$

- $X \sim \mathbb{N}(0,1)$, $(\varepsilon_1, \varepsilon_2)'$ joint normal with mean zero and variance one, $\rho = 0.5, 0.8$;
- $Z_1 \in \{0,1\}$, $Z_2 \in \{-3,-2,-1,0,1,2,3\}$, $\tilde{Z}_2 = 1[Z_2 > 0]$, Z_3 **irrelevant**;
- $\text{ATE}(x) = 0.341$.

Five sets of IV(s)

Table: Population Range of Conditional Propensity Score ($x = E[X]$)

				$IIP(x)$	
Sets	IVs	CPS definition	CPS Range	$\rho = 0.5$	$\rho = 0.8$
(1)	only Z_1	$\Pr(D=1\|X=x, Z_1)$	$[0.500, 0.682]$	0.305	0.232
(2)	only Z_2	$\Pr(D=1\|X=x, Z_2)$	$[0.367, 0.795]$	0.493	0.443
(3)	$Z_1, \widetilde{Z}_2$	$\Pr(D=1\|X=x, Z_1, \widetilde{Z}_2)$	$[0.410, 0.799]$	0.456	0.403
(4)	Z_1, Z_2	$\Pr(D=1\|X=x, Z_1, Z_2)$	$[0.274, 0.864]$	0.625	0.594
(5)	Z_1, Z_2, Z_3	$\Pr(D=1\|X=x, Z_1, Z_2, Z_3)$	$[0.274, 0.864]$	0.625	0.594

Table: True and Estimated Bounds ($\rho = 0.5, x = E[X]$)

		Bounds			
		Manski		SV	
		$[L^M, U^M]$	d_H	$[L^{SV}, U^{SV}]$	d_H
n	True Z_1, Z_2	$[-0.179, 0.821]$		$[0.341, 0.341]$	
$0.5k$	(1) only Z_1			[0.120,0.777]	0.436
	(2) only Z_2			[0.240,0.580]	0.244
	(3) $Z_1, \widetilde{Z}_2$	[-0.250,0.880]	0.083	[0.182,0.758]	0.417
	(4) Z_1, Z_2			[0.287,0.462]	0.128
	(5) Z_1, Z_2, Z_3			[0.296,0.452]	0.117
$5k$	(1) only Z_1			[0.121,0.769]	0.428
	(2) only Z_2			[0.263,0.374]	0.082
	(3) $Z_1, \widetilde{Z}_2$	[-0.200,0.838]	0.025	[0.220,0.757]	0.416
	(4) Z_1, Z_2			[0.310,0.378]	0.044
	(5) Z_1, Z_2, Z_3			[0.316,0.373]	0.038
$10k$	(1) only Z_1			[0.121,0.768]	0.426
	(2) only Z_2			[0.262,0.365]	0.081
	(3) $Z_1, \widetilde{Z}_2$	[-0.197,0.840]	0.022	[0.221,0.756]	0.414
	(4) Z_1, Z_2			[0.315,0.367]	0.033
	(5) Z_1, Z_2, Z_3			[0.321,0.362]	0.027

Table: Decomposition and IIP ($\rho = 0.5, x = E[X]$)

		Decomposition				
		C_1	C_2	C_3	C_4	IIP
n	True Z_1, Z_2	0.198	0.427	0.375	0.000	0.625
$0.5k$	(1) only Z_1	0.204	0.080	0.058	0.657	0.284
	(2) only Z_2	0.194	0.247	0.219	0.340	0.441
	(3) $Z_1, \widetilde{Z}_2$	0.202	0.118	0.104	0.576	0.320
	(4) Z_1, Z_2	0.196	0.335	0.294	0.175	0.531
	(5) Z_1, Z_2, Z_3	0.196	0.334	0.315	0.156	0.529
$5k$	(1) only Z_1	0.206	0.092	0.054	0.648	0.298
	(2) only Z_2	0.196	0.290	0.402	0.111	0.486
	(3) $Z_1, \widetilde{Z}_2$	0.204	0.145	0.115	0.537	0.348
	(4) Z_1, Z_2	0.198	0.401	0.333	0.068	0.599
	(5) Z_1, Z_2, Z_3	0.198	0.401	0.345	0.056	0.598
$10k$	(1) only Z_1	0.205	0.096	0.052	0.647	0.301
	(2) only Z_2	0.196	0.295	0.406	0.103	0.491
	(3) $Z_1, \widetilde{Z}_2$	0.204	0.147	0.115	0.535	0.351
	(4) Z_1, Z_2	0.198	0.408	0.341	0.053	0.606
	(5) Z_1, Z_2, Z_3	0.198	0.407	0.354	0.042	0.605

Table: True and Estimated Bounds ($\rho = 0.8, x = E[X]$)

		Bounds			
		Manski		SV	
		$[L^M, U^M]$	d_H	$[L^{SV}, U^{SV}]$	d_H
n	true Z_1, Z_2	$[-0.096, 0.904]$		$[0.341, 0.341]$	
	(1) only Z_1			[0.118,0.871]	0.530
	(2) only Z_2			[0.230,0.561]	0.230
$0.5k$	(3) $Z_1, \widetilde{Z}_2$	[-0.141,0.960]	0.066	[0.179,0.853]	0.513
	(4) Z_1, Z_2			[0.291,0.462]	0.127
	(5) Z_1, Z_2, Z_3			[0.295,0.447]	0.115
	(1) only Z_1			[0.127,0.861]	0.519
	(2) only Z_2			[0.253,0.359]	0.089
$5k$	(3) $Z_1, \widetilde{Z}_2$	[-0.117,0.925]	0.026	[0.205,0.853]	0.512
	(4) Z_1, Z_2			[0.311,0.378]	0.044
	(5) Z_1, Z_2, Z_3			[0.314,0.373]	0.039
	(1) only Z_1			[0.128,0.860]	0.519
	(2) only Z_2			[0.257,0.359]	0.084
$10k$	(3) $Z_1, \widetilde{Z}_2$	[-0.111,0.918]	0.018	[0.208,0.851]	0.510
	(4) Z_1, Z_2			[0.315,0.370]	0.035
	(5) Z_1, Z_2, Z_3			[0.318,0.365]	0.030

Table: Decomposition and IIP ($\rho = 0.8, x = E[X]$)

		Decomposition				
		C_1	C_2	C_3	C_4	IIP
n	true Z_1, Z_2	0.108	0.486	0.406	0.000	0.594
$0.5k$	(1) only Z_1	0.120	0.090	0.037	0.753	0.210
	(2) only Z_2	0.104	0.272	0.292	0.331	0.376
	(3) $Z_1, \widetilde{Z}_2$	0.117	0.133	0.077	0.674	0.249
	(4) Z_1, Z_2	0.104	0.388	0.336	0.171	0.493
	(5) Z_1, Z_2, Z_3	0.106	0.386	0.356	0.152	0.492
$5k$	(1) only Z_1	0.123	0.103	0.041	0.734	0.225
	(2) only Z_2	0.105	0.316	0.473	0.106	0.421
	(3) $Z_1, \widetilde{Z}_2$	0.119	0.166	0.067	0.648	0.285
	(4) Z_1, Z_2	0.108	0.457	0.368	0.068	0.565
	(5) Z_1, Z_2, Z_3	0.108	0.454	0.380	0.059	0.562
$10k$	(1) only Z_1	0.122	0.105	0.041	0.732	0.227
	(2) only Z_2	0.105	0.321	0.472	0.102	0.426
	(3) $Z_1, \widetilde{Z}_2$	0.119	0.170	0.067	0.643	0.290
	(4) Z_1, Z_2	0.108	0.465	0.372	0.055	0.573
	(5) Z_1, Z_2, Z_3	0.108	0.463	0.383	0.046	0.571

Empirical Example: Womens Childbearing and LFP

Women's LFP and Childbearing Angrist and Evans [1998]

- Outcome: $Y = 1$ if paid for work;
- Treatment: $D = 1$ if more than two children;
- Three alternative IV sets:
  - **Samesex**=1, if first two children are same sex;
  - **Twins**=1, if the second birth was a twin;
  - **Both**={Samesex, Twins}.

Table: Average of the Estimated Bounds

(a) IV: Samesex

	Manski	HV	Chesher	SV
HMUE	[-0.560,0.439]	[-0.537,0.402]	[-0.537,-0.011] ∪ [0.011,0.402]	[-0.537,-0.031]
95% CI	[-0.567,0.446]	[-0.547,0.412]	[-0.547,-0.003] ∪ [0.003,0.412]	[-0.548,-0.023]

(b) IV: Twins

	Manski	HV	Chesher	SV
HMUE	[-0.560,0.439]	[-0.305,0.118]	[-0.305,-0.057]	[-0.182,-0.095]
95% CI	[-0.567,0.446]	[-0.356,0.170]	[-0.356,-0.007]	[-0.276,-0.020]

(c) IV: Both={Samesex,Twins}

	Manski	HV	Chesher	SV
HMUE	[-0.560,0.439]	[-0.294,0.100]	[-0.294,-0.064]	[-0.189,-0.103]
95% CI	[-0.567,0.446]	[-0.335,0.142]	[-0.336,-0.022]	[-0.263,-0.038]

Table: Decomposition of Identification Gains and Instrument Identification Power

(a) IV: Samesex

	C_1	C_2	C_3	C_4	IIP
Based on HMUE	0.439	0.034	0.020	0.507	0.473
Based on 95% CI	0.447	0.023	0.020	0.525	0.470

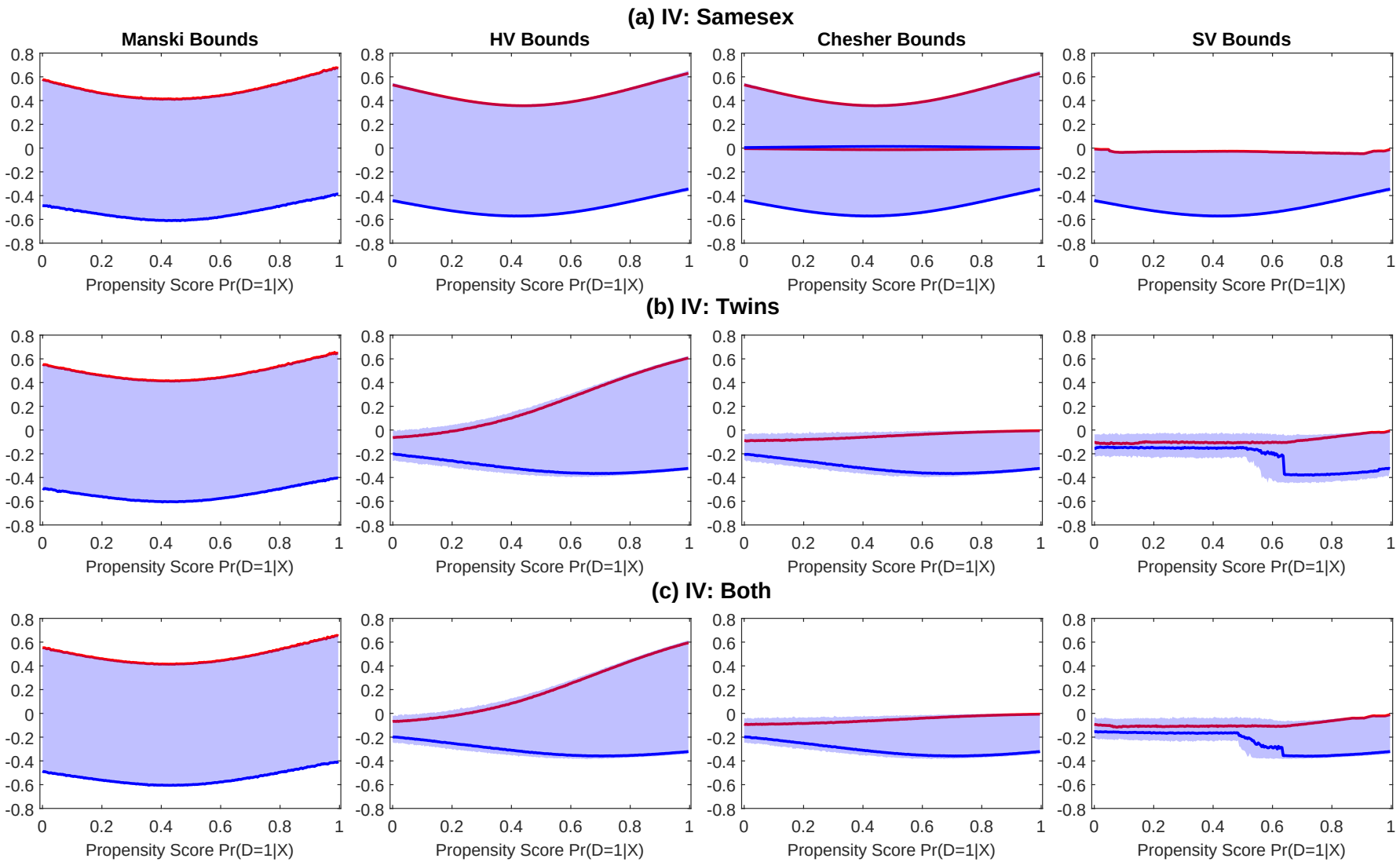
(b) IV: Twins

	C_1	C_2	C_3	C_4	IIP
Based on HMUE	0.439	0.312	0.162	0.086	0.751
Based on 95% CI	0.447	0.219	0.094	0.256	0.666

(c) IV: Both={Samesex,Twins}

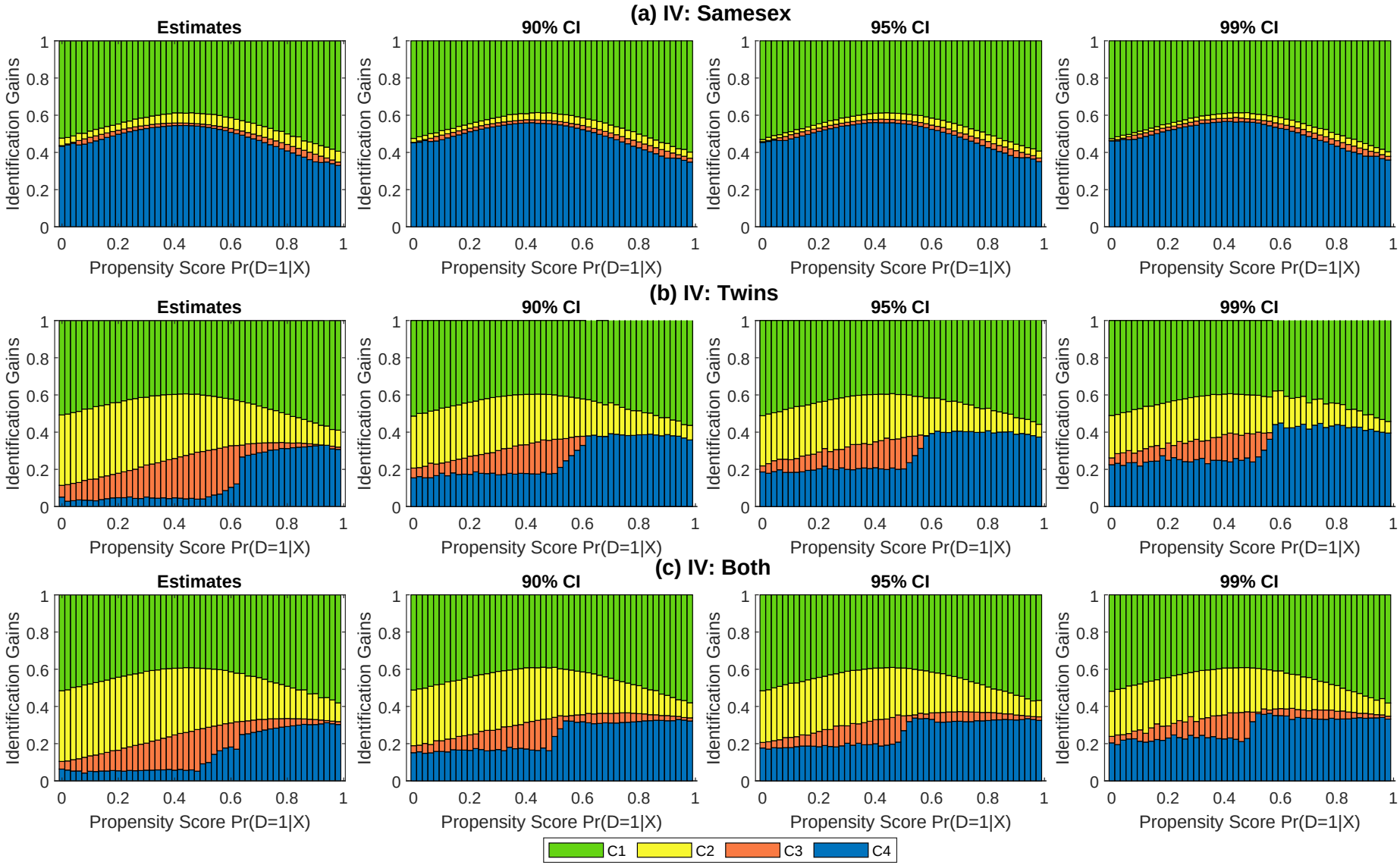
	C_1	C_2	C_3	C_4	IIP
Based on HMUE	0.439	0.330	0.144	0.086	0.769
Based on 95% CI	0.447	0.254	0.088	0.225	0.701

Figure: Estimated Bounds of ATE(x)



Angrist & Evans 2SLS: Samesex -0.123 with 95% CI [-0.178,-0.068]; Twins -0.087 with 95% CI [-0.120,-0.054].

Figure: Decomposition of Identification Gains



Summary

Summary:

- Conventional F-stat, R^2, pseudo-R^2 is not the sole arbiter of instrument usefulness;
- The proposed IIP in partially identified models quantifies the IV identification power;
- IIP also sheds new lights on IVs relevancy, IV weakness, and IV identification power comparison in empirical studies.
- The analysis in this paper is potentially useful for future instrument selection or treatment allocation.

The End! Thank You!

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