Asset Pricing with Free Entry and Exit of Firms*

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Abstract

We study the asset-pricing implications of changes in the variety of consumption goods which happens through free entry and exit of firms. Fluctuations in varieties drive a wedge between the measured and model-based (including variety growth) consumer price index making the pricing kernel as well as asset prices more volatile without driving up the volatility of consumption growth. Different from earlier endowment economy models of variety growth our model contains production which i) generates the correlations important for the explanation of the high mean and volatility of equity premium endogenously, and ii) leads to an increase of about 140 basis points in the risk-premia relative to the endowment model.

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Keywords: firm entry-exit, risk premium

*For the list of equilibrium conditions and further discussion on GHH preferences see the Online Appendix. We thank the editor and an anonymous referee for helpful comments.

We study the asset pricing implications of *free* entry-exit of firms (no entry costs) whereby a new firm is associated with a new product (variety) as in Bilbiie (2021). In the presence of fixed production costs changes in the number of active firms (extensive margin) dominate input adjustment (intensive margin) in response to shocks: firms enter (leave) the market following positive (negative) productivity shocks. On the one hand, our model is motivated by Bernard et al. (2010) who use US firm level micro data and show that product creation and destruction accounts for important shares of total production. Scanlon (2019) provides empirical evidence on the importance of changing varieties for asset prices and calibrates an endowment economy model to match asset pricing facts. Different from Scanlon (2019) our model contains production and generates the features important for good asset pricing performance, i.e., the positive correlation between variety growth and the risky return, endogenously. Further, the introduction of production adds about 140 basis points to the risk premia relative to an endowment model with variety effect. In our model firm entry and exit causes endogenous changes in productivity magnifying the effects of exogenous shocks. As a result, consumption becomes more procyclical and carries higher price of risk.

First, we take a simplified version of the endowment economy model calibrated by Scanlon (2019) to US data, and show that the variety effect induces additional volatility in the pricing kernel and contributes to the model-implied equity premium. To show this we consider a lognormal approximation to the pricing kernel and the return, and solve for the excess return analytically. In the absence of the variety effect the excess return is solely governed by the risk-aversion coefficient times the covariance of consumption growth and the asset return.

With fluctuating varieties model-based consumption growth can be decomposed into a data-consistent consumption growth term without the variety effect (statistical agencies rarely change the consumption basket), and the growth rate of varieties (firms). The covariance between the growth rate of varieties, and the return on the asset is a new positive term that appears due to variety growth and, thus, raises the excess return on the asset. Further, variety effect implies higher uncertainty and stronger precautionary savings effect keeping the risk-free rate low.

Second, we contribute to the macro-finance literature by introducing production with fixed costs into the variety effect model, and, find that it generates higher excess returns relative to the endowment economy model. In the model with production positive (negative) productivity shock implies firm entry (exit), and, leads to an endogenous change in productivity, and a rise in uncertainty which is reflected in the higher mean and volatility of the excess return on the consumption claim. More elastic labour supply can further raise consumption risk as long as labour is procyclical.

1 The model

First, we describe the non-linear free entry-exit model of Bilbiie (2021). Firm level variables are denoted with small-case letters. Capital letters denote aggregate variables. The representative household consumes a continuum of goods defined over the measure N_t: $C_t = \left(\int_0^{N_t} c_t(\omega)^{\frac{\theta-1}{\theta}} d\omega\right)^{\frac{\theta}{\theta-1}}$ where the elasticity of substitution among goods is constant (CES) and given by $\theta > 1$. The net markup (market power) is given by $1/(\theta - 1)$. The household derives utility from consumption (C_t) and leisure ($1 - L_t$). To choose C_t and L_t optimally it maximises lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, 1 - L_t)$$

with respect to its budget constraint:

$$B_t + P_t C_t = R_{t-1} B_{t-1} + W_t L_t$$

where R_t is gross nominal interest (also the policy rate) on gross nominal one-period risk-free bonds (B_t). The household has labour income ($W_t L_t$) and interest earned on bonds from the previous period ($R_{t-1} B_{t-1}$). Profit income is zero due to free entry condition.

The period utility U is assumed to be non-separable between consumption and labour as in Greenwood et al. (1988):

$$U(C_t, L_t) = \frac{\left(C_t - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)^{1-\sigma}}{1-\sigma}. \tag{1}$$

In expression (1) σ is related to the intertemporal elasticity of substitution (IES) as $IES = \frac{\sigma^{-1}}{1+\varphi}$, where φ is the Frisch elasticity of labour supply to wages and $\chi > 0$ is a parameter that pins down hours worked in steady-state. The utility maximisation of the household leads to the bond Euler and intratemporal conditions, respectively:

$$1 = E_t \left[\tilde{M}_{t,t+1} \frac{R_t}{\Pi^C_{t+1}} \right],$$

$$\frac{W_t}{P_t} = \chi L_t^{1/\varphi},$$

where $\tilde{M}_{t,t+1} \equiv \beta \left[\left(C_{t+1} - \chi \frac{L_{t+1}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right) / \left(C_t - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right) \right]^{-\sigma}$ is the stochastic discount factor and $\Pi^C_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is consumer price inflation.

Firms. In our setup final goods producers bundle intermediary goods. The cost-minimisation problem of the final goods-producer leads to:

$$y_t(\omega) = \left(\frac{p_t(\omega)}{P_t} \right)^{-\theta} (C_t + PAC_t),$$

where the nominal price of a particular good ω is denoted as $p_t(\omega)$. The aggregation of individual goods results in the consumer price index: $P_t = \left(\int_0^{N_t} p_t(\omega)^{1-\theta} d\omega \right)^{\frac{1}{1-\theta}}$. The price adjustment costs, $PAC_t \equiv N_t pac_t(\omega)$, are defined below.

Producer prices ($p_t(\omega)$) and the consumer price index (P_t) differ due to variety growth in the former:

$$\rho_t(\omega) \equiv \frac{p_t(\omega)}{P_t} = N_t^{\frac{1}{\theta-1}}. \tag{2}$$

The production function of an intermediary firm producing product ω is given by:

$$y_t(\omega) = \begin{cases} Z_t l_t(\omega) - f & \text{if } Z_t l_t(\omega) - f > 0 \\ 0, \text{otherwise} \end{cases} \tag{3}$$

where f is a fixed cost denominated in labour units. Z_t is temporary technology shock:

$$\log Z_t = \rho_z \log Z_{t-1} + \sigma_Z \varepsilon_t.$$

where ε_t represents i.i.d. innovations with scaling σ_Z.

Intermediary firm ω maximises real profits of the form:

$$d_t(\omega) = \rho_t(\omega) y_t(\omega) - \frac{W_t}{P_t} l_t(\omega) - pac_t(\omega)$$

where pac_t are Rotemberg price adjustment costs in real terms:

$$pac_t(\omega) = \frac{\phi}{2} \left[\frac{p_t(\omega)}{p_{t-1}(\omega)} - 1 \right]^2 \rho_t(\omega) y_t(\omega).$$

Optimal choice of the firm leads to the pricing condition (ω is dropped due to symmetric choices):

$$\rho_t = \mu_t MC_t,$$

where $MC_t = \frac{W_t/P_t}{Z_t}$ is the real marginal cost. μ_t denotes the gross markup *function*:

$$\mu_t \equiv \frac{\theta}{(\theta - 1)\left(1 - \frac{\phi}{2}\pi_t^2\right) + \phi\pi_t(1 + \pi_t) - \beta E_t \left\{\Upsilon_{t+1}\right\}}$$
$$\text{with } \Upsilon_{t+1} \equiv \tilde{M}_{t,t+1} \phi \pi_{t+1} \frac{Y_{t+1}}{Y_t} \frac{N_t}{N_{t+1}} (1 + \pi_{t+1})$$

where $\pi_t \equiv \log(p_t/p_{t-1}) = \log(\Pi_t)$. In the zero inflation steady-state ($\pi = 0$) $\mu = \theta/(\theta - 1)$.

The connection between producer price (Π_t) and consumer price (Π_t^C) inflation comes from the variety effect:

$$\frac{\Pi_t}{\Pi_t^C} = \left(\frac{N_t}{N_{t-1}} \right)^{\frac{1}{\theta - 1}}. \quad (4)$$

The aggregate production function takes the form of

$$Y_t = N_t^{\frac{\theta}{\theta - 1}} \left(\frac{Z_t L_t}{N_t} - f \right).$$

The aggregate market clearing implies:

$$Y_t(1 - PAC_t) = C_t = \frac{W_t}{P_t} L_t \quad (5)$$

Monetary policy is described by a simple interest rate rule:

$$R_t = \frac{1}{\beta}(1 + \pi_t)^{\phi_\pi}.$$

2 Endowment economy

We consider an endowment economy version of our model (without labour and with flexible prices) and solve for the excess return on the consumption claim analytically by assuming that the stochastic discount factor and the returns are jointly conditionally log-normally distributed as in Scanlon (2019).

The consumption claim is priced as follows:

$$S_t^C = \sum_{i=0}^{\infty} E_t \left[M_{t,t+i} C_{t+i} \right] \tag{6}$$

where S_t^C is the price of the consumption claim. In the absence of labour the stochastic discount factor in equation (6) is given by $M_{t,t+1} \equiv \beta \left(C_{t+1}/C_t \right)^{-\sigma}$.

Equation (6) can, alternatively, be written in recursive form as:

$$S_t^C = C_t + E_t \left[M_{t,t+1} S_{t+1}^C \right].$$

which can also be displayed in Euler equation form:

$$\begin{aligned} 1 &= E_t \left[M_{t,t+1} \frac{S_{t+1}^C}{S_t^C - C_t} \right] \\ &= E_t \left[M_{t,t+1} R_{C,t+1} \right] \end{aligned}$$

where $R_{C,t+1} \equiv \frac{S_{t+1}^C}{S_t^C - C_t}$ is the return on the consumption claim.

Next we transform variables to data-consistent format (i.e. we divide each variable with ρ_t to remove the variety effect as in Bilbiie et al. (2012)) and rewrite Euler equation as:

$$\begin{aligned} 1 &= E_t \left[\beta \left(\frac{C_{t+1}/\rho_{t+1}}{C_t/\rho_t} \right)^{-\sigma} \left(\frac{\rho_{t+1}}{\rho_t} \right)^{1-\sigma} \frac{S_{t+1}^C/\rho_{t+1}}{S_t^C/\rho_t - C_t/\rho_t} \right] \\ &= E_t \left[\beta \left(\frac{C_{t+1}^R}{C_t^R} \right)^{-\sigma} \left(\frac{N_{t+1}}{N_t} \right)^{\frac{1-\sigma}{\theta-1}} \frac{S_{t+1}^{C,R}}{S_t^{C,R} - C_t^R} \right] \\ &= E_t \left[M_{t,t+1}^R R_{C,t+1}^R \right] \end{aligned}$$

where $C_t^R \equiv C_t/\rho_t$ and $S_t^{C,R} \equiv S_t^C/\rho_t$ are data-consistent variables. The second row made use of equation (2). Hence, the data-consistent stochastic discount factor and return are given, respectively, by:

$$M^R_{t,t+1} \equiv \beta \left(\frac{C^R_{t+1}}{C^R_t}\right)^{-\sigma} \underbrace{\left(\frac{N_{t+1}}{N_t}\right)^{\frac{1-\sigma}{\theta-1}}}_{\text{new term}},$$

$$R^R_{C,t+1} \equiv \frac{S^{C,R}_{t+1}}{S^{C,R}_t - C^R_t}.$$

Note the 'new term' appearing due to firm entry-exit in $M^R_{t,t+1}$.

Second we take logs of the Euler equation and apply the log-normality assumption to obtain:

$$E_t(m_{t+1})+E_t(r_{c,t+1})+\frac{1}{2}\left(var_t(m_{t+1}) + var_t(r_{c,t+1}) + 2cov_t(m_{t+1}, r_{c,t+1})\right) = 0, \tag{7}$$

where $m_{t+1} \equiv \ln(M^R_{t,t+1})$ and $r_{c,t+1} = \ln(R^R_{C,t+1})$. $var_t(m_{t+1})$ is the conditional variance of the natural log stochastic discount factor, $var_t(r_{c,t+1})$ is the conditional variance of the return on the consumption claim, $cov_t(m_{t+1}, r_{c,t+1})$ is the conditional covariance between the stochastic discount factor and the return on the consumption claim.

Using the log-normality assumption the risk-free rate can be also derived as $r_{f,t} \equiv -E_t m_{t+1} - (1/2)var_t(m_{t+1})$. Using the log version of equation (7) one can rewrite the risk-free rate as:

$$\begin{aligned} r_{f,t} = & -\ln(\beta) + \sigma E_t \Delta c_{t+1} + \underbrace{\frac{\sigma-1}{\theta-1} E_t \Delta n_{t+1}}_{\text{new term due to variety effect}} \\ & - \frac{1}{2}\left[\sigma^2 var_t(\Delta c_{t+1}) + \underbrace{\left(\frac{\sigma-1}{\theta-1}\right)^2 var_t(\Delta n_{t+1}) + 2\sigma\left(\frac{\sigma-1}{\theta-1}\right) cov_t(\Delta c_{t+1}, \Delta n_{t+1})}_{\text{new terms due to variety effect}}\right] \end{aligned} \tag{8}$$

where $\Delta c_{t+1} \equiv \ln(C^R_{t+1}/C^R_t)$ and $\Delta n_{t+1} \equiv \ln(N_{t+1}/N_t)$. In the absence of uncertainty in the steady-state we have $r_{f,t} = r_{c,t+1} = -\ln(\beta)$. In the no-entry model precautionary savings decrease the risk-free due to uncertainty about consumption growth (see the $-(1/2)\sigma^2 var_t(\Delta c_{t+1})$ term in equation (8)). $cov_t(\Delta c_{t+1}, \Delta n_{t+1}) > 0$ and $\sigma > 1$ imply procyclical variety, higher uncertainty and stronger precautionary savings effects (see the additional terms in the squared bracket of equation (8)) keeping the risk-free rate low.

Hence, the excess return on the consumption claim can be written as (the difference between (7) expressed for $E_t(r_{c,t+1})$ and equation (8)):

$$\begin{aligned} E_t(r_{c,t+1}) - r_{f,t} + \frac{1}{2}var_t(r_{c,t+1}) &= -cov_t(m_{t+1}, r_{c,t+1}) \\ &= \sigma cov_t(\Delta c_{t+1}, r_{c,t+1}) + \underbrace{\frac{\sigma-1}{\theta-1}cov_t(\Delta n_{t+1}, r_{c,t+1})}_{\text{new term due to variety effect}} \end{aligned}$$

where $\frac{1}{2}var_t(r_{c,t+1})$ is the usual Jensen's inequality term of lognormal approximations. The covariance terms can be decomposed into correlations and standard deviations which are set to the values in Scanlon (2019) and reasonably close to our data moments in Table (2). σ and θ are given by Table (1). The excess return is levered as in Croce (2014), and is given in annualised percentage by

$$\begin{aligned} EQPR_t &\equiv (1+\phi_{lev})\left[E_t(r_{c,t+1}) - r_{f,t} + \frac{1}{2}var_t(r_{c,t+1})\right] \qquad (9) \\ &= \underbrace{(1+\phi_{lev})}_{1.67}\underbrace{\sigma}_{3}\underbrace{corr_t(\Delta c_{t+1}, r_{c,t+1})}_{0.4} * \underbrace{std_t(\Delta c_{t+1})}_{0.022} * \underbrace{std_t(r_{c,t+1})}_{0.2} * 100 \\ &+ \underbrace{(1+\phi_{lev})}_{1.67}\underbrace{\frac{\sigma-1}{\theta-1}}_{0.7143}\underbrace{corr_t(\Delta n_{t+1}, r_{c,t+1})}_{0.5} * \underbrace{std_t(\Delta n_{t+1})}_{0.022} * \underbrace{std_t(r_{c,t+1})}_{0.2} * 100 \\ &= 0.89\% + 0.27\% = 1.16\% \end{aligned}$$

Thus, the equity premium is 1.16 percent with 89 basis points coming from no variety model and an additional 27 basis points due to the variety effect.[1] Next we show that the excess return can be higher in the model with production.

3 Results from the model with production

In this section we solve the production model numerically using a second-order perturbation. Table (1) summerises model parametrisation. To highlight the mechanisms in the entry-exit model we study the propagation of a

[1] Unlike our model Scanlon (2019) also allows for quality improvement in varieties which would further raise the excess return by 27 basis points.

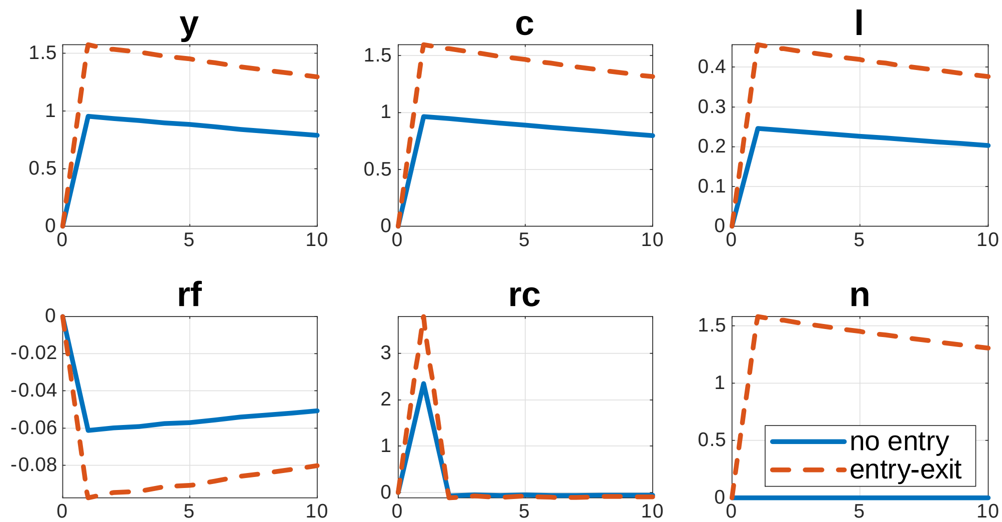

Figure 1: Impulse responses to a positive technology shock for the entry-exit (dashed) and no entry (solid) models. Notes: vertical axes show percentage deviation from steady-state. The risk-free rate (rf) and the return on the consumption claim (rc) are both annualised.

Table 1: Parametrisation

σ	3	φ	0.4	$\bar{L}$	1/3	σ_Z	0.0072	ϕ	77
β	0.99	ϕ_π	1.5	θ	3.80	ρ_Z	0.979	ϕ_{lev}	0.67

Notes: The choice of σ implies moderate amount of risk-aversion. ρ_Z, σ_Z, and θ are from Bilbiie et al. (2012). φ is chosen to match the standard deviation of consumption in the data. Price adjustment costs are chosen to imply a moderate degree of price rigidity ($\phi = 77$). The excess return on the consumption claim is levered by 2/3 as in Croce (2014).

positive technology shock which triggers firm entry due to profit opportunities (see figure (1)). Firms (n) enter until profits are driven to zero in line with the zero profit condition. The positive wealth effect of the productivity boosts consumption (c), output (y), labour (l) and also lead to a procyclical return (rc). Precautionary savings effect is stronger in the firm entry model and leading to larger drop in the risk-free rate (rf) relative to the no entry model.

Table (2) contains empirical and simulated moments. The first column contains data moments and columns A-B refer to simulated moments. Macroeconomic data and simulated model moments are both HP-filtered. In the top block we report the mean and standard deviation of the levered excess return on the consumption claim, the associated Sharpe ratio[2] as well as mean and standard deviation of the risk-free rate. The bottom part contains key macroeconomic moments such as the standard deviation of output, consumption and labour as well as relative standard deviations and correlation with output.

Columns A-B contain simulated moments from two parametrisations of the noentry (N) and entry (E) models. The column title indicates the implied IES of the particular parametrisation. Column A shows results with the baseline parametrisation. Column B displays the case of inelastic labour (setting Frisch elasticity close to zero) to illustrate the importance of labour elasticity in shaping our results.

The entry and exit model matches around half of the mean of the equity premium (143 basis points more than in the endowment model) while the no

[2]The Sharpe ratio is defined as the ratio of the mean and the standard deviation of the excess return on consumption.

	Data	A—baseline (IES=0.22) Entry-exit	No entry	B—zero Frisch (IES=0.33) Entry-exit	No entry
E(EQPR)	4.89	2.59	0.97	1.09	0.55
$\sigma(EQPR)$	17.92	26.26	16.18	17.02	12.28
Sharpe ratio	0.27	0.10	0.06	0.07	0.05
E(rf)	2.90	3.75	3.98	3.96	4.04
$\sigma(rf)$	3.00	0.55	0.34	0.35	0.25
$\sigma(y)$	1.81	1.36	1.27	0.89	0.99
$\sigma(c)$	1.35	1.37	1.28	0.89	0.99
$\sigma(l)$	1.79	0.60	0.33	–	–
$\sigma(c)/\sigma(y)$	0.74	1.00	1.00	1.00	1.00
$\sigma(l)/\sigma(y)$	0.99	0.44	0.26	–	–
$corr(y,c)$	0.88	0.99	1.00	1.00	1.00
$corr(y,l)$	0.88	1.00	1.00	–	–

Table 2: Data moments are from Bilbiie et al. (2012) and Donadelli and Grüning (2016). $E(x)$, $\sigma(x)$, and $corr(x,y)$ refer to unconditional mean and standard deviation of variable (x), as well as correlation of variables (x,y), respectively. Moments are obtained from models simulated for 10000 periods. Time is in quarters. Each moment is expressed in percentage form. Interest rates and excess returns are annualised.

entry model accounts for less than 1/5 of it (see column A). The entry and exit model generates excess volatility in equity returns but this is not coupled with excess volatility in the risk-free rate or consumption. The fact that even the noentry model performs relatively good in terms of finance moments is due to GHH preferences, which eliminate the negative wealth effect of the technology shock on labour supply and make labour highly procyclical. With inelastic labour (column B) variables are less procyclical, households dislike fluctuations in consumption less (higher IES), and asset returns carry lower risk-premia.

4 Conclusion

We have shown that expanding product variety that is coupled with free entry of firms in a production economy can produce high and volatile stock returns with smooth consumption. Future research could consider the introduction of capital which could help raise the standard deviations of output and labour and generate more realistic relative volatilities.

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