

# Misallocation under Heterogeneous Markups and Non-Constant Returns to Scale

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- How does equalizing the marginal revenue of capital and labor affect the labor share in developing countries?
- Recent studies suggest mixing evidence about correlations between markups and market shares (Edmond et al. (2019), Han et al. (2022), Kondziella (2022))
- How to introduce heterogeneous markups without imposing an ex ante correlation between markups and market shares?

## Key ideas

- We use nested CES with unobserved latent nests and idiosyncratic cost shocks to introduce varying markups.

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- We estimate industry-specific production elasticities, firm-specific distortions, nests-specific demand elasticities.

## Merits of our model and related literature

- can accommodate any pattern of firm-level markups (Atkeson and Burstein (2008), Haltiwanger et al. (2018), Peters (2020), Ruzic and Ho (2021), Liang (2021), and Gupta (2021))
- no need for a benchmark country to infer production elasticities (Hsieh and Klenow (2009), Ruzic and Ho (2021))
- calculate gains from reallocating resources both within and across industries (Hsieh and Klenow (2009), Ruzic and Ho (2021))
- easy to implement

## Main results

Applying this method to 2005 Chinese firm-level data, we find:

- the variation in markups does not affect predicted TFP gains, but it lowers the predicted labor income share by one-third;
- predicted TFP gains are sensitive to primitives. Not using estimated demand elasticities and production elasticities can cause predicted TFP gains to reach as high as 360%;
- our predicted TFP gains are 44% which is half of the previous findings;



## More literature

**Misallocation:** Restuccia and Rogerson (2008) and Hsieh and Klenow (2009).

**Labor share and markups:** Autor et al. (2020).

**Model misspecification and estimated distortions:** Haltiwanger et al. (2018).

**Sources of TFPR variation:** Haltiwanger et al. (2018), David and Venkateswaran (2019), and Bilal et al. (2020).

**Demand:** nested CES (Atkeson and Burstein (2008)) with unobserved demand-elasticities types within an industry nest.

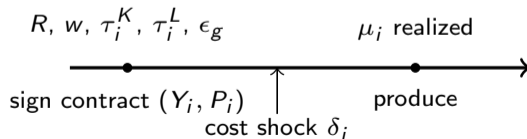
**Supply:** Cobb-Douglas production  $Y_i = A_i K_i^{\alpha_s^K} L_i^{\alpha_s^L}$  with

- industry-specific production elasticities,
- firm-specific productivity and cost shocks,
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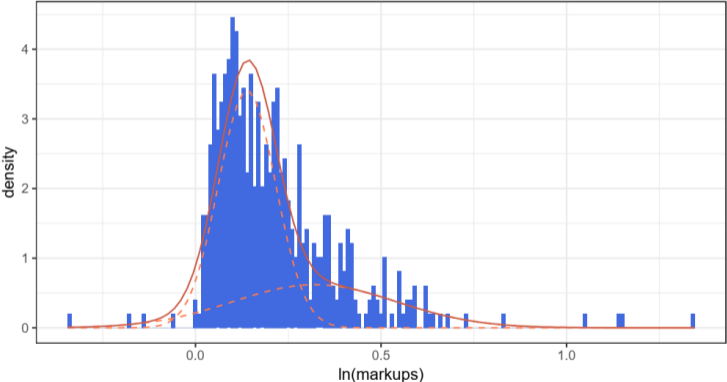
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# Markups distribution

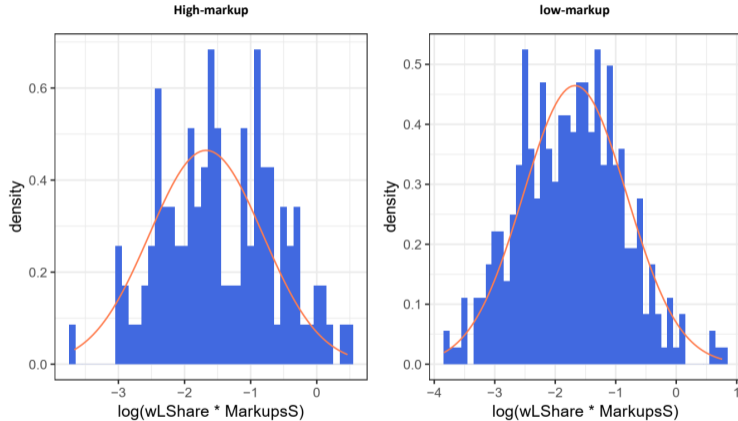
Tea refining industry



formula

# Labor share distribution

## Tea refining industry



$$\log(\text{labor share}_i * \text{Expected markups}_s) = \log(\alpha_s^L) - \log(1 + \tau_i^L)$$

## Identification: steps

Step 1: calculate markups using observed cost and sales.

Step 2: estimate group identity within each industry and group-level demand elasticities. Distribution parameters of cost shocks,  $\sigma_g$ , are also estimated.

Step 3: estimate industry-specific production elasticities,  $\alpha_s^K$ ,  $\alpha_s^L$ , firm-specific distortions  $\tau_i^K$  and  $\tau_i^L$ , and the distribution parameters of  $\tau_i^K$  and  $\tau_i^L$ .

## Chinese Annual Firm-Level Survey Data (2005) from NBS.

Statistic	N	Mean	Min	Pctl(25)	Median	Pctl(75)	Max
value added	229,241	13,814.46	122	2,517	5,377	13,250	277,908
K	229,241	16,366.41	83.76	1,620.23	4,211.66	12,151.88	515,954.20
wL	229,241	2,730.73	80	583	1,188	2,665	78,956
revenue	229,241	50,184.74	2	9,500	19,457	45,994	11,041,153
cost	229,241	43,075.61	1	7,935	16,481	39,072	10,757,115
profits	229,241	2,370.47	-292,087	72	480	1,815	415,879
revenue/cost	229,241	1.21	0.81	1.08	1.14	1.25	4.68
wL/value added	229,241	0.32	0.01	0.12	0.23	0.42	3.15

## Results: estimated parameters

two types	N	Mean	Min	Pctl(25)	Median	Pctl(75)	Max
No	61	23	2	6	15	27	237
Yes	462	494	12	118	256	544.500	9,947

	N	Mean	St. Dev.	Pctl(10)	Pctl(25)	Median	Pctl(75)	Pctl(90)
$\mathbb{E}_g[\mu_i + 1]$	985	1.30	0.25	1.11	1.14	1.22	1.39	1.57
$\sigma_g$	985	6.33	3.64	2.77	3.59	5.45	8.32	10.48
$\mathbb{E}_g[e_i^\delta]$	985	1.01	0.02	1	1	1.01	1.02	1.03
ex-ante $P_g[\bar{s}]$	928	0.66	0.22	0.27	0.59	0.73	0.82	0.88
$\alpha_K$	523	0.16	0.17	0.04	0.06	0.09	0.19	0.36
$\alpha_L$	523	0.39	0.23	0.13	0.21	0.33	0.57	0.76
scale	523	0.55	0.31	0.22	0.32	0.48	0.75	0.95



## Sizes and markups

	Dependent variable			
	$\ln(\mathbb{E}_g[\mu_i + 1])$ full sample (1)	$\ln(\mu_i + 1)$ (2)	$\ln(\mathbb{E}_g[\mu_i + 1])$ no SOEs (3)	$\ln(\mu_i + 1)$ (4)
ln(sales)	-0.003*** (0.0001)	-0.005*** (0.0003)	-0.002*** (0.0001)	-0.004*** (0.0003)
Constant	0.005*** (0.0002)	0.009*** (0.0004)	0.004*** (0.0002)	0.008*** (0.0004)
Observations	229,410	229,410	217,835	217,835
R <sup>2</sup>	0.001	0.001	0.001	0.001
Adjusted R <sup>2</sup>	0.001	0.001	0.001	0.001
Residual Std. Error	0.078 (df = 229408)	0.153 (df = 229408)	0.076 (df = 217833)	0.148 (df = 217833)
F Statistic	318.612*** (df = 1; 229408)	259.458*** (df = 1; 229408)	194.828*** (df = 1; 217833)	165.349*** (df = 1; 217833)

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

All the variables are in relative values, i.e. they are normalized by industry-type averages.

$\ln(\mathbb{E}_g[\mu_i + 1])$  is expected markups in log.  $\ln(\mu_i + 1)$  is the log of revenue-cost ratio.

## Sizes and markups: smaller industries

	Dependent variable			
	$\ln(\mathbb{E}_g[\mu_i + 1])$ small industries (1)	$\ln(\mu_i + 1)$ (2)	$\ln(\mathbb{E}_g[\mu_i + 1])$ small industries and no SOEs (3)	$\ln(\mu_i + 1)$ (4)
ln(sales)	-0.008*** (0.003)	-0.023*** (0.004)	-0.007*** (0.003)	-0.022*** (0.005)
Constant	0.127*** (0.004)	0.180*** (0.006)	0.128*** (0.004)	0.180*** (0.007)
Observations	2,652	2,652	2,397	2,397
R <sup>2</sup>	0.004	0.011	0.003	0.009
Adjusted R <sup>2</sup>	0.004	0.010	0.002	0.008
Residual Std. Error	0.158 (df = 2650)	0.273 (df = 2650)	0.155 (df = 2395)	0.269 (df = 2395)
F Statistic	10.858*** (df = 1; 2650)	28.277*** (df = 1; 2650)	6.836*** (df = 1; 2395)	21.050*** (df = 1; 2395)

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

All the variables are in relative values, i.e. they are normalized by industry-type averages.

$\ln(\mathbb{E}_g[\mu_i + 1])$  is expected markups in log.  $\ln(\mu_i + 1)$  is the log of revenue-cost ratio.

## Results: TFP gains (%) within industries

Figure: TFP gains in China (2005)

within industry (%)	across industry (%)	total (%)
43.9	4.7	50.6

Figure: Labor and capital income share (%)

(a) Heterogeneous markups

(b) Homogeneous markups=8.5

	observed	predicted	change
L	19.76	27.2	7.44
K	11.86	10.77	-1.09
L+K	31.62	37.97	6.35

	observed	predicted	change
L	20.72	32.15	11.44
K	12.85	12.98	0.13
L+K	33.56	45.13	11.57

## Results: labor and capital reallocation

Figure: Changes in type-level labor and capital

Statistic	Mean	Min	Pctl(25)	Median	Pctl(75)	Max
$\frac{l^*}{l}$	1.03	0.08	0.62	0.86	1.32	6.91
$\frac{k^*}{k}$	1.12	0.05	0.51	0.82	1.33	10.49

Figure: Estimated distortions for different firm types

	firm type	N	Mean	Min	Pctl(25)	Median	Pctl(75)	Max
$\tau_i^K$	domestic priv	164396	1.36	-0.99	-0.50	0.08	1.41	305.22
	SOE	10600	0.41	-1.00	-0.73	-0.38	0.33	147.12
	all	174996	1.31	-1.00	-0.52	0.05	1.34	305.22
$\tau_i^L$	domestic priv	164396	0.94	-0.98	-0.35	0.16	1.18	54.49
	SOE	10600	0.33	-0.99	-0.53	-0.13	0.54	26.08
	all	174996	0.91	-0.99	-0.36	0.13	1.13	54.49

## Importance of using estimated parameters

Figure: Within-type TFP gains in China (2005) comparison across models

Data	$\alpha$	$\sigma$	TFP gains (%)
HK	calibrated using US firms (HK)	3	86.6
HK	calibrated using US firms (HK)	8.5	362.3
HK	calibrated using US firms (HK)	heterogeneous (one type)	298.6
HK	Our estimators	3	51.5
HK	Our estimators	8.5	63.8
HK	Our estimators	heterogeneous (one type)	59.2
Our	Our estimators	8.5	46.3
Our	Our estimators	heterogeneous (two types)	43.9

# Robustness and possibility of alternative model choices

Reduced-form analysis: RF main RF L

Markups:

- compare to estimates from other papers, compare
- our estimates are close to those in the reduced form analysis when using L,
- robustness under possible corrections. Bias correction and robustness

$\alpha_L$ : robustness under possible corrections. Bias correction and robustness

A model with intangible asset.

## Conclusion

- We develop a flexible framework that can account for an arbitrary pattern of firm-level heterogeneous markups, variation of demand elasticities within industries, non-constant returns to scale (at least with regard to K and L).
- Our framework does not require using a benchmark economy to calibrate production elasticities.
- We find predicted TFP gains for 2005 Chinese firms to be 44%, about half of the previous findings.
- While the variation in markups does not affect predicted TFP gains, it lowers the predicted labor income share by one-third.
- We show that not using estimated demand elasticities and production elasticities can cause predicted TFP gains to reach as high as 360%.
- Assuming ex-ante a positive correlation between firms sizes and markups among Chinese firms may be inappropriate.

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*Thank you!*

## Model: demand

A representative consumer's utility:

$$y = \prod_{s=1}^S Y_s^{\beta_s}$$

where

$$Y_s = Y_{\bar{s}}^{\gamma_s} Y_{\underline{s}}^{1-\gamma_s}$$

and

$$Y_g = \left( \sum_{i \in g} Y_{ig}^{\frac{\epsilon_g - 1}{\epsilon_g}} \right)^{\frac{\epsilon_g}{\epsilon_g - 1}}, \text{ where } g \in \{\bar{s}, \underline{s}\}$$

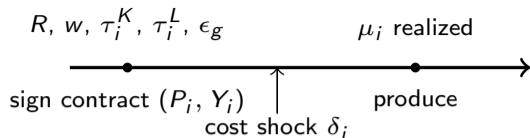
$Y_s$  is industry  $s$ 's production.  $Y_{\bar{s}}$  and  $Y_{\underline{s}}$  are groups inside industry  $s$ . Groups inside an industry differ by their products' demand elasticities,  $\epsilon_{\bar{s}} > \epsilon_{\underline{s}}$ .  $Y_{ig}$  is the production of firm  $i$  in group  $g$ . [More info](#)

## Model: firms

Firms inside industry  $s$  share the same labor and capital intensity,  $\alpha_s^L$  and  $\alpha_s^K$  but face two possible value of demand elasticities  $\epsilon_{\bar{s}}$  and  $\epsilon_{\underline{s}}$ .

Firms' production function:

$$Y_i = A_i K_i^{\alpha_s^K} L_i^{\alpha_s^L}$$



The ex-ante expected profits:

$$\mathbb{E}[\Pi_i] = P_i Y_i - (R(1 + \tau_i^K)K_i + w(1 + \tau_i^L)L_i)\mathbb{E}[e^{\delta_i}]$$

The ex-post profits:

$$\Pi_i = P_i Y_i - (R(1 + \tau_i^K)K_i + w(1 + \tau_i^L)L_i)e^{\delta_i}$$

Pricing rule:

$$P_i = \frac{\epsilon_g}{\epsilon_g - 1} C_i$$

Realized markups:

$$\mu_i = \frac{\epsilon_g}{\epsilon_g - 1} \frac{\mathbb{E}[e^{\delta_i}]}{e^{\delta_{ig}}} - 1$$

## Model: general EQ

General EQ: given the exogenous aggregate capital  $K$ , aggregate labor  $L$ , and  $\{A_i, \tau_i^K, \tau_i^L, \alpha_S^K, \alpha_S^L, \delta_i, \epsilon_g\}_{i,g,s}$ , there exist  $\{P_i, K_i, L_i, w, R\}_{i \in I}$  such that:

- firms' profits maximization conditions and the representative consumer's utility maximization conditions are met;
- firms' production decisions ensure goods market clears because firms set prices;
- wage  $w$  and capital rental rate  $R$  clear labor market and capital market.

$I$  denotes the set of all the firms in the economy.

Aggregate capital and labor is assumed to be fixed.

## Model: TFP gains

Aggregate TFP gains are:

$$\frac{y^*}{y} = \prod_g \underbrace{\left[ \frac{\text{TFP}_g^*}{\text{TFP}_g} \right]^{\beta_g}}_{\text{gains within Ind}} \cdot \underbrace{\left[ \left( \frac{L_g^*}{L_g} \right)^{\alpha_g^L} \left( \frac{K_g^*}{K_g} \right)^{\alpha_g^K} \right]^{\beta_g}}_{\text{gains across Ind}}$$

$\beta_g = \beta_s \gamma_s$  if  $g = \bar{s}$ ;  $\beta_g = \beta_s(1 - \gamma_s)$  if  $g = \underline{s}$ .  $\alpha_{\bar{s}}^f = \alpha_{\underline{s}}^f = \alpha_s^f$  for  $f \in \{K, L\}$ .  $\text{TFP}_g$ ,  $g \in \{\bar{s}, \underline{s}\}$ , is the group-level TFP, defined as:

$$\text{TFP}_g \equiv \frac{Y_g}{K_g^{\alpha_g^K} L_g^{\alpha_g^L}}$$

$\text{TFP}_g^*$ ,  $g \in \{\bar{s}, \underline{s}\}$ , is the industry-level TFP when  $\tau_i^K = \tau_i^L = 0$ .  $L_g^*$  and  $K_g^*$  are capital and labor used in group  $g$  when  $\tau_i^K = \tau_i^L = 0$ . [More info](#)

## Distributions of exogenous variables

Cost shock are i.i.d:

$$\delta_{ig} \sim \mathcal{N}(0, \sigma_g)$$

$\tau_i^K$  and  $\tau_i^L$  are i.i.d within industry and independent across industries:

$$\log(\tau_i^K + 1) \sim \begin{cases} 2\kappa^K \mathcal{N}(0, \sigma_-^K) , & \text{if } \tau_i^K < 0 \\ (2 - 2\kappa^K) \mathcal{N}(0, \sigma_+^K) , & \text{if } \tau_i^K > 0 \end{cases}$$

$$\log(\tau_i^L + 1) \sim \begin{cases} 2\kappa^L \mathcal{N}(0, \sigma_-^L) , & \text{if } \tau_i^L < 0 \\ (2 - 2\kappa^L) \mathcal{N}(0, \sigma_+^L) , & \text{if } \tau_i^L > 0 \end{cases}$$

$\tau_i^f > 0$  for  $f \in K, L$ : firms hire capital or labor at a price higher than market price.

$\tau_i^f < 0$  for  $f \in K, L$ : firms hire at a price lower than market price.

$\kappa^K, \kappa^L, \sigma_+^K, \sigma_-^K, \sigma_+^L,$  and  $\sigma_-^L$  are distribution parameters. [Return](#)

## Step 2: groups and demand elasticities

Realized markups:

$$\mu_{ig} = \frac{\epsilon_g}{\epsilon_g - 1} \frac{\mathbb{E}[e^{\delta_i}]}{e^{\delta_{ig}}} - 1$$

Distribution of markups observed in an industry is a mixture of two distributions, one from the group with higher demand elasticities,  $\epsilon_{\bar{S}}$ , and one from the group with lower demand elasticities,  $\epsilon_{\underline{S}}$ .

$$\log(\mu_i + 1) \sim w_S \cdot \underbrace{\mathcal{N}\left(\log \frac{\epsilon_{\underline{S}} e^{\sigma_{\epsilon_{\underline{S}}}^2/2}}{\epsilon_{\underline{S}} - 1}, \sigma_{\epsilon_{\underline{S}}}\right)}_{\text{low-demand-elasticities group}} + (1 - w_S) \cdot \underbrace{\mathcal{N}\left(\log \frac{\epsilon_{\bar{S}} e^{\sigma_{\epsilon_{\bar{S}}}^2/2}}{\epsilon_{\bar{S}} - 1}, \sigma_{\epsilon_{\bar{S}}}\right)}_{\text{high-demand-elasticities group}}$$

First test whether there exists more than one distribution. Then use the EM algorithm to estimate  $w_S$ ,  $\epsilon_{\bar{S}}$ ,  $\epsilon_{\underline{S}}$ ,  $\sigma_{\epsilon_{\bar{S}}}$ , and  $\sigma_{\epsilon_{\underline{S}}}$ .



## Step 1: infer firm-level markups

We use observed sale-cost ratio to calculate firm-level markups and then correct the possible bias caused by non-constant returns to scale.

We do not use the method developed by De Loecker and Warzynski (2012) because:

- Estimators obtained from production function estimators are biased when replacing physical production by revenue production and when markups are not random or constant.
- We do not observe a credible variable input such as electricity usage.

Correcting the bias in sale-cost ratio is easier.

## Discussion of possible bias: production elasticities

We may underestimate  $\alpha_L$  as the reported labor expenditure does not include non-wage labor expenditure. The reduced form analysis show  $\alpha_L$  is on average 0.45 while our structural estimator is on average 0.32.

Bias correction and robustness

## Estimated markups in literature

- cost-weighted average markups: ours (1.15), Edmond et al. (2019) (1.15), Baqaee and Farhi (2020) (De Loecker and Warzynski (2012)'s method 1.15), De Loecker and Warzynski (2012) (1.10-1.28), Peters (2020) (1.12).
- sales-weighted average markups: ours (1.17), De Loecker et al. (2020) (1.20 for 1980 and 1.60 for 2012).
- median markups: ours (1.24), Feenstra and Weinstein (2017) (1.30).

Return

## Discussion of possible bias: returns to scale and demand elasticities

Both our reduced form analysis and structural model find on average decreasing returns to scale. Are estimated demand elasticities biased downward?

Probably no, because our model are consistent with those found in existing literature.

- cost-weighted average markups: ours (1.15), Edmond et al. (2019) (1.15), Baqaee and Farhi (2020) (De Loecker and Warzynski (2012)'s method 1.15), De Loecker and Warzynski (2012) (1.10-1.28), Peters (2020) (1.12).
- sales-weighted average markups: ours (1.17), De Loecker et al. (2020) (1.20 for 1980 and 1.60 for 2012).
- median markups: ours (1.24), Feenstra and Weinstein (2017) (1.30).

and because our estimates are very close to those in the reduced form analysis.

## Model: demand

The representative consumer's utility maximization gives:

$$\frac{P_{ig} Y_{ig}}{P_g Y_g} = \frac{P_{ig}^{1-\epsilon_g}}{\sum_{i \in g} P_{ig}^{1-\epsilon_g}} = \left( \frac{P_{ig}}{P_g} \right)^{1-\epsilon_g}$$

$$\frac{P_g Y_g}{P_s Y_s} = \gamma_g$$

$$\frac{P_s Y_s}{y \mathcal{P}} = \beta_s$$

where  $P_g = \left( \sum_{i \in g} P_{ig}^{1-\epsilon_g} \right)^{1/(1-\epsilon_g)}$ ,  $P_s = \left( \frac{P_{\bar{s}}}{\gamma_{\bar{s}}} \cdot \frac{P_{\underline{s}}}{\gamma_{\underline{s}}} \right)^{1/2}$ ,  $\mathcal{P} = \prod_{s=1}^S \left( \frac{P_s}{\beta_s} \right)^{1/S}$ ,  $\gamma_{\bar{s}} = \gamma_s$ , and

$\gamma_{\underline{s}} = 1 - \gamma_s$ . [Return](#)

## Model: firms' profit maximization

The expected marginal cost:

$$C(Y_i) = \left(\frac{1}{A_i}\right)^{\frac{1}{\alpha_s^L + \alpha_s^K}} Y_i^{\frac{1 - \alpha_s^L - \alpha_s^K}{\alpha_s^L + \alpha_s^K}} \left(\frac{R(1 + \tau_i^K)}{\alpha_s^K}\right)^{\frac{\alpha_s^K}{\alpha_s^L + \alpha_s^K}} \left(\frac{w(1 + \tau_i^L)}{\alpha_s^L}\right)^{\frac{\alpha_s^L}{\alpha_s^L + \alpha_s^K}} \mathbb{E}[e^{\delta_i}]$$

Pricing rule:

$$P_i = \frac{\epsilon_g}{\epsilon_g - 1} C_i$$

Realized marginal cost is:

$$MC_i = \left(\frac{1}{A_i}\right)^{\frac{1}{\alpha_s^L + \alpha_s^K}} Y_i^{\frac{1 - \alpha_s^L - \alpha_s^K}{\alpha_s^L + \alpha_s^K}} \left(\frac{R(1 + \tau_i^K)}{\alpha_s^K}\right)^{\frac{\alpha_s^K}{\alpha_s^L + \alpha_s^K}} \left(\frac{w(1 + \tau_i^L)}{\alpha_s^L}\right)^{\frac{\alpha_s^L}{\alpha_s^L + \alpha_s^K}} e^{\delta_i}$$

Realized markups:

$$\mu_{ig} = \frac{\epsilon_g}{\epsilon_g - 1} \frac{\mathbb{E}[e^{\delta_i}]}{e^{\delta_{ig}}} - 1$$

Labor-capital ratio:

$$\frac{L_i}{K_i} = \frac{\alpha_s^L R(1 + \tau_i^K)}{\alpha_s^K w(1 + \tau_i^L)}$$

Labor expenditure share:

$$\frac{wL_i\mathbb{E}[e^{\delta_i}]}{P_i Y_i(\epsilon_g - 1)/\epsilon_g} = \frac{\alpha_s^L}{1 + \tau_i^L}$$

Capital expenditure share:

$$\frac{RK_i\mathbb{E}[e^{\delta_i}]}{P_i Y_i(\epsilon_g - 1)/\epsilon_g} = \frac{\alpha_s^K}{1 + \tau_i^K}$$

Return

## Model: TFP gains within Industries

I follow the method in Hsieh and Klenow (2009) to calculate TFP gains within industries.

$$\text{TFP}_g = \left( \sum_{i \in g} \left( A_i \cdot \frac{\text{TFPR}_g}{\text{TFPR}_i} \right)^{\epsilon_g - 1} \right)^{\frac{1}{\epsilon_g - 1}}$$

where

$$A_i \equiv \frac{Y_i}{K_i^{\alpha_K^g} L_i^{\alpha_L^g}} \propto \frac{(P_{ig} Y_{ig})^{\epsilon_g / (\epsilon_g - 1)}}{K_{ig}^{\alpha_K^g} (wL_{ig})^{\alpha_L^g}}, \quad \text{TFPR}_{ig} \equiv \frac{P_{ig} Y_{ig}}{K_{ig}^{\alpha_K^g} L_{ig}^{\alpha_L^g}}$$

$$\frac{\text{TFPR}_i}{\text{TFPR}_g} = \underbrace{(1 + \tau_i^K)^{\alpha_s^K} (1 + \tau_i^L)^{\alpha_s^L} \left( \sum_{i \in g} \frac{P_i Y_i}{P_g Y_g (1 + \tau_i^K)} \right)^{\alpha_s^K} \left( \sum_{i \in g} \frac{P_i Y_i}{P_g Y_g (1 + \tau_i^L)} \right)^{\alpha_s^L}}_{\text{Same as CRS}} \cdot \left( \frac{P_i Y_i}{P_g Y_g} \right)^{1 - \alpha_s^K - \alpha_s^L}$$



$$\text{TFP}_g^* = \left( \sum_{i \in g} \left( A_i \cdot \left( \frac{P_i^* Y_i^*}{P_g^* Y_g^*} \right)^{1 - \alpha_K - \alpha_L} \right)^{\epsilon_g - 1} \right)^{\frac{1}{\epsilon_g - 1}}$$

where

$$\frac{P_i^* Y_i^*}{P_g^* Y_g^*} = \frac{\sum_{i \in g} A_i^{\frac{\epsilon_g - 1}{\epsilon_g + (\epsilon_g - 1)(\alpha_L + \alpha_K)}}}{A_i^{\frac{\epsilon_g - 1}{\epsilon_g + (\epsilon_g - 1)(\alpha_L + \alpha_K)}}$$

The value of R and w affect within-industry TFP gains only via estimated  $\alpha_K$  and  $\alpha_L$ !

## Model: TFP gains across industries

$$\frac{L_g^*}{L_g} = \frac{w^* L_g^* / (w^* L)}{w L_g / (w L)}$$
$$\frac{K_g^*}{K_g} = \frac{K_g^* / K}{K_g / K}$$

$w L_g / (w L)$  and  $K_g / K$  are directly observed.

$$\frac{w^* L_g^*}{w^* L} = \frac{P_g^* Y_g^* \frac{\alpha_g^L}{\epsilon_g / (\epsilon_g - 1) \mathbb{E}[e^{\delta_i}]}}{\sum_g P_g^* Y_g^* \frac{\alpha_g^L}{\epsilon_g / (\epsilon_g - 1) \mathbb{E}[e^{\delta_i}]}} = \frac{\beta_g \cdot \frac{\alpha_g^L}{\epsilon_g / (\epsilon_g - 1) \mathbb{E}[e^{\delta_i}]}}{\sum_g \beta_g \cdot \frac{\alpha_g^L}{\epsilon_g / (\epsilon_g - 1) \mathbb{E}[e^{\delta_i}]}}$$

$$\frac{K_g^*}{K} = \frac{P_g^* Y_g^* \frac{\alpha_g^K}{\epsilon_g / (\epsilon_g - 1) \mathbb{E}[e^{\delta_i}]}}{\sum_g P_g^* Y_g^* \frac{\alpha_g^K}{\epsilon_g / (\epsilon_g - 1) \mathbb{E}[e^{\delta_i}]}} = \frac{\beta_g \cdot \frac{\alpha_g^K}{\epsilon_g / (\epsilon_g - 1) \mathbb{E}[e^{\delta_i}]}}{\sum_g \beta_g \cdot \frac{\alpha_g^K}{\epsilon_g / (\epsilon_g - 1) \mathbb{E}[e^{\delta_i}]}}$$

## Likelihood function:

$$\ell(P_i Y_i, K_i, L_i | \epsilon, \alpha, \sigma, \kappa, R, w) = \ell(P_i Y_i, K_i | \epsilon, \alpha, \sigma, \kappa, R, w) + \ell(P_i Y_i, L_i | \epsilon, \alpha, \sigma, \kappa, R, w)$$

$$\begin{aligned} \ell(P_i Y_i, K_i | \epsilon, \alpha, \sigma, \kappa, R, w) &\propto \left[ \log(2\kappa^K) - \log \sigma_+^K - \frac{1}{2} \left( \frac{\log(\frac{\alpha_s^K P_i Y_i (\epsilon - 1)/\epsilon}{RK_i \mathbb{E}[e^{\delta_i}]})}{\sigma_+^K} \right)^2 \right] \mathbb{1} \left[ \frac{\alpha_s^K P_i Y_i (\epsilon - 1)/\epsilon}{RK_i \mathbb{E}[e^{\delta_i}]} > 1 \right] \\ &+ \left[ \log(2 - 2\kappa^K) - \log \sigma_-^K - \frac{1}{2} \left( \frac{\log(\frac{\alpha_s^K P_i Y_i (\epsilon - 1)/\epsilon}{RK_i \mathbb{E}[e^{\delta_i}]})}{\sigma_-^K} \right)^2 \right] \mathbb{1} \left[ \frac{\alpha_s^K P_i Y_i (\epsilon - 1)/\epsilon}{RK_i \mathbb{E}[e^{\delta_i}]} < 1 \right] \\ \ell(P_i Y_i, L_i | \epsilon, \alpha, \sigma, \kappa, R, w) &\propto \left[ \log(2\kappa^L) - \log \sigma_+^L - \frac{1}{2} \left( \frac{\log(\frac{\alpha_s^L P_i Y_i (\epsilon - 1)/\epsilon}{wL_i \mathbb{E}[e^{\delta_i}]})}{\sigma_+^L} \right)^2 \right] \mathbb{1} \left[ \frac{\alpha_s^L P_i Y_i (\epsilon - 1)/\epsilon}{wL_i \mathbb{E}[e^{\delta_i}]} > 1 \right] \\ &+ \left[ \log(2 - 2\kappa^L) - \log \sigma_-^L - \frac{1}{2} \left( \frac{\log(\frac{\alpha_s^L P_i Y_i (\epsilon - 1)/\epsilon}{wL_i \mathbb{E}[e^{\delta_i}]})}{\sigma_-^L} \right)^2 \right] \mathbb{1} \left[ \frac{\alpha_s^L P_i Y_i (\epsilon - 1)/\epsilon}{wL_i \mathbb{E}[e^{\delta_i}]} < 1 \right] \end{aligned}$$

Return

## Extension: estimate $\alpha_s^K$ , $\alpha_s^L$

1. Draw a grid on the domain of  $\alpha_s^K$  and  $\alpha_s^L$ ,  $(0, 1) \times (0, 1)$ . The density of this grid affects the accuracy of estimated  $\alpha_s^K$  and  $\alpha_s^L$ .
2. For each point on the grid, set  $\alpha_s^K$  and  $\alpha_s^L$  equal to the value of this point, i.e. a guess of  $\alpha_s^K$  and  $\alpha_s^L$ .
3. For each industry, estimate  $\widehat{\sigma}_+^K, \widehat{\sigma}_-^K, \widehat{\sigma}_+^L, \widehat{\sigma}_-^L, \widehat{\kappa}^K, \widehat{\kappa}^L$  according to the equations above.
4. Calculate log-likelihood for each industry at the guessed  $\alpha_s^K$  and  $\alpha_s^L$ .
5. Find the  $\alpha_s^K$  and  $\alpha_s^L$  that give the highest log-likelihood, which is the estimated capital intensity of this industry.

Return

## Validity of my data's macro aggregates

	year	Number of firms	Sales	Output	Value added	Employment	Net value of fixed assets	Net value of fixed assets*	Export
1	1998	5e-4	0.02	4e-3	4e-3	-0.09	0.01	0.01	0.01
2	1999	4e-4	0.02	0.01	0.01	5e-3	0.02	-0.02	0.01
3	2000	6e-4	-8e-4	5e-3	5e-3	4e-3	0.01	-0.01	1e-3
4	2001	6e-4	6e-3	0.01	0.01	5e-3	0.01	-0.02	0.01
5	2002	7e-4	1e-4	0.01	0.01	4e-3	0.01	-0.02	2e-3
6	2003	1e-3	-0.01	0.02	0.02	0.01	0.02	-0.01	0.02
7	2004	-5e-3	1e-3	0.01	0.05	0.01	2e-3	-0.03	0.01
8	2005	1e-3	0.02	0.01	0.01	0.01	8e-3	-0.03	0.01
9	2006	1e-3	3e-3	0.01	0.01	0.01	0.01	-0.03	0.03
10	2007	1e-3	0.01	0.02	0.02	0.02	0.01	-0.03	0.02
11	2008	-0.03	-0.02	-0.01		-0.03	-0.06	-0.06	5e-3

Notes: "Net value of fixed assets" use data from the yearbook 2009. "Net value of fixed assets\*" use yearbook 2011.

The rest of yearbook data is from the latest available issue.

Export data of China Statistical Yearbook is from Brandt et al. (2014).

**Figure:** My data statistics in comparison with China Statistical Yearbooks: ratio

Return

Figure: Within-type TFP gains in China (2005): robustness analysis over  $\alpha_L$

var of interest	target	target source	mean type	unscaled mean	TFP gains (%)
$\alpha_L$	0.50	HK's guess	va-based	0.32	48.40
$\alpha_L$	0.50	HK's guess	sales-based	0.32	43.29
$\alpha_L$	0.50	HK's guess	cost-based	0.32	43.81
$\alpha_L$	0.44	RF main	va-based	0.32	55.72
$\alpha_L$	0.44	RF main	sales-based	0.32	56.72
$\alpha_L$	0.44	RF main	cost-based	0.32	53.29
$\alpha_L$	0.46	RF using L	va-based	0.32	48.80
$\alpha_L$	0.46	RF using L	sales-based	0.32	48.06
$\alpha_L$	0.46	RF using L	cost-based	0.32	48.12

Notes: var of interest indicates on which variables the robustness analysis is about.

RF main means estimates from reduced form analysis.

RF using L is the reduced form estimates using the number of employees.

unscaled mean is the mean from structural estimates.

Figure: Within-type TFP gains in China (2005): robustness analysis over  $\sigma$

var of interest	target	target source	mean type	unscaled mean	TFP gains (%)
$\sigma$	12.90	RF main	va-based	8.48	46.55
$\sigma$	12.90	RF main	sales-based	9.07	46.63
$\sigma$	12.90	RF main	cost-based	9.37	46.72

Notes: var of interest indicates on which variables the robustness analysis is about.

RF main means estimates from reduced form analysis.

RF using L is the reduced form estimates using the number of employees.

unscaled mean is the mean from structural estimates.

Return

## Reduced form analysis (Klette and Griliches (1996))

$$r_{it} = \beta_0 + \beta_1 k_{it} + \beta_2 l_{it} + \beta_3 y_{st} + v_{it}$$

$\beta_0 = \frac{\epsilon-1}{\epsilon} \log(A_i)$ ,  $\beta_1 = \frac{\epsilon-1}{\epsilon} \alpha^K$ ,  $\beta_2 = \frac{\epsilon-1}{\epsilon} \alpha^L$ , and  $\beta_3 = \frac{1}{\epsilon}$ .  $v_{it}$  is a combination of demand shocks and supply shocks, i.e.  $\frac{\epsilon-1}{\epsilon} u_{it}^y + \frac{u_{it}^d}{\epsilon}$ .

All the variables are in logs.

$r_{it}$ : value added deflated by two-digit industry price index.

$y_{st}$ : the production of two-digit industry.

$k_{it}$  and  $l_{it}$ : capital and labor, not deflated.

The estimated demand elasticities and returns to scale are:

$$\hat{\epsilon} = \frac{1}{\hat{\beta}_3}$$
$$\hat{\alpha}^K + \hat{\alpha}^L = \frac{\hat{\beta}_1 + \hat{\beta}_2}{1 - \hat{\beta}_3}$$



## Reduced form (Klette and Griliches (1996))

	Dependent variable	
	OLS (1)	IV (2)
$l_{it}$	0.343*** (0.001)	0.406*** (0.002)
$k_{it}$	0.134*** (0.001)	0.156*** (0.002)
$y_{st}$	0.104*** (0.003)	0.077*** (0.003)
constant	0.086*** (0.001)	0.048*** (0.001)
Observations	1,182,562	815,546
R <sup>2</sup>	0.099	0.070
Adjusted R <sup>2</sup>	0.099	0.070
Residual Std. Error	0.660 (df = 1182558)	0.615 (df = 815542)
F Statistic	43,471.580*** (df = 3; 1182558)	

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

$r_{it}$  is deflated firm-level value added,  $VA_s$  is industry  $s$ 's aggregate VA.

$l_{it}$  is deflated observed labor expenditure

Average returns to scale is 0.61  $((0.41+0.16)/(1-0.08))$ , average  $\alpha_L$  is 0.44  $(0.41/(1-0.08))$ , and average demand elasticities are 12.90  $(1/0.08)$ .

formula

Return

## Reduced form (Klette and Griliches (1996)): using L

	Dependent variable	
	OLS (1)	IV (2)
$l_{it}$	0.322*** (0.002)	0.410*** (0.003)
$k_{it}$	0.151*** (0.001)	0.173*** (0.002)
$y_{st}$	0.130*** (0.003)	0.114*** (0.003)
constant	0.117*** (0.001)	0.080*** (0.001)
Observations	1,186,861	819,923
R <sup>2</sup>	0.056	0.042
Adjusted R <sup>2</sup>	0.056	0.042
Residual Std. Error	0.686 (df = 1186857)	0.635 (df = 819919)
F Statistic	23,537.110*** (df = 3; 1186857)	

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

$r_{it}$  is deflated firm-level value added,  $VA_s$  is industry s's aggregate VA.

Average returns to scale is 0.66  $((0.41+0.17)/(1-0.11))$ , average  $\alpha_L$  is 0.46  $(0.41/(1-0.11))$ , and average demand elasticities are 8.80  $(1/0.11)$ .

formula

Return

## Reconcile the results by introducing intangible asset

Our data suggests on average firms behave like constant returns to scale but the sum of  $\alpha_K$  and  $\alpha_L$  is below 1. In other words, there are other production factors besides the observed K and L, such as intangible assets.

Proper interpretation of the results require a richer model with intangible assets included. Results are the same if tangible assets are treated as state variables when firm choose the optimal price, K, and L.

return