Introduction / Motivation

Focus on information in sequential games.

Traditional approach: stocks of information/information sets.

Chance a ◦ a b

Bob

Ann

But impossible to disentangle aspects of the rules of the game from cognitive features.
Introduction / Motivation

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• Focus on information in sequential games.
• Traditional approach: stocks of information/information sets.

But impossible to disentangle aspects of the rules of the game from cognitive features.
• Our methodological tenet: the formal description of the rules of the game should be independent of the personal features of those who happen to play the game.

• Our idea: flows of information.

• Note: this is determined by the rules of the game.
Introduction / Flows of information

- Our methodological tenet: the formal description of the rules of the game should be independent of the personal features of those who happen to play the game.

Chance chose \( a \), it's your turn

It's your turn

Note: this is determined by the rules of the game.
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Our methodological tenet: the formal description of the rules of the game should be independent of the personal features of those who happen to play the game.

Our idea: flows of information.

Note: this is determined by the rules of the game.
We can add descriptions of cognitive features and retrieve information sets. For example, if Ann has “good” memory:

"Chance chose $a \circ b$, it's your turn"
We can add descriptions of **cognitive features** and retrieve information sets.
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• For example, if Ann has “good” memory:

“Chance chose $a^\circ$, it’s your turn”

“It’s your turn”

\[
\begin{array}{c}
\text{Ann} \\
\text{Chance} \\
\text{Ann} \\
\text{Bob} \\
\text{Ann} \\
\text{Bob}
\end{array}
\]
But Ann may be forgetful: "Chance chose a°, it's your turn." It's your turn.

Modeling cognitive limitations explicitly is important: a designer has control over the rules of interaction only!
But Ann may be forgetful:

“Chance chose \( a^o \), it’s your turn”

“It’s your turn”
But Ann may be forgetful:

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Modeling cognitive limitations explicitly is important: a designer has control over the rules of interaction only!
Roadmap

1 Introduction

2 Flows of information

3 Memory

4 Conclusion

5 Bonus: An illustration
Roadmap

1 Introduction

2 Flows of information

3 Memory

4 Conclusion

5 Bonus: An illustration
Flows of information / Key ingredients

- \( I \) players
- \( A_i \) actions potentially available to \( i \in I \)
- \( M_i \) messages potentially observable by \( i \in I \)

For \( J \subseteq I \),

\[
A_J := \times_{j \in J} A_j \quad \text{and} \quad M_J := \times_{j \in J} M_j.
\]

\[
A := \bigcup_{J \in \mathcal{P}(I) \setminus \{\emptyset\}} A_J \quad \text{and} \quad M := \bigcup_{J \in \mathcal{P}(I) \setminus \{\emptyset\}} M_J.
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\(*\) Note: we allow for situations where not every player moves or receives messages.
Flows of information / Key ingredients

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Flows of information / Key ingredients

- $I$: players
- $A_i$: actions potentially available to $i \in I$.
- $M_i$: messages potentially observable by $i \in I$.
- For $J \subseteq I$, $A_J := \times_{j \in J} A_j$ and $M_J := \times_{j \in J} M_j$.

$A := \bigcup_{J \in 2^I \setminus \{\emptyset\}} A_J$ and $M := \bigcup_{J \in 2^I \setminus \{\emptyset\}} M_J \rightarrow$ sets of action and message profiles.

★ Note: we allow for situations where not every player moves or receives messages.
Flows of information / Key ingredients

- $A_i$: $M_i \Rightarrow A_i$ action feasibility correspondence of $i \in I$.
  
  $A_i(m_i) \subseteq A_i$ set of actions available to $i$ after message $m_i$.

  ⋆ Note: messages "explain" what actions can be chosen.

- $f$: $A \leq L \rightarrow M$ feedback function.
  
  $f(a_\ell)$ message profile generated after sequence of action profiles $a_\ell$.

  ⋆ Notation: $X \leq L = \text{set of sequences in } X \text{ of length } L \text{ or less, } x_\ell = \text{sequence of length } \ell$.

- $A$ game structure is $\Gamma = \langle I, f, (A_i, A_i, M_i) \rangle_{i \in I}$.

  ⋆ Note: this describes the rules of the game only.
Flows of information / Key ingredients

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- A **game structure** is \( \Gamma = \langle I, f, (A_i, A_i, M_i)_{i \in I} \rangle \).
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Flows of information / The game tree

- A play is $p = (a_1, a_2, \ldots, a_\ell)$.
- Note: plays may be partial, different terminology from the usual one.

- A history is $h = (m_0, a_1, m_1, \ldots, a_\ell, m_\ell)$.
- Interpretation: a message profile informs the players that the game is starting, then a message profile is generated after each action profile.

- We derive the trees of feasible plays and histories ($P$ and $H$) from the rules $\Gamma$.
- Feasible = action profiles are allowed, and message profiles are generated according to the feedback function.
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Roadmap

1 Introduction
2 Flows of information
3 Memory
4 Conclusion
5 Bonus: An illustration
Memory / A game-independent description

Consider a set $X$ and $X \leq N$ (with $N \in \mathbb{N}$).

⋆ Interpretation: $x \in X \leq N$ is a sequence of pieces of information ($\approx$ a database).

• A memory correspondence for agent $i$ is a nonempty-valued correspondence $M_i: X \leq N \Rightarrow X \leq N$.

⋆ Interpretation: $M_i(x)$ is the set of information streams/databases that are consistent with what $i$ remembers of $x$ ($\approx$ how precisely $x$ is stored/retained).

• Example: $x = \text{Ann tells Bob she liked the movie} = (\text{Ann}, \text{liked}, \text{movie})$. Bob remembers $(??, \text{liked}, \text{movie})$. So $(\text{Chloe}, \text{liked}, \text{movie}) \in M_{\text{Bob}}(x)$. 
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Memory / Some ideas

• Perfect memory: retain/retrieve the exact sequence of pieces of information observed or actions taken.

• Bounded memory: retain only the sequence of the most recent pieces of information.

• Statistical memory: remember which pieces of information were observed and their frequencies, but not their order.

• Approximate memory: remember something that is "similar" to the observed sequence.

• Context-dependent memory: remember pieces of information observed in contexts similar to the one being experienced.
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Memory / Memory in games

• How are streams of information treated during a game?
• Each play \( p \) induces a unique history \( h \) with the feedback function. But player \( i \) has access only to the actions he played and messages observed during \( h \).
• This is the personal history \( h_i \) induced by \( h \).
• \( F_i(p) = \) personal history of \( i \) induced by play \( p \). This is the relevant stream of information for \( i \) when \( p \) has realized.

\[
p = (a_1, \ldots, a_\ell)
\]

\[
h = (m_0, a_1, f(a_1), a_2, f(a_1, a_2), \ldots, a_\ell, f(a_1, \ldots, a_\ell))
\]

\[
h_i = (m_0, i, a_1, i, m_1, i, \ldots, a_k, i, m_k, i)
\]

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Memory / Memory in games

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\[ p = (a_1, \ldots, a_\ell) \rightarrow h = (m_0, a_1, f(a_1), a_2, f(a_1, a_2), \ldots, a_\ell, f(a_1, \ldots, a_\ell)) \]

\[ h_i = (m_{0i}, a_{1i}, m_{1i}, \ldots, a_{ki}, m_{ki}) \]
Memory / Memory in games

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$$F_i \rightarrow h_i = (m_{0i}, a_{1i}, m_{1i}, \ldots, a_{ki}, m_{ki})$$
• Assume player $i$’s perspective.

The set of plays indistinguishable from $p$ is $F_i(p) = \{p': F_i(p') = F_i(p)\} = F_{i-1}(F_i(p))$.

⋆ Interpretation: given the actual stream of information $F_i(p)$, plays in $F_i(p)$ cannot be ruled out (objective notion).

• What about (imperfect) memory $M_i$? The set of plays possible given $p$ is $P_i(p) = \{p': F_i(p') \in M_i(F_i(p))\} = F_{i-1}(M_i(F_i(p)))$.

⋆ Interpretation: given the remembered stream(s) of information $M_i(F_i(p))$, plays in $P_i(p)$ cannot be ruled out (subjective notion).
Memory / Memory in games

• Assume $p$ has realized and take player $i$'s perspective.

$F_i(p) = \{p': F_i(p') = F_i(p)\} = F^{-1}_i(M_i(F_i(p)))$.

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Memory / Memory in games

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Memory / Memory in games

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"Chance chose $a$, it's your turn"
Memory / Back to the example

Indistinguishability ($\mathcal{F}_{Ann}$):

“Chance chose $a^o$, it’s your turn”

Possibility ($\mathcal{P}_{Ann}$):

“Chance chose $a^o$, it’s your turn”
How does our approach compare to the usual one?

Two (intuitive) results.

Terminology: information partition = collection of information sets = partition of plays.

1. An information partition satisfies perfect recall iff it is induced by some flows of information and players have perfect memory.

2. A simple characterization of the perfect recall property.

Every information partition can be obtained using flows of information and some profile of memory correspondences.

Note: our approach is more expressive → it allows for cognitive limitations that do not induce information partitions (e.g., ruling out the actual play).
Memory / Some comments

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Roadmap

1. Introduction

2. Flows of information

3. Memory

4. Conclusion

5. Bonus: An illustration
What’s next?

In this paper:
• A framework to disentangle the objective information and subjective retention/retrieval in games.
• A rudimentary theory of memory.

Next steps:
• A theory of imperfect memory (or bounded rationality) and strategic thinking.
• A more structured theory of memory.
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Next steps:

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• A more structured theory of memory.
Thank you

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Roadmap

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2 Flows of information

3 Memory

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5 Bonus: An illustration
An illustration / The absentminded driver

A driver has to take the second exit. But when she gets to an exit she cannot remember if she already encountered an exit or not (Piccione & Rubinstein 1997):

Driver
Exit

Not
Driver
Exit

The information set $\{\emptyset, (\text{Not})\}$ is often thought to be problematic (e.g., Alos-Ferrer & Ritzberger 2016).
An illustration / The absentminded driver

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\[
\text{Driver} \rightarrow \text{Exit} \quad \text{Not} \quad \text{Driver} \rightarrow \text{Exit} \quad \text{Not}
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An illustration / The absentminded driver

We can retrieve absentmindedness with suitable mnemonic failures. The information flow:

- You reached an exit
- You missed your exit
- You took the second exit
- You took the first exit

Interpretation: the driver is just told (or, she can observe while driving) that she encountered an exit, but she is not reminded of whether it is the first or the second one.
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```
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```
```
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The failure to distinguish the plays $\emptyset$ and $(\text{Not })$ is due to two factors:

1. Driver does not remember if she took an action.
2. The message received after $(\text{Not })$ does not remind the driver of her previous action.

The memory correspondence: the driver retains only the last message observed.

The empty play $\emptyset$ induces the personal history $(m^*)$, where $m^* = \text{"You reached an exit".}$

The play $(\text{Not })$ induces the personal history $(m^*, \text{Not }, m^*)$.

So, they can be confused.

Absentmindedness emerges as a natural cognitive limitation: the agent remembers only the information materially available in that moment.

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- The empty play $\emptyset$ induces the personal history ($m^*$), where $m^*$ = "You reached an exit".
- The play ($Not$) induces the personal history ($m^*$, $Not$, $m^*$).

So, they can be confused.

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Pierpaolo Battigalli, Nicolò Generoso
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An illustration / The absentminded driver

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