Buy-and-resell Overpricing and Investor Experience

Bogdan Klishchuk

Humboldt University of Berlin

EEA ESEM 2022

Empirical Motivation

- The Chinese warrants bubble (Xiong and Yu 2011)
  - Fundamental values ≈ 0, almost certainly
  - Substantially inflated prices
  - Banned short-selling
  - The bubble lasted for three years
- No evidence of investor learning
- One "possibility is that the steady *inflow of naïve investors* to the warrants market was able to sustain the warrants bubble despite the learning of early arrived investors"

My Model

- An asset resembling a credit default swap
  - Personalized fundamental values ≈ 0, but may differ
  - Price $\geq$ any reasonable fundamental value
  - Prohibited short-selling
  - Overpricing lasts until maturity
- Investor learning assumed to suffer from imperviousness to information that is not experience-based (Malmendier 2021)
- Inflow of new investors

Model Basics

- Stripped-down overlapping generations
- Uncertainty only about the terminal time, e.g., default time
- A one-time divident of $1 at the terminal time
- Investor learning restricted to start at birth
- They update some common beliefs from birth only

Primitives

- *Terminal time*: $\tau \in (-\infty, \infty]$
- *Time till τ as perceived at birth*: random variable $0 < W \leq \infty$
- *Terminal age*: $T \in (0, \infty]$
- *Trade-frequency parameter*: $\Delta \in (0, T)$
- *Interest rate*: $r \in (0, \infty)$

Timing

- One investor is born each time point
- Continuous time
- Investors can trade every Δ time units at the time points

$$\{0, \Delta, -\Delta, 2\Delta, -2\Delta, 3\Delta, -3\Delta, \dots\} \cap (-\infty, \tau)$$

- An endogenous steady-state price $p^*_\Delta \in [0, 1]$

Equilibrium Concept

- The equilibrium concept of Harrison and Kreps (1978)
- Risk neutrality
- Investors decide to buy/sell or not
- *The equilibrium price*
  is one that makes the most optimistic investor(s) break even

  $\Big\downarrow\Big\uparrow$ fixed point

- Who the most optimistic are depends on *the equilibrium price*
  via expected resale proceeds
- Only the most optimistic investors buy the asset

Equilibrium Definition

Assumption

Nonzero perceived prob. at birth of survival to the terminal age T:
$\mathrm{P}(W > x) \neq 0$ for all $x \in [0, T] \backslash \{\infty\}$

Definition

We say that a price $p^*_\Delta \in [0, 1]$ is an equilibrium price if it equals the expected discounted return from the best holding duration y within lifetime and trade-frequency constraints
across all investors in terms of their age x in the sense that

$$p^*_\Delta = \max_{\substack{x \in [0, T-\Delta] \backslash \{\infty\} \\ y \in \{\Delta, 2\Delta, \ldots, \infty\} \\ x+y \leq T}} \mathrm{E}\left(e^{-r(W-x)} I_{\{W \leq x+y\}} + p^*_\Delta e^{-ry} I_{\{W > x+y\}} \middle| W > x \right) \tag{1}$$

and the maximum exists

Price $\geq$ Any Reasonable Fundamental Value

- Suppose $W < \infty$ and $W \sim \text{Gamma}(\alpha, \lambda)$ with $\alpha < 1$

- The equilibrium price $\geq$ any reasonable fundamental value in the continuous-trade limit:

$$\lim_{\Delta \to 0^+} p^*_\Delta = 1$$

- As if the one-time dividend of \$1 were imminent and there were no uncertainty about its timing

Personalized Fundamental Values ≈ 0

- Still let $W < \infty$ and $W \sim \text{Gamma}(\alpha, \lambda)$ with $\alpha < 1$
- Posteriors of older investors first-order stochastically dominate posteriors of younger investors (strictly)
- The *expected discounted dividend* depends only on the investors' age
- It is a strictly decreasing function of age
- Newborn investors' expected discounted dividend is $\mathsf{E}\left(e^{-rW}\right)$
- Everyone's expected discounted dividend is

$$\leq \mathsf{E}\left(e^{-rW}\right) = \frac{1}{(1 + r/\lambda)^{\alpha}}$$

- Everyone's expected discounted dividend $\to 0$ as $\lambda \to 0^+$
- In contrast to the equilibrium price $\to 1$ as $\Delta \to 0^+$

Buy-and-resell Premium

- Investors live through different histories
  $\Longrightarrow$ belief heterogeneity
- Most investors' histories differ only in the birth date
  $\Longrightarrow$ minimal room for belief heterogeneity
- Prohibited short-selling
  $\Longrightarrow$ expect overpricing regardless of how high the price is (Harrison and Kreps 1978, Morris 1996, Werner 2020)
- Expect buy-and-resell premium relative to buy-and-hold price
- *Buy-and-hold price* is what an equilibrium price would be if buyers had to meet a life-long no-retrade constraint and only then could resell at this price (if the dividend were still unpaid)

No-retrade Constraint for Buy-and-hold Equilibrium*

Definition

The correspondence from the participating investors' age interval

$$Y_\Delta : [0, T - \Delta] \backslash \{\infty\} \twoheadrightarrow \{\Delta, 2\Delta, 3\Delta, \ldots, \infty\}$$

defined by (longer no-retrade durations for younger investors)

$$Y_\Delta(x) = \begin{cases} \{\Delta\} & \text{if } T - 2\Delta < x \leq T - \Delta, \\ \{\Delta, 2\Delta\} & \text{if } T - 2\Delta = x, \\ \{2\Delta\} & \text{if } T - 3\Delta < x < T - 2\Delta, \\ \{2\Delta, 3\Delta\} & \text{if } T - 3\Delta = x, \\ \{3\Delta\} & \text{if } T - 4\Delta < x < T - 3\Delta, \\ & \vdots \\ \{\infty\} & \text{if } T = \infty \end{cases}$$

is a no-retrade constraint

Buy-and-hold Price*

*To be skipped

Definition

We say that a price $\bar{p}_\Delta \in [0,1]$ is a buy-and-hold price if it equals the most optimistic expected discounted return from restricted trading under the no-retrade constraint
across all investors in terms of their age x in the sense that

$$\bar{p}_\Delta = \max_{\substack{x \in [0, T-\Delta] \setminus \{\infty\} \\ y \in Y_\Delta(x)}} \mathrm{E}\left(e^{-r(W-x)} I_{\{W \leq x+y\}} + \bar{p}_\Delta e^{-ry} I_{\{W > x+y\}} \middle| W > x \right)$$

and the maximum exists

Reselling Early within Investment Horizon*

Proposition

An equilibrium price p^*_Δ coincides with a buy-and-hold price if in this equilibrium it is optimal to meet the no-retrade constraint starting from some age $x \in [0, T - \Delta] \backslash \{\infty\}$,

i.e., if for every $w \in \{0, \Delta, 2\Delta, 3\Delta, \dots\} \cap [0, T - \Delta - x]$ there is a $y \in \{\Delta, 2\Delta, 3\Delta, \dots, \infty\} \cap [\Delta, T - x - w]$ such that $(x + w, y)$ is in the argmax in (1)

Investors' Posteriors on Reaching Their Age*

Definition

The function $F(\cdot|\cdot) : \mathbb{R} \times [0, T) \to \mathbb{R}$ defined by

$$F(w|x) = \mathrm{P}(W - x \leq w | W > x)$$

is in the first variable the posterior distribution function given the second variable (age, x)

For $x = 0$, we denote $F(\cdot|0)$ by F and its limit at infinity by $F(\infty)$

Extended First-order Stochastic Dominance*

Definition

For posteriors given survival up to any two $x, x' \in [0, T)$,
we say that $F(\cdot|x)$ first-order stochastically dominates $F(\cdot|x')$
if for every (weakly) increasing function $u : (-\infty, \infty] \to \mathbb{R}$ we have

$$\int_{-\infty}^{\infty} u|_{\mathbb{R}}(w)\, dF(w|x) + u(\infty)\left(1 - \lim_{w\to\infty} F(w|x)\right)$$
$$\geq \int_{-\infty}^{\infty} u|_{\mathbb{R}}(w)\, dF(w|x') + u(\infty)\left(1 - \lim_{w\to\infty} F(w|x')\right)$$

Existence and Uniqueness*

Assumption

Continuity of F on $[0, T] \setminus \{\infty\}$

Assumption

Either $T < \infty$ or eventual first-order stochastic dominance in the sense that there exists an age threshold $\tilde{x} \in [0, T)$ such that $F(\cdot|x)$ first-order stochastically dominates $F(\cdot|\tilde{x})$ for all $x \in [\tilde{x}, T)$

Proposition

There exist:

(i) a unique equilibrium price;

(ii) a unique buy-and-hold price

Hazard Rate and Hazard Weight*

Assumption

A density $f : \mathbb{R} \to [0, \infty)$ for F exists

Assumption

Continuity of f on $(0, T)$

Definition

The hazard rate is $h : (0, T) \to [0, \infty)$ defined by $h(x) = \frac{f(x)}{1-F(x)}$

Definition

The hazard weight is $\tilde{h} : (0, T) \to [0, 1]$ defined by $\tilde{h}(x) = \frac{h(x)}{h(x)+r}$

Continuous-trade Limits*

Assumption

One-sided limits of h at the endpoints of its domain or else divergence to infinity:

(i) h has a limit or tends to ∞ as $x \to 0^+$;

(ii) h has a limit or tends to ∞ as $x \to T^-$

Proposition

Consider the equilibrium price p^*_Δ and buy-and-hold price $\bar{p}_\Delta$ as functions of the trade-frequency parameter Δ on $(0, T)$

They have right-hand limits at 0

Let $p^*_0 = \lim_{\Delta \to 0^+} p^*_\Delta$ and $\bar{p}_0 = \lim_{\Delta \to 0^+} \bar{p}_\Delta$

Fundamental Valuation*

Definition

The fundamental valuation
is the function of age $V : [0, T] \setminus \{\infty\} \to \mathbb{R}$ defined by

$$V(x) = \mathrm{E}\left(e^{-r(W-x)} I_{\{W \leq T\}} + \bar{p}_0 e^{-r(T-x)} I_{\{W > T\}} \middle| W > x\right)$$

Proposition

The buy-and-hold price's continuous-trade limit $\bar{p}_0$
equals the most optimistic fundamental valuation:

$$\bar{p}_0 = \max_{x \in [0,T] \setminus \{\infty\}} V(x)$$

and the maximum exists.

Sufficient Condition for Overpricing*

Definition (Valuation-switching Condition)

The fundamental valuation V exhibits switching
if some (relatively young) age $\hat{x} \in [0, T)$ is a maximizer of V
but another (older) age $\check{x} \in (\hat{x}, T)$ is not

Proposition

If the fundamental valuation V exhibits switching, then $p_0^* > \bar{p}_0$

Characterization of Buy-and-resell Overpricing*

Definition (End-of-life Hazard Switching)

The hazard rate h exhibits end-of-life switching if

$$\lim_{x \to T^-} h(x) < \sup_{x \in (0,T)} h(x)$$

Proposition

If the hazard rate h exhibits end-of-life switching, then $p_0^* > \bar{p}_0$

Proposition

If the hazard rate h does not exhibit end-of-life switching, then $p_0^* = \bar{p}_0$

Necessary and Sufficient Condition

In the continuous-trade limit,

buy-and-resell premium $\neq 0$

$$\iff \lim_{x \to T^-} \frac{f(x)}{1 - F(x)} < \sup_{x \in (0,T)} \frac{f(x)}{1 - F(x)}$$

"$<$" says that the most senior investors are not instantaneously the most optimistic

More on Price $\geq$ Any Reasonable Fundamental Value

Sufficient Condition

$$\lim_{x \to 0^+} \frac{f(x)}{1 - F(x)} = \infty \implies \lim_{\Delta \to 0^+} p_\Delta^* = 1$$

"∞" says that newborn investors' beliefs are instantaneously sufficiently optimistic

$1 \geq$ any reasonable fundamental value

Conclusion

- The first model where inflow of new investors keeps the price $\geq$ any reasonable fundamental value
- It occurs when new investors' beliefs are sufficiently optimistic and impervious to information that is not experience-based
- Empirical evidence and neuroscience foundation (Malmendier 2021)
- Shared features with the Chinese warrants bubble (Xiong and Yu 2011)

Conclusion

- The first model where inflow of new investors keeps the price $\geq$ any reasonable fundamental value
- It occurs when new investors' beliefs are sufficiently optimistic and impervious to information that is not experience-based
- Empirical evidence and neuroscience foundation (Malmendier 2021)
- Shared features with the Chinese warrants bubble (Xiong and Yu 2011)

Thank You!