Make-up Strategies with Finite Planning Horizons but Forward-Looking Asset Prices

Stéphane Dupraz

Banque de France

Hervé Le Bihan

Banque de France and Banco de España

Julien Matheron[*]

Banque de France

July 8, 2022

Abstract

How effective make-up strategies are depends heavily on how forward-looking agents are. Workhorse monetary models, which are much forward-looking, find them so effective that they run into the forward-guidance puzzle. Models that discount the future further find them much less effective, but imply that agents discount the very perception of future policy rates. This only evaluates make-up strategies when financial markets do not notice them, or deem them non-credible. We amend one leading solution to the forward-guidance puzzle—Woodford's finite planning horizons—to the assumption that financial markets have rational expectations on policy rates, and incorporate them into the long-term nominal interest rates faced by all. Agents still have a limited ability to foresee the *consequences* of monetary policy on output and inflation, making the model still immune to the forward-guidance puzzle. First, we find that make-up strategies that compensate for a past deficit of accommodation after an ELB episode have sizably better stabilization properties than inflation targeting. Second, we find that make-up strategies that *always* respond to past economic conditions, such as average inflation targeting, do too but that their stabilization benefits over IT can be *reduced* by the existence of the ELB.

Keywords: Make-up strategies, forward-guidance puzzle, finite planning horizons.

JEL Classification: E31, E52, E58.

1 Introduction

With short-term interest rates stuck at or near their effective lower bound (ELB) during a large part of the last decade, central banks have increasingly engaged in policies seeking to affect aggregate demand through expectations of future interest rates. Such policies promising "low-for-long" policy rates have found their way into the regular toolbox of central banks in the form of forward guidance. Recently, several central banks including the Federal Reserve and the European Central Bank (ECB) have considered whether to make this feature of policy a systematic part of the way they conduct monetary policy by adopting a "make-up" strategy.[1] The strategy the Fed adopted in August 2020 as an outcome of its strategic review is commonly considered to be a form of *flexible average-inflation targeting* (AIT), one such make-up strategy. The strategy the ECB adopted in July 2021 insists on the need, when the economy is close to the lower bound, for *"especially forceful or persistent monetary policy measures [which] may also imply a transitory period in which inflation is moderately above target* (ECB, 2021a).

Both forward-guidance and make-up strategies rely on managing expectations. As such, their effectiveness depends critically on economic agents forming forward-looking expectations and making decisions in a forward-looking manner. In the workhorse New Keynesian (NK) model, which assumes fully forward-looking agents, such policies are highly effective (Eggertsson and Woodford, 2003). Even suspiciously so: they result in the "forward guidance puzzle" (Del Negro, Giannoni, and Patterson (2012)). The announcement of policy changes in the far future have an unrealistically large effect on current output and inflation.

On the empirical side, how forward-looking private agents are in real life is subject to conflicting evidence. On the one hand, survey evidence of households and firms suggest that they are far from being fully forward-looking. For instance, Coibion, Georgarakos, Gorodnichenko, and Weber (2020) document that consumer expectations about inflation and other macro variables are only weakly sensitive to announcements of policy interest rate trajectories at a long horizon. Similarly, in direct application to make-up strategies Coibion, Gorodnichenko, Knotek, and Schoenle (2020) find that US households paid little attention to the new monetary policy strategy announced by the Fed in August 2020 nor change their expectations when given information about it, casting doubts on the real-life effectiveness of make-up strategies.[2] Yet, on the other hand, a large body of empirical evidence documents that asset prices are very responsive to news about future economy developments, which requires at least some financial-market participants to be forward-looking—consistent with the strong incentives they face. In particular, asset prices respond strongly and quickly to forward guidance announcement—e.g. Swanson (2021) and Bhattarai and Neely (2022). Noticeably, these asset prices include interest rates that households and firms directly face: Bhattarai and Neely (2022) document the response of corporate bond yields to forward guidance announcements, while Rostagno, Altavilla, Carboni, Lemke, Motto, and Saint Guilhem (2021) document the response of bank lending rates for firms and households to forward guidance shocks in the euro area.

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[1] Whether to adopt a make-up strategy was among the main issues considered in the strategic reviews of both the Fed and the ECB. See, e.g. Hebden, Herbst, Tang, Topa, and Winkler (2020) for the Fed and ECB (2021b) for the ECB.

[2] Households' expectation responses can be heterogeneous both within and across countries. Hoffmann, Moench, Pavlova, and Schultefrankenfeld (2022) subject German households to a hypothetical shift by the ECB to an AIT strategy and find that their inflation expectations increase.

We study make-up strategies and forward-guidance in a NK model amended along two dimensions that we deem necessary in view of both the existence of the forward-guidance puzzle and the evidence on the extent of forward-lookingness of real-life agents. First, following Woodford (2019), we assume that consumers and firms have finite planning horizons (FPH), a form of bounded rationality. They are not able to optimally forecast economic variables into a distant future. As Woodford shows, finite planning horizons reduce the degree of forward-lookingness of the NK model, providing a solution to the forward-guidance puzzle. It also aligns well with survey evidence on the extent of forward-lookingness of firms and households (Coibion, Georgarakos, Gorodnichenko, and Weber (2020)).

Second, and in contrast to Woodford (2019), we assume that, while consumers and firms have finite planning horizons, financial markets participants have fully forward-looking—fully rational—expectations. In particular, they fully take into account future monetary policy in their expectations of future interest rates, and incorporate them into the long-term nominal interest rates faced by all agents. We call this assumption forward-looking asset prices (FLAP). It is similar to the assumption of model-consistent asset prices made in Bernanke, Kiley, and Roberts (2019) and Hebden, Herbst, Tang, Topa, and Winkler (2020) within the large semi-structural model of the Fed—FRB-US.

We show how to solve the NK model under our joint FPH-FLAP assumption. We show that it results in a parsimonious extension of Woodford (2019)'s plain FPH model that can be solved using standard DSGE techniques. We use our FPH-FLAP model to evaluate the effect of forward-guidance announcements and the stabilization properties of various make-up strategies.

We reach two main conclusions. First, when asset prices are forward-looking, lower-for-longer policies at the ELB remain effective even when households and firms have short planning horizons. Their effect is in particular much stronger than in the plain FPH model where all agents (including financial-market participants) have finite planning horizons. These results are consistent with those obtained by Bernanke, Kiley, and Roberts (2019) and Hebden, Herbst, Tang, Topa, and Winkler (2020) through semi-structural models. This qualifies concerns on the efficacy of make-up strategies that can arise from Coibion, Gorodnichenko, Knotek, and Schoenle (2020)'s evidence that households do not seem to change their expectations in reaction to the announcement of a shift to such a policy. At the same time, owing to the short planning horizons of households and firms, the model is immune to the forward-guidance puzzle.

Second, we find that while systematic make-up strategies—which embed history-dependence both at and away from the ELB, such as AIT—have better stabilization properties than IT, their extra performance over IT can actually be *reduced* by the presence of the ELB. Indeed, absent the ELB constraint, an AIT strategy would be able to provide lower rates in the near future, not just rates at the ELB for longer in the distant future. This contrasts with the case of rational expectations, in which the existence of the ELB strengthens the benefits of AIT over IT because switching to AIT allows to mitigate the extremely—unrealistically— large deflationary spirals that occur at the ELB under IT. Our results thus bring nuances to a traditional argument for switching to an AIT strategy, which insists on the constraint posed by the ELB as an important motivation for moving beyond IT.

Our FPH-FLAP model also provides a methodological contribution, of interest in itself: the assumption of forward looking asset prices allows to easily simulate the FPH model under the ELB, a non trivial achievement. Indeed, it circumvent aggregation issues that are particularly intricate in a FPH set-up subject to the zero lower bound.

This paper is related to several branches of literature. We build crucially on Woodford (2019)'s solution to the FG puzzle by amending his assumption of finite planning horizons to allow for forward-looking asset prices. We thus follow up on a sparse recent literature that has adopted Woodford (2019)'s framework. Gust, Herbst, and Lopez-Salido (2022) estimate agents' planning horizons in the version of the model that includes learning, and study to what extent this assumption can generate macroeconomic persistence. Woodford and Xie (2022) and Xie (2020) investigate the monetary/fiscal interaction in a set-up with FPH agents. More broadly, our paper also connects to the set of recent works that investigate solutions to the FG puzzle based on bounded rationality—departures from rational expectations—such as Gabaix (2020) and Farhi and Werning (2019), or departure from common knowledge such as Angeletos and Lian (2018). Our assumption of forward-looking asset prices could be investigated in these alternative models as well.[3] Also departing from rational expectations, Eusepi, Gibbs, and Preston (2022) analyze forward-guidance in a model where agents form expectations by learning the data-generating process from recent data.

Other papers consider alternative solutions to the FG puzzle which do not relax rational expectations. MacKay, Nakamura, and Steinsson (2016) show that in a model with household heterogeneity and incomplete markets, households' precautionary savings motive can add discounting to the Euler equation and provide a solution to the FG puzzle.[4] As Bilbiie (2020), Werning (2015) and Acharya and Dogra (2020) show however, household heterogeneity can either attenuate of amplify the FG puzzle, and solving the FG puzzle through household heterogeneity requires further assumptions on the cyclicality of idiosyncratic income.

A distinct solution to the FG puzzle is to assume that forward-guidance announcements are only imperfectly credible, and all the more so that they concern policy rates further into the future (Campbell, Ferroni, Fisher, and Melosi, 2019; Coenen, Montes-Galdon, and Smets, 2019).[5] Such assumptions can be seen as the exact mirror to the assumptions we make in our FPH-FLAP model: they attenuate how much an announcement of future policy affects expectations of future policy but make no change to how much expectations of future policy affect inflation and output. Since credibility is key to the success of forward guidance, such approaches that relax perfect credibility are of much practical relevance. Yet they answer a distinct policy question: We focus on the question of the effect of forward guidance and make-up strategies when they are perceived as credible by financial markets. Besides, while imperfect credibility is a very relevant assumption empirically, it can be imperfect as a solution to the FG puzzle, which is theoretical in nature. If the effects of credible forward guidance found by a model are deemed unrealistic, it is questionable how much faith to put

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[3]The assumption that financial markets feed households and firms with their rational expectations of future policy rates would make most of a difference in Gabaix (2020)'s cognitive-discounting model. In their model of k-level thinking, Farhi and Werning (2019) assume that all households perfectly observe future policy rates.

[4]Although, unlike MacKay, Nakamura, and Steinsson (2016), we do not consider the effect of the precautionary savings motive induced by borrowing constraints, in section 3.3 we consider the effect of the shorter horizon induced by borrowing constraints.

[5]ECB (2022) also reviews several solutions to the FG puzzle and develops a set-up in which a fraction of agents are inattentive to FG announcements.

in the effects the same model finds for imperfectly credible forward guidance. If it overstates the former, it is likely to overstate the latter as well. Finally, imperfect credibility is more difficult to specify for make-up strategies than it is for forward-guidance announcements.

Our paper belongs to the recent expanding literature comparing alternative policy rules and strategies in a low real rate environment, such as Bernanke, Kiley, and Roberts (2019), Coenen, Montes-Galdon, and Smets (2019) , Coenen, Montes-Galdon, and Schmidt (2021), Busetti, Neri, Notarpietro, and Pisani (2020), Hebden, Herbst, Tang, Topa, and Winkler (2020), Erceg, Jakab, and Lindé (2021). Within this literature, we connect in particular to Bernanke, Kiley, and Roberts (2019) and Hebden, Herbst, Tang, Topa, and Winkler (2020), whose assumption of model-consistent asset prices is similar to our assumption of forward-looking asset prices. We show how to incorporate this assumption in a DSGE model, when Bernanke, Kiley, and Roberts (2019) and Hebden, Herbst, Tang, Topa, and Winkler (2020) consider it in a semi-structural model.

The paper is structured as follows. Section 2 introduces and motivates the two main assumptions of our set-up: finite planning horizons (FPH) and forward-looking asset prices (FLAP). Section 3 derives the baseline New-Keynesian model under both assumptions for a fixed planning horizon. Section 4 further assumes a geometric distribution of planning horizons in the population and obtains a parsimonious system for the baseline FPH-FLAP New-Keynesian model. Section 5 evaluates the power of forward-guidance announcements in the FPH-FLAP New-Keynesian model, and determines under which conditions it runs into the forward-guidance puzzle. Section 6 evaluates the stabilization properties of various make-up strategies in the FPH-FLAP New-Keynesian model through simulations.

2 Finite Planning Horizons but Forward Looking Asset Prices

In this section, we summarize Woodford's approach to generating extra-discounting of the future through finite planning horizons, point out that it implies that asset prices similarly discount the future path of monetary policy, and introduce our assumption of forward-looking asset prices.

2.1 Finite Planning Horizons

Motivated by the need to provide a solution to the forward-guidance puzzle, several rationales for generating extra discounting of the future in macroeconomic models have been proposed, such as heterogeneity and incomplete markets, imperfect central credibility, or bounded rationality. In this paper, we focus on one such solution: Woodford (2019)'s finite planning horizons (FPH), a form of bounded rationality.[6] Under finite planning horizons, agents are assumed to be able to form rational expectations about future variables and shocks, as well as to reason through their consequences on endogenous economic variables only until h periods ahead. To evaluate the consequences of their choices beyond their planning horizon h however, they rely on an approximate value function under which all variables that they take as exogenous are back to their

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[6]Woodford (2019) also consider an extension of the FPH model where agents gradually update the approximate steady-state value function they use, through learning. We abstract from learning.

steady-state values. Finite planning horizons constitute a departure from rational expectations—the latter obtain as the limit when agents' planning horizon h tends to infinity.

Under the additional assumption that agents are heterogeneous in their forecast horizon h and that forecast horizons are distributed in the population according to a geometric distribution, Woodford (2019) shows that the basic NK model under finite planning horizons reduces to the following equations:[7]

$$y_t = \nu_t^y - \sigma(r_t - \boldsymbol{\rho}\mathrm{E}_t[\pi_{t+1}]) + \boldsymbol{\rho}\mathrm{E}_t[y_{t+1} - \nu_{t+1}^y], \tag{1}$$

$$\pi_t = \nu_t^p + \kappa(y_t - y_t^e) + \beta\boldsymbol{\rho}\mathrm{E}_t[\pi_{t+1}]. \tag{2}$$

where π_t is the deviation of inflation from its target, y_t is the deviation of output from its steady-state level, y_t^e is the deviation from steady-state of efficient output which fluctuates with productivity shocks, ν_t^y is a preference demand shock, and ν_t^p is a cost-push shock, and $\mathrm{E}_t[\cdot]$ is the standard, model-consistent, expectations operator. The model must be closed by a third, monetary-policy equation, but this last one is not affected by the assumption of finite planning horizons (at least provided that monetary policy does not respond to past economic conditions).[8]

Finite planning horizons parsimoniously add a single discounting parameter $\rho \in [0, 1]$ in front of future expected variables of the standard NK model.[9] This parameter captures the extent of foresight of agents and is increasing with the average planning horizon in the population: to a value ρ corresponds an average planning horizon in the population of $\rho/(1-\rho)$ periods ahead. As ρ tends to 1, the average planning horizon tends to infinity, and the model tends to the model under rational expectations.

2.2   Discounting Future Policy vs. the Consequences of Future Policy

The extra-discounting at the discount rate ρ mitigates the forward-looking channels embedded in the Euler equation and New-Keynesian Phillips curve. Iterating the system forward makes more apparent how extra discounting affects the model:

$$y_t = \mathrm{E}_t \sum_{k=0}^{\infty} \rho^k(\nu_{t+k}^y - \sigma(r_{t+k} - \pi_{t+k+1})), \tag{3}$$

$$\pi_t = \mathrm{E}_t \sum_{k=0}^{\infty} (\beta\rho)^k(\nu_{t+k}^p + \kappa(y_{t+k} - y_{t+k}^e)). \tag{4}$$

Future shocks ν_{t+k}^y and ν_{t+k}^p and future endogenous variables π_{t+k} and y_{t+k} are discounted with the extra discount factor ρ. The discounting of future inflation and future output is central to resolving the forward-guidance puzzle. It mitigates the amplifying spiral embedded in the basic New-Keynesian model, whereby expectations of low inflation tomorrow increase the real interest rate today, generating even lower aggregate

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[7]These expressions are valid under the assumption of no government debt.

[8]The equations (1) and (2) are valid in any model with no state variable—i.e. any purely forward-looking model. They are therefore valid at least when combined with any interest-rate rule that does not include state-variables.

[9]We emphasize this by displaying the discounting parameter in bold in the equations

demand today, while expectations of low output tomorrow decrease expected future marginal cost, generating even lower inflation today.

But note that future policy rates r_{t+k} are also discounted with the same extra discount factor ρ. The model implies that a pre-announced future interest rate cut k periods ahead has an effect on aggregate demand today that is discounted by a factor ρ^k. This is a logical consequence of the assumption that agents make economic decisions with a foresight of $N = \rho/(1-\rho)$ periods on average: as a consequence they do not take into account future interest-rates changes more that $N = \rho/(1-\rho)$ periods ahead on average. This discounting of future policy rates is a property of the FPH model conceptually distinct from the discounting of future inflation and future output. The latter attenuates the general-equilibrium mechanisms that amplify the effect of a future interest-rate cut, while the former attenuates the very perception of the future interest-rate cut.

2.3   Forward-Looking Asset Prices

The distinct assumption that households and firms with finite planning horizons discount future policy rates may not be the best description of reality. In order to intertemporally substitute consumption far into the future, households do not need to roll over the short-term nominal interest rate and hence do not need to form expectations on its future path. They can instead borrow and save through long-term contracts at long-term rates. When they do so, they face the long-term interest rates offered by banks or available on financial markets. These rates are directly observable (and likely something households pay attention to before contracting a loan). The expectations of future nominal interest rates that determine firms and households' decisions are therefore the ones of financial market participants. Provided financial participants foresee the future path of policy rates and incorporate them into long-term interest rates through arbitrage, firms and households will behave as if they did so themselves.

We assume that participants in financial markets have an infinite-horizon foresight on future policy rates—i.e. model-consistent, or rational, expectations on future policy rates—and incorporate these expectations into the nominal interest rates of all maturities faced by all agents. We refer to this assumption as forward-looking asset prices (FLAP). This assumption is similar to the one used by Bernanke, Kiley, and Roberts (2019).[10] We amend Woodford (2019)'s finite-planning horizon to this assumption, while retaining the assumption that expectations of other variables remain formed under finite planning horizons.[11] We refer to this joint assumption as finite planning horizons with forward-looking asset prices (FPH-FLAP). The FPH-FLAP framework effectively restricts the problem of forecasting future variables faced by firms and households to forecasting future inflation and future activity, ie. the general-equilibrium consequences of future monetary policy on future inflation and future activity, but not future monetary policy itself.

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[10]In that paper, and other ones using the FRB-US model, this assumption is labeled MCAP, for "model consistent asset prices".

[11]While only a small fraction of the population works in financial markets—in particular in jobs that require forecasting the economic outlook very accurately—this small fraction still consumes. That they too have finite planning horizons and forget what they knew during their day job when they come home and consume would appear schizophrenic. However, given the assumption of a geometric distribution of planning horizons among households, there is always a positive fraction of households with arbitrary long planning horizons. Households that work in financial markets can fit into those.

Formally, at t all agents perfectly observe the entire infinite sequence of nominal interest rate of maturity n, $r_{t,n}$. Financial markets participants have rational expectations and financial markets are efficient in the sense that financial market participants make the law of one price hold by arbitrage. At first-order it gives the expectation hypothesis, where expectations are rational:

$$r_{t,n} = \frac{1}{n} \sum_{k=0}^{n-1} \mathrm{E}_t[r_{t+k}]. \tag{5}$$

As a consequence, all households can substitute consumption between any two dates t and $t + n$ at the long-term rate $r_{t,n}$ consistent with rational expectations on the sequence of future short-term nominal rates $(r_{t+i})_i$. Households' situation is therefore equivalent to having rational expectations on the future path of nominal interest rates. Appendix A makes this point formally.

The assumption that financial markets have perfectly rational expectations can seem extreme. Coibion and Gorodnichenko (2015) reject the null of full-information rational expectations in the US Survey of Professional Forecasters, as their forecasts underreact to new information. Farmer, Nakamura, and Steinsson (2021) show that these anomalies are consistent with agents not having rational expectations on the data-generating process but instead learning it very slowly because its low-frequency behavior takes a very long time to learn. We assume financial markets have perfectly rational expectations mainly because it brings much tractability to the model. A natural extension would be to consider the case where financial markets have expectations more forward-looking than those of households and firms, yet less than infinitely forward-looking. Interestingly, notice that how strong the RE assumption on financial markets is depends on the kind of lower-for-longer policy considered. For time-dependent FG like the one we consider in the FG experiments of section 5, the assumption consists only in assuming that financial markets believe the communication of the central bank on its future policy. In contrast, for state-dependent FG (and in most proposals for make-up strategies), financial markets need to form expectations on the future state of the economy, so that the forward-lookingness of their expectations of the future state of the economy matters. In this case however, central banks can still help financial markets to form expectations by providing their own expectations on the state of the economy and/or the future path of policy rates (either in policy statements or in the form of dot plots).

3 Derivation of the NK Model under FPH-FLAP

In this section, we derive the baseline New-Keynesian model under our joint assumption of finite planning horizons and forward-looking asset prices. Throughout the section we restrict to the problems faced by agents with a given planning horizons h. The aggregation across agents with different planning horizons is left to the following section.

3.1 Notations

We denote $x_{t+j}^{h-j}(h)$ the perception of a variable x_{t+j} by an agent with planning horizon h, formed in period t. The superscript notation $h-j$ takes note of the number of periods remaining until the end of the agent's planning horizon.[12] The notation (h) keeps track of the agent's overall planning horizon. Because exogenous shocks until planning horizon h and the nominal interest rate r_t are assumed to be expected with rational expectations, we do not index them by $h-j$ and (h).

3.2 The Consumption Equation

Start with households. We consider here solely their consumption/saving decision; the labor-supply decision will be considered when deriving the supply-side of the economy. An individual household i with planning horizon h maximizes the finite-horizon objective:

$$\tilde{E}_t \sum_{j=0}^{h} \beta^j e^{-\frac{1}{\sigma}\nu_{t+j}^y} u(C_{t+j,i}) + \beta^{h+1} v_H \big(B_{t+h+1,i}, (R_{t+j})_{j\geq h+1}\big), \tag{6}$$

$$\text{s.t. } \forall j = 0,\ldots,h, \; \frac{B_{t+j+1,i}}{R_{t+j}} + C_{t+j,i} = \frac{B_{t+j,i}}{\Pi_{t+j,i}} + \Omega_{t+j,i}, \tag{7}$$

where C_i is household i's consumption, B_i its real holding of bonds, Ω_i its real income, and where R is the nominal interest rate, Π inflation, and ν^y is a demand preference shock, rescaled with $-\sigma^{-1}$. We assume that firms' shares are not traded. Real income Ω_i includes both labor income and profits, which household takes as exogenous. The only financial asset is nominal bonds, of which household i starts period t with a predetermined quantity B_t^i. We assume throughout that government debt is in zero net supply at all dates and that all households start period t with zero wealth. They are however free to save and borrow at market interest rates.

The tilded expectation operator $\tilde{E}$ denotes the distorted expectations of the household, and v_H is the terminal value function that the household uses to estimate the continuation value of its problem past its planning horizon. Following Woodford (2019), we assume that the terminal value function v_H is the one that obtains in an environment where the variables Π, Ω_i, and ν^y that the household takes as exogenous are back to steady-state. In departure from Woodford however, we assume that the household continues to perceive nominal interest rates R past its planning horizon—our FLAP assumption. Under our FLAP assumption, the terminal value function v_H is therefore a function not only of bond holdings $t+h+1$, but also of the path of nominal interest rates from date $t+h+1$ to infinity. We highlight this by making the dependence of v_H in $(R_{t+j})_{j\geq h+1}$ explicit in equation (6).

Taking first-order conditions, household i is on a standard Euler equation from t to the end of its planning

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[12]An agent with a planning horizon of h periods contemplates that when date $t+j$ comes, she will have a planning horizon of $h-j$ periods.

horizon $t + h$. In loglinear form (lower-cased variables denote log-deviations):

$$\forall j = 0, \ldots, h-1, c_{t+j,i}^{h-j}(h) = \nu_{t+j}^y - \sigma(r_{t+j} - \mathrm{E}_{t+j}(\pi_{t+j+1}^{h-j-1}(h))) + \mathrm{E}_t(c_{t+j+1,i}^{h-j-1}(h) - \nu_{t+j+1}^y), \quad (8)$$

where E_t designates the standard, model-consistent, expectations operator and distorted beliefs are now taken note of through the $h - j$ and (j) indexes, as in Woodford (2019). At the end of its planning horizon however, the optimality condition equates the marginal value of consumption at $t + h$ to the marginal value of entering period $t + h + 1$ with wealth $B_{t+h+1,i}$:[13]

$$c_{t+h,i}^0(h) = \nu_{t+h}^y - \sigma r_{t+h} - \sigma \mathrm{E}_{t+h}(\hat{v}_H'(b_{t+h+1,i}^0(h), (r_{t+j})_{j \geq h+1})). \quad (9)$$

Equations (8) and (9) can be iterated forward to give:

$$c_{t,i}^h(h) = \nu_t^y - \sigma \left(\sum_{j=0}^h \mathrm{E}_t(r_{t+j}) - \sum_{j=0}^{h-1} \mathrm{E}_t(\pi_{t+j+1}^{h-j-1}(h)) \right) - \sigma \mathrm{E}_{t+h}(\hat{v}_H'(b_{t+h+1}^0(h), (r_{t+j})_{j \geq h+1})). \quad (10)$$

To express the terminal value function $\hat{v}_H'$ we need to consider the world perceived by household i past its horizon h. It perceives that all exogenous variables Π, Ω, ν^y—although not R—will be back to their steady-state values Π^*, Ω^* and 0.[14] The value function v is therefore defined as:

$$v\big(B_{t,i}, (R_{t+j})_{j \geq 0}\big) = \max_{(C_{t+j,i})_{j \geq 0}, (B_{t+j,i})_{j \geq 0}} \mathrm{E}_t \sum_{j=0}^\infty \beta^j u(C_{t+j,i}), \quad (11)$$

$$\text{s.t. } \forall j \geq 0, \frac{B_{t+j+1,i}}{R_{t+j}} + C_{t+j,i} = \frac{B_{t+j,i}}{\Pi^*} + \Omega^*. \quad (12)$$

We take this terminal value to remain unchanged over time, i.e. we do not consider the possibility that households could use past data to update their approximation of their terminal value function. Woodford (2019) considers an extension of the plain FPH model with such learning, which adds a backward-looking component to agents' expectations. We restrict to the version of the model without learning, i.e. to what makes agents' expectations less forward-looking yet not backward-looking, as in other main models of bounded rationality, e.g. Gabaix (2020) and Farhi and Werning (2019).

The envelope theorem gives, in log-linear form:

$$\hat{v}_H'\big(b_{t,i}, (r_{t+j})_{j \geq 0}\big) = -\frac{1}{\sigma} c_{t,i}^*, \quad (13)$$

where $c_{t,i}^*$ can be solved to be given, around a steady-state with no public debt $B^* = 0$, by the consumption

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[13]The lower-cased variable b_t^i is defined as $b_t = B_t^i / Y^* \Pi^*$, since the usual log-deviation definition is inapplicable to a zero level of debt in steady-state.

[14]We assume the household perceives all these exogenous variables to be back to steady state as soon as $t + h + 1$, regardless of the value of possible state variables at $t + h$. Although we consider an essentially forward-looking model, such state variables can exist in the model if the monetary policy rule responds to past macroeconomic conditions.

function

$$c^*_{t,i} = (1 - \beta)b_{t,i} - \sigma \mathrm{E}_t \left(\sum_{j=0}^{\infty} \beta^{j+1} r_{t+j} \right). \tag{14}$$

Combining equations (13) and (14):

$$\hat{v}'_H \left(b_{t,i}, (r_{t+j})_{j \geq 0} \right) = -\frac{1}{\sigma}(1 - \beta)b_{t,i} + \mathrm{E}_t \left(\sum_{j=0}^{\infty} \beta^{j+1} r_{t+j} \right). \tag{15}$$

Plugging this expression into equation (10), we get the expression for the consumption of household i in period t

$$c^h_{t,i}(h) = \nu^y_t - \sigma \left(\sum_{j=0}^{h} \mathrm{E}_t(r_{t+j}) - \sum_{j=0}^{h-1} \mathrm{E}_t(\pi^{h-j-1}_{t+j+1}(h)) + \sum_{j=h+1}^{\infty} \beta^{j-h} \mathrm{E}_t(r_{t+j}) \right) + (1 - \beta)E_t(b^0_{t+h+1,i}). \tag{16}$$

All agents with planning horizon h are assumed to perceive that all agents in the model face the same shocks, entertain the same preferences and have the same planning horizon as they do, whether or not it is actually the case. Aggregating across households, agents with planning horizon h therefore perceive aggregate consumption $c^h_t(h)$, which they understand to be equal to aggregate production $y^h_t(h)$, to be given by:

$$y^h_t(h) = \nu^y_t - \sigma \left(\sum_{j=0}^{h} \mathrm{E}_t(r_{t+j}) - \sum_{j=0}^{h-1} \mathrm{E}_t(\pi^{h-j-1}_{t+j+1}(h)) + \sum_{j=h+1}^{\infty} \beta^{j-h} \mathrm{E}_t(r_{t+j}) \right). \tag{17}$$

This is the expression of aggregate consumption perceived by all agents with planning horizon h in the model. To get this last equation, we used the fact that agents with planning horizon h correctly understand all the general-equilibrium implications of the model until their planning horizon h. They therefore understand that public debt is in zero net supply until the end of their planning horizon, $b^0_{t+h+1} = 0$.[15]

In the expression for aggregate consumption (17), nominal interest rates past horizon h are discounted with the discount factor β. By contrast, nominal interest rates before horizon h, enter with no discount. The mechanics behind this different degrees of discounting is that, to form expectations on the response of aggregate consumption to an expected interest rate change before horizon h, an agent with planning horizon h endogenously adjusts its expectations of aggregate income and therefore of aggregate consumption. The agent reasons that interest rates will impact aggregate demand which will impact aggregate income, which will impact aggregate demand, and so on—i.e. the Keynesian cross. The general-equilibrium effect of the Keynesian cross amplifies the purely decision-theoretical (or partial-equilibrium) effect of nominal interest rates r_{t+j} on consumption c_t. It increases from $-\sigma \beta^{j+1}$ to $-\sigma$.

When forming expectations on the response of aggregate consumption to an expected interest rate change past horizon h however, an agent with planning horizon h fails to reason through the Keynesian cross. It only takes into account the interest rate change keeping aggregate income fixed, unable to reason how aggregate

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[15] Note that household i does not necessarily expect that its own level of bond holding b_i will be zero past horizon h, but we do not need to keep track of its expectation for the path of its individual wealth in order to solve for aggregate consumption.

income will respond to the change in nominal interest rates. It only takes into account the decision-theoretical effect embedded in the consumption function and abstracts from general-equilibrium amplifying effects. As a result, nominal interest rates past horizon h are discounted with the discount rate β.

3.3 Stronger Discounting through Financial Constraints

Since the parameter β is one determinant of the discounting of future policy rates in the model, its calibration will be an important determinant of the power of forward guidance and make-up strategies. In equation (17), β is simply a preference discount factor and as such cannot be argued to be very much below one. Some proposed solutions to the forward guidance puzzle argue however that incomplete markets, e.g. in the form of households' borrowing constraints, can generate a stronger discounting of future interest rates. To take into account such arguments, in Appendix B we consider a setup that replaces the representative agent assumed so far with a perpetual-youth set-up in which a fraction λ of households die and are born every period (Blanchard, 1985). Following Nistico (2016); Del Negro, Giannoni, and Patterson (2012); Farhi and Werning (2019), we interpret the probability of death λ as the probability for a household of hitting its borrowing constraint.[16]

Appendix B shows that the model generates exactly the same equation for consumption as equation (17), up to replacing β with:

$$\bar{\beta} = \beta(1 - \lambda). \tag{18}$$

In what follows, we will consider lower calibrations of β in equation (17) in line with this version of the model with financial constraints. To keep track of this alternative interpretation of β, from now on we use the notation $\bar{\beta}$ to denote the discount factor in equation (17). We distinguish the notation because the discount factor that enters the Phillips curve will always be β, with or without financial constraints on the household's side.

Note that under rational expectations $h = \infty$, financial constraints in the form of a perpetual youth model make strictly no difference to the final consumption equation (17), as shown by Farhi and Werning (2019).[17] Financial frictions by themselves do not increase the extent of discounting of future interest rates in aggregate consumption. While financial frictions increase the extent of discounting in the consumption function, they also increase the slope of the Keynesian cross and the two effects exactly offset each other. However, under the combined assumptions of financial constraints, finite-planning horizons and forward-looking asset prices, consumption dynamics is affected. This is akin to the result in Farhi and Werning (2019) that the combined assumptions of financial constraints and bounded rationality generate discounting of future interest rates.[18]

Be it in its standard interpretation or its interpretation with financial constraints, the consumption bloc

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[16]This way of introducing discounting from borrowing constraints is distinct from the solution proposed by MacKay, Nakamura, and Steinsson (2016), which relies instead on precautionary savings. The perpetual-youth model abstracts from the precautionary savings motive.

[17]The strict equivalence only holds under our assumption that public debt is in zero net supply. It is of course lost with a positive net supply of public debt, since the Ricardian equivalence does not hold in the perpetual youth model.

[18]In the finite planning horizon set-up however, only under the assumption of forward-looking asset prices do financial constraints increase the extent of discounting of interest rates. Without forward-looking asset prices, the finite-planning horizon model reduces to (17) regardless of whether there are financial constraints.

of our economy assumes that, regardless of their planning horizons, households consume a small fraction of their transitory income: their marginal propensity to consume is $1 - \bar{\beta}$, which even when $\bar{\beta}$ is interpreted through the lens of financial constraints, is difficult to match with empirical estimates of the average marginal propensity to consume in the data. Therefore, both in its standard interpretation and its interpretation with financial constraints, our model tends to underestimate the slope of the Keynesian cross. This can raise the question of whether a model with a lower average marginal propensity to consume—and therefore a steeper Keynesian cross—could increase the extent of discounting $\bar{\beta}$ in the expression of aggregate consumption (17) under FPH-FLAP. Appendix C considers a simple extension to a model that generates a steeper Keynesian cross—a Two-Agent New-Keynesian (TANK) model—under FPH-FLAP and shows that it does not affect the extent of discounting of interest rates in equation (17).

3.4 The Phillips Curve

Because the Phillips curve does not include any nominal interest rate, its expression under our FPH-FLAP assumption remains unchanged relative to the plain FPH case where agents do not observe nominal interest rates considered by Woodford (2019). As re-derived in appendix D, agents with planning horizon h perceive inflation to solve:

$$\forall j = 0, \ldots, h-1, \pi_{t+j}^{h-j}(h) = \kappa(y_{t+j}^{h-j}(h) - y_{t+j}^e) + \beta \mathrm{E}_{t+j}(\pi_{t+j+1}^{h-j-1}(h)) + \nu_{t+j}^p, \tag{19}$$

and

$$\pi_{t+h}^0(h) = \kappa(y_{t+h}^0(h) - y_{t+h}^e) + \nu_{t+h}^p, \tag{20}$$

where β is the usual preference-based discount factor, y^e is the efficient level of production that fluctuates with exogenous productivity shocks, and ν^p is a cost-push shock. Equations (19) and (20) can be iterated forward to give:

$$\pi_t^h(h) = \mathrm{E}_t \sum_{j=0}^{h} \beta^j (\kappa(y_{t+j}^{h-j}(h) - y_{t+j}^e) + \nu_{t+j}^p). \tag{21}$$

4 Aggregation across Heterogeneous Planning Horizons

In this section, we make the additional assumption that planning horizons are distributed geometrically into the population and derive a simple expression for the FPH-FLAP baseline NK model in this case.

4.1 Assumptions on the Distribution of Planning Horizons

We aggregate the consumption and price-setting decisions of agents with different planning horizons h. We assume that all households and all firms face the same shocks and entertain the same preferences, and differ only through their planning horizons. Following Woodford (2019), we assume that planning horizons are distributed geometrically, with a fraction $(1-\rho)\rho^h$ of agents having a planning horizon h. Because the actual

realization at t of a variable x by agents with planning horizon h is $x_t^h(h)$, the aggregate variable x is given by:

$$x_t = (1 - \rho) \sum_{h=0}^{\infty} \rho^h x_t^h(h). \tag{22}$$

4.2 Matrix form of the model

Note that the expression of consumption (17) includes expectations of future inflation, while the expression of inflation (21) includes expectations of future consumption. This makes it impossible, in general, to aggregate each equation independently of the entire system.[19] In economic terms, an agent with planning horizon h still needs to have a model of the entire economy, even if incorrect, in order to form expectations.

Therefore we consider the system (17)-(21) as a block formed by the two endogenous variables (y, π), treating r_t like an exogenous variable for the moment. We can write the system perceived by agents with planning horizon h in recursive form, provided we specify two regimes. First, before horizon h

$$\forall j \leq h - 1, \ y_{t+j}^{h-j}(h) = \nu_{t+j}^y - \sigma(r_{t+j} - \mathrm{E}_{t+j}(\pi_{t+j+1}^{h-j-1}(h))) + \mathrm{E}_{t+j}(y_{t+j+1}^{h-j-1}(h) - \nu_{t+j+1}^y), \tag{23}$$

$$\pi_{t+j}^{h-j}(h) = \kappa(y_{t+j}^{h-j}(h) - y_{t+j}^e) + \nu_{t+j}^p + \beta \mathrm{E}_{t+j}(\pi_{t+j+1}^{h-j-1}(h)), \tag{24}$$

and at h

$$y_{t+h}^0(h) = \nu_{t+h}^y - \sigma r_{t+h} + \mathrm{E}_{t+h}(y_{t+h+1}(h)), \tag{25}$$

$$\pi_{t+h}^0(h) = \kappa(y_{t+h}^0(h) - y_{t+h}^e) + \nu_{t+h}^p, \tag{26}$$

while, after horizon h[20]

$$\forall j \geq h + 1, \ y_{t+j}(h) = -\sigma\bar{\beta} r_{t+j} + \bar{\beta} \mathrm{E}_{t+j}(y_{t+j+1}(h)), \tag{27}$$

$$\pi_{t+j}(h) = 0. \tag{28}$$

The first system is the same recursion as would hold under rational expectations, while the second system is a new recursion.

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[19] Aggregating equation by equation turns out to give the same result in purely forward-looking models such as the present one. Yet, such an equation-by-equation derivation would rely on properties of expectations that only hold in purely forward-looking models, in particular the property that expectations formed with the same number of periods left until the end of the planning horizon are the same across households with different planning horizons.

[20] We keep track of the superscript $h - j$ for horizons $j \leq h$ but drop it for horizons $j > h$. The variable $h - j$ turns negative past horizon $j = h$. While it remains meaningful—it counts how far beyond the agent's planning horizon h the horizon j is—keeping track of it is also unnecessary.

We can write the two systems (as well as the intermediate case at horizon h) in matrix form as:

$$\forall j \leq h-1, BY_{t+j}^{h-j}(h) = A\mathrm{E}_{t+j}(Y_{t+j+1}^{h-j-1}(h)) + f_a a_{t+j} + f_{a+}\mathrm{E}_{t+j}(a_{t+j+1}) + f_r r_{t+j}, \tag{29}$$

$$BY_{t+h}^0(h) = A\mathrm{E}_{t+h}(Y_{t+h+1}(h)) + f_a a_{t+j} + f_r r_{t+j}, \tag{30}$$

$$\forall j \geq h+1, B_2 Y_{t+j}(h) = A_2 \mathrm{E}_{t+j}(Y_{t+j+1}(h)) + f_{r,2} r_{t+j}, \tag{31}$$

where $Y_t = (y_t, \pi_t)'$, $a_t = (\nu_t^y, \nu_t^p - \kappa y_t^e)$ and the matrices are:

$$B = \begin{pmatrix} 1 & 0 \\ -\kappa & 1 \end{pmatrix}, A = \begin{pmatrix} 1 & \sigma \\ 0 & \beta \end{pmatrix}, f_a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, f_{a+} = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}, f_r = \begin{pmatrix} -\sigma \\ 0 \end{pmatrix}, \tag{32}$$

$$B_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, A_2 = \begin{pmatrix} \bar{\beta} & 0 \\ 0 & 0 \end{pmatrix}, f_{r,2} = \begin{pmatrix} -\bar{\beta}\sigma \\ 0 \end{pmatrix}. \tag{33}$$

Note that the system under the rational expectation assumption (infinite h) is formally identical to the first system, described by equation (29).

Since B and B_2 are invertible, the systems can be rewritten:

$$\forall j \leq h-1, \ Y_{t+j}^{h-j}(h) = C\mathrm{E}_{t+j}(Y_{t+j+1}^{h-j-1}(h)) + D^r r_{t+j} + D^a a_{t+j} + D^{a+}\mathrm{E}_{t+j}(a_{t+j+1}), \tag{34}$$

$$Y_{t+h}^0(h) = C\mathrm{E}_{t+h}(Y_{t+h+1}(h)) + D^r r_{t+h} + D^a a_{t+h}, \tag{35}$$

$$\forall j \geq h+1, \ Y_{t+j}(h) = C_2 \mathrm{E}_{t+j}(Y_{t+j+1}(h)) + D_2^r r_{t+h}, \tag{36}$$

where $C = B^{-1}A$, $D^r = B^{-1}f_r$, $D^a = B^{-1}f_a$, $D^{a+} = B^{-1}f_{a+}$, $C_2 = B_2^{-1}A_2$, and $D_2^r = B_2^{-1}f_{r,2}$.

4.3 Solution Procedure

To solve the baseline NK model under FPH-FLAP, we now derive our solution procedure. This solution procedure can be more generally applied to solve any model under FPH-FLAP, provided the model does not contain endogenous state variables—except possibly in the monetary-policy equation. The requirement is that the model can be written as a first recursion (34) until horizon h (which corresponds in principle to the rational-expectations recursion), then a second recursion (36) past horizon h, for a vector of variables of interest y_t that are forward-looking. Although the solution procedure requires the variables Y_t to be forward-looking, the model can possibly include state variables in the monetary-policy equation that describes the interest-rate rule for setting policy rates. It can also include exogenous state variables in a_t.

The system (34)-(36) can be iterated forward:

$$Y_t^h(h) = \mathrm{E}_t \left(\sum_{j=0}^{h} C^j D^r r_{t+j} + \sum_{j=h+1}^{\infty} C^{h+1} C_2^{j-(h+1)} D_2^r r_{t+j} + \sum_{j=0}^{h} C^j D^a a_{t+j} + \sum_{j=0}^{h-1} C^j D^{a+} a_{t+1+j} \right). \quad (37)$$

This block-expression has the advantage of including only exogenous shocks and the nominal interest rate on the right-hand side, so that it lends itself to aggregation. Aggregating according to the geometric distribution of forecast horizons, we have:

$$Y_t = \mathrm{E}_t \sum_{j=0}^{\infty} \left(\sum_{h=0}^{j-1} (1-\rho)\rho^h C^{h+1} C_2^{j-(h+1)} D_2^r + \sum_{h=j}^{\infty} (1-\rho)\rho^h C^j D^r \right) r_{t+j}$$

$$+ \mathrm{E}_t \sum_{j=0}^{\infty} (\rho C)^j D^a a_{t+j} + \mathrm{E}_t \sum_{j=0}^{\infty} (\rho C)^j \rho D^{a+} a_{t+j+1} \quad (38)$$

Denote:

$$\gamma_j = \sum_{h=j}^{\infty} (1-\rho)\rho^h C^j D^r = (\rho C)^j D^r, \quad (39)$$

$$\delta_j = \sum_{h=0}^{j-1} (1-\rho)\rho^h C^{h+1} C_2^{j-(h+1)} D_2^r. \quad (40)$$

These coefficients solve the recursions:

$$\gamma_j = \rho C \gamma_{j-1}, \text{with } \gamma_0 = D^r, \quad (41)$$

$$\delta_j = \rho C \delta_{j-1} + (1-\rho) C C_2^{j-1} D_2^r, \text{with } \delta_0 = 0. \quad (42)$$

So:

$$Y_t = \mathrm{E}_t \sum_{j=0}^{\infty} (\gamma_j + \delta_j) r_{t+j} + \mathrm{E}_t \sum_{j=0}^{\infty} (\rho C)^j D^a a_{t+j} + \mathrm{E}_t \sum_{j=0}^{\infty} (\rho C)^j \rho D^{a+} a_{t+j+1}$$

$$Y_t = D^r r_t + D^a a_t + \rho D^{a+} \mathrm{E}_t(a_{t+1})$$

$$+ (\rho C) \left(\mathrm{E}_t \sum_{j=1}^{\infty} (\gamma_{j-1} + \delta_{j-1}) r_{t+j} + \sum_{j=1}^{\infty} (\rho C)^j D^a a_{t+j} + \mathrm{E}_t \sum_{j=1}^{\infty} (\rho C)^j \rho D^{a+} a_{t+j+1} \right)$$

$$+ \mathrm{E}_t \sum_{j=1}^{\infty} (1-\rho) C C_2^{j-1} D_2^r r_{t+j} \quad (43)$$

$$Y_t = D^r r_t + D^a a_t + \rho D^{a+} \mathrm{E}_t(a_{t+1}) + \rho C \mathrm{E}_t(Y_{t+1}) + (1-\rho) C \mathrm{E}_t(z_{t+1}), \quad (44)$$

where

$$z_{t+1} = \mathrm{E}_{t+1} \sum_{j=1}^{\infty} C_2^{j-1} D_2^r r_{t+j}. \tag{45}$$

Premultiplying by B, equation (46) can be written:

$$BY_t = \rho A \mathrm{E}_t(Y_{t+1}) + f_a a_t + f_r r_t + \rho f_{a+} \mathrm{E}_t(a_{t+1}) + (1-\rho) A \mathrm{E}_t(z_{t+1}), \tag{46}$$

where z_t solves the recursion:

$$z_t = C_2 \mathrm{E}_t(z_{t+1}) + D_2^r r_t \tag{47}$$

This system can be solved using standard procedures, when complemented with an interest rate rule.

The recursion (46) can be compared to the recursion under rational expectations (29). A first difference is the appearance of the discount factor ρ in front of future expected variables. It is due to the assumption of finite planning horizons and is already present in Woodford (2019). But a second difference is the addition of the variable z_t which solves the recursion (47). It is how the assumption of forward-looking asset prices manifests itself. We restrict back to the baseline New Keynesian model to look at it more closely.

4.4   A Simple Expression for the FPH-FLAP Baseline NK Model

Applying the results to the baseline New Keynesian model, we get the following simple recursive writing for the FPH-FLAP New Keynesian model.

Proposition 1 *The FPH-FLAP New-Keynesian model solves the recursive system:*

$$y_t = \nu_t^y - \sigma(r_t - \rho E_t(\pi_{t+1})) + \rho E_t(y_{t+1} - \nu_{t+1}^y) - \sigma(1-\rho)E_t(\xi_{t+1}), \tag{48}$$

$$\pi_t = \nu_t^p + \kappa(y_t - y_t^e) + \beta\rho E_t(\pi_{t+1}), \tag{49}$$

$$\xi_t = \bar{\beta} r_t + \bar{\beta} E_t(\xi_{t+1}). \tag{50}$$

The additional discounting of the future at rate ρ is due to finite planning horizons, and is already present in the plain FPH model (1)-(2). Our FLAP assumption adds the new term in ξ. It captures the effect on consumption of nominal interest beyond the planning horizon of agents. While interest rates through this channel are not discounted at the discount rate ρ, they are still discounted but at the rate $\bar{\beta}$. This $\bar{\beta}$-discounting comes from the $\bar{\beta}$-discounting of rates beyond the planning horizon in equation (17), which itself comes from the $\bar{\beta}$-discounting of interest rates in the decision-theoretic consumption function.

The *consequences* of these future interest rate changes on future inflation and future activity (as well as the fundamental socks ν_t^c, ν_t^p and y_t^e) are however still discounted at the discount rate ρ. The FPH-FLAP model allows to distinguish between the discounting of future interest rates in the consumption function of agents—a decision-theoretic, or partial equilibrium feature—and the discounting of the future general-

equilibrium consequences of interest rate changes. Note that the model under FLAP still reduces to the rational expectations model when ρ tends toward 1.

5   Forward-Guidance Shocks and the Forward-Guidance Puzzle

In this section, we evaluate the power of forward guidance in the FPH-FLAP New-Keynesian model. We provide a formal definition of the forward-guidance puzzle and show that the plain FPH model and the FPH-FLAP avoid the forward-guidance puzzle under the same conditions on their calibrations. We then show that forward-guidance is much more powerful in the FPH-FLAP model.

5.1   Forward-Guidance Shocks Experiments

To analyze the power of forward guidance in the FPH-FLAP NK model (48)-(49)-(50), we consider the following experiment similar to the one considered by MacKay, Nakamura, and Steinsson (2016). We assume, when the economy is initially at its steady-state, the central bank makes the following forward-guidance announcement at t: it will cut its policy rate by 100 basis points in quarter $t + n$, while pegging policy rates to their steady-state value from t to $t + n - 1$ (not responding to inflation and output like its standard policy rule would prescribe). In period $t + n + 1$, the central bank reverts to its standard policy rule. We assume that the standard policy rule of the central bank does not respond to any lagged variable and satisfies the Taylor principle.[21] Under this assumption, the model is entirely forward-looking and the economy is back to steady-state at $t + n + 1$. For our forward-guidance experiments, we do not need to specify the standard monetary-policy rule any further.

While in practice the motivation of central banks for doing such forward-guidance announcements lies mainly in the existence of the effective lower bound (ELB) constraining current policy rates, in the present forward-guidance experiments we abstract from the ELB. We will explicitly take the ELB into account in the next section.[22]

5.2   Condition for Ruling Out the Forward-Guidance Puzzle

Before looking at numerical results for these forward-guidance experiments, we study analytically under which conditions the FPH-FLAP model is subject to the forward-guidance puzzle, and compare them to the conditions for the plain FPH model. As pointed out by Del Negro, Giannoni, and Patterson (2012) and Carlstrom, Fuerst, and Paustian (2015), standard DSGE models under rational expectations, including the baseline NK model make a peculiar prediction. They predict that the size of the stimulative effect to the

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[21]That the standard policy rule satisfies the Taylor principle is actually unnecessary to ensure the determinacy of the equilibrium when condition (51) below is satisfied, since in this case the FPH and FPH-FLAP models are determined under a peg.

[22]An advantage of abstracting from the ELB constraint is that in this case we do not face the problem of dealing with the ELB constraint in the plain FPH model. As we explain below, this problem only occurs with the plain FPH model, and not with the FPH-FLAP model.

an announcement to cut interest rates n periods into the future increases and explodes into infinity as the horizon n of the interest rate cut increases and tends to infinity.

To analyze whether the same issue arises with the plain FPH-FLAP model, we first introduce a precise definition of what we call (not) being subject to the forward-guidance puzzle.

Definition 1 *A model is not subject to the forward-guidance puzzle if the impact response of all endogenous variables to the forward-guidance announcement of an interest-rate cut in n periods converges to 0 as the horizon n of the interest rate cut increases to infinity.*

Because, bar knife-edge calibrations, when a DSGE model does not converges back to steady-state it typically explodes, our definition is essentially equivalent to saying that a model is subject to the forward-guidance puzzle if the impact response of endogenous variables explodes to infinity with the horizon n of the announcement of an interest-rate cut. Note that in our definition, not being subject to the forward-guidance puzzle still allows the impact effect of the announcement to increase with the horizon n over some values of n, provided it then decreases back to zero as n tends to infinity.

Proposition 2 *The FPH-FLAP model is not subject to the forward-guidance puzzle under the condition $\rho < \rho^\star$, where :*

$$\rho^\star = \frac{1 + \sigma\kappa + \beta - \sqrt{(1 + \sigma\kappa + \beta)^2 - 4\beta}}{2\beta}. \tag{51}$$

The same condition holds to rule out the FG puzzle in the plain FPH model.

Proposition 2 captures first that Woodford (2019)'s assumption of finite-planning horizons provides a solution to the forward-guidance puzzle. Provided the average planning horizon of agents (and so ρ) is small enough, the plain FPH model does not run into the forward-guidance puzzle. Second, and more relevant to our analysis, proposition 2 states that the FPH-FLAP model is equally immune to the forward-guidance puzzle: it runs into the forward-guidance puzzle exactly when the plain FPH model does.

Intuitively, the existence of the FG puzzle depends only on the strength of the general-equilibrium amplifying effects embedded in the model, not on the initial, decision-theoretic reaction of households and firms to interest rates. Because the general-equilibrium amplifying effects are the same in the FPH-FLAP and plain FPH models with same ρ, the FPH-FLAP and plain FPH models run into the FG puzzle for exactly the same values of ρ. Consistently, note that the parameter $\bar{\beta}$ does not intervene in the condition of proposition 2. While the power of forward guidance in the FPH-FLAP model depends on both discount rates ρ and $\bar{\beta}$, only the discount rate ρ that controls the discounting on the amplifying general equilibrium effects determines whether the model the FPH-FLAP model is subject to the forward-guidance puzzle.

Note that since the thought experiment used in the definition of being subject to the forward-guidance puzzle effectively corresponds to a peg until period $t + n - 1$, condition (51) also guarantees determinacy of the FPH-FLAP model under a peg.[23]

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[23]In our model with no learning, this further implies that the central bank could permanently set the nominal interest rate to a value distinct from the one consistent with the long-run Fisher equation (the steady-state real interest rate plus the current

Corollary 1 *If condition* (51) *is satisfied, both the FPH model and the FPH-FLAP model are determined under a peg, and more generally under any exogenous path for the nominal interest rate.*

5.3 Calibration

To consider the quantitative predictions of the FPH-FLAP NK model with respect to forward-guidance announcements—and later on make-up strategies—we set its parameters according to the following approach. We consider a quarterly calibration of the model. In our main specification, we set $\rho = 0.5$, i.e. an average planning horizon of one quarter. We do so for two reasons. First, this is in line with what Gust, Herbst, and Lopez-Salido (2022) estimate for ρ when estimating Woodford (2019)'s plain FPH model on US data.[24] Second, this strong extent of cognitive discounting means that we are considering an economy where firms and households are on average little forward-looking, i.e. an economy that does not in principle set the stage for large effects of forward-guidance.[25] In this section, we also provide the results on the effect of forward-guidance announcements for $\rho = 0.8$, i.e. an average planning horizon of 4 quarters (keeping all other parameters to the same values).

Other structural parameters are set to standard values. We set the value of β to correspond to a real annualized interest rate of 0.5%, consistent with recent evidence on the low steady state value of the natural real rate of interest. In our main specification we impose the representative-agent version of the model without financial frictions $\bar{\beta} = \beta$. We set the labor share to 0.7, i.e. $\phi = 1/0.7$. We set the elasticity of substitution between goods to $\theta = 6$. We set the inverse of the Frisch elasticity on labor supply ψ to 2. We set the intertemporal elasticity of substitution σ to 0.5. We set the Calvo probability α of not resetting one's price to correspond to an average duration of three quarters.

To use a calibration of the shocks that broadly matches US business cycles, we estimate them through Bayesian methods on US data. We use PCE inflation, output growth and the Fed Funds rate over the period 1985-2009. When doing so, we assume that the monetary policy is the following inflation-targeting rule with inertia:[26]

$$r_t = \rho_{TR} r_{t-1} + (1 - \rho_{TR})(\phi_\pi \pi_t^{1a} + \phi_y(y_t - y_t^e)) + \nu_t^r, \tag{52}$$

where π_t^{1a} is inflation over the past year, or 4 quarters. We calibrate $\rho_{TR} = 0.85$, $\phi_\pi = 1.5$ and $\phi_y = 0.125$. We assume that the three shocks ν_t^y, ν_t^p, ν_t^r follow AR(1) shocks with persistence ρ^y, ρ^p, ρ^r and standard deviations σ^y, σ^p, σ^r. We assume that technology shocks are zero, as they are redundant with cost-push shocks. We impose $\rho^r = 0$ and estimate all other shock parameters. Table 1 reports the calibration we obtain.

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inflation target) without the economy diverging into hyperinflation or hyperdeflation. Woodford (2019) shows that in a version of the plain FPH model with learning, if the nominal interest rate is pegged to a value inconsistent with the long-run Fisher equation, the economy does not converge to a stationary equilibrium but diverges into a cumulative process, as in Friedman (1968) and in contrast to neo-Fisherian reasoning.

[24]Gust, Herbst, and Lopez-Salido (2022) estimate an extension by Woodford of the plain FPH model that also features learning.

[25]Note that one quarter is only the *average* planning horizon. Planning horizons are heterogeneous in the model, so that even with $\rho = 0.5$ there are still some agents with arbitrarily large planning horizons.

[26]As mentioned above, in the forward-guidance experiments of this section, we then set the inertia parameter ρ_{RT} back to zero.

Table 1: Calibration

Parameter	Interpretation	Value
ρ	FPH	0.5000
β	Subjective discount factor	0.9988
θ	Substitution elasticity between intermediate goods	6.0000
ϕ	Inverse elasticity of production wrt labor	1.4286
ψ	Inverse Frisch elasticity	2.0000
σ	Intertemporal substitution elasticity	0.5000
α	Probability of not resetting prices	0.6600
ρ_{TR}	Interest rate smoothing	0.8500
ϕ_π	Policy response to inflation	1.5000
ϕ_y	Policy response to the output gap	0.1250
ρ_y	Persistence of preference shocks	0.9641
$100 \times \sigma_y$	Standard deviation of preference shocks	1.1490
ρ_r	Persistence of monetary policy shocks	0.0000
$100 \times \sigma_r$	Standard deviation of monetary policy shocks	0.1460
ρ_p	Persistence of cost-push shocks	0.3830
$100 \times \sigma_p$	Standard deviation of cost-push shocks	0.2443

Note: The calibration is quarterly.

For this calibration, the threshold value in Proposition 2 above which the FLAP and plain FPH models are subject to the forward-guidance puzzle is $\rho^\star = 0.86$. The values $\rho = 0.5$ and $\rho = 0.8$ we consider are therefore below $\rho^\star$ and do not subject the model to the forward-guidance puzzle.

5.4 Response to Forward-Guidance Shocks

Figure 1 plots the effect on output (top panel) and inflation (bottom panel) on impact at date t of an announcement to cut policy rates by 100p in n quarters, as a function of the horizon n. As mentioned above, in these forward-guidance experiments we want to consider a purely forward-looking interest-rate rule so that the economy returns to steady-state after the interest-rate peg, so we set $\rho_{TR} = 0$. Each panel plots the result in both the FPH-FLAP and the plain FPH versions of the model, and under both $\rho = 0.5$ and $\rho = 0.8$.

The comparison of the FPH-FLAP and plain FPH versions of the model under $\rho = 0.5$ provides a clear illustration of the importance of our FLAP assumption that asset prices are forward-looking. In the plain FPH version of the model (displayed with the plain blue line), the impact effect of FG announcements on output and inflation decreases very quickly with the horizon of the announcement, reaching practically zero for a horizon of 10 quarters. In the FPH-FLAP version of the model (displayed with plain red line) in contrast, the effect of forward guidance announcements decreases almost imperceptibly within the 50-quarter horizons plotted on Figure 1. (It does ultimately decrease to zero, as proposition 2 guarantees).

The effect is actually increasing in the horizon of the announcement at small horizons, particularly so for inflation. This initially increasing pattern is a meaningful consequence of the NKPC. Because a contemporaneous interest-rate cut today at t stimulates aggregate demand and increases marginal costs today only, firms that can reset their prices at t have little incentive to increase their prices in response. But because an interest-rate cut n quarters in the future stimulates aggregate demand and increases marginal costs from t to $t+n$, firms that can reset their prices at t have a stronger incentive to increase their prices today in response to a forward-guidance announcement at relatively short horizons. Yet, the FPH-FLAP model (with a value of ρ lower than $\rho^\star$) does not feature the exploding property of the forward-guidance puzzle: ultimately, the effect on inflation an output shrinks to zero as the horizon n tends to the infinite future.
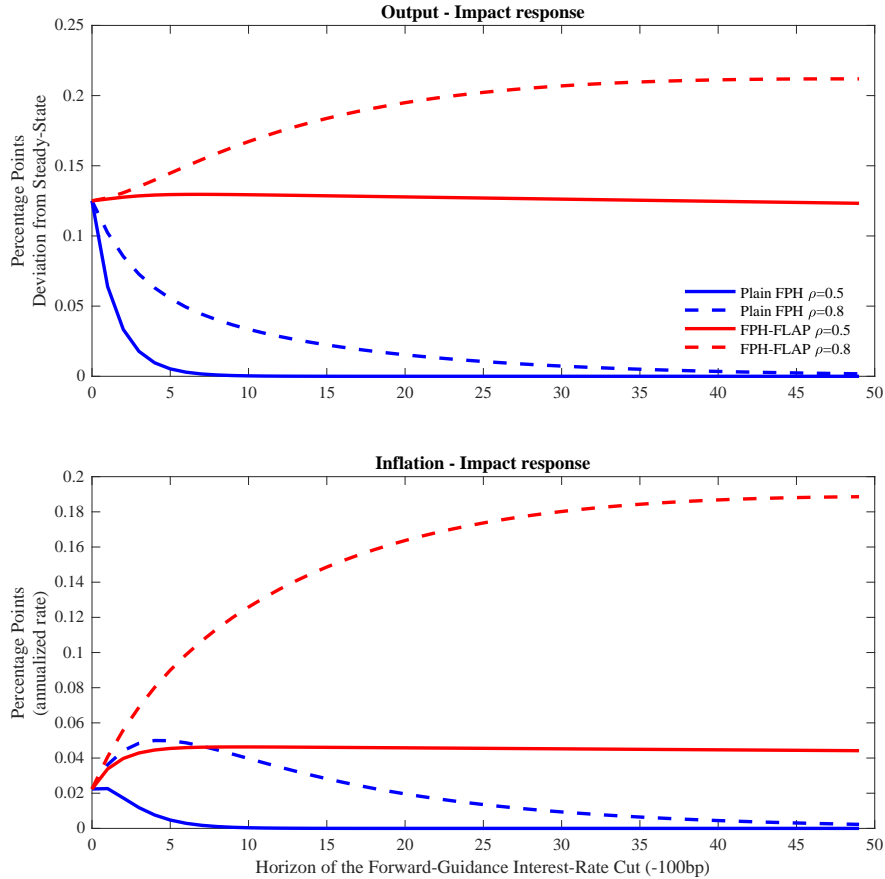
Figure 1: Impact Effects of a FG Announcement as a Function of the Horizon of the Announcement

Note: A FG announcement corresponds to the announcement of the following policy: cutting the policy rate by 100bp basis points in quarter $t+n$, while pegging the policy rate to its steady-state value from t to $t+n-1$ and reverting to the standard policy rule at $t+n+1$. "Plain FPH" refers to the version of the model without rational expectations on nominal interest rates. "FPH-FLAP" refers to the version of the model with rational expectations on nominal interest rates.

The results for $\rho = 0.8$ plotted (in dashed lines) on Figure 1 show that the increasing impact of forward-guidance announcements with the horizon of the announcement at short horizons is more pronounced as ρ increases and approaches the threshold value $\rho^\star$. At such a value of ρ, even in the plain FPH model the effect of a forward-guidance announcement on inflation is increasing in the horizon of the announcement at first, although only until an announcement of 5 quarters ahead.

To look at the effect of a forward-guidance announcement beyond its effect on impact, Figure 2 plots the entire impulse response function of output (top panel) and inflation (bottom panel) to the forward guidance shock experiment described above, for the specific announcement of a future cut in rates in $n = 5$ quarters. In the plain FPH model (displayed with the plain blue curve), the effect on output and inflation starts from virtually zero on impact and increases to peak at the time of the interest-rate cut. In the FPH-FLAP model in contrast, the effect on output is about the same from the date of the announcement t to the date of the interest-rate cut $t+5$, as the implicit long-term interest rate is reduced by virtually the same amount from
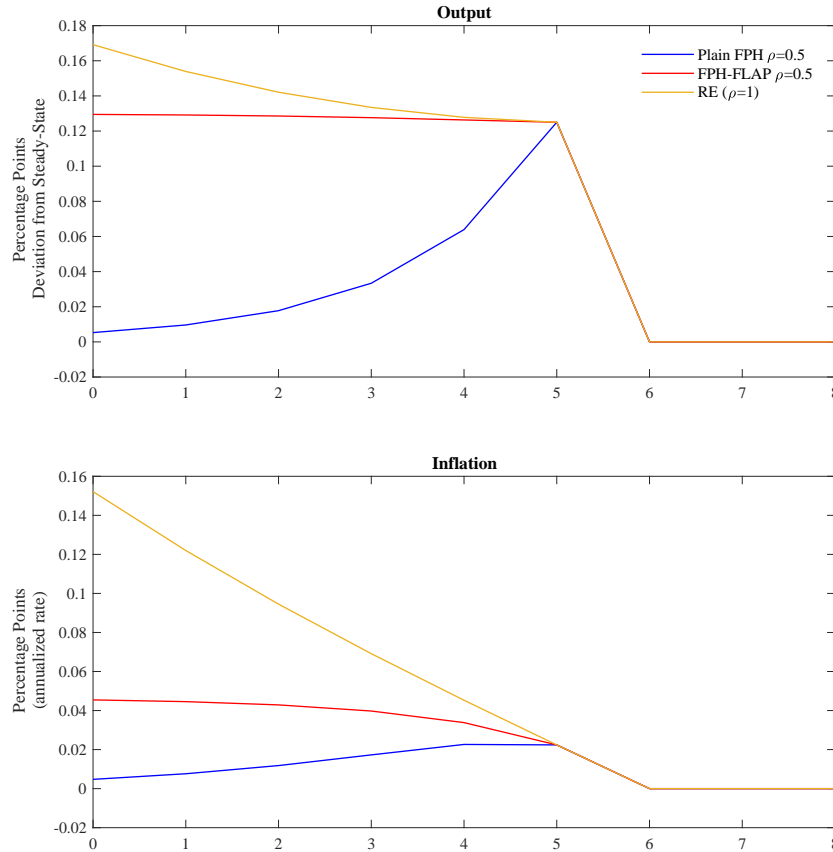
Output

Inflation

Figure 2: IRF to a FG Announcement to Cut Policy Rates Five Quarters in the Future

Note: The FG announcement corresponds to the announcement of the following policy: cutting the policy rate by 100 basis points in quarter $t + 5$, while pegging the policy rate to its steady-state value from t to $t + 4$ and reverting to the standard policy rule at $t + 6$. "Plain FPH" refers to the version of the model without rational expectations on nominal interest rates. "FPH-FLAP" refers to the version of the model with rational expectations on nominal interest rates. "RE" refers to the version of the model under rational expectations (which coincides with both the FPH and FPH-FLAP models for $\rho = 1$).

t to $t + 5$. The effect is therefore about the same as the one a contemporaneous interest rate cut at t would have. Figure 2 also plots for comparison the impulse response functions in the version of the model under rational expectations. Under rational expectations, the responses of output to the announcement of the future interest rate cut is larger, and the response of inflation much larger, than the responses to a current interest rate cut—the forward guidance puzzle. Overall, these FG experiments show that FG is effective in our set up even with finite planning horizons owing to the forward looking asset-price channel, but without running back into the unrealistic predictions of the rational-expectations model.
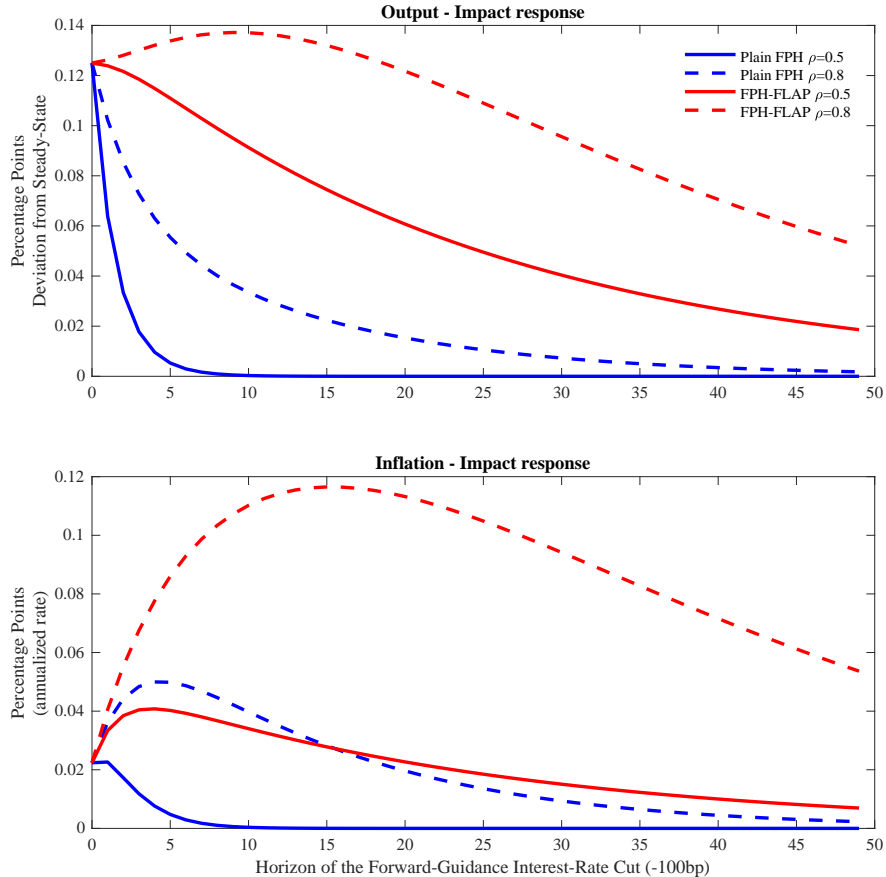
23

Figure 3: Impact Effects of a FG Announcement as a Function of the Horizon of the Announcement, with Financial Frictions

Note: A FG announcement corresponds to the announcement of the following policy: cutting the policy rate by 100 basis points in quarter $t + n$, while pegging the policy rate to its steady-state value from t to $t + n - 1$ and reverting to the standard policy rule at $t + n + 1$. "FPH" refers to the version of the model without rational expectations on nominal interest rates. "FPH-FLAP" refers to the version of the model with rational expectations on nominal interest rates.

5.5 Robustness to Stronger Discounting from Financial Frictions

Figures 1 and 2 consider the effect of forward-guidance announcements in our main model, where the extent of discounting $\bar{\beta}$ in equation (50) corresponds to the preference discount factor β of households, and as such is very close to one. (We calibrate it to correspond to an annual real interest rate of 0.5%). As shown in section (3.3), an extension of the model with perpetual youth yields the same equations (48)-(49)-(50)- as our main model, except $\bar{\beta}$ is now the product of the preference discount factor β and the survival probability of a household $1 - \lambda$. This leaves the scope for $\bar{\beta}$ to be calibrated to a lower value, attenuating the effect of forward-guidance far into the future. As argued by Nistico (2016); Del Negro, Giannoni, and Patterson (2012); Farhi and Werning (2019), one minus the survival probability can be interpreted as the probability for a household of hitting its borrowing constraint, in which case it is financial frictions that attenuate the effect of forward guidance.

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To see to what extent financial frictions attenuate the strong effects of forward guidance announcement that we find in our main model, Figure 3 plots the same impact effect of FG announcements as on Figure 1 for the version of the model with financial frictions. To calibrate the latter, we follow Del Negro, Giannoni, and Patterson (2012) and Farhi and Werning (2019) and set $\bar{\beta} = 0.96$ quarterly, to correspond to a probability of hitting the borrowing constraint of about 15% annually. As can be seen on the figure, the decline of the impact effect with the horizon of forward-guidance is naturally stronger in the calibration of the FPH-FLAP model. Yet, it remains substantial even at long horizons and much stronger than in the plain FPH model. Because the FPH-FLAP model discounts strongly only the general-equilibrium consequences of the future interest rate cut, but only moderately the interest rate cut itself (in a plausible calibration), forward guidance retains much effect even with extra discounting arising from financial frictions.

6 Evaluating Make-Up Strategies

In this section, we assess the effect of various make-up strategies in our FPH-FLAP New-Keynesian model through stochastic simulations. We simulate the model under various make-up strategies and report their implications for the averages and standard deviations of inflation, the output gap, and the policy rate, and the frequency and average duration of ELB episodes.

6.1 Specifying Make-up Strategies

The forward-guidance announcements we considered in the previous section seek to address the (implicit) constraint of the ELB by promising lower-for-longer policy rates when the economy is at the ELB. They do so in a specific way: by deviating occasionally from an otherwise regular inflation-targeting strategy. Make-up strategies offer an alternative way of promising lower-for-longer policy rates when the economy is at the ELB: by changing instead the monetary policy strategy. Lower-for-longer policy rates at the ELB are then a feature of the strategy and requires no deviation from it.[27]

Proposals for make-up strategies are numerous. They include average-inflation targeting (AIT) and price-level targeting (PLT). Plain vanilla versions of AIT and PLT commit to history-dependence at all times (regardless of whether the economy is at the ELB or not) and in a symmetric way (both when inflation has undershot or overshot its target in the past). Yet, a make-up strategy does not need to be history-dependent outside the ELB nor to be symmetric to qualify as a make-up strategy. For instance, Bernanke (2017)'s Temporary Price-Level Targeting (TPLT) is a make-up strategy that only promises to remain accommodative when inflation has undershot its target and the economy has hit the ELB.[28] It does not promise to remain accommodative when the economy has not hit the ELB, nor does it symmetrically promise to remain restrictive when inflation has overshot its inflation target. Similarly, the current monetary policy strategy of the

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[27]In the parlance of Campbell, Evans, Fisher, and Justiniano (2012), these forward-guidance shocks are more strictly speaking *Odyssean* forward-guidance—a commitment to deviate in the future from the central bank's usual strategy. Make-up strategies turn the communication of a lower-for-longer policy at the ELB into *Delphic* forward-guidance—a feature of policy that can be inferred from the knowledge of the future state of the economy and the knowledge of the strategy of the central bank. See e.g. Evans (2017).

[28]Evans (2010) defended such a policy to the FOMC during the Great Recession.

Federal Reserve adopted in 2020, which is widely regarded as a make-up strategy, is only a commitment to remain accommodative when inflation has undershot its target, not to remain restrictive when the economy has overshot it (Federal Reserve, 2020).[29]

In our simulations, we consider both types of make-up strategies, which we define through interest-rate rules that describe how the target policy rate r_t^* is set as a function of endogenous variables. Monetary policy sets the policy rate equal to the target rate assigned by the interest-rate rule, unless the ELB prevents it from doing so:

$$r_t = \max\{\text{elb}, r_t^*\}. \tag{53}$$

Since the calibration of our model is informed by US data, we calibrate the ELB to 0%. This implies that in log-deviations terms, the ELB is binding when r_t is by r^* percentage points below its steady-state value of r^*. The steady-state value of r^* is the sum of the steady-state value of the natural real rate, which we calibrated to 0.5%, and the inflation target, which we calibrate to 2%.

We compare make-up strategies to an inflation-targeting (IT) benchmark which we define as the interest-rate (Taylor) rule (52), except that it now defines only the target policy-rate r_t^*:

$$r_t^* = \rho_{TR} r_{t-1} + (1 - \rho_{TR})(\phi_\pi \pi_t^{1a} + \phi_y(y_t - y_t^e)) + \nu_t^r, . \tag{54}$$

Note that the inertia assumed in this Taylor rule, while most standard, implies that the IT benchmark already embeds some history-dependence, at least when away from the ELB. Indeed, iterating (54) backward abstracting from the constraint of the ELB (53) we get:

$$r_t^* = \phi_\pi \bar{\pi}_t + \phi_y \bar{x}_t + \sum_{k=0}^{\infty} \rho_{TR}^k \nu_{t-k}^r, \tag{55}$$

where $\bar{\pi}_t = \sum_{k=0}^{\infty}(1 - \rho_{TR})\rho_{TR}^k \pi_{t-k}^{1a}$ and $\bar{x}_t = \sum_{k=0}^{\infty}(1 - \rho_{TR})\rho_{TR}^k x_{t-k}$.

The expression (55) stresses that the IT rule (54) can be seen as responding to an average of lagged inflation rates and an average of lagged output gaps, calculated over an infinite past with geometrically declining weights. As such, the IT rule (54) can be seen as already embedding some elements of a make-up strategy.[30] The equivalence between history-dependence and inertial policy rules is indeed emphasized in Woodford (2003)'s initial case for make-up strategies. In what follows however, we follow the literature on the evaluation of make-up strategies and reserve the term *make-up strategies* to policy rules that are more history-dependent than the IT rule (54).

To define AIT and PLT, we follow the specifications used in the evaluation of make-up strategies in the strategic reviews of the Fed (Hebden, Herbst, Tang, Topa, and Winkler, 2020) and the ECB (ECB, 2021b).

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[29] *"The Committee [...] judges that, following periods when inflation has been running persistently below 2 percent, appropriate monetary policy will likely aim to achieve inflation moderately above 2 percent for some time.", (Federal Reserve, 2020).*

[30] The equivalence between the policies (54) and (55) is broken when taking into account the ELB constraint (53), since the lagged interest r_{t-1} then no longer records past inflation and output gap misses. We consider the properties of the policy-rule (55) and variants of it in appendix (F).

AIT is defined as the interest-rate rule:

$$r_t^* = \rho_{TR}r_{t-1} + (1 - \rho_{TR})(\pi_t^{1a} + \phi_y(y_t - y_t^e) + (\phi_\pi - 1) \times T \times \pi_t^{Ta}) + \nu_t^r. \tag{56}$$

where π_t^{Ta} is average inflation over the past T years:

$$\pi_t^{Ta} = \frac{1}{T}\sum_{k=0}^{4T-1}\pi_{t-k}. \tag{57}$$

Note that this AIT rule falls back on the inflation targeting rule (54) for $T = 1$.[31] We consider this AIT strategy for $T = 4$ and $T = 8$ years.

PLT is defined as the interest-rate rule:

$$r_t^* = \rho_{TR}r_{t-1} + (1 - \rho_{TR})(\pi_t^{1a} + \phi_y(y_t - y_t^e) + (\phi_\pi - 1) \times p_t) + \nu_t^r. \tag{58}$$

where p_t is the price level:

$$p_t = \pi_t + p_{t-1}. \tag{59}$$

Note that the AIT rule (56) converges to the PLT rule (58) when T tends to infinity.

Note that these specifications of the AIT and PLT rules assign a particular strength with which monetary policy responds to average inflation and to the price level, uniquely pinned down for a given parameterization of ρ_{TR}, ϕ_y and ϕ_π in the inflation targeting rule (54). Our results on the stabilization properties of AIT and PLT are specific to this particular specification used in the Fed's and ECB's strategic reviews.

While we wish to stick to the specifications used in the Fed's and ECB's strategic reviews, we do consider one variant specification of the AIT rule (56). The AIT rule (56) (and PLT rule (58)) do not simply introduce history-dependence in the interest-rate rule relative to IT. They are also overall more responsive to economic conditions than IT, since the sum of the coefficients on π_{1a} and π_{Ta} is greater than ϕ_π. The AIT rules are also all the more responsive as the window T over which average inflation is calculated increases.

To isolate the sole effect of history-dependence, we also consider the following AIT rules where the overall coefficient on inflation remains constant across AIT rules with different windows T, and so in particular is the same as the in the IT rule (54).[32]

$$r_t^* = \rho_{TR}r_{t-1} + (1 - \rho_{TR})(\pi_t^{1a} + \phi_y(y_t - y_t^e) + (\phi_\pi - 1) \times \pi_t^{Ta}) + \nu_t^r. \tag{60}$$

We refer to the rule (60) as the AIT rule with fixed coefficient. We again consider it for $T = 4$ and $T = 8$

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[31]We use a weight $(\phi_\pi - 1) \times T = 0.5 \times T$ on π^{1a}, when the strategic reviews of the Fed and the ECB use a weight T instead. We do so in order for the AIT rule to fall back on the inflation targeting rule (54) for $T = 1$. Accordingly, we also use a weight $(\phi_\pi - 1) = 0.5$ on p_t in our PLT rule (58) when the strategic reviews of the Fed and the ECB use a weight of 1.

[32]At bottom, there is no obvious way to translate a strategy of average inflation targeting or price level targeting into an instrument (interest-rate) rule. Indeed, AIT and PLT are initially defined as *target rules*–an objective of stabilizing average inflation or the price level to a target. There is no obvious mapping between these objectives and how to set instruments (here the policy rate) in order to achieve them. The same remark applies to standard inflation targeting. On the difference between target rules and instrument rules, see e.g. Svensson (2020).

years. Considering the PLT rule (58) under the same overall responsiveness to inflation is not feasible.

As an example of a make-up strategy that is not symmetric, and only provides lower-for-longer policy rates when the economy has hit the ELB but not otherwise, we consider the Reifschneider Williams (RW) rule proposed by Reifschneider and Williams (2000):

$$r_t^* = r_t^{IT} - \alpha_{RW} Z_t, \tag{61}$$

$$Z_t = Z_{t-1} + d_t, \tag{62}$$

$$d_t = r_t - r_t^{IT}, \tag{63}$$

where r^{IT} is the interest rate given by the IT rule (54). We follow the parameterization in Bernanke, Kiley, and Roberts (2019) and set $\alpha_{RW} = 1$. This rule stipulates that the target policy rate is the same as under the IT policy (54) when the economy has been away from the ELB for a long time, but adds history-dependent when the economy hits the ELB, recording past deficits of accommodation in the variable Z_t. It effectively promises lower-for-longer policy rates when the economy hits the ELB, but is otherwise equivalent to IT.

6.2 Simulation Design and Solution Method

We simulate the model under these various monetary policy rules for 500 simulations of 300 quarters each (after discarding a burn-in sample of 200 quarters). We calibrate the model according to Table 1, except that we set monetary policy shocks to zero $\sigma_r = 0$ and multiply the standard deviation of demand shocks σ_y by 1.5. We do the latter in order to take into account greater demand shocks relative to the 1985-2009 sample coinciding to a large extent with the Great Moderation period. We take explicitly into account the ELB in our main simulations, which makes the model piece-wise linear. We solve it through a variant of the Occbin approach (Guerrieri and Iacoviello, 2015).

We provide simulation results for the FPH-FLAP model, as well as for the plain-FPH and rational expectations versions of the model. When simulating the latter two, calibrating the same volatility of the shocks as in the FPH-FLAP model would result in unrealistically volatile business cycles. Therefore, when simulating them, we first rescale the standard deviations of the shocks σ_r and σ_p in the following way. We multiply both standard deviations by a factor ζ chosen so that the standard deviation of inflation remains the same as in the FPH-FLAP model when simulating the models without the ELB. The rescaling factor ζ can be obtained analytically.[33]

Note that an advantage of our FPH-FLAP model is that under FPH-FLAP the inclusion of state variables in the policy rules adds no complication. The system (48)-(49)-(50) is indeed valid regardless of the monetary-policy rule, as its derivation—which made no assumption on the determinants of interest rates—showed. This is not the case of the plain FPH system (1)-(2), which is only valid under a monetary policy rule featuring only forward-looking variables. In addition, in the plain FPH model taking into account the ELB adds

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[33]See Appendix G.

complications, since agents with different planning horizons do not in general perceive the ELB to be binding in the same periods of the future. In the FPH-FLAP model in contrast, only one expectation of future policy rates is relevant—the rational expectation formed by financial-market participants—so that the ELB constraints can be handled with standard Occbin-like approaches.

Because we wish to compare our simulation results for the FPH-FLAP model to the results under plain FPH, we rely on the following approximation to simulate the plain FPH model: we simulate it by considering the system (1)-(2) and (53), although the interest-rate rules of the make-up strategies do include state variables and equation (53) only approximates the constraint of the ELB under plain FPH.

6.3 Stabilization Properties of Alternative Strategies

Panel A of Table 2 documents the stabilization properties of our various make-up strategies in the FPH-FLAP model by reporting the averages and root mean square deviations (RMSD) from their steady-state levels of inflation, the output gap and policy rate (with in the case of inflation the steady-state corresponding to the inflation target). The Table reports as well the frequency and average duration of ELB episodes. Consider first the benchmark of inflation targeting. Under inflation targeting, the economy is at the ELB about 12% of the time, through episodes of 10.6 quarters on average. Because of the downward constraint on policy rates, policy rates are above natural rates on average, by 26 basis points (bp). As a result, the ELB creates a deflationary bias, with inflation 9bp below its intended target on average, which also manifests itself as a negative bias on the output gap, at -26bp. As a result also, inflation and output are more volatile than they would be absent the ELB constraint. The RMSD of inflation is for instance 1.36pp, while it would be 1.18pp absent the ELB (results in the no-ELB case are reported in Table 4).

Turning to make-up strategies, consider first (in the second row of Table 2) the RW rule, which makes lower-for-longer policy rates at the ELB a systematic feature of monetary policy. The lower-for-longer feature of the RW policy at the ELB reduces the RMSD of inflation and the output gap relative to IT, by about 7%. The policy eliminates the deflationary bias caused by the ELB and, for our calibration of the parameters, even turns the bias positive, as it leads to inflation overshoots after ELB episodes. This improved performance is achieved by spending more time at the ELB: the share of the time spent at the ELB increases to about 20%, and the average duration of ELB episodes increases sharply to 26 quarters, i.e. 6.5 years. The RMSD of policy rates increases as a result relative to IT, from 2.11pp to 2.17pp. The better stabilization properties of the RW rule are also apparent from Figure 4, which plots the response of inflation and the interest rate to a large demand shock that brings the economy to the ELB under the various policy rules. Under the RW rule, the economy spends about 35 quarters at the ELB, against about 25 under IT. Because this future accommodation (implicitly) produces lower long-term interest rates in the FPH-FLAP model, the departure of inflation from target is smaller throughout all the simulation, consistent with the lower RMSD of inflation.

Consider now the AIT and PLT rules, which introduce history-dependence both at and away from the ELB. They reduce the RMSD of inflation and the output gap further down, to 1.20pp under AIT-4-year to 1.08pp under PLT for inflation. Notice however that the better stabilization properties of these rules is

Table 2: Policy Simulation Results

Panel A: Model FLAP

	ELB frequency (percent)	Mean duration of ELB (quarters)	Mean output gap	Mean inflation gap	Mean nominal interest rate gap	RMSD. output gap	RMSD. inflation	RMSD. nominal interest rate
IT	12.34	10.59	-0.26	-0.09	0.26	3.43	1.36	2.11
RW	19.58	26.40	0.16	0.06	0.17	3.11	1.26	2.17
AIT 4 years	14.29	14.97	-0.31	-0.11	0.26	2.98	1.20	2.17
AIT 8 years	14.63	15.96	-0.26	-0.09	0.24	2.76	1.12	2.15
Price level targeting	13.26	13.39	-0.02	-0.00	0.21	2.64	1.08	2.07
AIT 4 years − Fixed coeff	12.07	11.84	-0.24	-0.08	0.24	3.43	1.36	2.06
AIT 8 years − Fixed coeff	11.50	11.16	-0.20	-0.07	0.23	3.43	1.36	2.03

Panel B: Model FPH (Rescaled Shocks)

	ELB frequency (percent)	Mean duration of ELB (quarters)	Mean output gap	Mean inflation gap	Mean nominal interest rate gap	RMSD. output gap	RMSD. inflation	RMSD. nominal interest rate
IT	16.91	14.60	-0.02	-0.01	0.46	3.58	1.22	2.53
RW	33.80	57.95	0.07	0.02	0.10	3.63	1.24	2.63
AIT 4 years	23.85	21.68	-0.11	-0.04	0.80	3.49	1.19	3.19
AIT 8 years	28.89	31.78	-0.19	-0.07	1.10	3.45	1.17	3.75
Price level targeting	-	-	-	-	-	-	-	-
AIT 4 years − Fixed coeff	16.87	15.71	-0.02	-0.01	0.44	3.60	1.23	2.51
AIT 8 years − Fixed coeff	16.46	15.41	-0.01	-0.01	0.42	3.61	1.23	2.46

Panel C: Model RE (Rescaled Shocks)

	ELB frequency (percent)	Mean duration of ELB (quarters)	Mean output gap	Mean inflation gap	Mean nominal interest rate gap	RMSD. output gap	RMSD. inflation	RMSD. nominal interest rate
IT	6.67	11.94	-0.20	-0.20	0.11	1.30	1.82	1.64
RW	7.97	15.79	0.01	0.06	0.10	0.69	1.13	1.64
AIT 4 years	1.61	10.42	-0.01	-0.00	0.04	0.52	0.50	1.15
AIT 8 years	0.50	8.36	0.00	0.00	0.03	0.50	0.36	0.97
Price level targeting	0.15	5.52	0.00	-0.00	0.02	0.50	0.36	0.85
AIT 4 years − Fixed coeff	5.17	12.52	-0.10	-0.09	0.08	1.08	1.45	1.51
AIT 8 years − Fixed coeff	3.67	9.67	-0.03	-0.01	0.07	0.91	1.18	1.41

Note: Results from the simulations of the FPH-FLAP model, plain FPH model, and rational expectations (RE) model under alternative policy rules, based on 500 simulations of 300 quarters each (after having discarded a burn-in sample of 200 quarters). RMSD stands for Root Mean Square Deviation, IT for Inflation Targeting, AIT for Average Inflation Targeting, PLT for Price Level Targeting, RW for the Reifschneider-Williams rule. Results are expressed in percentage points of annualized rates. The FPH-FLAP models and plain FPH models are simulated under the calibration of table 1. The plain FPH and RE models are simulated under the same calibration, up to rescaling the shocks in order to yield the same standard deviation of inflation as in the FPH-FLAP model when simulated without the ELB constraint. We do not provide results for the PLT rule in the plain FPH model as the simulations do not converge in less than 72 hours on a computer with 54 cores, while they converge within a few minutes for all other policies.

not a feature of their make-up nature alone. Indeed, as mentioned above, these rules are also overall more responsive to economic conditions, which mechanically makes them more stabilizing. The AIT rules with fixed coefficients, which are designed to remain as responsive to inflation as the IT rule and to only embed more history-dependence, reduce the RMSD of inflation and the output gaps by less than one basis point. If assessed through the RMSD of inflation and the output gap, whether AIT outperforms an asymmetric strategy like the RW rule can therefore depend on the precise parameterization of AIT.

The lower RMSD of inflation and the output gap under the RW, AIT and PLT rules is in contrast with the results in the plain FPH version of the model, shown in Panel B of Table 2. In the plain FPH version of the model, while the economy spends about the same time at the ELB under IT as in the FPH-FLAP version, the deflationary spirals at the ELB are much less potent, and as a result the deflationary gap on inflation and output gap is much smaller, at a couple of basis points. Furthermore, both the RW rule and the AIT rules with fixed coefficients have an almost imperceptible effect on inflation and output gap RMSE relative to IT, despite imposing longer ELB episodes. The RMSD even slightly increases under the RW rule, as the overshooting after ELB episodes more than compensates for the higher inflation it induces during the ELB episode. Note that for the RW rule, this allows to eliminate the negative bias on mean inflation and mean output gap. Yet whether eliminating this bias is valuable is debatable, since it only comes through adding departures from target from above after the ELB episodes, with no reduction of the departures from target from below during the ELB episodes.

These poor stabilization properties of make-up strategies in the plain FPH model can also be seen through the IRF to a demand shock on Figure 4. In the plain FPH model, the RW and AIT-4-year rule make the interest rate stay at the ELB for even longer than in the FPH-FLAP model. But since this future accommodation has almost no impact on inflation today, the inflation trajectory is almost indistinguishable from the one under IT. Under the RW rule, the main effect is to create some inflation overshooting after the ELB episode without increasing inflation much during the recession, resulting in a larger RMSD of inflation under the RW rule in table 2.

At the same time, the stabilization benefits of make-up strategies in the FPH-FLAP models are substantially lower than found in the RE version of the model, reported in Panel C of Table 2. In the version of the model under rational expectations, switching to the RW rule reduces the RMSD of inflation by about at third relative to IT, and shifting to AIT-4-year divides it by close to 4. The AIT policies are even so powerful that the time spent at the ELB is *lower* under AIT than under IT in equilibrium. Under RE, even the AIT rules with fixed coefficients, which had virtually no effect in the FPH-FLAP model, significantly reduce the RMSD of inflation and the output gap relative to IT.

These very large stabilization benefits can also be seen through the IRF to a negative demand shock on Figure 4. Because of the strength of the amplifying general equilibrium effects in the RE version of the model, when large negative shocks occur, the deflationary spiral under IT is much stronger in the RE model than in the FPH-FLAP model (despite the shocks being rescaled to reproduce the same volatility of inflation absent the ELB). As a result, shifting to either the RW rule or the AIT-4-year rule considerably reduces the size of deflationary spirals. Under AIT-4-year, the future accommodation even allows to avoid hitting the ELB
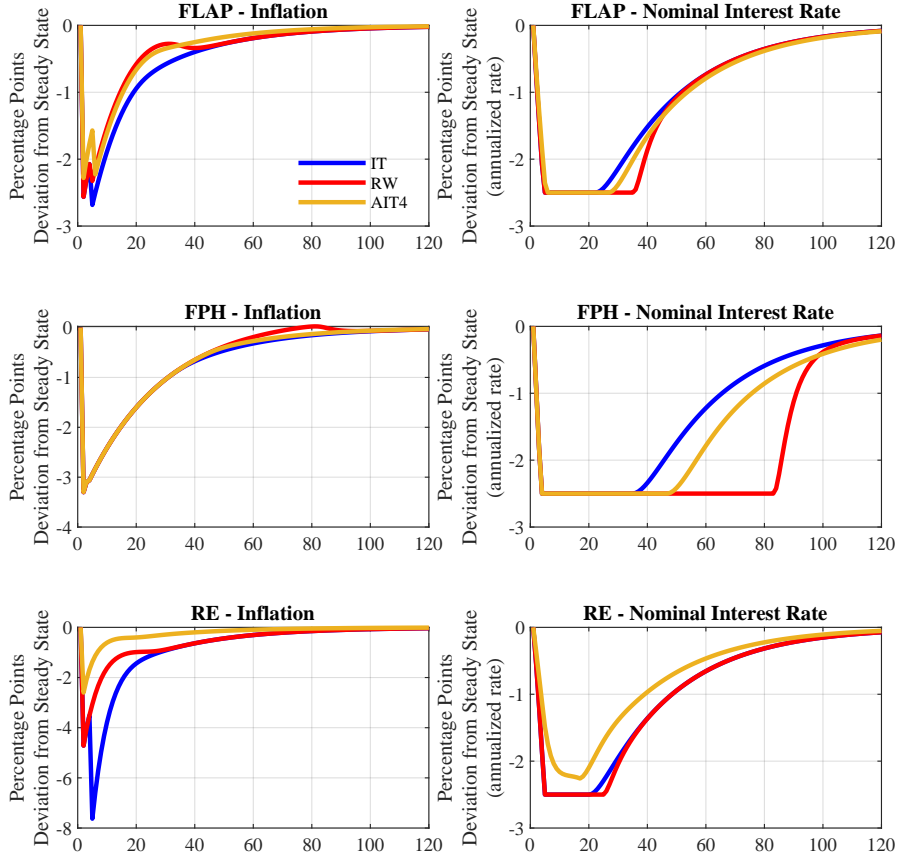
Figure 4: IRF to a Demand Shock Bringing the Economy to the ELB

Note: "FPH" refers to the version of the model without rational expectations on nominal interest rates. "FLAP" refers to the version of the model with rational expectations on nominal interest rates. "RE" refers to the version of the model with rational expectations. In each model, the size of the shock is taken to be $2/\sqrt{1-\rho_y^2}$ times the standard deviation of demand shocks in the simulations.

constraint. These benefits of makeup strategies under RE are however suspiciously large, as the RE model is subject to the FG puzzle.

Overall, the plain FPH model would lead one to conclude that make-up strategies make no significant improvement over IT, while the RE model would lead one to be extremely optimistic about their performance. The FPH-FLAP model finds that they make an improvement, although a more moderate—and arguably more realistic—one.

6.4 Extra Performance of AIT over IT with and without the ELB Constraint

The current interest in make-up strategies has been largely driven by their ability to mitigate the deflationary spirals that can occur when the economy reaches the ELB under inflation targeting. The FPH-FLAP model confirms that both the RW rule and the AIT rules are indeed able to mitigate the fall in inflation and the output gap during an ELB episode, as is illustrated by the IRF in Figure 4. Since make-up strategies are

Table 3: Policy Simulation Results—Conditional Moments

Panel A: Model FLAP

	Mean				RMSD			
	$\pi\|ELB$	$\pi\|\overline{ELB}$	$r\|ELB$	$r\|\overline{ELB}$	$\pi\|ELB$	$\pi\|\overline{ELB}$	$r\|ELB$	$r\|\overline{ELB}$
IT	-2.23	0.23	-2.50	0.60	2.42	1.07	2.50	2.02
RW	-1.05	0.30	-2.50	0.75	1.68	1.09	2.50	2.03
AIT 4 years	-1.69	0.18	-2.50	0.67	1.99	0.94	2.50	2.08
AIT 8 years	-1.35	0.14	-2.50	0.65	1.74	0.92	2.50	2.05
AIT 4 years − Fixed coeff	-2.17	0.22	-2.50	0.57	2.39	1.09	2.50	1.97
AIT 8 years − Fixed coeff	-2.20	0.21	-2.50	0.53	2.40	1.10	2.50	1.94

Panel B: Model FPH (Rescaled Shocks)

	Mean				RMSD			
	$\pi\|ELB$	$\pi\|\overline{ELB}$	$r\|ELB$	$r\|\overline{ELB}$	$\pi\|ELB$	$\pi\|\overline{ELB}$	$r\|ELB$	$r\|\overline{ELB}$
IT	-1.76	0.34	-2.50	0.98	1.84	0.99	2.50	2.48
RW	-0.95	0.47	-2.50	1.27	1.44	1.05	2.50	2.57
AIT 4 years	-1.44	0.40	-2.50	1.69	1.58	0.95	2.50	3.27
AIT 8 years	-1.19	0.39	-2.50	2.34	1.43	0.97	2.50	3.98
AIT 4 years − Fixed coeff	-1.75	0.34	-2.50	0.96	1.83	1.00	2.50	2.45
AIT 8 years − Fixed coeff	-1.77	0.33	-2.50	0.92	1.86	1.01	2.50	2.40

Panel C: Model RE (Rescaled Shocks)

	Mean				RMSD			
	$\pi\|ELB$	$\pi\|\overline{ELB}$	$r\|ELB$	$r\|\overline{ELB}$	$\pi\|ELB$	$\pi\|\overline{ELB}$	$r\|ELB$	$r\|\overline{ELB}$
IT	-4.23	0.17	-2.50	0.27	4.89	1.06	2.50	1.55
RW	-1.62	0.19	-2.50	0.30	1.74	1.06	2.50	1.54
AIT 4 years	-1.32	0.02	-2.50	0.07	1.44	0.46	2.50	1.12
AIT 8 years	-0.74	0.01	-2.50	0.04	0.80	0.35	2.50	0.96
AIT 4 years − Fixed coeff	-3.43	0.13	-2.50	0.20	3.89	1.03	2.50	1.44
AIT 8 years − Fixed coeff	-2.75	0.10	-2.50	0.15	2.89	1.04	2.50	1.36

Note: Results from the simulations of the FPH-FLAP model, plain FPH model, and rational expectations (RE) model under alternative policy rules, based on 500 simulations of 300 quarters each (after having discarded a burn-in sample of 200 quarters). For any generic variable x, $x|ELB$ stands for x conditional on the nominal interest rate being at the ELB and $x|\overline{ELB}$ stands for x conditional on the nominal interest rate not being at the ELB, where π and r denote inflation and the nominal interest rate, respectively. IT stands for Inflation Targeting, AIT for Average Inflation Targeting, PLT for Price Level Targeting, RW for the Reifschneider-Williams rule. Results are expressed in percentage points of annualized rates. The FPH-FLAP models and plain FPH models are simulated under the calibration of table 1. The plain FPH and RE models are simulated under the same calibration, up to rescaling the shocks in order to yield the same standard deviation of inflation as in the FPH-FLAP model when simulated without the ELB constraint.

Table 4: Policy Simulation Results without ELB

Panel A: Model FLAP

	ELB frequency (percent)	Mean duration of ELB (quar-ters)	Mean output gap	Mean inflation gap	Mean nominal interest rate gap	RMSD. output gap	RMSD. inflation	RMSD. nominal interest rate
IT	0.00	-	0.06	0.02	0.06	2.81	1.18	2.41
RW	0.00	-	0.06	0.02	0.06	2.81	1.18	2.41
AIT 4 years	0.00	-	0.04	0.01	0.06	2.28	1.00	2.48
AIT 8 years	0.00	-	0.03	0.01	0.06	2.12	0.94	2.44
Price level targeting	0.00	-	0.01	-0.00	0.06	2.18	0.96	2.33
AIT 4 years − Fixed coeff	0.00	-	0.06	0.02	0.06	2.84	1.19	2.34
AIT 8 years − Fixed coeff	0.00	-	0.06	0.02	0.06	2.89	1.21	2.29

Panel B: Model FPH (Rescaled Shocks)

	ELB frequency (percent)	Mean duration of ELB (quar-ters)	Mean output gap	Mean inflation gap	Mean nominal interest rate gap	RMSD. output gap	RMSD. inflation	RMSD. nominal interest rate
IT	0.00	-	0.08	0.03	0.08	3.44	1.17	3.05
RW	0.00	-	0.08	0.03	0.08	3.44	1.17	3.05
AIT 4 years	0.00	-	0.07	0.02	0.11	3.26	1.10	4.06
AIT 8 years	0.00	-	0.06	0.02	0.14	3.15	1.06	4.95
Price level targeting	0.00	-	-0.01	-0.00	0.39	3.24	1.10	7.17
AIT 4 years − Fixed coeff	0.00	-	0.08	0.03	0.08	3.46	1.18	3.01
AIT 8 years − Fixed coeff	0.00	-	0.08	0.03	0.08	3.48	1.19	2.93

Panel C: Model RE (Rescaled Shocks)

	ELB frequency (percent)	Mean duration of ELB (quar-ters)	Mean output gap	Mean inflation gap	Mean nominal interest rate gap	RMSD. output gap	RMSD. inflation	RMSD. nominal interest rate
IT	0.00	-	0.01	0.03	0.04	0.66	1.17	1.75
RW	0.00	-	0.01	0.03	0.04	0.66	1.17	1.75
AIT 4 years	0.00	-	0.00	0.01	0.03	0.49	0.47	1.17
AIT 8 years	0.00	-	0.00	0.00	0.03	0.50	0.35	0.98
Price level targeting	0.00	-	0.00	-0.00	0.02	0.50	0.36	0.86
AIT 4 years − Fixed coeff	0.00	-	0.01	0.02	0.04	0.74	1.11	1.59
AIT 8 years − Fixed coeff	0.00	-	0.01	0.02	0.04	0.81	1.10	1.47

Note: Results from the simulations of the FPH-FLAP model simulated without taking into account the ELB constraint, plain FPH model, and rational expectations (RE) model under alternative policy rules, based on 500 simulations of 300 quarters each (after having discarded a burn-in sample of 200 quarters).RMSD stands for Root Mean Square Deviation, IT for Inflation Targeting, AIT for Average Inflation Targeting, PLT for Price Level Targeting, RW for the Reifschneider-Williams rule. Results are expressed in percentage points of annualized rates. The FPH-FLAP models and plain FPH models are simulated under the calibration of table 1. The plain FPH and RE models are simulated under the same calibration, up to rescaling the shocks in order to yield the same standard deviation of inflation as in the FPH-FLAP model when simulated without the ELB constraint.
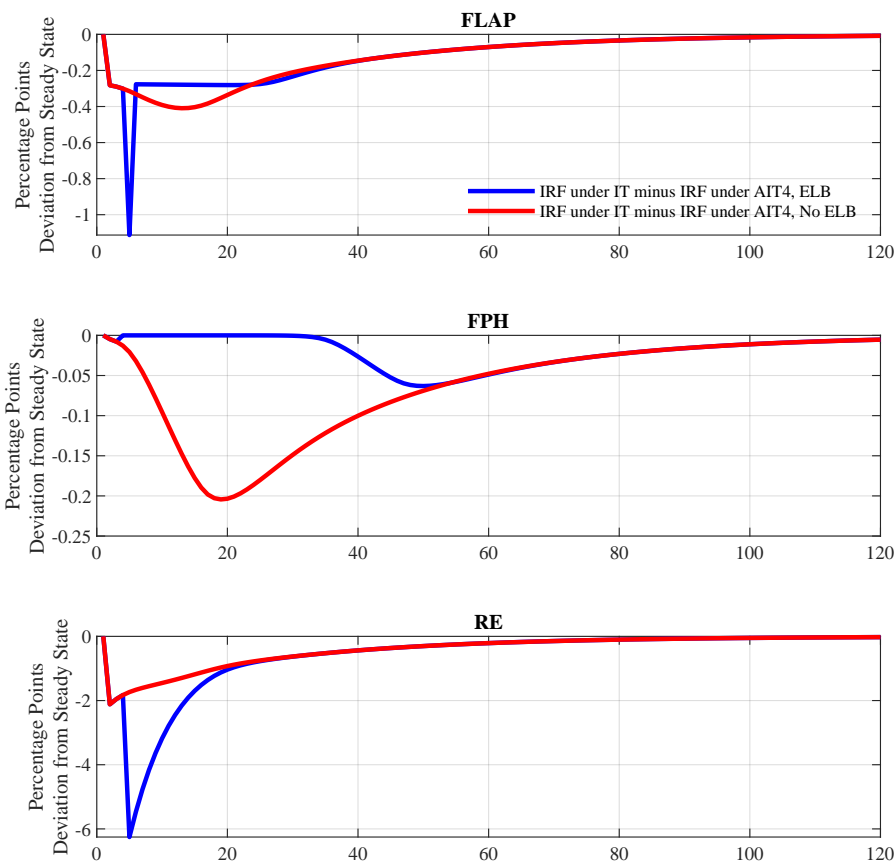
Figure 5: Benefits of AIT over IT in Stabilizing Inflation, with and without the ELB

Note: The figure plots the IRF of inflation to a demand shock under IT net of the IRF to the same demand shock under AIT4. "FPH" refers to the version of the model without rational expectations on nominal interest rates. "FLAP" refers to the version of the model with rational expectations on nominal interest rates. "RE" refers to the version of the model with rational expectations. In each model, the size of the shock is taken to be $2/\sqrt{1-\rho_y^2}$ times the standard deviation of demand shocks in the simulations.

expected to primarily mitigate such departures of inflation from target from below, the improved stabilization they provide—as measured by the RMSD of inflation and the output gap relative to IT—is expected to come with a reduction in the negative bias on mean inflation and mean output gap. However, while the RW rule does indeed eliminate this negative bias—it even leads to a positive bias—Table 2 reports that the negative bias is not reduced under the AIT-8-year rule and even *increases* under the AIT-4-year rule, from -9bp to -11bp on average inflation.

To understand this outcome, Table 3 provides the average and RMSD of inflation and the interest rate under the various policy rules, conditional on being at the ELB or not. Looking at the results for the AIT rules, it appears that they only moderately reduce the size of the deflationary spirals during an ELB episodes. At any rate, they decrease it much less than under the RW rule, despite providing a lager reduction in the overall RMSD of inflation. For instance, average inflation—in deviation from target–conditional on being at the ELB increases in absolute terms from -2.2pp under IT to -1.7pp under AIT-4-year, while it increases

to -1.1pp under the RW rule. However, contrary to the RW rule where inflation overshooting after ELB episodes increases mean inflation outside the ELB, under AIT-4-year average inflation *decreases* outside of ELB episodes, from 23bp under IT to 18bp.

This is because, while the AIT rules occasionally lead to some overshooting after an ELB episodes, they are also more stabilizing outside the ELB because they counter positive inflationary shocks better. Indeed, history-dependent strategies such as AIT are typically found to have better stabilization properties than IT even absent the constraint of the ELB Svensson (1999); Woodford (2003). It is the case in the FPH-FLAP model, as can be seen in Panel A of Table 4 which gives the stabilization properties of the various policies in simulations of the model that abstract from the constraint of the ELB. For instance, the AIT-4-year rule decreases the RMSD of inflation from 1.18pp under IT to 1.00pp.

Even more, while the presence of the ELB makes the extra performance of AIT over IT much *larger* under rational expectations, the ELB can make it *smaller* in the FPH-FLAP model. This lower extra performance is apparent comparing the reduction in the RMSD of inflation and the output gap brought by AIT in the model with and without the ELB. Under rational expectations, when abstracting from the ELB (Panel C of Table 4), moving from IT to AIT-4-year decreases the RMSD of inflation by 60% (from 1.17pp to 0.47pp). When taking into account the ELB (panel C of table 2) the reduction reaches 73% (from 1.82pp to 0.50pp). By contrast, in the FPH-FLAP model, the reduction is of 15% (from 1.18pp to 1.00pp) without the ELB, and of only 12% (from 1.36pp to 1.20pp) with the ELB.

To further investigate these features, Figure 5 plots the difference between the IRF to a large demand shock under IT and the IRF to the same demand shock under AIT-4-year in the model with ELB, superimposed with the same difference in the model without the ELB. Under rational expectations, the ELB considerably increases the stabilization benefits of AIT over IT. This is because under rational expectations the deflationary spiral at the ELB is extremely—unrealistically—large under IT, so that there is much deflation to eliminate when switching to AIT.

In the FPH-FLAP model however, the stabilization benefits of AIT over IT are *smaller* when taking into account the ELB constraint. On Figure 5, once interest rates are at the ELB under both the IT and AIT rules, the benefits of AIT over IT are less than they would be absent the ELB.[34] Indeed, absent the ELB, the history-dependence of the AIT rule promises lower interest rates in the near future than under IT, and therefore stabilizes inflation better. Taking into account the constraint of the ELB however, this stabilizing feature of the AIT rule loses its potency since interest rates in the near future are constrained by the ELB just as they are constrained today. The AIT rule still improves over IT thanks to the lower interest rates it provides in the far future—which under FLAP affects aggregate demand today—but less so than if the ELB constraint were absent. Notice that the decrease in the relative stabilization benefits of AIT over IT at the ELB is even larger in the plain FPH version of the model. There, the AIT rule provides virtually no benefit over IT during most of the time the economy is at the ELB, since the lower interest rates provided by the AIT rule in the far future have almost no impact on long-term interest rates today, and therefore almost no

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[34]Because, for the shock considered in Figure 5, interest rates reach the ELB in period 4 under IT but only in period 5 under AIT, the ELB constraint improves the stabilization benefits of AIT in the single period 5.

impact on aggregate demand and inflation today.

The above results explain why the AIT-4-year rule fails to decrease the bias on mean inflation and mean output gap in the FPH-FLAP model simulated under the ELB. Because the ELB does not constraint the response of AIT to positive, inflationary, shocks, the AIT-4-year rule reduces the departures of inflation above its target more than it reduces the departures of inflation below its target. As a result, average inflation is lower than under IT.

Ultimately, in the FPH-FLAP the ELB *reduces* the advantage of AIT over IT, in contrast to what happens in the RE model. This brings some nuances to conventional arguments in favor of switching to an AIT strategy, that typically insist on the constraint posed by the ELB to make the case for make-up strategies (e.g. ECB, 2021b). Results in our set-up do support that AIT improves over IT when the ELB constrains policy rates, but they point that it would improve over IT even more in the absence of the ELB. The FPH-FLAP model also finds that switching to an AIT rule has little effect on the downward bias on average inflation, and can even increase it. If having average inflation on target is seen as an objective in itself, asymmetric make-up strategies such as the RW rule, which only become more history-dependent after an ELB episode, provide an advantage over symmetric make-up strategies such as AIT.[35]

7   Concluding Remarks

One important conclusion of this paper is that lower-for-longer policies retain a significant degree of efficacy even when firms and households have limited planning horizons, provided the information on the future trajectory of interest rates is channeled to them by forward-looking financial-market participants through asset prices. Another finding is that in such a set-up, the improvement of AIT over IT is reduced when the ELB is a relevant constraint: while AIT is a useful strategy when the ELB is a relevant threat, the benefits brought by adopting AIT over IT would actually be larger in a world without the ELB.

One relevant extension of our work would be to embed our FPH-FLAP assumption in a larger DSGE model that includes more frictions and more inertia, such as habit formation, wage rigidity, and price and wage indexation. Adapting the FPH-FLAP set-up to such a model that contains endogenous state variables raises substantial technical issues which we leave for future research to address. Adapting the FPH-FLAP model to a model that contains state variables could also allow to consider the case where financial markets have less than fully forward-looking expectations. Yet another extension would be to characterize the optimal monetary policy rule in our FPH FLAP New Keynesian model.

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[35]To what extent having average inflation on target is valuable in itself remains debatable. In standard formal models, the loss function of the central bank can be summed up by the RMSD of inflation and the output gap from their targets, regardless of whether a smaller RMSD corresponds to a lower (squared) mean departure from target, or a lower standard deviation.

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A Long-Term Bonds and the Equivalence with Having Rational Expectations on Future Short-Term Rates under FLAP

Consider a household i deciding of its portfolio allocation at t. We assume that the household has access to zero-coupon bonds of all maturities j, which can be bought at price $(\mathcal{R}_{t,j})^{-j}$, where $\mathcal{R}_{t,j}$ is the yield on the bond of maturity j (so that $\mathcal{R}_{t,1} = r_t$).[36] Its budget constraint at t is therefore:

$$\sum_{j=1}^{\infty}(\mathcal{R}_{t,j})^{-j}B_{t,i}^{j} + C_{t,i} = \Omega_{t,i} + A_{t,i}, \tag{64}$$

where $B_{t,i}^{j}$ is household i's real holding of bond of maturity j, $C_{t,i}$ its consumption at t, $\Omega_{t,i}$ its real income at t, and $A_{t,i}$ is the real wealth it inherited from the previous period.

To buttress the point that expectations of future asset prices and interest rates do not affect the household's consumption decision above and beyond the long-term yields it observes at t, we assume that the household restricts itself to transferring wealth across periods using only these long-term bonds. The household's perceived flow budget constraint at $t + j$ will therefore be:

$$\forall j \geq 1, C_{t+j,i} = \Omega_{t+j,i} + \frac{B_{t,i}^{j}}{\Pi_{t,t+j}}, \tag{65}$$

where $\Pi_{t,t+j} = P_{t+j}/P_t$ is cumulative inflation from t to $t + j$.

The budget constraint (64) and (65) can be combined into the intertemporal budget constraint:

$$\sum_{j=0}^{\infty}\Pi_{t,t+j}(\mathcal{R}_{t,j})^{-j}C_{t+j,i} = \sum_{j=0}^{\infty}\Pi_{t,t+j}(\mathcal{R}_{t,j})^{-j}\Omega_{t+j,i} + A_{t,i}, \tag{66}$$

with the convention that $\mathcal{R}_{t,0} = 1$.

The household maximizes its intertemporal utility

$$\tilde{\mathrm{E}}_t \sum_{j=0}^{\infty}\beta^j e^{-\frac{1}{\sigma}\nu_{t+j}^y}u(C_{t+j,i}) \tag{67}$$

under the constraint of its intertemporal budget constraint (66). The operator $\tilde{\mathrm{E}}_t$ denotes the household's subjective expectations, which is a probability distribution not restricted to being the rational-expectations one. The first-order conditions give:

$$\forall j \geq 1, e^{-\frac{1}{\sigma}\nu_t^y}u'(C_{t,i}) = \beta^j \frac{(\mathcal{R}_{t,j})^j}{\Pi_{t,t+j}}\tilde{\mathrm{E}}_t(e^{-\frac{1}{\sigma}\nu_{t+j}^y}u'(C_{t+j,i})), \tag{68}$$

or in log-linear form:

$$c_{t,i} - \nu_t^y = -\sigma(j \times r_{t,j} - \tilde{\mathrm{E}}_t(\pi_{t,t+j})) + \tilde{\mathrm{E}}_t(c_{t+j,i} - \nu_{t+j}^y). \tag{69}$$

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[36]We could further assume that the household has access to all state-contingent assets. Given that we consider the linearized model however, this would make no difference.

Under our FLAP assumption that asset prices satisfy no-arbitrage and that financial-market participants have rational expectations, long-term interest rates satisfy (at first order) the expectations hypothesis

$$r_{t,j} = \frac{1}{j} \sum_{k=0}^{j-1} \mathrm{E}_t[r_{t+k}], \tag{70}$$

the first-order condition (69) can be rewritten:

$$c_{t,i} - \nu_t^y = -\sigma \sum_{i=0}^{j-1} \left(\mathrm{E}_t(r_{t+i}) - \tilde{\mathrm{E}}_t(\pi_{t+i+1}) \right) + \tilde{\mathrm{E}}_t(c_{t+j,i} - \nu_{t+j}^y). \tag{71}$$

This is the standard Euler equation iterated forward from t to $t+j$, except that expectations of future short-term interest rates are the financial markets' rational expectations, while expectations of inflation and future consumption are the household's subjective expectations.

We can similarly derive the household's consumption function. Combining the log-linearized intertemporal budget constraint (66) and the first-order condition (69) gives:

$$c_{t,i} = \nu_t^y - \sigma \left((1-\beta) \sum_{j=1}^{\infty} \beta^j \times j \times r_{t,j} - \sum_{j=1}^{\infty} \beta^j \tilde{\mathrm{E}}_t(\pi_{t+j}) \right) + (1-\beta) \left(a_{t,i} + \sum_{j=0}^{\infty} \beta^j \tilde{\mathrm{E}}_t(\omega_{t+j,i} - \nu_{t+j}^y) \right). \tag{72}$$

Under our FLAP assumption (70), this can be rewritten:

$$c_{t,i} = \nu_t^y - \sigma \sum_{j=0}^{\infty} \beta^{j+1} \left(\mathrm{E}_t(r_{t+j}) - \tilde{\mathrm{E}}(\pi_{t+j+1}) \right) + (1-\beta) \left(a_{t,i} + \sum_{j=0}^{\infty} \beta^j \tilde{\mathrm{E}}_t(\omega_{t+j,i} - \nu_{t+j}^y) \right). \tag{73}$$

This is the standard consumption function, except that expectations of future short-term interest rates are the financial markets' rational expectations, while expectations of inflation and future consumption are the household's subjective expectations. While the expressions (72) and (73) are equivalent under our FLAP assumption, expression (72) highlights that the household's consumption behavior does not require it to form expectations of future short-term rates. In the particular case of FPH households, how FPH households form their subjective expectations of their future incomes in (73) can be solved for, resulting in equation (16).

While the derivation assumes that the household does not roll over short-term bonds in order to buttress the point that it does not *need* to form expectations on future short-term rates, the household can be allowed to do so. In this case, the household's expectations of future short-term rates must be consistent with markets' expectations, e.g. because the household applies the expectations hypothesis to infer future short-term rates from present long-term rates.

B Derivation of the Consumption Function in the Perpetual Youth Model

Households differ in their age. The size of the population is normalized to one. Each period λ new households are born, and λ die. The probability of dying is independent of the household's age. Households therefore face the risk of dying with positive wealth. Insurance companies provide them with actuarially fair contracts to insure them against this risk: The wealth of households who die at t is redistributed to households still alive, in proportion to their financial wealth. The redistributed amount λB_t is redistributed to the savings $(1 - \lambda)B_t$ of the surviving households. Each dollar of saving therefore receives $\lambda/(1 - \lambda)$ dollar or annuity. A household i with real wealth $B_{t,i}$ saved from period $t - 1$ therefore starts period t with wealth $(1 + \lambda/(1 - \lambda))B_{t,i} = 1/(1 - \lambda)B_{t,i}$. The flow budget constraint of a household i of age n is therefore:

$$\frac{B_{t+1,i}^n}{R_t} + C_{t,i}^n = \frac{1}{1 - \lambda}\frac{B_{t,i}^n}{\Pi_{t+1}} + \Omega_{t,i}^n. \tag{74}$$

A household of age n maximizes:

$$\max \tilde{E}_t \sum_{j=0}^{h}(\beta(1 - \lambda))^j e^{\frac{-1}{\sigma}\nu_{t+j}^y}u(C_{t+j,i}^n) + \beta^{h+1}v_H\big(B_{t+h+1,i}^n, (R_{t+j})_{j\geq h+1}\big), \tag{75}$$

subject to the flow budget constraint (74). Optimization still gives the Euler equation until horizon h (in loglinear form):[37]

$$\forall j = 0,\ldots,h - 1\, c_{t,i}^{h-j,n} = \nu_{t+j}^y - \sigma(r_{t+j} - E_{t+j}(\pi_{t++1}^{h-j-1})) + E_{t+j}(c_{t+j+1,i}^{h-j-1,n} - \nu_{t+j+1}^y). \tag{76}$$

And at the end of the planning horizon the optimality condition is still:

$$c_{t+h,i}^{0,n}(h) = \nu_{t+h}^y - \sigma r_{t+h} - \sigma E_{t+h}(\hat{v}_H'(b_{t+h+1,i}^{0,n}(h), (r_{t+j})_{j\geq h+1})). \tag{77}$$

Equations (8) and (9) can still be iterated forward to give:

$$c_{t,i}^{h,n}(h) = \nu_t^y - \sigma\left(\sum_{j=0}^{h}E_t(r_{t+j}) - \sum_{j=0}^{h-1}E_t(\pi_{t+j+1}^{h-j-1}(h))\right) - \sigma E_{t+h}(\hat{v}_H'(b_{t+h+1}^{0,n}(h), (r_{t+j})_{j\geq h+1})). \tag{78}$$

To express the terminal value function $\hat{v}_H'$ consider the world perceived by household i of generation n

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[37]We loglinearize consumption and output around the steady-state for *aggregate* consumption (scaled for the household size). $C_{t,i}^n$ and $C_{t+1,i}^n$ are therefore loglinearized around the same value, even if the consumption profile of an individual household is not constant in steady-state, which is the case when $R \neq 1/\beta$ in steady-state.

past its horizon h. Its value function v_H is defined as:

$$v_H\big(B_{t,i}^n, (R_{t+j})_{j\geq 0}\big) = \max_{(C_{t+j,i}^n)_{j\geq 0}, (B_{t+j,i}^n)_{j\geq 0}} E_t \sum_{j=0}^{\infty} (\beta(1-\lambda))^j u(C_{t+j,i}^n), \tag{79}$$

$$\text{s.t.}\forall j \geq 0, \frac{B_{t+j+1,i}^n}{R_{t+j}} + C_{t+j,i}^n = \frac{1}{1-\lambda}\frac{B_{t+j,i}^n}{\Pi^*} + \Omega^*. \tag{80}$$

The envelope theorem gives, in log-linear form:

$$\hat{v}_H'\big(b_{t,i}^n, (r_{t+j})_{j\geq 0}\big) = -\frac{1}{\sigma}c_{t,i}^n, \tag{81}$$

where $c_{t,i}^n$ can be solved to be given, around a steady-state with no public debt $B^* = 0$, by the consumption function

$$c_{t,i}^n = (1-(1-\lambda)/R^*)b_{t,i}^n - \sigma E_t\left(\sum_{j=0}^{\infty}((1-\lambda)/R^*)^{j+1}r_{t+j}\right), \tag{82}$$

where R^* is the steady-state real interest rate. Interest rates in the consumption function are now discounted both by R^* and by $(1-\lambda)$, because the household values less future consumption knowing that it may no longer be in this world to enjoy it. Combining the two:

$$\hat{v}_H'\left(b_{t,i}^n, (r_{t+j})_{j\geq 0}\right) = -\frac{1}{\sigma}(1-(1-\lambda)/R^*)b_{t,i}^n + E_t\left(\sum_{j=0}^{\infty}((1-\lambda)/R^*)^{j+1}r_{t+j}\right). \tag{83}$$

Plugging this expression into equation (10), and aggregating across all households and generations we get aggregate consumption $c_t^h(h)$ as perceived by agents with planning horizon h, perceived to be equal to aggregate production $y_t^h(h)$:

$$y_t^h(h) = \nu_t^y - \sigma\left(\sum_{j=0}^{h}E_t(r_{t+j}) - \sum_{j=0}^{h-1}E_t(\pi_{t+j+1}^{h-j-1}(h)) + \sum_{j=h+1}^{\infty}((1-\lambda)/R)^{j-h}E_t(r_{t+j})\right) + (1-(1-\lambda)/R^*)b_{t+h+1}^0. \tag{84}$$

We are restricting to the case where aggregate public debt is zero $B_t = 0$ at all dates. We show that this implies that the steady-state interest rate $R = 1/\beta$. In a non-stochastic steady-state, all aggregate variables are constant over time (although not necessarily the individual variables of a generation). The real interest rate R^* in particular is constant. The consumption of a generation n absent shocks is:

$$C^n = \left(1 - \frac{1-\lambda}{R}(\beta R)^\sigma\right)\left(B^n + \frac{1}{1-\frac{1-\lambda}{R}}\Omega^n\right). \tag{85}$$

Aggregating across generations, and using the fact that aggregate debt is zero:

$$Y = \left(1 - \frac{1-\lambda}{R}(\beta R)^\sigma\right)\frac{1}{1-\frac{1-\lambda}{R}}\Omega. \tag{86}$$

In equilibrium $Y = \Omega$, so it must be that $\beta R = 1$.

Using the fact that $1/R = \beta$ and that agents of planning horizon h understand that public debt will be in zero net supply at $t + h$, aggregate consumption as perceived by agents of planning horizon h (84) writes:

$$y_t^h(h) = \nu_t^y - \sigma \left(\sum_{j=0}^{h} E_t(r_{t+j}) - \sum_{j=0}^{h-1} E_t(\pi_{t+j+1}^{h-j-1}(h)) + \sum_{j=h+1}^{\infty} (\beta(1-\lambda))^{j-h} E_t(r_{t+j}) \right). \tag{87}$$

This is exactly the same expression as under infinitely-lived households, except β is now $\beta' = \beta(1 - \lambda)$.

C   Irrelevance of a Steeper Keynesian Cross

The permanent-incomer assumption in the consumption bloc of the baseline New-Keynesian model implies that, regardless of their planning horizons, households consume a small fraction of their income: their marginal propensity to consume is $1 - \beta$, and tied to the high value of the preference discount factor β. As a result, the slope of the Keynesian cross is equally small, at $(1 - \beta)$. This property of the permanent-incomer model is at odds with empirical estimates of the average marginal propensity to consume in the data, and can therefore raise the question of whether a model with a higher average marginal propensity to consumer—and therefore a steeper Keynesian cross—could increase the extent of discounting β in the expression of aggregate consumption (17) under FPH-FLAP. This appendix considers a simple extension to a model that generates a steeper Keynesian cross—a Two-Agent New-Keynesian (TANK) model—under FPH-FLAP and shows that it does not affect the extent of discounting of interest rates in equation (17).

Assume a fraction λ of households are hands-to-mouth (HTM), who simply eat their incomes every period. The remaining $1 - \lambda$ households are still permanent incomers, with finite planning horizon h. (HTM households do not need to expect any future variable, so their planning horizon is irrelevant). Denote $\Omega t, i^H$ the income of a HTM household, and $\Omega_{t,i}^P$ the income of a permanent-incomer household. Note that:

$$\Omega_t = \lambda \Omega_{t,i}^H + (1 - \lambda)\Omega_{t,i}^P, \tag{88}$$

or once loglinearized:

$$\omega_t = s\omega_{t,i}^H + (1 - s)\omega_{t,i}^P, \tag{89}$$

where:

$$s \equiv \lambda \frac{\Omega^{H*}}{\Omega^*} \tag{90}$$

is the share of aggregate income that goes to HTM households in steady-state. Similarly:

$$y_t = c_t = sc_{t,i}^H + (1 - s)c_{t,i}^P. \tag{91}$$

We assume that the income of HTM households and permanent-income household have the same elasticity to aggregate income. This implies that this elasticity is equal to 1, so that $\omega_{t,i}^H = \omega_{t,i}^P = \omega_t$.

The consumption function of permanent-income households is:

$$c_{t,i}^P = \tilde{E}_t \left[\nu_t^y + (1-\beta)b_{t,i} + (1-\beta)\sum_{j=0}^{\infty}\beta^j(\omega_{t+j}^P - \nu_{t+j}^y) - \sigma\sum_{j=0}^{\infty}\beta^{j+1}(r_{t+j} - \pi_{t+j+1}) \right]. \tag{92}$$

where $\tilde{E}$ denotes the possibly distorted (under FPH) expectations of permanent incomers.

The Keynesian cross gives how much aggregate demand (here, aggregate consumption) increases with aggregate income in the aggregate consumption function. Using the fact that agents of planning horizon h correctly understand that $y_t = sc_{t,i}^H + (1-s)c_{t,i}^P$, that the consumption function of hand-to-mouth households is $c_t^H = \omega_t^H$, that the one of permanent incomers is (92), and that $\omega_t^P = \omega_t^H = \omega_t$, we get that agents of planning horizon h expect consumption to solve:

$$y_t^h(h) = [s + (1-\beta)(1-s)]\omega_t^h(h)$$

$$+ (1-s)\tilde{E}_t \left[\beta\nu_t^y + (1-\beta)b_{t,i} + (1-\beta)\sum_{j=1}^{\infty}\beta^j(\omega_{t+j}^P - \nu_{t+j}^y) - \sigma\sum_{j=0}^{\infty}\beta^{j+1}(r_{t+j} - \pi_{t+j+1}) \right]. \tag{93}$$

So the slope of the Keynesian cross is $s + (1-s)(1-\beta)$. It reduces to $(1-\beta)$ when there is no HTM households, but is now free to vary between $(1-\beta)$ and 1 depending on the new parameter s, which can be calibrated independently of the preference discount factor β.

The expression for the consumption of permanent-income households is exactly the same as the expressions of aggregate consumption (17) in the main model with only permanent incomers:

$$c_{t,i}^{P,h}(h) = E_t \left[\nu_t^y - \sigma\left(\sum_{j=0}^{h}r_{t+j} - \sum_{j=0}^{h-1}(\pi_{t+j+1}^{h-j-1}(h)) + \sum_{j=h+1}^{\infty}\beta^{j-h}(r_{t+j}) \right) \right] \tag{94}$$

Aggregate consumption is therefore:

$$y_t^h(h) = s\omega_{t,H}^h(h) + (1-s)E_t \left[\nu_t^y - \sigma\left(\sum_{j=0}^{h}r_{t+j} - \sum_{j=0}^{h-1}(\pi_{t+j+1}^{h-j-1}(h)) + \sum_{j=h+1}^{\infty}\beta^{j-h}r_{t+j} \right) \right]. \tag{95}$$

Using the fact that agents of planning horizon h correctly understand that $c_t = \omega_{t,H}$, we get that aggregate consumption is given by the same equation (17) as in the case with only permanent-income households.

D   Derivation of the NKPC under Finite Planning Horizons

Firm i's demand is given by the standard Dixit-Stglitz aggregator:

$$Y_t^i = (q_t^i)^{-\theta_t}Y_t, \tag{96}$$

where $q_t^i = P_t^i/P_t$ is firm i's relative price, and θ_t is the elasticity of demand, which we allow to exogenously fluctuate over time to generate cost-push shocks. Denote $C(Y_t^i, Y_t, A_t)$ firm i's real cost function. Firm i's instantaneous real profits are:

$$\Gamma(q_t^i, Y_t, A_t, \theta_t) = (q_t^i)^{1-\theta_t} Y_t - C((q_t^i)^{-\theta_t} Y_t, Y_t, A_t), \tag{97}$$

Firm i values future profits according to the average marginal utility real income of its shareholders, which since shares are not traded and equally distributed across households is the average marginal utility of real income across households:

$$\lambda_t = e^{\frac{-1}{\sigma}\nu_t^y} \int_i u'(C_t^i) di. \tag{98}$$

Under Calvo pricing, firm i gets to reset its price with a probability α every period. We assume that when firm i does not reset its price, its price is automatically increased at the steady-state inflation rate Π^*. Firm i's relative price j periods after resetting it is therefore $q_t^i \frac{\Pi^{*j}}{\Pi_{t,t+j}}$, where $\Pi_{t,t+j}$ is the inflation rate from t to $t+j$.

Firm i's objective when resetting its price is to maximize its expected sum of discounted future real profits. Assume firm i has a planning horizon h. Its objective is to maximize:

$$\max_{q_t^i} \tilde{\mathrm{E}}_t \sum_{j=0}^{h} (\alpha\beta)^j \lambda_{t+j} \Gamma\left(q_t^i \frac{\Pi^{*j}}{\Pi_{t,t+j}}, Y_{t+j}, A_{t+j}, \theta_{t+j}\right) + (\alpha\beta)^{h+1} v_F\left(q_t^i \frac{\Pi^{*h}}{\Pi_{t,t+h}}\right), \tag{99}$$

where $\tilde{\mathrm{E}}$ again denotes the distorted expectations of agents with planning horizon h, and v^F is the terminal value function that firm i uses to estimate the continuation value of its problem past its planning horizon:. Past its planning horizon, firm i expects Y^*, A^*, λ^* and θ^* to be back to their steady-state value. The terminal value function v_F therefore writes:

$$v_F(q) = \sum_{j=0}^{\infty} (\alpha\beta)^j \lambda^* \Gamma(q, Y^*, A^*, \theta^*) = \frac{1}{1-\alpha\beta} \lambda^* \Gamma(q, Y^*, A^*, \theta^*). \tag{100}$$

Plugging it in into the objective (99):

$$\max_{q_t^i} \tilde{\mathrm{E}}_t \sum_{j=0}^{h} (\alpha\beta)^j \lambda_{t+j} \Gamma\left(q_t^i \frac{\Pi^{*j}}{\Pi_{t,t+j}}, Y_{t+j}, A_{t+j}, \theta_{t+j}\right) + \frac{(\alpha\beta)^{h+1}}{1-\alpha\beta} \lambda^* \Gamma\left(q_t^i \frac{\Pi^{*h}}{\Pi_{t,t+h}}, Y^*, A^*, \theta^*\right). \tag{101}$$

The first-order condition gives:

$$\tilde{\mathrm{E}}_t \sum_{j=0}^{h} (\alpha\beta)^j \lambda_{t+j} \frac{\Pi^{*j}}{\Pi_{t,t+j}} \Gamma_q\left(q_t^i \frac{\Pi^{*j}}{\Pi_{t,t+j}}, Y_{t+j}, A_{t+j}, \theta_{t+j}\right)$$

$$+ \frac{(\alpha\beta)^{h+1}}{1-\alpha\beta} \lambda^* \frac{\Pi^{*h}}{\Pi_{t,t+h}} \Gamma_q\left(q_t^i \frac{\Pi^{*h}}{\Pi_{t,t+h}}, Y^*, A^*, \theta^*\right) = 0. \tag{102}$$

Using equation (97) to express Γ_q:

$$\tilde{E}_t \sum_{j=0}^{h} (\alpha\beta)^j \lambda_{t+j} Y_{t+j}^i \left(q_t^i \frac{\Pi^{*j}}{\Pi_{t,t+j}} (\theta_{t+j} - 1) - \theta_{t+j} C_{Y^i} \left(Y_{t+j}^i, Y_{t+j}, A_{t+j} \right) \right)$$

$$+ \frac{(\alpha\beta)^{h+1}}{1-\alpha\beta} \lambda^* Y_{t+h+1}^i \left(q_t^i \frac{\Pi^{*h}}{\Pi_{t,t+h}} (\theta^* - 1) - \theta^* C_{Y^i} \left(Y_{t+h+1}^i, Y^*, A^* \right) \right) = 0. \quad (103)$$

Log-linearizing:

$$\hat{q}_t^i = (1-\alpha\beta)\tilde{E}_t \sum_{j=0}^{h} (\alpha\beta)^j [\pi_{t,t+j} + mc(y_{t+j}^i, y_{t+j}, a_{t+j}) + \mu_{t+j}] + (\alpha\beta)^{h+1} [\pi_{t,t+h} + mc(y_{t+h+1}^i, 0, 0)], \quad (104)$$

where μ_t is the log-deviation of the markup $\theta_t/(\theta_t - 1)$.

We specify the supply side further in order to get the marginal cost function. Firm i produces good i with the production function:

$$Y_t^i = A_t (N_t^i)^{\frac{1}{\phi}}, \quad (105)$$

The real marginal cost of a firm i at t is:

$$MC_t^i = \frac{w_t^i}{A_t F'(N_t^i)}, \quad (106)$$

where w_t^i is the real wage it pays its employees. In log-linear form and using the production function:

$$mc_t^i = w_t^i - \phi a_t + (\phi - 1)y_t^i. \quad (107)$$

We assume segmented labor markets: firm i uses as input a type of labor i that is not perfectly substitutable with labor types used by other firms. The disutility of the different types of labor to a household is $\int_i v(N_t^i) di = \int_i \frac{N_t^{i^{\psi+1}}}{\psi+1} di$. We assume that hours N_t^i are supplied by competitive intermediaries on behalf of representative samples of households. Intermediairies choose the number of hours to supply at a given wage w_t^i in order to maximize the average utility of all households in the sample they represent. The supply of labor to firm i by households is:

$$w_t^i = \frac{v'(N_t^i)}{u'(Y_t)} = (N_t^i)^{\psi} (Y_t)^{\frac{1}{\sigma}}. \quad (108)$$

where v is the dis-utility of providing labor type N_t^i. The marginal cost of firm i therefore rewrites:

$$mc_t^i = \omega y_t^i - (\omega + 1)a_t + \frac{1}{\sigma} y_t., \quad (109)$$

where

$$\omega = \phi(\psi + 1) - 1. \quad (110)$$

We assume that the steady-state is efficient. The efficient allocation y_t^e is characterized by $mc_t^i = 0$ and no

price or production dispersion $y_t^e = y_t^i = y_t$. So:

$$y_t^e = \frac{\omega + 1}{\omega + \frac{1}{\sigma}} a_t. \tag{111}$$

So marginal cost can be rewritten:

$$mc_t^i = \omega(y_t^i - y_t^e) + \frac{1}{\sigma}(y_t - y_t^e). \tag{112}$$

Therefore:

$$mc_{t+j}^i = -\theta\omega(q_t^i - \pi_{t,t+j}) + \left(\omega + \frac{1}{\sigma}\right)(y_{t+j} - y_{t+j}^e), \tag{113}$$

$$mc_{t+h+1}^i = -\theta\omega(q_t^i - \pi_{t,t+h}). \tag{114}$$

Plugging it in into (104):

$$\hat{q}_t^i = (1 - \alpha\beta)\tilde{E}_t \sum_{j=0}^{h}(\alpha\beta)^j \left[\pi_{t,t+j} + \zeta(y_{t+j} - y_{t+j}^e) + \frac{\mu_{t+j}}{1 + \theta\omega}\right] + (\alpha\beta)^{h+1}\pi_{t,t+h}, \tag{115}$$

where $\zeta = \dfrac{\omega + \frac{1}{\sigma}}{1 + \theta\omega}$ is the slope of the Short-Run Aggregate Supply curve. $\quad(116)$

From the definition of the price index, $\pi_t = (1 - \alpha)q_t^i$. Replacing it into (115), and rewriting the inflation terms as a function of one-period inflation rates only:

$$\pi_t^h(h) = (1 - \alpha)E_t \sum_{j=0}^{h}(\alpha\beta)^j \left[\pi_{t+j}^{h-j}(h) + (1 - \alpha\beta)\zeta(y_{t+j}^{h-j}(h) - y_{t+j}^e) + \frac{\mu_{t+j}}{1 + \theta(\omega - 1)}\right] \tag{117}$$

It can be written recursively as equations (19)-(20) with:

$$\kappa = \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha}\zeta, \tag{118}$$

and

$$\nu_t^p = \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} \frac{\mu_{t+j}}{1 + \theta(\omega - 1)}. \tag{119}$$

E Proof of Proposition 2

When agents do not observe nominal interest rates at all horizons, inflation and output $Y_t = (y_t, \pi_t)'$ depend on nominal interest rates through (we do not keep track of the exogenous shocks which are irrelevant to the issue):

$$Y_t = \rho C E_t(Y_{t+1}) + D^r r_t. \tag{120}$$

Iterating forward:

$$Y_t = \sum_{k=0}^{\infty} (\rho C)^k D^r E_t(r_{t+k}). \tag{121}$$

The coefficient $(\rho C)^k D^r$ on r_{t+K} converges to zero if and only if both eigenvalues of ρC are less than one in modulus. The two roots are the solution to the quadratic equation:

$$P(\lambda) = \lambda^2 - \rho(1 + \sigma\kappa + \beta)\lambda + \rho^2\beta. \tag{122}$$

For all values of ρ, both eigenvalues are positive with one less than one. The larger one is less than one if and only if P(1)>0, i.e.

$$Q(\rho) = \beta\rho^2 - (1 + \sigma\kappa + \beta)\rho + 1 > 0, \tag{123}$$

which gives the threshold in equation (51).

When agents do observe nominal interest rates at all horizons, inflation and output $Y_t = (y_t, \pi_t)'$ depend on nominal interest rates through:

$$Y_t = \rho C E_t(Y_{t+1}) + D^r r_t + D^\xi E_t(\xi_{t+1}), \tag{124}$$

where $D^\xi = B^{-1}(-\sigma(1 - \rho), 0)'$. Iterating forward:

$$Y_t = \sum_{k=0}^{\infty} (\rho C)^k D^r E_t(r_{t+k}) + \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} (\rho C)^k \beta^{j+1} D^\xi E_t(r_{t+1+k+j}). \tag{125}$$

It can be rewritten:

$$Y_t = D^r r_t + \sum_{k=1}^{\infty} \left[(\rho C)^k D^r + (I - \frac{\rho}{\beta}C)^{-1}(\beta^k I - (\rho C)^k)D^\xi \right] E - t(r_{t+k}). \tag{126}$$

The term on r_{t+k} converges again if and only if $(\rho C)^k$ converges, which proves the result.

F   Simulation Results under Additional Make-Up Strategies

In this appendix, we present simulation results for additional interest-rate rules that build on the observation that our IT benchmark—the Taylor rule with inertia (54)—already embeds history-dependence and so already embeds elements of a make-up strategy. In the main text we emphasized this point by iterating equation (54) backward abstracting from the constraint of the ELB (53), to get to equation (55). The equivalence between the policies (54) and (55) is however broken when taking into account the ELB constraint (53), since the lagged interest r_{t-1} then no longer records past inflation and output gap misses. The policy rule (55) then differs from the policy rule (54) since it continues to be history-dependent even when r_t has reached the

ELB. Note that the rule (55) can equivalently be written recursively as:

$$r_t^* = \rho_{TR} r_{t-1}^* + (1 - \rho_{TR})(\phi_\pi \pi_t^{1a} + \phi_y(y_t - y_t^e)) + \nu_t^r, \tag{127}$$

where the difference with the policy (54) is in the presence of the lagged target rate r_{t-1}^* instead of the lagged effective rate r_{t-1}. This simple change implies that the rule (127) records past deficits of accommodation at the ELB, while the IT-benchmark (54) does not. We call the policy rule (55)-(127) the Kiley-Roberts rule (KR), in reference to Kiley and Roberts (2017).[38]

Because the KR rule (55) responds to a weighted average of past inflation, it can be related to an AIT policy. Yet, it also responds to average lagged output gaps. To assess whether this difference matters, we also consider the policy:

$$r_t^* = \phi_\pi \bar{\pi}_t + \phi_y x_t + \sum_{k=0}^{\infty} \rho_{TR}^k \nu_{t-k}^r, \tag{128}$$

which responds to the same weighted average of lagged inflation as KR rule (55), but responds to the contemporaneous output gap only. We call the rule (128) *KR inflation only*. We consider both the KR and the KR inflation only rules calibrating $\rho_{TR} = 0.85$. Finally, we also consider the KR rule (55) calibrating $\rho_{TR} = 0.95$.

Table 5 provides the results of simulations under these three rules, as well as under inflation targeting and the Reifschneider-Williams rule for comparison. In the FPH-FLAP model, the baseline KR rule (55) turns out to only slightly reduce the RMSD of inflation and the output gap. This weak effect occurs because, while this rule does increase the time spend at the ELB (to about 20%) and the mean duration of ELB episodes (to about 4.5 years), ultimately it only slightly reduces the average level of policy rates, from 26bp above steady-state under IT to 23bp under the KR rule. In turn, the small change in the average level of policy rates owes to the fact that, while the KR rule promises to keep rates at the ELB for longer than under IT, it does not promise significantly lower rates once interest rates have been lifted above the ELB, in contrast to the RW rule. As for the KR rule that responds to lagged inflation only, it has even worse stabilization properties than IT. The same is true of the KR rule with $\rho_{TR} = 0.95$. In the plain FPH model, all three KR rules appear unsurprisingly ineffective, just like all other make-up strategies.

The RE model would in contrast be much more optimistic in its assessment of the KR rules. There, all three specifications considerably reduce the RMSD of inflation and the output gap. The KR rule with $\rho_{TR} = 0.95$ even yields a RMSD of inflation about the same as under the RW rule. In addition, it reduces the time spent at the ELB much more than the RW rule. This strong effects owes to the fact that the KR rule with $\rho_{TR} = 0.95$ also embeds more history-dependence outside the ELB, which prevents the economy from reaching the ELB to start with.

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[38]Coibion, Gorodnichenko, and Wieland (2012) use a similar rule, although they do not emphasize its ability to record and make up for past deficits of accommodation.

Table 5: Policy Simulation Results: Kiley-Roberts Rules

Panel A: Model FLAP

	ELB frequency (percent)	Mean duration of ELB (quarters)	Mean output gap	Mean inflation gap	Mean nominal interest rate gap	RMSD. output gap	RMSD. inflation	RMSD. nominal interest rate
IT	12.34	10.59	-0.26	-0.09	0.26	3.43	1.36	2.11
KR	16.01	18.50	-0.21	-0.07	0.23	3.39	1.35	2.13
KR inflation only	17.15	10.26	-0.22	-0.07	0.27	3.45	1.38	2.23
KR r_{TR}=0.95	14.97	8.24	-0.12	-0.04	0.22	3.51	1.40	2.09
RW	19.58	26.40	0.16	0.06	0.17	3.11	1.26	2.17

Panel B: Model FPH (Rescaled Shocks)

	ELB frequency (percent)	Mean duration of ELB (quarters)	Mean output gap	Mean inflation gap	Mean nominal interest rate gap	RMSD. output gap	RMSD. inflation	RMSD. nominal interest rate
IT	16.91	14.60	-0.02	-0.01	0.46	3.58	1.22	2.53
KR	21.08	24.03	-0.01	-0.01	0.43	3.58	1.22	2.55
KR inflation only	21.43	12.65	-0.02	-0.01	0.45	3.54	1.22	2.60
KR r_{TR}=0.95	19.30	11.51	0.01	0.00	0.36	3.59	1.24	2.40
RW	33.80	57.95	0.07	0.02	0.10	3.63	1.24	2.63

Panel C: Model RE (Rescaled Shocks)

	ELB frequency (percent)	Mean duration of ELB (quarters)	Mean output gap	Mean inflation gap	Mean nominal interest rate gap	RMSD. output gap	RMSD. inflation	RMSD. nominal interest rate
IT	6.67	11.94	-0.20	-0.20	0.11	1.30	1.82	1.64
KR	8.23	16.25	-0.09	-0.06	0.10	0.97	1.44	1.64
KR inflation only	8.84	13.00	-0.11	-0.08	0.11	1.02	1.53	1.69
KR r_{TR}=0.95	2.57	5.87	0.00	0.01	0.04	1.00	1.16	1.24
RW	7.97	15.79	0.01	0.06	0.10	0.69	1.13	1.64

Note: Results from the simulations of the FPH-FLAP model, plain FPH model, and rational expectations (RE) model under alternative policy rules, based on 500 simulations of 300 quarters each (after having discarded a burn-in sample of 200 quarters). RMSD stands for Root Mean Square Deviation, IT for Inflation Targeting, KR for the Kiley-Roberts rule. Results are expressed in percentage points of annualized rates. The FPH-FLAP models and plain FPH models are simulated under the calibration of table 1. The plain FPH and RE models are simulated under the same calibration, up to rescaling the shocks in order to yield the same standard deviation of inflation as in the FPH-FLAP model when simulated without the ELB constraint.

G   Rescaling Shocks

This section details how the rescaling factor ζ is computed. Letting s_t denote the $n_s \times 1$ vector of variables, absent the ELB constraint, the FLAP model solution is

$$s_t = P s_{t-1} + Q \epsilon_t.$$

The unconditional variance-covariance matrix of s_t obeys

$$\text{vec}(\Sigma_s) = (I_{n_s^2} - P \otimes P)^{-1} \text{vec}(QQ').$$

Letting e_π denote the selection vector retrieving the variance of π_t^{1a} from $\text{vec}(\Sigma_s)$, the rescaling factor associated with model M, $M \in \{FPH, RE\}$, obeys

$$\zeta^M = \sqrt{\frac{e_\pi \text{vec}(\Sigma_s)}{e_\pi \text{vec}(\Sigma_s^M)}},$$

where Σ_s^M is the variance-covariance matrix of s_t under model M.