Competitive Search and the Social Value of Public Information

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EEA, 23 August 2022

Motivation

- Policy uncertainty/communication is important
  - monetary surprises (Lucas (1972, 1973, 1975))
  - global games (Moris and Shin (2002))
  - usually: frictionless trade
- Competitive search has strong efficiency properties
  - market prices the good and the likelihood of trade
  - usually: market conditions are known/no aggregate risk

This paper

- Competitive (posting + commitment + directed) search,
- aggregate risk, and
- public information.

- Characterize equilibria.
    - What does aggregate risk imply for competitive search equilibrium?
    - How do search frictions interact with information friction?
- Study the effects of information on welfare.

Results

- Arbitrary marginal effects of public information.
- Price dispersion even with identical sellers and buyers.
- Market freezes (no trade in some states).
- Less efficient single-price than price-dispersed equilibria.
- Entry is generally inefficient.
- When/if more general mechanism improves upon price posting.

Related Literature

- Price setting under incomplete information (e.g. Keller and Rady, 1999, Hellwig and Venkateswaran, 2009): frictionless trading
- Search and aggregate uncertainty & incomplete information: Mauring (2017), Lauermann et al. (2018), Shneyerov and Wong (2020): random search
- Competitive search (Moen, 1997) with incomplete information (e.g. Guerrieri et al., 2010, Moen and Rosén, 2011, Julien and Roger, 2019, and Mayr-Dorn, 2020): uncertainty about private/individual state

Model

Sellers and buyers

- Competitive search model with unknown demand state.
- Sellers
  - fixed mass $\mathcal{S} = 1$,
  - one unit of indivisible good each,
  - post prices (can mix).
- Buyers
  - mass $\mathcal{B}_i$,
  - unit demand,
  - each values good at v_i,
  - see prices and decide which firm to contact (can mix),
  - trade off price and probability of getting good.

Matching

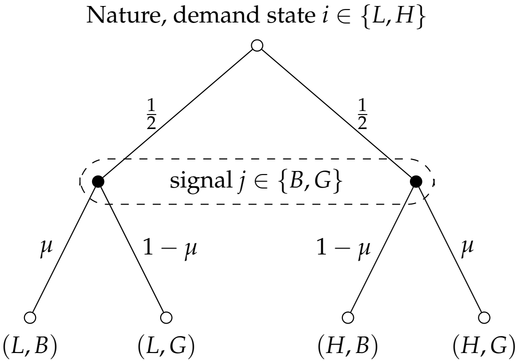
- Matching function $M(\mathcal{B}, \mathcal{S})$.
- Buyer-seller ratio: $x = \frac{\mathcal{B}}{\mathcal{S}}$.
- Probability of selling: $\lambda(x)$ with $\lambda' > 0$, $\lambda'' < 0$.
- Probability of buying: $\eta(x)$ with $\eta' < 0$, $\eta'' > 0$.
- Bilateral meetings.

Uncertainty and information

- Uncertainty about state of demand $i \in \{L, H\}$ with $(\mathcal{B}_i, v_i)$
  - uncertainty about buyer-seller ratio: $\mathcal{B}_H \geq \mathcal{B}_L$, (tightness uncertainty)
  - uncertainty about valuation: $v_H \geq v_L$ (surplus uncertainty)
  - Today: $\mathcal{B}_H > \mathcal{B}_L$ and $v_H = v_L = v$.
- Information
  - Buyers know state.
  - Sellers get public signal $j \in \{G, B\}$ before setting prices

$$\mu = Pr(j = G | i = H) = P(j = B | i = L) \in \left[\frac{1}{2}, 1\right].$$

States and signals



Timing

1. Nature draws state $i \in \{H, L\}$.
2. Public signal is realised and sellers see outcome $j \in \{G, B\}$.
3. Sellers post prices.
4. Buyers contact sellers.
5. A buyer can buy at each seller that meets at least one buyer.
6. Trade.
7. Utilities are realised.

Equilibrium

- Strategies
  - firms see signal and post prices (can mix),
  - buyers see prices and choose prices to contact (can mix).
- Equilibrium
  - optimal prices p^j for firms, $j = \{G, B\}$,
  - contacting probabilities for buyers: buyer-seller ratios x_i^j, $i = \{H, L\}$ and $j = \{G, B\}$,
  - market clearing: buyer-seller ratios consistent with total measures of buyers and sellers.
- Symmetric equilibria.

Perfect information

Perfect information problem

- Sellers know state is $i \in \{L, H\}$.
- Sellers compete against market utility of buyers $U_i \leq v$ in state i:

$$\max_{p_i} \pi_i(p_i) := \lambda(x_i) p_i,$$

$$\text{subject to: } \eta(x_i)(v - p_i) - U_i = 0.$$

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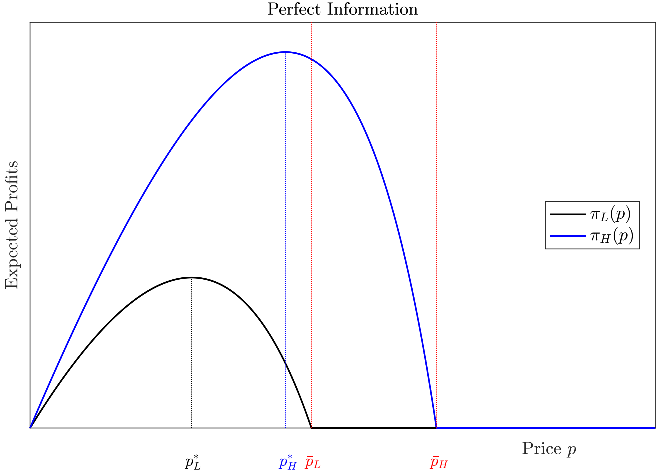
- Solution:

$$p_i^* = \phi(x_i) v.$$

  - $\phi(x_i) = -\frac{x_i \eta'(x_i)}{\eta(x_i)}$: elasticity of buying probability.
- Perfect information: single price in each state.

Perfect information profits



Imperfect information

Imperfect information problem

- Sellers do not know state $i \in \{L, H\}$, see $j = G$ or B.
- Seller who sees $j = G$ chooses p^G to

$$\max_{p^G} \pi^G(p^G) := \left[\mu\lambda\left(x_H^G\right) + (1-\mu)\lambda\left(x_L^G\right)\right] p^G \tag{1}$$

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- If p^G is acceptable in state i, $x_i^G > 0$.
- If p^G is unacceptable to buyers in state i, $x_i^G = 0$.

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- If p^G is acceptable in state i, $x_i^G > 0$.
- If p^G is unacceptable to buyers in state i, $x_i^G = 0$.
- Similar problem for sellers who see $j = B$.

Equilibrium

Pricing under imperfect information

- Pricing for both states, or pricing for high state only. Details

Pricing under imperfect information

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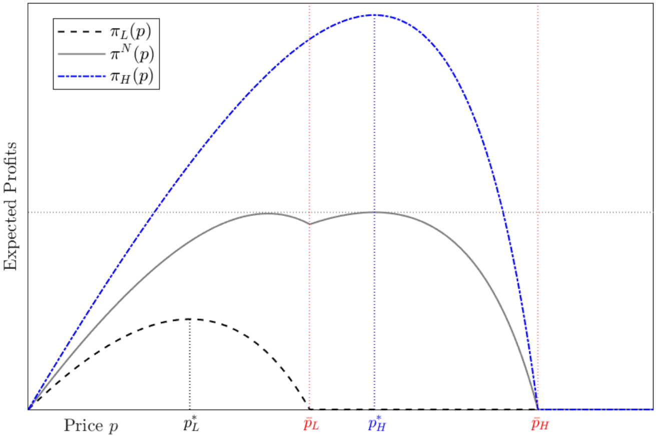
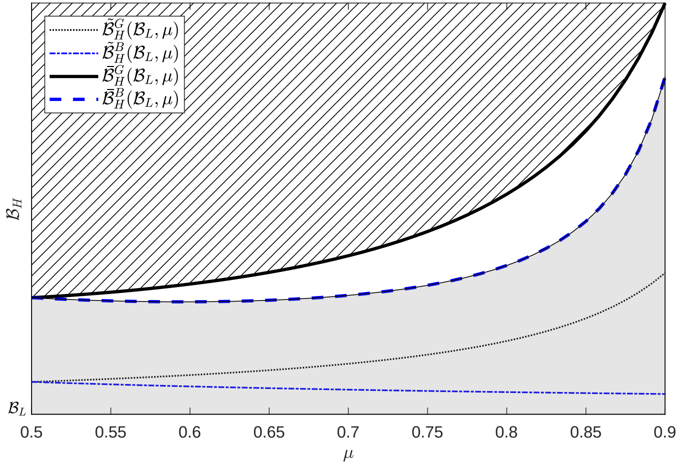


Theorem (Equilibria, tightness risk)

1. *There exist thresholds* $\tilde{\mathcal{B}}_H^j := \tilde{\mathcal{B}}_H^j(\mathcal{B}_L, \mu)$, $\mathcal{B}_L < \tilde{\mathcal{B}}_H^j < \infty$ *such that for* $\mathcal{B}_H \in \left(\mathcal{B}_L, \tilde{\mathcal{B}}_H^j\right]$ *the equilibrium profit function* $\pi^j(p)$ *is unimodal. Furthermore,* $\partial\tilde{\mathcal{B}}_H^j/\partial\mathcal{B}_L > 0$, $\partial\tilde{\mathcal{B}}_H^G/\partial\mu > 0$, $\partial\tilde{\mathcal{B}}_H^B/\partial\mu < 0$ *and* $\tilde{\mathcal{B}}_H^G \geq \tilde{\mathcal{B}}_H^B$ *with equality only if* $\mu = 1/2$,
2. *there exist thresholds* $\bar{\mathcal{B}}_H^j := \bar{\mathcal{B}}_H^j(\mathcal{B}_L, \mu)$; $\tilde{\mathcal{B}}_H^j < \bar{\mathcal{B}}_H^j \leq \infty$ *such that for* $\mathcal{B}_H \in \left(\tilde{\mathcal{B}}_H^j, \bar{\mathcal{B}}_H^j\right)$ *the equilibrium profit function* $\pi^j(p)$ *is bimodal, but pricing for both states maximizes profits. Furthermore,* $\lim_{\mu\to 1} \bar{\mathcal{B}}_H^j(\mathcal{B}_L, \mu) = \infty$, *and* $\bar{\mathcal{B}}_H^j(\mathcal{B}_L, \mu) < \infty$ *for* $\mathcal{B}_L$ *and* μ *small enough,*
3. *a unique PSE exists iff* $\mathcal{B}_H \leq \bar{\mathcal{B}}_H^j(\mathcal{B}_L, \mu)$ *and a unique MSE exists iff* $\mathcal{B}_H > \bar{\mathcal{B}}_H^j(\mathcal{B}_L, \mu)$.

Equilibria



Both signals mix in hatched and post single price in shaded area.

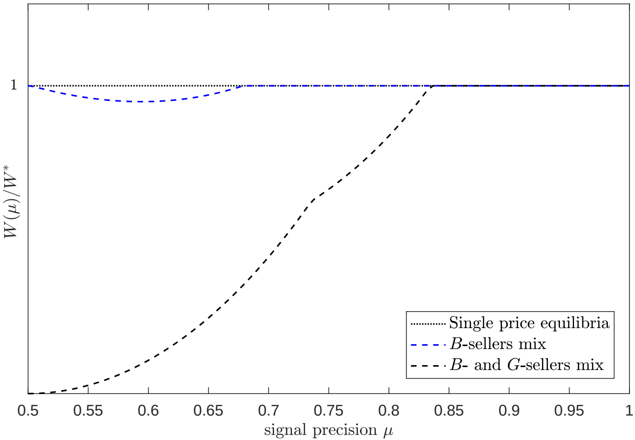
Welfare measure

- We measure welfare as expected value of trades:

$$W(\mu) = \frac{1}{2}\left[\mu \sum_{k=1}^{K^G} \kappa^{G,k} \lambda(x_H^{G,k}) + (1-\mu) \sum_{k=1}^{K^B} \kappa^{B,k} \lambda(x_H^{B,k})\right] v_H$$
$$+ \frac{1}{2}\left[\mu \sum_{k=1}^{K^B} \kappa^{B,k} \lambda(x_L^{B,k}) + (1-\mu) \sum_{k=1}^{K^G} \kappa^{G,k} \lambda(x_L^{G,k})\right] v_L$$

- Price level does not matter.
- Price dispersion is inefficient.

Information and welfare



Normalised welfare for three different $\mathcal{B}_H$ (fixed v and $\mathcal{B}_L$).

Conclusions

- Competitive search and unknown aggregate state.
- Type of aggregate uncertainty matters for ▸ Surplus uncertainty
  - how information affects trade volume.
  - what is optimal trading mechanism.
- Provision of incomplete information might harm welfare.
- Some implications find support in empirical literature.
  ▸ Details
- Extensions: entry and more general trading mechanisms.
  ▸ Entry ▸ Lotteries

Appendix

Model predictions

Model predictions and evidence ◂ Go back

- Bond markets
  - Increase in transparency → decrease in price dispersion: municipal bonds in US (Schultz, 2012).
  - More risky markets → more dispersed prices: OTC corporate bonds in US (Jankowitsch et al., 2011, Uslu and Velioglu, 2021).
  - Decrease in transparency → more market freezes possible; in contrast to Zou (2021).
  - States more different → more market freezes; as in Chiu and Koeppl, (2016).
- Labour markets
  - Increase in uncertainty → decrease in hiring; as in den Haan, Freund, Rendahl (2021).

US corporate bonds: dispersion and beta

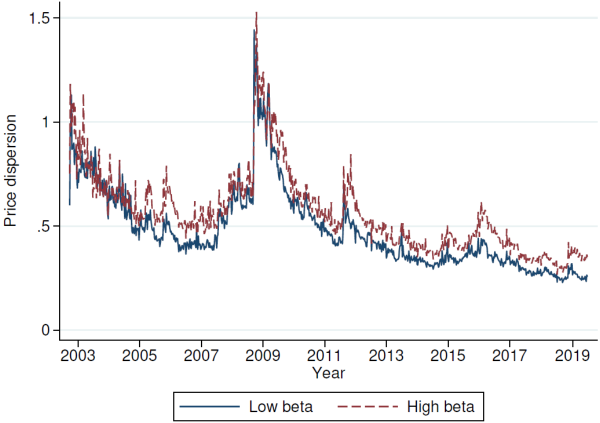


Figure: Uslu and Velioglu (2021)

US corporate bonds: dispersion and rating

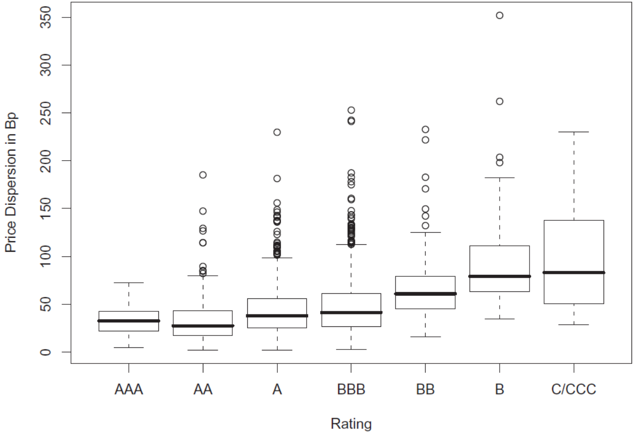


Figure: Jankowitsch et al (2011)

Equilibrium

Equilibrium conditions

- Focus on symmetric Nash equilibria $\left\{ \left\{ \kappa^{j,k}, x_i^{j,k}, p^{j,k} \right\}_{k=1}^{K^j}, U_i^j \right\}_j$, for $i \in \{L, H\}, j \in \{B, G\}$.
- Submarkets indexed by prices $p^{j,k}$.
- Buyers in state i for signal j indifferent between $p^{j,k}$:

$$U_i^{j,k} = (1 - \lambda(x_i^{j,k})) \left(v_i - p^{j,k} \right) \quad \text{for all } x_i^{j,k} > 0. \tag{3}$$

- Buyer-seller ratios consistent with measures of sellers and buyers: in state i, if $\sum_k^{K^j} x_i^{j,k} > 0$,

$$\sum_k^{K^j} \kappa^{j,k} x_i^{j,k} = \mathcal{B}_i. \tag{4}$$

Definition (Equilibrium)

We will say that a tuple $\left\{\left\{\kappa^{j,k}, x_i^{j,k}, p^{j,k}\right\}_{k=1}^{K^j}, U_i^j\right\}_j$ is an equilibrium for exogenous parameters $\Theta = (v_i, \mathcal{B}_i)$ with $i \in \{H, L\}$ and signal precision μ if for each $j \in \{G, B\}$:

1. given market utilities U_i^j, a tuple $\left\{x_i^{j,k}, p^{j,k}\right\}$ solves (1) and (2) for each k,
2. market utilities satisfy (3) given $\left\{x_i^{j,k}, p^{j,k}\right\}_{k=1}^{K^j}$,
3. buyer-seller ratios and probability weights $\left\{x_i^{j,k}, \kappa^{j,k}\right\}_{k=1}^{K^j}$ are consistent with (4).

Go back

Pricing for both states

Lemma

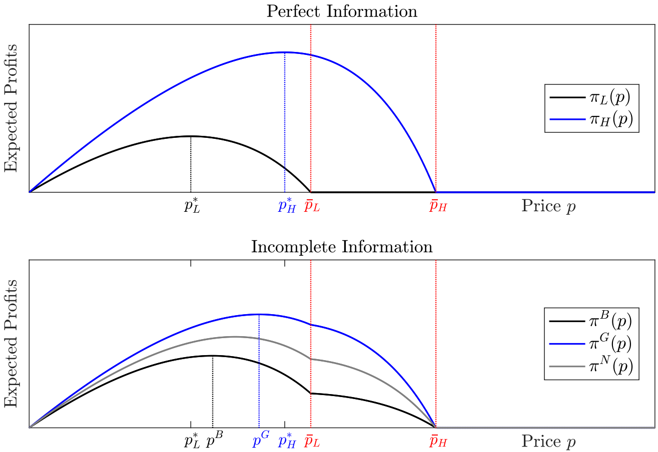
If the constrained profit maximisation on $[0, \min_i \bar{p}_i]$ *has an interior solution* p^j*, then unique buyer-seller ratios* $x_i^j > 0$ *exist which together with* p^j *jointly solve:*

$$\left(1 - \lambda\left(x_i^j\right)\right)\left(v_i - p^j\right) = U_i^j, \text{ for } i \in \{L, H\}, j \in \{B, G\}, \text{ and}$$

$$\mu\lambda(x_H^G)\left[\frac{\phi(x_H^G)v_H - p^G}{\phi(x_H^G)\left(v_H - p^G\right)}\right] + (1-\mu)\lambda(x_L^G)\left[\frac{\phi(x_L^G)v_L - p^G}{\phi(x_L^G)\left(v_L - p^G\right)}\right] = 0,$$

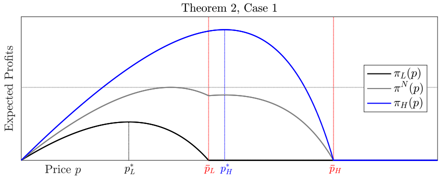
for $j = G$ and analogously for $j = B$. ▸ Simplifying assumptions

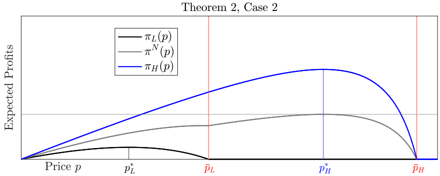
Small demand difference

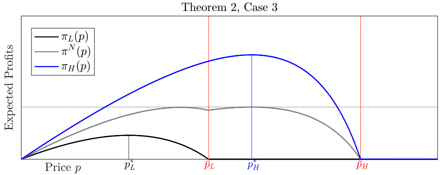


Large demand difference

◂ Go back







Entry

Entry

Cost of setting up a trading post c:

- First best: entry depends on *demand state* and not signal.
- Second best: entry depends on signal, "pricing" as in perfect information, planner maximizes expected trades net of setup costs:

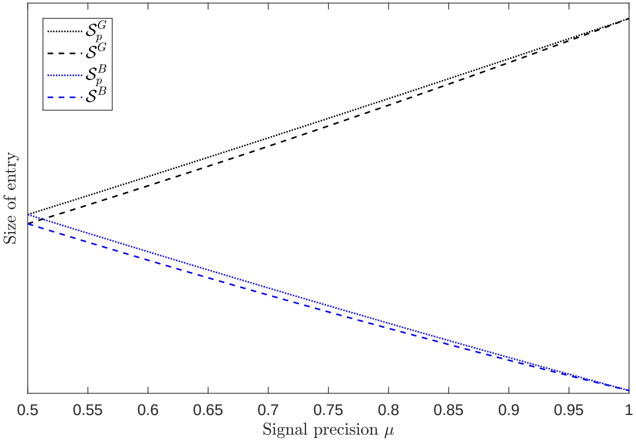
$$\mu\lambda(x_H^G)\phi(x_H^G)v + (1-\mu)\lambda(x_L^G)\phi(x_L^G)v = c,$$
$$(1-\mu)\lambda(x_H^B)\phi(x_H^B)v + \mu\lambda(x_L^B)\phi(x_L^B)v = c.$$

- Free entry conditions:

$$\left[\mu\lambda(x_H^G) + (1-\mu)\lambda(x_L^G)\right] p^G = c,$$
$$\left[(1-\mu)\lambda(x_H^B) + \mu\lambda(x_L^B)\right] p^B = c.$$

Information distorts entry

Go back



Entry and information

Simplifying assumption

Assumption (Particular matching function)

Let $M(\mathcal{B}, \mathcal{S}) = \frac{\mathcal{B}\mathcal{S}}{\mathcal{B}+\mathcal{S}}$*. Then* $\lambda(x) = \phi(x) = \frac{x}{1+x}$*,* $\eta(x) = 1 - \lambda(x)$*.*

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- Under tightness risk: for $i \in \{L, H\}$ and $j = G$

$$\left(1 - \lambda\left(x_i^G\right)\right)\left(v - p^G\right) = U_i^G, \text{ and}$$

$$\mu\lambda(x_H^G)\left[\frac{\lambda(x_H^G)v - p^G}{\lambda(x_H^G)\left(v - p^G\right)}\right] + (1-\mu)\lambda(x_L^G)\left[\frac{\lambda(x_L^G)v - p^G}{\lambda(x_L^G)\left(v - p^G\right)}\right] = 0.$$

Go back

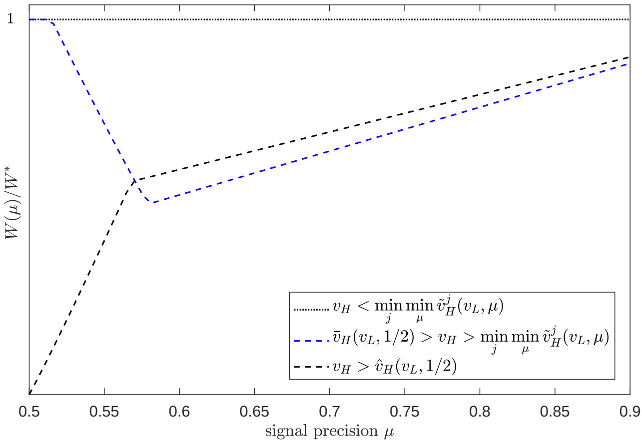
Surplus risk

Surplus risk

- If $v_H - v_L$ is small, only single price equilibria exist.
- If $v_H - v_L$ is large, equilibrium under no information features $p^N = \phi(\mathcal{B})v_H > v_L$ and no trade in L-state:
  - interval $\left(\underline{\mu}, \overline{\mu}\right)$ exists where sellers mix and expected value of trades is greater than under no information.
  - qualitatively similar to tightness risk otherwise.
- Numerically find thresholds for each type of equilibrium.

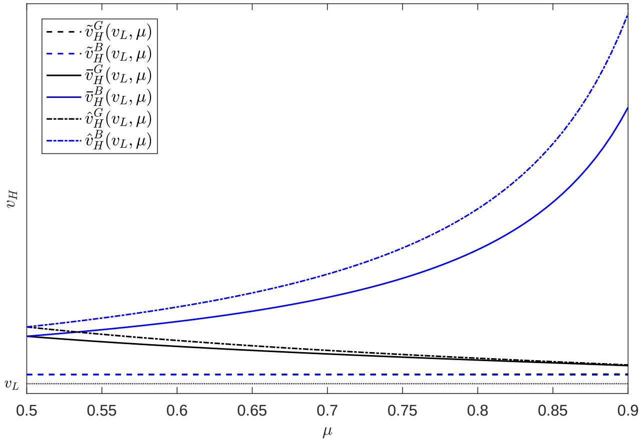
Information and welfare: surplus risk

◂ Go back



Normalised welfare for three different v_H (fixed v_L and $\mathcal{B}$).

Existence thresholds: surplus risk



Lotteries

Lotteries

Go back

- Suppose sellers can post lotteries $\left(\theta_i^j, p_i^j\right)$:
  - lottery price p_i^j, and
  - probability of getting the good θ_i^j.
- Lotteries screen states if

$$\theta_i^j v_i - p_i^j \geq \theta_{-i}^j v_i - p_{-i}^j.$$

- We show under
  - tightness risk, price posting is optimal: $\theta_i^j = 1$ and $p_i^j = p^j$.
  - surplus risk, lotteries weakly dominate price posting: zero trade in low state no longer equilibrium outcome.