Naked Exclusion with Heterogeneous Buyers

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Abstract

We investigate the effect of buyer heterogeneity in a model in which an incumbent firm prevents a competitor's entry when it signs enough exclusionary contracts with buyers. With heterogeneous buyers several well-known results in exclusionary contracting with homogenous buyers are overturned and novel ones emerge. a) Equilibria exist in which contracts are signed by some buyers but entry still occurs, rendering contract ratification an insufficient evidence of exclusion. These equilibria show that signing an exclusionary contract can generate positive externality by being pro-competitive. b) Sequential contracting may be more pro-competitive than simultaneous contracting in the sense that entry occurs under sequential contracting but not under simultaneous contracting. When that happens, sequential contracting Pareto dominates simultaneous contracting. c) Equilibrium outcome may change from exclusion to entry either when the monopoly profit of the incumbent increases or when the market share necessary for exclusion decreases.

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1 Introduction

In a seminal contribution to antitrust economics, Rasmusen, Ramseyer, and Wiley [1991] and Segal and Whinston [2000] introduce a canonical model of exclusionary contracting. In the model, an incumbent firm signs exclusionary contracts with buyers. Signing a contract binds a buyer to purchase from the incumbent firm. When the incumbent firm signs exclusionary contracts with a sufficient number of buyers, entry is not profitable for a potential entrant, and exclusion ensues. The key economic insight of their analysis is that exclusionary contracting generates negative externalities and can result in social inefficiency. Another finding is that sequential contracting is more anti-competitive than simultaneous contracting because whenever exclusion happens in equilibrium with simultaneous contracting it also happens with sequential contracting.

One important assumption in Rasmusen et al. [1991], Segal and Whinston [2000] and most of the subsequent exclusionary contracting literature is that buyers are homogeneous. Although a very useful starting point, there are many industries in which the market consists of heterogeneous buyers. How important is buyer heterogeneity in determining whether exclusion is successful? Is it still true that sequential contracting is anti-competitive? New questions arise with buyer heterogeneity as well, for example, with whom and in which order contracts are signed.

We address these question by building upon the model in Segal and Whinston [2000], allowing for buyers of heterogeneous size. This enables us to consider a range of market structures, including the special case of homogeneous buyers and markets with veto buyers, that is, buyers whose signing of a contract is necessary for successful exclusion of the competitor. When the market consists of buyers not too heterogeneous in size so that no buyer is a veto buyer, the equilibrium outcomes resemble the homogeneous-buyer case: when the incumbent's monopoly profit is small, no exclusionary contract is signed and entry happens, and when it is large, the incumbent firm is able to achieve exclusion at zero cost.

Novel equilibrium outcomes arise in the presence of veto buyers when the incumbent's monopoly profit is large. In this case in any equilibrium all non-veto buyer sign exclusionary contracts in return for zero transfers, no veto buyer contracts with the incumbent, and entry happens. The non-veto buyers are willing to sign because if they were to reject, the incumbent firm would contract with buyers sufficient for exclusion. Somewhat paradoxically, signing an exclusionary contract by a non-veto buyer is pro-competitive because it leads to entry in equilibrium and generates positive externality on the other buyers. Interestingly, exclusionary contracts are signed but entry still occurs in equilibrium. Entry occurs not because the incumbent firm is unable to achieve exclusion at a profit, but because it is even more profitable to allow for entry, since preventing entry requires full compensation of the veto buyers in return for exclusionary contracts.

It is well known that with homogeneous buyers the divide-and-conquer mechanism enabled by sequential contracting allows the incumbent firm to achieve exclusion at a low cost, which makes sequential contracting more anti-competitive than simultaneous contracting. However, with heterogeneous buyers, sequential contracting may be more pro-competitive. The veto buyers do not sign exclusionary contracts unless fully compensated even under sequential contracting. The divide-and-conquer strategy is limited to the non-veto buyers, which results in entry in the presence of veto buyers.

Under buyer heterogeneity equilibrium outcome may change from exclusion to entry either when the monopoly profit of the incumbent increases or when the market share necessary for exclusion decreases. Each change makes it easier for the incumbent firm to achieve exclusion, which makes non-veto buyers willing to sign an exclusionary contract in return for zero transfer, and, seemingly paradoxically, makes it more profitable for the incumbent firm to contract only with the non-veto buyers and allow for entry than to pursue exclusion.

Related literature: This paper builds on the seminal work on exclusionary contracting by Rasmusen et al. [1991] and Segal and Whinston [2000]. Subsequent work includes number of theoretical papers, which extend the original work in several directions [see, for example, Chen and Shaffer 2014; Miklós-Thal and Shaffer 2016], as well as experimental and empirical papers [see, for example, Landeo and Spier 2009; Boone, Müller, and Suetens 2014; Asker and Bar-Isaac 2014; Asker 2016]. Whinston [2006, chapter 4] and Fumagalli, Motta, and Calcagno [2018, chapter 3] provide exhaustive surveys of this literature. Our contribution to this literature is incorporating heterogeneous buyers to the study of exclusionary contracting, which has remained unexplored. One exception we know of is Fumagalli et al. [2018, chapter 3], who extend their two-buyer model to the case of heterogeneous buyers. Their discussion does not note that entry might be more profitable than exclusion, which we discuss at length in Section 4. Also related are Fumagalli and Motta [2006]. In their model two buyers compete in downstream market and hence their size is endogenous.

2 Model

There are two firms, an incumbent I and a potential entrant E, and $n \geq 1$ buyers. The game proceeds in three periods. In period 1, the incumbent offers buyers exclusionary contracts. An exclusionary contract commits the buyer to purchasing only from the incumbent. For most part of our paper, we consider *sequential offers* and as a comparison, we also consider *simultaneous offers*. With sequential offers, in each round of period 1, firm I either decides to stop, in which case the game proceeds to period 2, or approaches an unapproached buyer i with offer $t_i \geq 0$, after which i either accepts (in which case i becomes contracted and transfer t_i is made) or rejects (in which case i remains uncontracted and no transfer is made), and the game proceeds to another round of period 1. All actions are publicly observable. With simultaneous offers, firm I approaches all buyer simultaneously with a profile of offers $(t_i)_{i \in N} \in \mathbb{R}^n_+$, and then buyers simultaneously respond with acceptance or rejection, and the game proceeds to period 2.

In period 2, firm E decides whether to enter or not. In period 3, active firms set prices. If firm E does not enter, firm I acts as a monopolist in the market. If firm E enters, firm I still acts as a monopolist with those buyers who have signed exclusionary contracts, but engages in Bertrand competition with firm E in the market for those buyers who have not signed exclusionary contracts (the "free" buyers). The firms produce at a constant marginal cost $c_I > c_E > 0$. Firm E pays $f > 0$ if it enters.

Buyers are heterogeneous in terms of the size of their demand. Specifically, we assume that each buyer $i \in N = \{1, \ldots, n\}$ has a demand function d_i such that given price p, she demands $d_i(p) = s_i d(p)$ units, where $d : \mathbb{R}_+ \to \mathbb{R}_+$ and $s_i \in \mathbb{R}_{++}$. Hence, we can think of s_i as the "size" of buyer i. To make comparison with the canonical model of Segal and Whinston [2000], we have kept most of the ingredients of their model unchanged and focus on the effects of buyer heterogeneity; their model is a special case of ours when $s_i = 1$ for all buyers.

Denote by $C \subseteq N$ the set of buyers who have signed exclusionary contracts with I in period 1 and by $F = N \setminus C$ the set of free buyers.

In period 3, firm I sells to each buyer $i \in C$ at price p^m, where p^m is the maximizer of $(p - c_I)s_i d(p)$, which we assume is unique.[1] Note that $p^m > c_I$ and p^m is independent of i. Let $\pi = (p^m - c_I)d(p^m)$, and $\pi_i = s_i \pi$ be the monopoly profit of I from selling to buyer $i \in N$. If firm E does not enter, then firm I also

[1] This can be derived from a model where I offers a price to $i \in C$ in period 3, which i either accepts or rejects, and acceptance results in trade while rejection results in no trade.

sells to each buyer $i \in F$ at price p^m.

If firm E enters, then it sells to each buyer $i \in F$ at price c_I, bringing profit $(c_I - c_E)s_i d(c_I)$ to firm E and profit 0 to firm I.[2] Hence, the total profit of firm E if it enters is $\sum_{i\in F}(c_I - c_E)s_i d(c_I) - f = (c_I - c_E)d(c_I)\sum_{i\in F} s_i - f$ and E enters if and only if its profit is strictly positive: $\sum_{i\in F} s_i > \frac{f}{(c_I-c_E)d(c_I)}$. Let $\sum_{i\in N} s_i$ be the total size of the market, and m be the size of the market that firm I has to capture in order to deter entry. Since $\sum_{i\in N} s_i = \sum_{i\in C} s_i + \sum_{i\in F} s_i$, firm E does not enter if and only if

$$\sum\nolimits_{i\in C} s_i \geq \sum\nolimits_{i\in N} s_i - \tfrac{f}{(c_I-c_E)d(c_I)}. \tag{1}$$

Hence, $m = \sum_{i\in N} s_i - \frac{f}{(c_I-c_E)d(c_I)}$. Note that $m < \sum_{i\in N} s_i$, implying that contracting with all buyers leads to exclusion. Assume that $m > 0$, so that exclusion requires contracting with at least one buyer.

Let $x = \int_{c_I}^{p^m} d(p)dp$ and $x_i = \int_{c_I}^{p^m} d_i(p)dp = s_i x$ be the loss in consumer surplus of buyer $i \in N$ from buying at the monopoly rather than the competitive price. Because of deadweight loss from monopoly pricing, we have $x > \pi$.

The solution concept we use is subgame perfect equilibrium in pure strategies in which an indifferent buyer accepts. For the simultaneous offer game, we additionally require that buyers' strategies when they respond to I's offer do not admit profitable self-enforcing coalitional deviations.[3] This refinement rules out equilibria supported by mis-coordination of buyers. For the rest of the paper, we simply use the term equilibrium to refer to this solution concept.

Incorporating buyer heterogeneity allows our model to encompass a number of different market structures. If the buyers are homogeneous, then they are perfectly substitutable. As they become somewhat heterogeneous in size, they are still substitutable for the purpose of exclusion. Specifically, given a set of buyers, we say that they are *substitutable* if there exists a threshold k such that firm I needs any arbitrary k or more buyers to prevent entry. We call buyer j a *veto* buyer if $s_j > \sum_{i\in N} s_i - m = \frac{f}{(c_I-c_E)d(c_I)}$. That is, without contracting with buyer j, firm I is unable to deter entry since the size of buyer j is large enough to generate

[2] This can be derived from Bertrand price-competition game between E and I, which admits a Nash equilibrium in which E charges price c_I and captures the entire (uncontracted) market.

[3] Formally, consider a history after I has offered a profile of transfers to the buyers. A profile of strategies is an equilibrium if i) for every buyer $i \in N$, there is no profitable deviation, and ii) for any coalition of buyers $D \subseteq N$, there is no deviation by all buyers in D that a) would be profitable for all the deviating buyers in D and b) for each deviating buyer $i \in D$ would be best-response given that buyers in $D \setminus \{i\}$ deviate. A similar refinement is used in both Segal and Whinston [2000] and Genicot and Ray [2006].

enough profit to attract firm E to enter. There could be market structures in which there is no veto buyer and the buyers are not substitutable. For example, there could be a market with one large buyer and two small buyers. In order to deter entry, firm I needs to contract with either the large buyer or both small buyers.

3 Simultaneous offers

This section studies the simultaneous-offer game. We consider two variants of this game that differ in whether firm I is able to offer different transfers to buyers, or not.

Consider a subgame after firm I offers a profile of transfers to the buyers. In this subgame, each buyer decides either to accept or rejects I's offer. In any equilibrium of this subgame, any buyer $i \in N$ with offer weakly above $s_i x$ accepts. This is because signing an exclusionary contract results in a loss of consumer surplus of at most $s_i x$, which the offer fully compensates for. When the sizes of the fully compensated buyers sum to at least m, then in the unique equilibrium of the subgame all buyers accepts. Because buyers who are not fully compensated know that exclusion happens irrespective of their actions, they accept as well. When the sizes of the fully compensated buyers sum to strictly less than m, in the unique equilibrium of the subgame only the fully compensated buyers accept. Other buyers reject because either a) their acceptance turns entry into exclusion and they are not fully compensated for the implied loss in consumer surplus, or b) exclusion arises irrespective of their responses, in which case there is a coalition of such buyers who all gain by collectively rejecting to induce entry. We formalize this in Lemma A1 in the Appendix. Thus, to prevent entry using simultaneous offers firm I has to offer a profile of transfers $(t_1, \ldots, t_n)$ such that the sizes of the fully compensated buyers sum to at least m, that is, $\sum_{j \in \{i \in N | t_i \geq s_i x\}} s_j \geq m$.

We first characterize equilibrium when firm I cannot discriminate. Let $t^* = \min_{t \in \mathbb{R}} t$ subject to $\sum_{j \in \{i \in N | t \geq s_i x\}} s_j \geq m$. In words, t^* is the lowest nondiscriminatory transfer that fully compensates buyers with sizes that sum to at least m.

Proposition 1. *In the simultaneous-offer game, when firm I cannot discriminate, equilibrium exists and in any equilibrium:*

1. *if $\pi \sum_{i \in N} s_i < nt^*$, then I offers $t < \min_{i \in N} s_i x$ to the buyers, no buyer accepts, and entry happens,*

2. *if* $\pi \sum_{i \in N} s_i > nt^*$*, then* I *offers* t^* *to the buyers, all buyers accept, and exclusion happens.*

The cost of exclusion for firm I that is unable to discriminate is an offer of t^* to all buyers. When the cost of exclusion, nt^*, exceeds I's profit from exclusion, $\pi \sum_{i \in N} s_i$, firm I in equilibrium offers a transfer no buyer accepts. In the opposite case, it offers t^* and all buyers accept.

The latter case is only possible when buyers are heterogeneous. It requires that the losses I incurs by fully compensating some buyers be offset by profits from the other buyers. Because the fully compensated buyers are small, while the other buyers are large, exclusion requires a market to have enough large buyers in order for firm I to recover the cost of exclusion. This is impossible with homogeneous buyers. Formally, if $s_i = \bar{s}$ for each buyer $i \in N$, then $t^* = \bar{s}x$ and exclusion requires that $n\bar{s}(\pi - x) > 0$, which fails.

Markets with heterogeneous buyers, however, present a different obstacle to exclusion: veto buyers. Recall that a buyer is a veto buyer if exclusion cannot happen without him signing an exclusionary contract. Note that any buyer larger than a veto buyer is also a veto buyer. Hence, if there is a veto buyer in the market, then $t^* = \max_{i \in N} s_i x$, and exclusion requires that $\pi \sum_{i \in N} s_i - n \max_{i \in N} s_i x > 0$, which fails.

We now characterize equilibrium when firm I can discriminate. Let $m^* = \min_{T \subseteq N} \sum_{i \in T} s_i$ subject to $\sum_{i \in T} s_i \geq m$. In words, m^* is the minimal sum of buyers' sizes that is at least m.

Proposition 2. *In the simultaneous-offer game, when firm* I *can discriminate, equilibrium exists and in any equilibrium:*

1. *if* $\pi \sum_{i \in N} s_i < xm^*$*, then* I *offers* $t_i < s_i x$ *to each buyer* i*, no buyer accepts, and entry happens,*

2. *if* $\pi \sum_{i \in N} s_i > xm^*$*, then* I *offers* $t_i = s_i x$ *to all buyers in some* $T^* \subseteq N$ *that satisfies* $m^* = \sum_{i \in T^*} s_i$ *and* $t_i = 0$ *otherwise, all buyers accept, and exclusion happens.*

The cost of exclusion for a discriminating firm I is the cost of fully compensating buyers whose sizes sum to m^*. Firm I prevents entry in equilibrium whenever the profit from doing so, $\pi \sum_{i \in N} s_i$, exceeds the cost, xm^*.

The ability to discriminate enables firm I to exclude even in cases when it would not be able to without discrimination. Discriminatory offers allow firm I

to offset the loss from full compensation it offers to some buyers by the profit it makes on the other buyers. For example, consider a market with three buyers such that $s_1 = s_2 = 1$. When $s_3 = 1$ and $m = \frac{3}{2}$, buyers are homogeneous, we have $m^* = 2$ and exclusion with discriminatory offers requires $3\pi > 2x$, which holds if $\pi > \frac{2}{3}x$. When $s_3 = 3$ and $m = 3$, buyer 3 is a veto buyer, we have $m^* = 3$ and exclusion with discriminatory offers requires $5\pi > 3x$, which holds if $\pi > \frac{3}{5}x$. In both examples exclusion does not happen when firm I cannot discriminate.

The ability to discriminate also changes the set of fully compensated buyers firm I targets in equilibrium. Without discriminatory offers, small buyers are fully compensated. With discriminatory offers, large buyers might be fully compensated. Continuing with the example of three buyers, suppose their sizes are $s_1 = s_2 = 1$, $s_3 = \frac{3}{2}$, and that $m = \frac{3}{2}$. Without discrimination, $t^* = x$ and buyers 1 and 2 are fully compensated. With discrimination, $m^* = \frac{3}{2}$ and buyer 3 is fully compensated. Full compensation of large buyers with discriminatory offers is, however, suboptimal when buyers are substitutable. In this case firm I needs to contract with arbitrary k or more buyers in order to exclude and if exclusion occurs in equilibrium the set of fully compensated buyers consists of the k smallest buyers.

We call buyer $j \in S$ *redundant* when j can be dropped from S without lowering the sum of buyers' sizes in the set below m, that is, if $\sum_{i \in S \setminus \{j\}} s_i \geq m$. The set of fully compensated buyers in equilibrium never includes redundant buyers. When the sum of fully compensated buyers' sizes strictly exceeds m (like in the example with homogeneous buyers above), it is because for each buyer firm I faces a discrete problem of contracting with that buyer or not. We highlight here that redundant buyers are not fully compensated in equilibrium with simultaneous offers. As we show later, this might occur in equilibrium when offers are sequential.

4 Sequential offers

This section studies exclusionary practices when firm I approaches buyers sequentially. To facilitate the discussion of the results we introduce the notions of "(in)dispensability". Fixing the strategies of the other players and a history, we say that a buyer is *indispensable* when his rejection leads to entry and that a buyer is *dispensable* when his rejection leads to exclusion.[4] (In)dispensability of buyer i captures his strategic position: an indispensable i has to be fully compensated

[4] We suppress the history and strategy dependence when no confusion arises.

in order to accept and has a strong position, because he best-responds to offer t_i with acceptance if and only if $t_i \geq s_i x$, whereas an dispensable i best-responds with acceptance to any non-negative offer and has a weak position.[5]

Throughout the section we assume that for any $A, B, C \subseteq N$ with $A \neq \varnothing$, $\pi \sum_{i \in A} s_i \neq x(\sum_{i \in B} s_i - \sum_{i \in C} s_i)$. The assumption holds *generically* as it requires that $\frac{\pi}{x}$ differs from a finite set of values. Note that in any equilibrium any approached buyer is either dispensable and offered zero or indispensable and offered full compensation. Thus firm I's equilibrium payoff from exclusion achieved by fully compensating buyers in B is $\pi \sum_{i \in N} s_i - x \sum_{i \in B} s_i$ and its payoff from entry achieved by fully compensating buyers in C is $\pi \sum_{i \in N \setminus A} s_i - x \sum_{i \in C} s_i$ where A is the set of buyers served by firm E. Under this genericity assumption, these two payoffs are different and equilibria with entry and exclusion cannot exist simultaneously.

Lemma 1. *An equilibrium exists in the sequential-offer game. Either exclusion happens in any equilibrium, or entry happens in any equilibrium.*

Our main interest is in understanding if and how firm I achieves exclusion and how it behaves when entry occurs. For comparison, we sum key results in Segal and Whinston [2000], whose model is a special case of ours with homogeneous buyers each of unit size. They show that cutoffs c^* and c' exist such that a) exclusion happens in equilibrium if and only if $\frac{\pi}{x} \geq c^*$, b) any equilibrium with exclusion goes through two phases: in the first phase, nonempty only when $\frac{\pi}{x} < c'$, approached buyers are indispensable and firm I fully compensates them, and in the second phase, approached buyers are dispensable and sign exclusionary contracts in return for zero transfers, and c) in any equilibrium with entry, no buyer signs an exclusionary contract. The condition ensuring exclusion with sequential offers, $\frac{\pi}{x} \geq c^*$, is weaker than the condition ensuring exclusion with simultaneous (discriminatory) offers, $\frac{\pi}{x} \geq \frac{m}{n}$, since $c^* \leq \frac{m}{n}$. This implies that sequential offers make exclusion more likely relative to simultaneous offers when buyer are homogeneous.[6]

Our results below show that the equilibrium properties in Segal and Whinston [2000] do not hold in general with heterogeneous buyers. These properties still arise with heterogeneous buyers in certain special cases, as discussed in the first two parts of the following proposition. The third part of the proposition establishes

[5] These notions are adapted from Chen and Zápal [2022].

[6] Here we summarize Proposition 4, Corollary 1, its proof, which implies existence of the c' cutoff, and the discussion that precedes the corollary in Segal and Whinston [2000].

some novel equilibrium properties that arise from buyer heterogeneity.[7]

Proposition 3. *Cutoffs $\underline{\pi} > 0$ and $\overline{\pi} < x$ exists such that in any equilibrium of the sequential-offer game we have:*

1. *if $\pi < \underline{\pi}$, then entry happens because no buyer signs an exclusionary contract,*
2. *if $\pi > \overline{\pi}$ and no buyer is a veto buyer, then exclusion happens and each buyer who signs an exclusionary contract receives zero transfer,*
3. *if $\pi > \overline{\pi}$ and at least one buyer is a veto buyer, then entry happens, no veto buyer signs an exclusionary contract, but every non-veto buyer signs an exclusionary contract and receives zero transfer.*

Part 3 shows that equilibrium outcomes unlike those in Segal and Whinston [2000] arise when π is large and at least one buyer has veto power. Specifically, all non-veto buyers sign exclusionary contracts with firm I and receive zero transfers, but entry still happens. To illustrate the mechanism underlying the result, consider an example with two buyers, a small buyer 1 of size s_1 and a large buyer 2 of size $s_2 > s_1$. Setting $m \in (s_1, s_2]$ implies that firm I deters entry if and only if it contracts with the large buyer, making him a veto buyer. Because any veto buyer is always indispensable, the large buyer will not contract unless fully compensated with offer of at least $s_2 x$. By contracting with the large buyer and inducing exclusion, firm I obtains payoff $\pi(s_1 + s_2) - s_2 x$. Suppose this payoff is strictly positive. Then it is optimal for firm I to contract with the large buyer and induce exclusion in the subgame after the small buyer rejects I's offer, making the small buyer dispensable at the beginning of the game. Firm I can thus achieve payoff πs_1 by contracting with the small buyer for zero transfer in the first round and then stopping. This payoff is strictly larger than $\pi(s_1+s_2)-s_2 x$ because $s_2(\pi - x) < 0$ and hence in equilibrium the small buyer signs an exclusionary contract and entry happens.[8]

The example points to three observations. First, rejection by the small buyer in the first round leads to exclusion in the resulting subgame whereas his acceptance leads to entry. This may seem counter-intuitive and the reason is acceptance by a buyer induces two countervailing forces. First, acceptance by a buyer lowers

[7] The set of equilibria does not change when π and x change but $\frac{\pi}{x}$ does not. In Proposition 3, we consider π below or above certain cutoffs, which is equivalent to considering x above or below certain cutoffs.

[8] Proposition 3.1 in Fumagalli et al. [2018] discusses two heterogeneous buyers. Their discussion does not note that firm I can do better to allow entry than to exclude when the small buyer is dispensable.

the share of the market firm I has to capture in order to deter entry. Second, acceptance by a buyer lowers the additional payoff of firm I from exclusion beyond what has already been secured and hence lowers the incentive of firm I to seek exclusion. The former force makes exclusion post-acceptance more likely, the latter less likely. In the example the former force is absent because firm E enters if and only if the large buyer does not contract, making the small buyer irrelevant for exclusion.

An act of signing an exclusionary contract by a buyer generates *positive* externalities on the other buyers when it leads to entry. This contrasts with the usual negative externalities found in the exclusionary contracting literature and highlighted in surveys [see Whinston 2006, page 144 or Fumagalli et al. 2018, page 245].

We have illustrated the positive externalities in an example in which the first force is absent and only the second force is at work. Because the first force dominates the second one with homogeneous buyers, it is tempting to conjecture that this is also true when buyers are substitutable. This turns out not to be the case. Consider a market composed of two small buyers and two large buyers when exclusion requires contracting with any two or more buyers.[9] In this market rejection by a large buyer in the first round leads to exclusion in the resulting subgame while acceptance leads to entry. This example further illustrates that it is the larger buyers whose contracting is more likely to generate positive externalities. This is because when buyers are substitutable, the first force is independent of buyer size, while the second force is stronger for larger buyers.

Second, note that firm I is able to induce exclusion at a positive profit, but chooses not to do so because entry is more profitable than exclusion. To see how this holds more generally with veto buyers, let $V \subseteq N$ be the set of veto buyers. Then I's payoff from exclusion is at most $\pi \sum_{i \in N} s_i - x \sum_{i \in V} s_i$, while I's payoff from entry if it contracts with the non-veto buyers at no cost is $\pi \sum_{i \in N \setminus V} s_i$. The difference between the exclusion and the entry payoff is $\sum_{i \in V}(\pi - x) < 0$. The joint implication of Proposition 3 parts 2 and 3 is that for high enough π, firm I achieves its best possible payoff: in the absence of veto buyers, it contracts with buyers sufficient for exclusion at no cost; in the presence of veto buyers, it contracts with non-veto buyers at no cost and lets entry happen.

[9] In particular, when $N = \{1, 2, 3, 4\}$, $s_1 = s_2 = 6$, $s_3 = s_4 = 9$, $m = 10$, $\pi = 21$ and $x = 80$. Entry happens in any equilibrium after one of the large buyers accepts if $\pi(s_1+s_2+s_4) < s_1 x$ and exclusion happens in any equilibrium after one of the large buyers rejects if $\pi(s_1+s_3+s_4) > s_1 x$. In every equilibrium of this game entry happens and one of the large buyers contracts in the first round in return for zero transfer. Example 3 in the Appendix elaborates on the details.

Third, entry happens in any equilibrium in a market with two heterogeneous buyers in which exclusion happens if and only if firm I contracts with the large buyer. The discussion of the example already explains that entry happens in any equilibrium when $\pi(s_1 + s_2) - s_2 x$ is strictly positive. Entry also happens in equilibrium when this payoff is strictly negative (equality is ruled out by genericity). The reason is that $\pi(s_1 + s_2) - s_2 x$ is the upper bound on the payoff of firm I from exclusion. When this upper bound is strictly negative, firm I is better off stopping immediately than inducing exclusion.

A natural conjecture is that entry happens in equilibrium in the presence of veto buyers for any value of π. This conjecture turns out to be true under a further condition. We call a set of buyers $M \subseteq N$ *minimal exclusionary* given N and m if $\sum_{i\in M} s_i \geq m$ and $\sum_{i\in M'} s_i < m$ for any $M' \subsetneq M$. Note that if M is the *unique* minimal exclusionary set given N and m, then $M = V$. In this case, firm I deters entry if and only if it contracts with all buyers in V. In the two-buyer example above with $m \in (s_1, s_2]$, $M = V = \{2\}$.[10]

Proposition 4. *Suppose there exists a unique minimal exclusionary set given N and m, denoted by M. Entry happens in any equilibrium of the sequential-offer game. Moreover, if $\pi \sum_{i\in N} s_i > x \sum_{i\in M} s_i$, then exclusion happens in any equilibrium of the simultaneous-discriminatory-offer game, but any equilibrium in the sequential-offer game Pareto dominates any equilibrium in the simultaneous-discriminatory-offer game.*

When the set of veto buyers present in the market is sufficient for exclusion, entry happens in any equilibrium of the sequential-offer game. The intuition is the same as in the two-buyer example above: firm I either does not profit from exclusion which comes at the cost of fully compensating the veto buyers, or it does, but in that case, entry is even more profitable than exclusion. The key difference between Proposition 4 and Proposition 3 part 3 is that Proposition 4 applies to arbitrary not just large π.

Moreover, sequential contracting is pro-competitive in the sense that entry happens under sequential but not under simultaneous (discriminatory) contracting when $\pi \sum_{i\in N} s_i > x \sum_{i\in M} s_i$. By Proposition 2, the condition ensures that exclusion happens in equilibrium of the simultaneous-discriminatory-offer game because $m^* = \sum_{i\in M} s_i$ when M is the unique exclusionary set. This contrasts

[10] An example of a market with *multiple* minimal exclusionary sets of buyers is a three-buyer market with $s_1 = s_2 = 1$, $s_3 = 3$ and $m = 4$. In this case both $\{1, 3\}$ and $\{2, 3\}$ are minimal exclusionary given N and m.

with the anti-competitive effect of sequential contracting with homogeneous buyers. To understand this, we note that the effect of the change from simultaneous to sequential offers on the equilibrium payoff of firm I is necessarily weakly positive. Consider firm I's strategy from the simultaneous-discriminatory-offer game, in which it offers full compensation to a set of buyers sufficient for exclusion. It can achieve the same payoff in the sequential-offer game, by sequentially approaching the same set of buyers offering full compensation to each. This implies that sequential contracting cannot be pro-competitive with homogeneous buyers: if exclusion happens in equilibrium with simultaneous offers and firm I's equilibrium payoff is strictly positive, then its equilibrium payoff is also strictly positive with sequential offers, which is incompatible with entry when buyers are homogeneous. With heterogeneous buyers, however, sequential contracting can be pro-competitive because firm I can achieve strictly positive equilibrium payoff with sequential offers even when entry happens.

Proposition 4 also shows that equilibria in the sequential-offer game Pareto dominate equilibria in the simultaneous-discriminatory-offer game. We have already shown that firm I has a higher equilibrium payoff in the sequential-offer game. Firm E is also better off with sequential offers because it enters when the profit from doing so positive. The non-veto buyers receive zero transfers when offers are simultaneous, and thus cannot be made worse off. And the veto buyers are either fully compensated or benefit from entry when offers are sequential, and hence cannot be worse off.

5 Two examples

This section includes two examples of the sequential-offer game to highlight a number of equilibrium outcomes that arise from buyer heterogeneity. Example 1 shows that both contracting with redundant buyers and rejection of firm I's offer can arise in equilibrium. Example 2 shows that increasing the monopoly profit π or decreasing the market share necessary for exclusion m can counter-intuitively turn equilibrium exclusion to entry.

Example 1. Consider the sequential offers-game with two small buyers of size $s_1 = s_2 = l$ and one large buyer $s_3 = h > l$. Assume that $m \in (l, \min\{2l, h\}]$, which implies that there are two minimal exclusionary sets (given N and m) of buyers $\{1, 2\}$ and $\{3\}$.

Throughout the example, suppose that any buyer is indispensable initially and thus does not contract unless fully compensated.[11] Suppose, moreover, that after one small buyer contracts, the remaining small buyer is still indispensable. Firm I can thus achieve exclusion either by fully compensating the large buyer, with payoff $\pi(2l + h) - hx$, or by fully compensating the small buyers, with payoff $\pi(2l + h) - 2lx$.

If $\pi(l + h) - lx > 0$, firm I can achieve a higher payoff of $\pi(2l + h) - lx$ by first fully compensating one small buyer and then offering zero to the large buyer, which he accepts. The large buyer accepts because in the subgame after a small buyer accepts, the large buyer becomes dispensable since even if he does not contract with firm I, it would find it profitable to contract with the remaining small buyer by compensating him fully and induce exclusion.

In the Appendix we show that it is indeed an equilibrium for firm I to first contract with a small buyer by fully compensating him and then contract with the large buyer with zero transfer. Somewhat paradoxically, by contracting with a set of buyers strictly larger than minimal exclusionary, firm I achieves a payoff larger than the payoff it would achieve by contracting with either minimal exclusionary set. The reason is the effect of contracting with the small buyer on the bargaining position of the large buyer: by securing acceptance of the small buyer, firm I makes the large buyer dispensable.[12]

If $\pi(l+h) - lx < 0$, the strategy described above is not part of an equilibrium because the large buyer is still indispensable after a small buyer contracts with firm I. Suppose that $\pi(2l+h) - hx > 0$, so that firm I can still profitably exclude by fully compensating the large buyer. In this case firm I can achieve a payoff of πl by first offering zero to one small buyer, which he rejects, and then offering zero to the remaining small buyer, which he accepts. The small buyer accepts because in the subgame after a small buyer rejects, the remaining small buyer becomes dispensable since even if he does not contract with firm I, it would find it profitable to contract with the large buyer by compensating him fully and induce exclusion.

In the Appendix we show that it is indeed an equilibrium for firm I to first offer zero to a small buyer, which is rejected, then contract with the remaining

[11]In the Appendix, we present conditions on the model parameters such that this, and any other property we impose below, is satisfied. And we provide a numerical example that shows that all the conditions can hold simultaneously.

[12]Contracting with redundant buyers might also arise because for any equilibrium with exclusion, there is an identical equilibrium except that after ensuring exclusion, firm I approaches the remaining buyers with zero offers, which the buyers accept. These equilibria are not robust to costs of contracting because of the zero payoff benefit firm I derives from contracting with the redundant buyers.

small buyer in return for zero transfer, and then let entry happen. Interestingly, rejection occurs on the equilibrium path because firm I uses rejection by one buyer strategically to weaken the bargaining positions of other buyers and make them dispensable.

We have provided an explanation for equilibrium rejection even without any incomplete information. Rejections of exclusionary contracts have been documented in case studies. For example, Fumagalli et al. [2018] provide an account of Norwegian postal service company contracting with retailers so that Posten Norge could exclusively offer its services at the retailers' stores. Posten Norge failed to reach an agreement with some of the retailers it approached. Because rejections arise on the equilibrium path with heterogeneous but not homogeneous buyers, our analysis incorporating buyer heterogeneity reconciles theoretical predictions with certain industry observations.[13]

Example 2. Consider the sequential-offer game with two small buyers of size $s_1 = s_2 = l$ and one large buyer $s_3 = h > l$. Assume that $m \in (\max\{2l, h\}, h+l\}]$, which implies that there are two minimal exclusionary sets (given N and m) of buyers $\{1, 3\}$ and $\{2, 3\}$.

Because the large buyer is a veto buyer he is always indispensable. Suppose that after the large buyer contracts with firm I both small buyers become dispensable. Then firm I can achieve exclusion by contracting with the large buyer first and a small buyer second. This results in payoff $\pi(2l + h) - hx$ we assume is strictly positive.

Suppose that $\pi(2l + h) - (l + h)x < 0$ and $\pi(l + h) - hx < 0$. As we show below under these conditions exclusion happens in any equilibrium with firm I contracting with the large buyer first and a small buyer second.

Condition $\pi(2l + h) - (l + h)x < 0$ ensures that both small buyers are indispensable initially. This is because if a small buyer rejects, both remaining buyers become indispensable making exclusion too costly. Condition $\pi(l + h) - hx < 0$ ensures that after one small buyer contracts, the remaining small buyer is still indispensable. This is because if one small buyer accepts and one small buyer rejects, the remaining large buyer is indispensable making exclusion too costly. The two conditions thus jointly imply that firm I cannot achieve strictly positive payoff by approaching a small buyer initially: if firm I eventually achieves exclusion then it comes at a cost of fully compensating one small and one large buyer, which

[13]Rejections might also arise because for any equilibrium with entry, there is an identical equilibrium except that firm I approaches members with offers the buyers reject. These equilibria are not robust to costs of contracting.

is not profitable by the first condition, and if firm I eventually does not achieve exclusion then since any buyer it contracts with must be fully compensated its payoff cannot be positive.

Proposition 3 part 3 implies that in this example entry happens in any equilibrium for sufficiently large π. Hence, increasing the monopoly profit π can turn equilibrium outcome from exclusion to entry, a comparative statics than does not arise with homogeneous buyers. Intuitively, when π is in an intermediate range it is optimal for firm I to contract with the veto buyer first and a small buyer second and thus induce exclusion. As π increases, it becomes feasible to contract with the small buyers in return for zero transfers because they are now dispensable. This is the most profitable strategy to pursue for firm I and it leads to entry.

Moreover, Proposition 4 implies that in this example entry happens in any equilibrium whenever m decreases to $m' \in (2l, h]$. This is because $\{3\}$ is the unique minimal exclusionary set given N and m'. Hence, decreasing the market share necessary for exclusion m can turn equilibrium outcome from exclusion to entry, a comparative statics than also does not arise with homogeneous buyers. To understand why, notice that with $m' \in (2l, h]$ the large buyer is still a veto buyer and hence the best payoff from exclusion for firm I is $\pi(2l + h) - hx$. Because this payoff is strictly positive, exclusion happens in any equilibrium in the subgame after both small buyers reject and thus, after a small buyer rejects initially, the other small buyer is dispensable. This implies that firm I can achieve payoff of πl by offering zero transfer to a small buyer, which he rejects, then offering another zero transfer to the remaining small buyer, which he accepts, and then letting entry occur. Since $\pi l > \pi(2l + h) - hx$, entry happens in any equilibrium. Similar to increasing π, decreasing m makes it feasible and profitable for firm I to sign contracts with small buyers at zero costs, even if it leads to entry.

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A Proofs

A.1 Proofs of Propositions 1 and 2

The proof of both propositions makes use of the following lemma. For each buyer $i \in N$, let $cs_i(p^m)$ and $cs_i(c_I) = cs_i(p^m) + s_i x > cs_i(p^m)$ be the consumer surplus of i from buying at the monopoly price p^m and the competitive price c_I, respectively. Let a_i be the response of buyer i to I's offer, where $a_i = 0$ and $a_i = 1$ denote rejection and acceptance, respectively.

Lemma A1. *Consider the response subgame of the simultaneous-offer game after I offers $(t_1, \dots, t_n) \in \mathbb{R}^n_+$ to the buyers. A profile of responses $(a_1, \dots, a_n)$ that constitutes a NE immune to self-enforcing coalitional deviations exists. In any equilibrium:*

1. *if $\sum_{j \in \{i \in N | t_i \geq s_i x\}} s_j \geq m$, then $a_i = 1$ for any buyer $i \in N$,*
2. *if $\sum_{j \in \{i \in N | t_i \geq s_i x\}} s_j < m$, then $a_i = 1$ for any buyer $i \in N$ with $t_i \geq s_i x$ and $a_i = 0$ for any buyer $i \in N$ with $t_i < s_i x$.*

Proof. Consider the response subgame of the simultaneous-offer game after I offers profile $(t_1, \ldots, t_n) \in \mathbb{R}^n_+$ to the buyers. Denote the set of buyers offered transfers above their $s_i x$ by $A = \{i \in N | t_i \geq s_i x\}$ and denote be the set of buyers offered transfers below their $s_i x$ by $B = \{i \in N | t_i < s_i x\}$.

Given a profile of responses $(a_1, \ldots, a_n)$, we say that $(a_1, \ldots, a_n)$ leads to exclusion if $\sum_{j \in \{i \in N | a_i = 1\}} s_j \geq m$, and that $(a_1, \ldots, a_n)$ leads to entry if $\sum_{j \in \{i \in N | a_i = 1\}} s_j < m$. Given $(a_1, \ldots, a_n)$, buyer $i \in N$ receives payoff $t_i + cs_i(p^m)$ if $a_i = 1$, while i's payoff from $a_i = 0$ is $cs_i(p^m)$ if $(a_1, \ldots, a_n)$ leads to exclusion, and is $cs_i(c_I) = cs_i(p^m) + s_i x > cs_i(p^m)$ if $(a_1, \ldots, a_n)$ leads to entry.

We remark that if $(a_1, \ldots, a_n)$ constitutes a NE, then $a_i = 1\ \forall i \in A$. This is because $a_i = 1$ provides i with payoff $t_i + cs_i(p^m)$ while $a'_i = 0$ provides i with payoff at most $cs_i(c_I)$, and we have $t_i \geq s_i x$ for any $i \in A$. This also implies that for any buyer $i \in A$, $a_i = 1$ is a best-response of i to any $(a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$, which implies that i's deviation from $a_i = 1$ to $a'_i = 0$ is not profitable, both when i deviates alone or as part of a deviating coalition.

Part 1: Suppose $\sum_{j \in A} s_j \geq m$. By the opening remark, if $(a_1, \ldots, a_n)$ constitutes a NE, then $a_i = 1\ \forall i \in A$ and hence $(a_1, \ldots, a_n)$ leads to exclusion. This implies that $a_i = 1\ \forall i \in B$: payoff of any $i \in B$ from $a'_i = 0$ is $cs_i(p^m)$, from $a_i = 1$ is $t_i + cs_i(p^m)$, and $t_i \geq 0$.

What remains is to show that $(a_1, \ldots, a_n) = (1, \ldots, 1)$ is a NE immune to self-enforcing coalitional deviations. By the opening remark, no buyer $i \in A$ has a profitable deviation, either individual or as part of a deviating coalition. Consider buyer $i \in B$ who deviates as part of coalition $C \subseteq B$ to $a'_j = 0\ \forall j \in C$. When $C = \{i\}$, this is individual deviation. When C includes other buyers, this is coalitional deviation. It suffices to restrict $C \subseteq B$ by the opening remark. The profile of responses induced by the deviation leads to exclusion because $C \subseteq B$ and $\sum_{i \in A} s_i \geq m$. Hence j's payoff decreases by $t_j \geq 0$ as a result of the deviation, and thus is not profitable.

Part 2: Suppose $\sum_{i \in A} s_i < m$. Suppose first that $(a_1, \ldots, a_n)$ constitutes a NE immune to self-enforcing coalitional deviations. We show that $a_i = 1\ \forall i \in A$ and $a_i = 0\ \forall i \in B$. That $a_i = 1\ \forall i \in A$ follows by the opening remark. Now suppose, towards a contradiction, that we have $a_i = 1$ for some $i \in B$. Then i's payoff is $t_i + cs_i(p^m)$. Deviation by i to $a'_i = 0$ induces a profile of responses that leads either to entry or to exclusion. In the former case, i's payoff from $a'_i = 0$ is $cs_i(c_I)$, making the deviation profitable because $t_i < s_i x$. In the latter case, let $C = \{j \in B | a_j = 1\}$. Note that $i \in C$ and that $a_j = 0\ \forall j \in B \setminus C$. This implies that when all buyers in C deviate to $a'_j = 0\ \forall j \in C$, the induced profile of responses leads to entry because all buyers in B reject and $\sum_{j\in A} s_j < m$. Thus, for each buyer $j \in C$, j's payoff from the coalitional deviation is $cs_j(c_I)$, whereas the payoff from $a_j = 1$ is $t_j + cs_j(c_I)$, making the coalitional deviation profitable and self enforcing. In either case, we reach a contradiction.

What remains is to show that $(a_1, \ldots, a_n)$ with $a_i = 1$ $\forall i \in A$ and $a_i = 0$ $\forall i \in B$ constitutes a NE immune to self-enforcing coalitional deviations. By the opening remark, no buyer $i \in A$ has a profitable deviation, either individual or as part of a deviating coalition. Consider buyer $i \in B$. His payoff from $a_i = 0$ is $cs_i(c_I)$ because $\sum_{j\in A} s_j < m$. Deviation to $a_i' = 1$, either individual or as a part of deviating coalition, results in payoff $t_i + cs_i(p^m)$, which is not profitable because $t_i < s_i x$. □

Proof of Proposition 1: Let $t' \geq 0$ be I's homogeneous offer to all buyers, and let $a(t') = \sum_{j\in\{i\in N|t'\geq s_i x\}} s_j$ be the sum of sizes of the buyers with $s_i x$ weakly greater than t'.

If $a(t') < m$, then, by Lemma A1 part 2, entry happens after I offers t' and her payoff is $\sum_{j\in\{i\in N|t'\geq s_i x\}} (\pi s_j - t')$. Because $t' \geq s_j x$ $\forall j \in \{i \in N|t' \geq s_i x\}$, the payoff is maximized by setting $t' < x \min_{i\in N} s_i$, in which case it is zero.

If $a(t') \geq m$, then, by Lemma A1 part 1, exclusion happens after I offers t' and her payoff is $\sum_{j\in N}(\pi s_j - t') = \pi \sum_{j\in N} s_j - nt'$, which is maximized by setting $t' = t^* = \min_{t\in\mathbb{R}} t$ subject to $a(t) = \sum_{j\in\{i\in N|t\geq s_i x\}} s_j \geq m$, in which case it equals $\pi \sum_{j\in N} s_j - nt^*$.

Characterization of the buyers who accept or rejects follows directly from Lemma A1. □

Proof of Proposition 2: By Lemma A1 part 2, I's payoff from any offer $(t_1, \ldots, t_n) \in \mathbb{R}^n_+$ with $\sum_{j\in\{i\in N|t_i\geq s_i x\}} s_j < m$ is $\sum_{j\in\{i\in N|t_i\geq s_i x\}} (\pi s_j - t_j)$, which is maximized by setting $t_j < s_j x$ $\forall j \in N$, in which case it equals zero.

By Lemma A1 part 1, I's payoff from any offer $(t_1, \ldots, t_n) \in \mathbb{R}^n_+$ with $\sum_{j\in\{i\in N|t_i\geq s_i x\}} s_j \geq m$ is $\sum_{j\in N}(\pi s_j - t_j)$, which is maximized by setting $t_j = s_j x$ $\forall j \in T^*$ and $t_j = 0$ $\forall j \in N \setminus T^*$, where T^* satisfies $m^* = \sum_{j\in T^*} s_j$, in which case it equals $\pi \sum_{j\in N} s_j - xm^*$.

Characterization of the buyers who accept or rejects follows directly from Lemma A1. □

A.2 Proofs of Lemma 1 and Propositions 3 and 4

Throughout the proofs of the results for the sequential-offer game, we use a notion of state. A state is (N', m', r, a), where $N' \subseteq N$ is the set of un-approached buyers,

$m' \in \mathbb{R}$ is the sum of buyers' shares required for exclusion, and r and a are the sums of shares of the buyers who already rejected and accepted, respectively. The entire game starts in state $(N, m, 0, 0)$. Notice that a state (N', m', r, a) arising in a game $(N, m, 0, 0)$ satisfies $m' = m - a$, $r + a = \sum_{i \in N \setminus N'} s_i$, $r = \sum_{i \in A} s_i \geq 0$ for some $A \subseteq N$, and $a = \sum_{i \in A} s_i \geq 0$ for some $A \subseteq N$. Formally, state is a collection of histories with certain properties. Subgames starting at histories within the same state are identical up to a constant in payoffs and hence admit identical set of equilibria.

Lemma 1 follows from Lemma A2. Proposition 3 follows from Lemma A3. The first part of Proposition 4, that entry happens in any equilibrium of the sequential-offer game, follows from Lemma A4. The moreover part of Proposition 4 follows from the discussion after the proposition.

Lemma A2. *Consider any state (N', m', r, a) of the sequential-offer game. In the subgame starting with (N', m', r, a): equilibrium exists, either exclusion happens in any equilibrium or entry happens in any equilibrium, and equilibria are payoff equivalent for I.*

Proof. Fix a state (N', m', r, a) of the sequential offers game. We proceed by induction on the size of N'. If $|N'| = 0$, a unique equilibrium exists in which I stops as stoping is her only available action. If $m' \leq 0$, exclusion happens in the equilibrium and I's payoff is $\pi(r + a + \sum_{i \in N'} s_i)$. If $m > 0$, entry happens in the equilibrium and I's payoff is πa.

Now suppose the lemma holds for all states with $|N'| = k - 1$, where $k \geq 1$. We need to show the lemma holds for any state with $|N'| = k$. Given $|N'| = k$, consider buyer $i \in N'$ approached with offer t_i. Acceptance by i means he will buy at price p^m in stage 3. Rejection by i moves the game to state $(N' \setminus \{i\}, m', r + s_i, a)$, with, by the induction hypothesis, either entry in any equilibrium or exclusion in any equilibrium. In the former case, i will buy at price c_I after rejecting, making his payoff gain from acceptance equal to $t_i - s_i x$. In the latter case, i will buy at price p^m after rejecting, making his payoff gain from acceptance equal to t_i. In either case, it is an equilibrium for i to accept if and only if $t_i \geq c_{(N',m',r,a),i} \in \{0, s_i x\}$, with $c_{(N',m',r,a),i}$ constant across equilibria.

Hence I in (N', m', r, a) faces finite choice problem: either stop, or approach $i \in N'$ with some $t_i < c_{(N',m',r,a),i}$ (any such t_i will be rejected and provides all players with the same payoff), or approach $i \in N'$ with $t_i = c_{(N',m',r,a),i}$ (any $t_i' > c_{(N',m',r,a),i}$ provides strictly lower payoff to I than $t_i = c_{(N',m',r,a),i}$ because both t_i and t_i' are accepted moving the game to the same state). Hence equilibrium exists.

We now argue that either exclusion happens in any equilibrium or entry happens in any equilibrium. This is obvious when $m' \leq 0$ or $m' > \sum_{i \in N'} s_i$, hence consider $m' \in (0, \sum_{i \in N'} s_i]$. Suppose, towards a contradiction, that I is indifferent between approaching $i \in N'$ and $j \in N'$, where the former leads to exclusion and the latter to entry. I's payoff from the former is

$$\pi(r + a + \sum_{i \in N'} s_i) - x \sum_{i \in T'} s_i \tag{A1}$$

for some $T' \subseteq N'$, while I's payoff from the latter is

$$\pi(a + \sum_{i \in N''} s_i) - x \sum_{i \in T''} s_i \tag{A2}$$

for some $N'', T'' \subseteq N'$. The two payoffs equal and hence

$$\pi(r + \sum_{i \in N' \setminus N''} s_i) = x(\sum_{i \in T'} s_i - \sum_{i \in T''} s_i). \tag{A3}$$

We have $r + \sum_{i \in N' \setminus N''} s_i = \sum_{i \in A} s_i$ for some $A \subseteq N$. Because N'' is the set of buyers I contracts with and $N'' = N'$ would lead to exclusion, we have $A \neq \varnothing$. Thus (A3) is a contradiction.

Finally, suppose multiple equilibria exist that are not payoff equivalent for I. By the induction hypothesis, these equilibria differ in the action I takes in state (N', m', r, a) and, thus, I has a profitable deviation in at least one of these equilibria. $\square$

Lemma A3. *Consider any state (N', m', r, a) of the sequential-offer game with $m' \in (0, \sum_{i \in N'} s_i]$. Cutoffs $\underline{\pi} > 0$ and $\overline{\pi} < x$ exist such that, in any equilibrium of the subgame starting with (N', m', r, a) we have:*

1. *if $\pi < \underline{\pi}$, then no buyer signs an exclusionary contract with I, and entry happens,*

2. *if $\pi > \overline{\pi}$, $r = 0$, and $v \in N'$ such that $\sum_{i \in N' \setminus \{v\}} s_i < m'$ exists, then no veto buyer signs an exclusionary contract with I, all non-veto buyers sign an exclusionary contract with I in return for zero transfer, and entry happens,*

3. *if $\pi > \overline{\pi}$, and if either $r > 0$ or no $v \in N'$ such that $\sum_{i \in N' \setminus \{v\}} s_i < m'$ exists, then a set of buyers sufficient for exclusion signs an exclusionary contract*

with I in return for zero transfer, except for each veto buyer i who receives $s_i x$, and exclusion happens.

Proof. Fix a state (N', m', r, a) of the sequential offers game with $m' \in (0, \sum_{i \in N'} s_i]$. If $N' = \varnothing$ then $\sum_{i \in N'} s_i = 0$ and the condition cannot be satisfied, hence we have $N' \neq \varnothing$. Moreover, $r \geq 0$ and $a \geq 0$.

We prove the lemma by induction on $|N'|$. Throughout, we use repeatedly the fact that equilibria exist and are payoff equivalent for I, without explicitly invoking Lemma A2.

Let $W' = \{C \in 2^{N'} | \sum_{i \in C} s_i \geq m'\}$ be the collection of winning coalitions induced by N' and m'. We have $N' \in W'$ and thus $W' \neq \varnothing$ from $m' \leq \sum_{i \in N'} s_i$, and $\varnothing \notin W'$ from $m' > 0$.[14] Let $V' = \{i \in N' | i \in C \ \forall C \in W'\}$ be the set of veto buyers in W'. Note that V' might be, but need not be, empty.

Rejection by buyer $j \in N'$ in state (N', m', r, a) moves the game to state $(N' \setminus \{j\}, m', r + s_j, a)$, in which the collection of winning coalitions is $W'_{j,r} = \{C \in 2^{N' \setminus \{j\}} | \sum_{i \in C} s_i \geq m'\}$. Note that $W'_{j,r} = \varnothing$ if and only if j is a veto buyer in W'. Moreover, $\varnothing \notin W'_{j,r}$.

Acceptance by buyer $j \in N'$ in state (N', m', r, a) moves the game to state $(N' \setminus \{j\}, m' - s_j, r, a + s_j)$, in which the collection of winning coalitions is $W'_{j,a} = \{C \in 2^{N' \setminus \{j\}} | \sum_{i \in C} s_i \geq m' - s_j\}$ and the set of veto buyers is $V'_{j,a} = \{i \in N' \setminus \{j\} | i \in C \ \forall C \in W'_{j,a}\}$. Note that $W'_{j,a} \neq \varnothing$ although $\varnothing \in W'_{j,a}$ is possible when j's acceptance leads to exclusion. Moreover, $V'_{j,a} = V' \setminus \{j\}$.[15]

Initial induction step: $|N'| = 1$. Because $N' = \{j\}$, $W' \neq \varnothing$, and $\varnothing \notin W'$, we have $W' = \{\{j\}\}$ and $V' = \{j\}$. Acceptance by j leads to exclusion and rejection by j leads to entry. Hence, in any equilibrium, j accepts I's offer if and only $t_j \geq s_j x$.

Suppose $\pi < \frac{x s_j}{r + s_j}$ and, towards a contradiction, that an equilibrium with exclusion exists. I's equilibrium payoff is $\pi(r + a + s_j) - x s_j$, which is strictly less than πa, which is the payoff I can attain by stopping in (N', m', r, a). Hence I has a profitable deviation, a contradiction. Thus entry happens in any equilibrium and no buyer signs an exclusionary contract. Note that $\frac{x s_j}{r + s_j} > 0$ and that $\pi < \frac{x s_j}{r + s_j}$ at $r = 0$ becomes $\pi < x$, which holds. Hence Parts 1 and 2 follow.

Suppose $\pi > \frac{x s_j}{r + s_j}$ and, towards a contradiction, than an equilibrium with entry

[14] $W' = \varnothing$ arises when exclusion cannot be achieved. $\varnothing \in W'$ arises when exclusion has been achieved. These cases are ruled out by $m' \in (0, \sum_{i \in N'} s_i]$ but arise from acceptance/rejection in (N', m', r, a).

[15] Denote $W' = \{C_1, \ldots, C_l\}$ and fix $j \in N'$. Then $W'_{j,a} = \{C_1 \setminus \{j\}, \ldots, C_l \setminus \{j\}\}$ and we have $V' \setminus \{j\} = (C_1 \cap \cdots \cap C_l) \setminus \{j\} = (C_1 \setminus \{j\}) \cap \ldots \cap (C_l \setminus \{j\}) = V'_{j,a}$, where the second equality follows from the fact that set difference is right distributive over set intersection.

exists. I's equilibrium payoff is πa, which is strictly less than $\pi(r+a+s_j)-xs_j$, which is the payoff I can attain by approaching j with $t_j = s_j x$ in (N', m', r, a). Hence I has a profitable deviation, a contradiction. Thus exclusion happens in any equilibrium and the veto buyer j signs an exclusionary contract in return for $t_j = s_j x$. Note that $\frac{xs_j}{r+s_j} < x$ when $r > 0$. Hence Part 3 follows.

Induction step: $|N'| = k$. Suppose the lemma holds for all $|N'| \leq k-1$, where $k \geq 2$. We need to prove the lemma for $|N'| = k$.

Rejection by any buyer $i \in N'$ leads to entry if $i \in V'$ and moves the game to a state for which the lemma holds, by the induction hypothesis, if $i \notin V'$. Thus $\underline{\pi} > 0$ and $\overline{\pi} < x$ exist such that, in any equilibrium, any buyer $i \in N'$ accepts I's offer if and only if $t_i \geq s_i x$, either when $\pi < \underline{\pi}$ or when $\pi > \overline{\pi}$ and $i \in V'$, and if and only if $t_i \geq 0$, when $\pi > \overline{\pi}$ and $i \notin V'$.

Part 1: Suppose $\pi < \underline{\pi}$ and, towards a contradiction, that an equilibrium with exclusion exists. I's equilibrium payoff is $\pi(r + a + \sum_{i \in N'} s_i) - x \sum_{i \in C} s_i$, for some $C \in W'$, where C is non-empty because $\varnothing \notin W'$. If $\pi < \frac{x \sum_{i \in C} s_i}{r + \sum_{i \in N'} s_i}$, this payoff is strictly smaller than πa, which is the payoff I can attain by stopping in (N', m', r, a). Hence I has a profitable deviation, a contradiction. Thus entry happens in any equilibrium. Because entry happens in any equilibrium, I's equilibrium payoff is $\pi(a + \sum_{i \in T'} s_i) - x \sum_{i \in T'} s_i$, for some $T' \subseteq N'$. If $T' \neq \varnothing$, then I stopping in (N', m', r, a) is a profitable deviation, hence $T' = \varnothing$. Thus entry happens in any equilibrium and no buyer signs an exclusionary contract. Note that $\frac{x \sum_{i \in C} s_i}{r + \sum_{i \in N'} s_i} > 0$ because C is non-empty.

Part 2: Suppose $\pi > \underline{\pi}$, $r = 0$, $V' \neq \varnothing$, and, towards a contradiction, that an equilibrium with exclusion exists. I's equilibrium payoff is at most $\pi(a + \sum_{i \in N'} s_i) - x \sum_{i \in V'} s_i$. This payoff is strictly smaller than $\pi(a + \sum_{i \in N' \setminus V'} s_i)$, which is the payoff I can attain by approaching $i \in N' \setminus V'$ with $t_i = 0$ in (N', m', r, a). This, by the induction hypothesis, leads to all remaining buyers in $N' \setminus V'$ and no buyer in V' subsequently approached. Hence I has a profitable deviation, a contradiction. Thus entry happens in any equilibrium. Because entry happens in any equilibrium, I's equilibrium payoff is $\pi(a + \sum_{i \in NV'' \cup V''} s_i) - x \sum_{i \in V''} s_i$, for some $NV'' \subseteq N' \setminus V'$ and some $V'' \subseteq V'$. If $V'' \neq \varnothing$ or $NV'' \neq N' \setminus V'$, then I approaching $i \in N' \setminus V'$ with $t_i = 0$ in (N', m', r, a) is a profitable deviation, hence $V'' = \varnothing$ and $NV'' = N' \setminus V'$. Thus entry happens in any equilibrium, no veto buyer signs an exclusionary contract, and all non-veto buyers sign an exclusionary contract in return for zero transfer.

Part 3: Suppose $\pi > \underline{\pi}$, either $r > 0$ or $V' = \varnothing$, and, towards a contradiction, that an equilibrium with entry exists. I's equilibrium payoff is $\pi(a +$

$\sum_{i\in NV''\cup V''} s_i) - x\sum_{i\in V''} s_i$, for some $NV'' \subseteq N' \setminus V'$ and some $V'' \subseteq V'$ such that $V'' \cup NV'' \notin W'$. When I in (N', m', r, a) deviates and approaches $i \in C$ with $t_i = s_i x$ if $i \in V'$ and with $t_i = 0$ if $i \notin V'$, for some $C \in W'$, she receives payoff $\pi(r + a + \sum_{i\in N'} s_i) - x\sum_{i\in V'} s_i$. This is because the deviation, by the induction hypothesis, leads to all remaining buyers in C subsequently approached. When $V' = \varnothing$, the entry payoff is maximized for $V'' = \varnothing$ and NV'' a solution to $\max_{C\subseteq N', C\notin W'} \sum_{i\in C} s_i < \sum_{i\in N'} s_i$, where the inequality follows from $N' \in W'$, and the maximized entry payoff is strictly smaller than the deviation payoff. When $V' \neq \varnothing$, the entry payoff is maximized for $V'' = \varnothing$ and $NV'' = N' \setminus V'$, and the maximized entry payoff is strictly smaller than the deviation payoff when $\pi > \frac{x\sum_{i\in V'} s_i}{r+\sum_{i\in V'} s_i}$. Thus exclusion happens in any equilibrium and a winning coalition of buyers signs an exclusionary contract in return for $s_i x$ and zero transfers for veto and non-veto buyers respectively. Note that $\frac{x\sum_{i\in V'} s_i}{r+\sum_{i\in V'} s_i} < x$ when $r > 0$. $\square$

Lemma A4. *Consider any state (N', m', r, a) of the sequential-offer game with $m' \in (0, \sum_{i\in N'} s_i]$. Suppose exactly one $V' \subseteq N'$ exists such that $\sum_{i\in V'} s_i \geq m'$ and $\sum_{i\in V''} s_i < m'$ for any $V'' \subsetneq V'$. We have:*

1. *if $\pi(r + \sum_{i\in V'} s_i) > x\sum_{i\in V'} s_i$, then exclusion happens in any equilibrium,*
2. *if $\pi(r + \sum_{i\in V'} s_i) < x\sum_{i\in V'} s_i$, then entry happens in any equilibrium.*

Proof. Fix a state (N', m', r, a) of the sequential offers game with $m' \in (0, \sum_{i\in N'} s_i]$. If $N' = \varnothing$ then $\sum_{i\in N'} s_i = 0$ and the condition cannot be satisfied, hence we have $N' \neq \varnothing$. Moreover, $r \geq 0$ and $a \geq 0$.

We call a set of buyers $V' \subseteq N'$ *minimal exclusionary given N' and m'* if $\sum_{i\in V'} s_i \geq m'$ and $\sum_{i\in V''} s_i < m'$ $\forall V'' \subsetneq V'$. Suppose there exists unique minimal exclusionary V' given N' and m' and let $NV' = N' \setminus V'$. Given (N', m', r, a), both acceptance and rejection by $i \in NV'$ moves the game to a state with unique minimal exclusionary V'. Rejection by $i \in V'$ results in exclusion because i is a veto buyer in (N', m', r, a), while acceptance by $i \in V'$ moves the game to a state with unique minimal exclusionary $V' \setminus \{i\}$.

We proceed by induction on $|NV'|$. Throughout, we use repeatedly the fact that equilibria exist and are payoff equivalent for I, without explicitly invoking Lemma A2.

Initial induction step: $|NV'| = 0$. Because $NV' = \varnothing$, $N' = V'$. Suppose $\pi(r + \sum_{i\in V'} s_i) > x\sum_{i\in V'} s_i$ and, towards a contradiction, that entry happens in equilibrium. The equilibrium payoff of I is $\pi(a + \sum_{i\in T'} s_i) - x\sum_{i\in T'} s_i$ for some $T' \subseteq V'$, and hence is at most $\pi(a)$. Consider I's deviation to a strategy of approaching all buyers in V', in some sequence, and offering $t_i = s_i x$ to each buyer $i \in V'$. The payoff from the deviation is $\pi(r + a + \sum_{i \in V'} s_i) - x \sum_{i \in V'} s_i > \pi(a)$, where the inequality follows from $\pi(r + \sum_{i \in V'} s_i) > x \sum_{i \in V'} s_i$, and hence is profitable, a contradiction.

Suppose $\pi(r + \sum_{i \in V'} s_i) < x \sum_{i \in V'} s_i$ and, towards a contradiction, that exclusion happens in equilibrium. The equilibrium payoff of I is $\pi(r + a + \sum_{i \in V'} s_i) - x \sum_{i \in V'} s_i < \pi(a)$, where the inequality follows from $\pi(r + \sum_{i \in V'} s_i) < x \sum_{i \in V'} s_i$. Hence stopping is a profitable deviation for I, a contradiction.

Induction step: $|NV'| = k$. Suppose the lemma holds for all $|NV'| \leq k - 1$, where $k \geq 1$. We need to prove the lemma for $|NV'| = k$.

Suppose $\pi(r + \sum_{i \in V'} s_i) > x \sum_{i \in V'} s_i$ and, towards a contradiction, that entry happens in equilibrium. The equilibrium payoff of I is at most $\pi(a + \sum_{i \in NV'} s_i)$. Consider I's deviation to a strategy of approaching all buyers in V', in some sequence, and offering $t_i = s_i x$ to each buyer $i \in V'$. The payoff from the deviation is $\pi(r + a + \sum_{i \in NV'} s_i + \sum_{i \in V'} s_i) - x \sum_{i \in V'} s_i > \pi(a + \sum_{i \in NV'} s_i)$, where the inequality follows from $\pi(r + \sum_{i \in V'} s_i) > x \sum_{i \in V'} s_i$, and hence is profitable, a contradiction.

Suppose $\pi(r + \sum_{i \in V'} s_i) < x \sum_{i \in V'} s_i$ and, towards a contradiction, that exclusion happens in equilibrium. The equilibrium payoff of I is $\pi(r + a + \sum_{i \in NV'} s_i + \sum_{i \in V'} s_i) - x \sum_{i \in V'} s_i$. Consider I's deviation to a strategy of approaching all buyers in NV', in some sequence, and offering $t_i = 0$ to each buyer $i \in NV'$. Partition the approached buyers in NV' such that $l \in NV'$ is the last buyer to be approached, and R' and A' are the buyers in $NV' \setminus \{l\}$ that reject and accept, respectively, I's offer. Note that $NV' = R' \cup A' \cup \{l\}$.

We first argue that l accepts. To see this, we have

$$\pi(r + a + \sum_{i \in NV'} s_i + \sum_{i \in V'} s_i) - x \sum_{i \in V'} s_i > \pi(a + \sum_{i \in A'} s_i) \tag{A4}$$

because the deviation is not profitable (equality is ruled out by genericity). By construction $NV' \setminus A' = R' \cup \{l\}$, and thus the inequality is equivalent to

$$\pi(r + s_l + \sum_{i \in R'} s_i + \sum_{i \in V'} s_i) > x \sum_{i \in V'} s_i \tag{A5}$$

which, by the induction hypothesis, implies that rejection by l leads to exclusion. Hence l accepts.

The payoff of I from the deviation thus equals $\pi(a + s_l + \sum_{i \in A'} s_i)$. Because the deviation is not profitable, we have (equality is ruled out by genericity)

$$\pi(r + \sum_{i\in R'} s_i + \sum_{i\in V'} s_i) > x \sum_{i\in V'} s_i. \tag{A6}$$

Because $\pi(r + \sum_{i\in V'} s_i) < x \sum_{i\in V'} s_i$, $R' \neq \varnothing$. Let $l_r \in R'$ be the last buyer to reject. If rejection by l_r led to exclusion he would accept, hence his rejection leads to entry. This, by the induction hypothesis, implies that $\pi(r + \sum_{i\in R'} s_i + \sum_{i\in V'} s_i) \leq x \sum_{i\in V'} s_i$, a contradiction. □

A.3 Formal details of examples

Below we provide formal details of Examples 1 and 2 and develop Example 3 mentioned in footnote 9. For all subgames with one or two buyers Lemma A5 lists the key equilibrium outcomes. Proof of the lemma is a routine backward induction argument and is omitted. We also omit the details of which buyers are approached with what offers; these details are immediate from I's payoff.[16]

Lemma A5. *Consider any state (N', m', r, a) of the sequential-offer game. If $|N'| = |\{i\}| = 1$, the equilibrium outcomes in the subgame starting with (N', m', r, a) are described in the following table.*

parameters	*equilibrium*	
	outcome	*I's payoff*
$m' \leq 0$	*exclusion*	$\pi(r + a + s_i)$
$m' \in (0, s_i]$, $\pi(r + s_i) > s_i x$	*exclusion*	$\pi(r + a + s_i) - s_i x$
$m' \in (0, s_i]$, $\pi(r + s_i) < s_i x$	*entry*	πa
$m' > s_i$	*entry*	πa

If $|N'| = |\{i, j\}| = 2$, with $s_i \leq s_j$, the equilibrium outcomes in the subgame starting with (N', m', r, a) are described, using shorthand $\overline{\pi} = \pi(r + a + s_i + s_j)$, in the following table.

[16] One exception is the second and the third case of $m' \in (0, s_i]$ in the second table. In both cases, exclusion is achieved by approaching a buyer with zero offer. When $\pi(r + s_i + s_j) \in (s_i x, s_j x)$, the approached buyer is the larger buyer j. When $\pi(r+s_i+s_j) > s_j x$, the approached buyer is i in some equilibria and is j in another.

parameters	*equilibrium*	
	outcome	*I's payoff*
$m' \leq 0$	*exclusion*	$\overline{\pi}$
$m' \in (0, s_i]$, $\pi(r + s_i + s_j) < s_i x$	*entry*	πa
$m' \in (0, s_i]$, $\pi(r + s_i + s_j) \in (s_i x, s_j x)$	*exclusion*	$\overline{\pi}$
$m' \in (0, s_i]$, $\pi(r + s_i + s_j) > s_j x$	*exclusion*	$\overline{\pi}$
$m' \in (s_i, s_j]$, $\pi(r + s_j) < s_j x$, $\pi(r + s_i + s_j) < s_j x$	*entry*	πa
$m' \in (s_i, s_j]$, $\pi(r + s_j) < s_j x$, $\pi(r + s_i + s_j) > s_j x$	*entry*	$\pi(a + s_i)$
$m' \in (s_i, s_j]$, $\pi(r + s_j) > s_j x$	*exclusion*	$\overline{\pi} - s_j x$
$m' \in (s_j, s_i + s_j]$, $\pi(r + s_i + s_j) < (s_i + s_j)x$	*entry*	πa
$m' \in (s_j, s_i + s_j]$, $\pi(r + s_i + s_j) > (s_i + s_j)x$	*exclusion*	$\overline{\pi} - (s_i + s_j)x$
$m' > s_i + s_j$	*entry*	πa

Example 1: In the example, $N = \{1, 2, 3\}$, $s_1 = s_2 = l < h = s_3$ and $m \in (l, \min\{2l, h\}]$. The minimal exclusionary sets of buyers given N and m are $\{1, 2\}$ and $\{3\}$.

The example assumes throughout that rejection by any of the buyers at the beginning of the game leads to a subgame in which entry happens in any equilibrium. The subgame after rejection by the large buyer starts with $(\{1, 2\}, m' \in (l, 2l], h, 0)$, in which entry happens in any equilibrium if, from Lemma A5,

$$\pi(2l + h) < 2lx. \tag{A7}$$

The subgame after rejection by the small buyer 1 starts with $(\{2, 3\}, m' \in (l, h], l, 0)$, in which entry happens in any equilibrium if, from Lemma A5,

$$\pi(l + h) < hx. \tag{A8}$$

Moreover, the example assumes that after one of the small buyers accepts, rejection by the remaining small buyer leads to a subgame in which entry happens in any equilibrium. This subgame starts with $(\{3\}, m' \in (0, h], l, l)$, in which entry happens in any equilibrium if, from Lemma A5, (A8) holds.

At the beginning of the game I has four possible actions to choose from: either approach one of the small buyers or approach the large buyer, and with an offer I knows the approached buyer would either accept or reject. The following table, derived from Lemma A5, disregarding the cases not possible under (A7) and (A8), shows the payoff of I from the four actions.

approached buyer	transfer t	I's payoff
1 or 2	$t < lx$	0 if $\pi(2l+h) < hx$, πl if $\pi(2l+h) > hx$
1 or 2	$t = lx$	$\pi l - lx$ if $\pi(l+h) < lx$, $\pi(2l+h) - lx$ if $\pi(l+h) > lx$
3	$t < hx$	0
3	$t = hx$	$\pi(2l+h) - hx$

If $\pi(l+h) > lx$, then $\pi(2l+h) - lx > 0$ and $\pi(2l+h) - lx > \pi l$ and hence fully compensating one of the small buyers maximizes I's payoff. Equilibrium construction is routine and confirms that in the subgame after acceptance by one of the small buyers I approaches the large buyer with zero offer, which he accepts, and exclusion happens in any equilibrium. Example of parameters satisfying (A7), (A8) and $\pi(l+h) > lx$ is $l = 1$, $h = 6/5$, $\pi = 1$ and $x = 21/10$.

If $\pi(l+h) < lx$ but $\pi(2l+h) > hx$, then we have $\pi(2l+h) - hx < \pi l$ because the inequality is equivalent to $\pi(l+h) - hx < 0$, which holds because $\pi(l+h) - hx < \pi(l+h) - lx < 0$. Thus approaching one of the small buyers with zero offer maximizes I's payoff. Equilibrium construction is routine and confirms that in the subgame after rejection by one of the small buyers I approaches the remaining small buyer with zero offer, which he accepts, and entry happens in any equilibrium. Example of parameters satisfying (A7), (A8), $\pi(l+h) < lx$ and $\pi(2l+h) > lx$ is $l = 1$, $h = 6/5$, $\pi = 1$ and $x = 12/5$.

Example 2: In the example, $N = \{1, 2, 3\}$, $s_1 = s_2 = l < h = s_3$ and $m \in (\max\{2l, h\}, l+h]$. The minimal exclusionary sets of buyers given N and m are $\{1, 3\}$ and $\{2, 3\}$.

The example assumes that after the large buyer accepts, rejection by one of the small buyers leads to a subgame in which exclusion happens in any equilibrium. This subgame starts with $(\{i\}, m' \in (0, s_j], l, h)$, where $i, j \in \{1, 2\}$ and $i \neq j$, in which exclusion happens in any equilibrium if, from Lemma A5,

$$\pi(2l) > lx. \tag{A9}$$

The example further assumes that rejection by any of the small buyers at the beginning of the game leads to a subgame in which entry happens in any equilibrium. The subgame after rejection by the small buyer 1 starts with $(\{2, 3\}, m' \in (h, l+h], l, 0)$, in which entry happens in any equilibrium if, from Lemma A5,

$$\pi(2l+h) < (l+h)x. \tag{A10}$$

Finally, the example assumes that after one of the small buyers accepts, rejection by the remaining small buyer leads to a subgame in which entry happens in any equilibrium. This subgame starts with $(\{3\}, m' \in (0, h], l, l)$, in which entry happens in any equilibrium if, from Lemma A5,

$$\pi(l + h) < hx. \tag{A11}$$

At the beginning of the game I has four possible actions to choose from: either approach one of the small buyers or approach the large buyer, and with an offer I knows the approached buyer would either accept or reject. The following table, derived from Lemma A5, disregarding the cases not possible under (A9), (A10) and (A11), shows the payoff of I from the four actions.

approached buyer	transfer t	I's payoff
1 or 2	$t < lx$	0
1 or 2	$t = lx$	$\pi l - lx$
3	$t < hx$	0
3	$t = hx$	$\pi(2l + h) - hx$

If $\pi(2l + h) - hx > 0$, fully compensating the large buyer maximizes I's payoff. Equilibrium construction is routine and confirms that in the subgame after acceptance by the large buyer I approaches one of the small buyers with zero offer, which he accepts, and exclusion happens in any equilibrium. Example of parameters satisfying (A9), (A10), (A11) and $\pi(2l + h) - hx > 0$ is $l = 1$, $h = 6$, $\pi = 1$ and $x = 5/4$. Note that $2l < h$ so that there exists $m' \in (2l, h]$.

Example 3. Consider the sequential offers-game with two identical small buyers of size $s_1 = s_2 = 6$ and two identical large buyers of size $s_3 = s_4 = 9$. Assume that in order to exclude, I needs to contract with two or more buyers. Let $\pi = 21$ and $x = 80$.

We use an alternative notation for states. A state is $((n_l, n_h), n', r, a)$, where n_l is the number of un-approached small buyers, n_h is the number of un-approached large buyers, and n' is the number of buyers I needs to contract with in order to exclude (r and a are as before). The game starts at $((2, 2), 2, 0, 0)$.

We first consider states with $n_l = n_h = 2$. Consider any subgame starting with state $((n_l, n_h), n', r, a)$ where $n_l + n_h = 2$.

1. If $n' = 0$, exclusion happens in any equilibrium of the subgame and the equilibrium payoff of I from this subgame is $21 \cdot (6 \cdot 2 + 9 \cdot 2) = 21 \cdot 30$.

2. If $n' = 1$, from Lemma A5, if $21 \cdot (r+6n_l+9n_h) < (6\mathbb{I}(n_l \neq 0)+9\mathbb{I}(n_l = 0)) \cdot 80$, then entry happens in any equilibrium of the subgame and the equilibrium payoff of I from this subgame is $21 \cdot a$, and if $21 \cdot (r + 6n_l + 9n_h) > (6\mathbb{I}(n_l \neq 0)+9\mathbb{I}(n_l = 0)) \cdot 80$, then exclusion happens in any equilibrium of the subgame and the equilibrium payoff of I from this subgame is $21 \cdot 30$.

3. If $n' = 2$, from Lemma A5, entry happens in any equilibrium of the subgame if $21 \cdot (r+6n_l+9n_h) < (6n_l+9n_h) \cdot 80$ and the equilibrium payoff of I from this subgame is $21 \cdot a$. Because $n' = 2$, we have $a = 0$ and $r = 6(2-n_l)+9(2-n_h)$, which implies that $r+6n_l+9n_h = 6 \cdot 2+9 \cdot 2 = 30$. Moreover, $6n_l+9n_h \geq 12$. Thus the inequality holds, entry happens in any equilibrium of the subgame and the equilibrium payoff of I from this subgame is 0.

Consider a subgame R after one of the buyers rejects at the beginning of the game. Rejection by any buyer in R leads to a subgame starting with state in which $n_l + n_h = 2$ and $n' = 2$. Entry happens in any equilibrium of this subgame and the equilibrium payoff of I from this subgame is 0. Acceptance by any buyer in R leads to a subgame starting with state in which $n_l + n_h = 2$ and $n' = 1$. If $21 \cdot (r + 6n_l + 9n_h) > (6\mathbb{I}(n_l \neq 0) + 9\mathbb{I}(n_l = 0)) \cdot 80$, then exclusion happens in any equilibrium of this subgame and the equilibrium payoff of I from this subgame is $21 \cdot 30$. If the inequality fails (equality is ruled out by genericity), then entry happens in any equilibrium of this subgame and the equilibrium payoff of I from this subgame is $21 \cdot a$. Because $r + 6n_l + 9n_h \in \{6 \cdot 2 + 9, 6 + 9 \cdot 2\} = \{21, 24\}$, the left hand side of the inequality is in $\{441, 504\}$. The right hand side of the inequality is in $\{480, 720\}$. The inequality thus fails if either $n_l = 0$, or $n_l = 2$, or $n_l = 1$ and $r = 6$. The inequality holds if $n_l = 1$ and $r = 9$.

Consider a subgame R_6 after one of the small buyers rejects at the beginning of the game. By the preceding paragraph with subgame R (the inequality fails because $r = 6$), entry happens in any equilibrium of R_6. Moreover, the equilibrium payoff of I from R_6 is 0. This is because, in R_6, I's payoff from approaching any buyer with offer strictly smaller than full compensation is 0 and the payoff from fully compensating buyer $i \in N$ is at most $21 \cdot s_i - s_i \cdot 80 < 0$.

Consider a subgame R_9 after one of the large buyers rejects at the beginning of the game. By the paragraph with subgame R, I's payoff is at most zero both from approaching any buyer with offer strictly smaller than full compensation and also from fully compensating the remaining large buyer. Moreover, I's payoff from fully compensating one of the small buyers is $21 \cdot 30 - 6 \cdot 80 = 150$. Thus exclusion happens in any equilibrium of R_9 and the equilibrium payoff of I from R_9 is 150.

Consider a subgame A_9 after one of the large buyers accepts at the beginning of the game. Rejection by any buyer in A_9 leads to a subgame starting with state in which $n_l + n_h = 2$ and $n' = 1$. The same condition as in the paragraph with subgame R determines the equilibrium outcome. Because $r = 9$ implies $n_l = 2$ now that one of the large buyers accepted at the beginning of the game, the condition fails. Hence rejection by any buyer in A_9 leads to a subgame in which entry happens in any equilibrium and the equilibrium payoff of I from this subgame is $21 \cdot 9$. Acceptance by any buyer in A_9 leads to a subgame in which exclusion happens in any equilibrium and the equilibrium payoff of I from this subgame is $21 \cdot 30$. Thus in A_9 the payoff of I from approaching any buyer with offer strictly smaller than full compensation is $21 \cdot 9$ while the payoff from fully compensating buyer $i \in N$ is at most $21 \cdot 30 - s_i 80$. Note that $21 \cdot 9 = 189 > 150 = 21 \cdot 30 - 6 \cdot 80 \geq 21 \cdot 30 - s_i 80 \; \forall i \in N$. Thus entry happens in any equilibrium of A_9 and the equilibrium payoff of I from A_9 is 189.

Finally, consider the initial history. Rejection by any of the small buyers leads to R_6 in which entry happens and hence none of the small buyers contracts at the beginning of the game unless fully compensated. The payoff of I from approaching one of the small buyers with offer strictly smaller than full compensation is 0 while the payoff from fully compensating one of the small buyers is at most $21 \cdot 30 - 6 \cdot 80 = 150$. Rejection by any of the large buyers leads to R_9 in which exclusion happens and hence both of the large buyers are willing to contract in return for zero transfer. The payoff of I from approaching one of the large buyers with zero offer is $21 \cdot 9 = 189$. Thus entry happens in any equilibrium of the entire game and I at the initial history approaches one of the large buyers with zero offer, which the buyer accepts. (Multiple equilibria exist in the subgame after the acceptance by the large buyer. Entry and no further acceptance happens in any equilibrium. Firm I stops in some equilibria, but might approach further buyers with offers that are rejected in other equilibria.)