Optimal delegated search with learning and no monetary transfers

Elnaz Bajoori[1] Julia Wirtz[2]

[1]University of Bath

[2]University of Bristol

Delegated Search

- Variation on standard search problem.
- Principal delegates search to an expert (agent).
- Only principal cares about characteristic (quality, price etc.) of object.
- Agent has private information about distribution of characteristic, wants to search as little as possible.

Application: Public Procurement

- Principal: Government/Institution
- Agent: Procurement officer
- Agent needs to procure an object (e.g. computer, vehicle) paid in full by principal.
- Agent has private information about price distribution (e.g. needs object of high or low quality)
- Agent searches for offers.
- Sellers not strategic, listed prices, small scale procurement.
- Principal can set search rules, cannot use conditional payments.

- Alternative applications: Hiring committee, real estate agents, R&D division, etc.

Contribution

- **Standard optimal search, known distribution** (McCall; 1970).
  - Stationary threshold.
  - Search until price below given threshold found, irrespective of number of previous searches.

- **Delegated search, asymmetric information**
  - Search rule using **increasing thresholds**
  - Simple, robust search rule
    - does not require outcome-dependent payments
    - requires minimal monitoring
  - Explanation of existing rules: Minimum number of offers, depending on cost of procured item.

Model

Market state can be high or low price:

- State $a \in \{H, L\}$, privately observed by agent.
- Principal's belief: $\Pr[a = H] = \rho_0 \in (0, 1)$.
- First order stochastic dominance of price distributions: $F_H(p) \leq F_L(p)$.
  $\rightarrow$ Price likely lower in state L than H.

- Search: draw from $F_a(p)$

Payoffs

- Agent: $-\sigma t$.
  - Search cost: σ.
  - Number of searches: t.
- Principal: $-ct - p$.
  - waiting cost c
  - additionally cares about price p.

First best search rule

Symmetric information

- Reduces to standard search problem.
- **Optimal Search rule:** Stationary threshold.
  - Myopic: As if only one additional search possible.
  - Cost of additional search = Expected saving.
  - First best thresholds: $\mathbf{y_H^*} > \mathbf{y_L^*}$

Asymmetric Information

- First best thresholds $y_H^* > y_L^*$ cannot be used.
- State L: Agent would pretend state is H to get higher threshold $y_H^* \geq y_L^* \Rightarrow$ fewer searches.

Increasing threshold rules

- In practice: agent submits (subset of) obtained offers.
- Search rule could depend on all of them.
- Incentives for manipulation, would require credible threat to verify **all prices**

$\Rightarrow$ We restrict attention to **increasing threshold rules** for **minimum price**

Increasing threshold rules $\{y_t\}$

- y_t: Threshold after t searches.
- Stop if **one** $p \leq y_t$
- Rule: sequence of thresholds $\{y_t\}$, with $y_1 \leq y_2 \ldots$.

- Require minimal monitoring, principal only verifies:
  - **at least** t offers received
  - Price of chosen offer **not higher** than reported.
- Robust to manipulating/ hiding/ inflating offers
- Do not require performance dependent payments

Pooling Rule

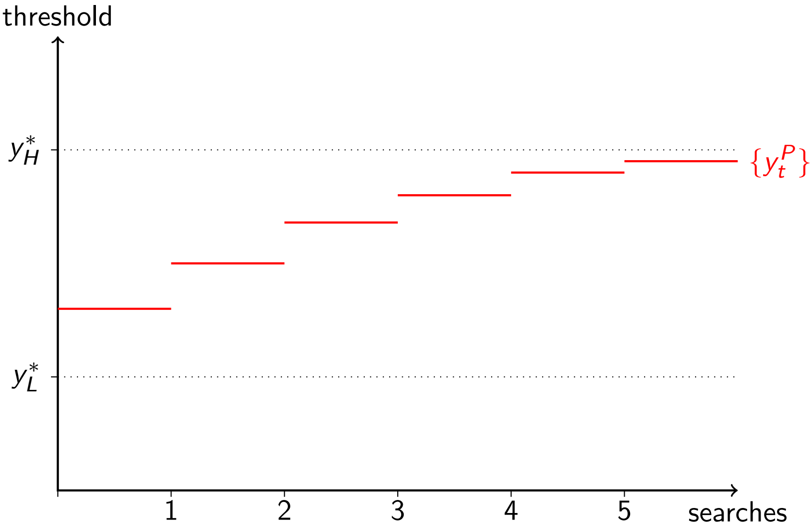
Pooling Rule

- Principal offers single rule $\{y_t^P\}$ for both states.
  $\Rightarrow$ cannot separate states, but updates over time.
- ρ_t: principal's posterior for H, given that agent has not found a price below threshold y_t after t searches.
- ρ_t increases over time.

Optimal Pooling Rule

- Adapting results on optimal single agent search with unknown distribution: Rothschild (1978), Bikhchandani and Sharma (1996).
- Optimal threshold with **uncertain** distribution after t searches = optimal (constant) threshold with **known** price distribution:
- $\hat{F}_t(p) = \rho_t F_H(p) + (1 - \rho_t) F_L(p)$.
- Expected **posterior** distribution given agent does not find price below y_t^P in t searches.
- Optimal thresholds myopic, increasing, and between first best thresholds: $y_L^* \leq \{y_t^P\} \leq y_H^*$.

Optimal Pooling Rule



Separating Rule

Separating rule with first best thresholds

- Principal offers different rules for each state.
- Principal can elicit the state from the agent in advance
- First best thresholds plus **minimum number of offers** k in state H (restriction):

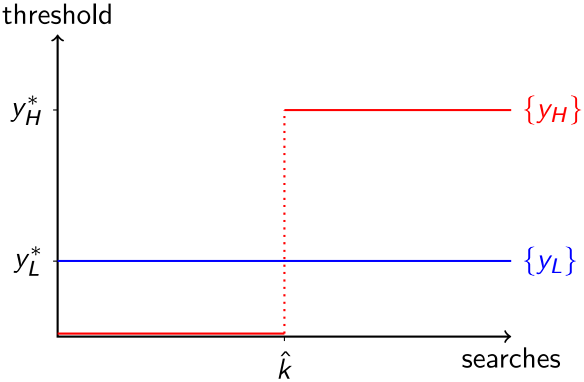
$$y_{L,t} = y_L^*$$
$$y_{H,t} = \begin{cases} 0 & \text{for } t < k \\ y_H^* & \text{for } t \geq k \end{cases}$$

Optimal minimum number of searches $\hat{k}$

- $\hat{k}$ optimally set as small as possible
- $\Rightarrow$ Incentive constraint in low state binding:
  Mis-reporting high state results in equal expected number of searches:

$$\mathbb{E}_L(t|y_L^*) = \mathbb{E}_L(t|\hat{k}, y_H^*)$$

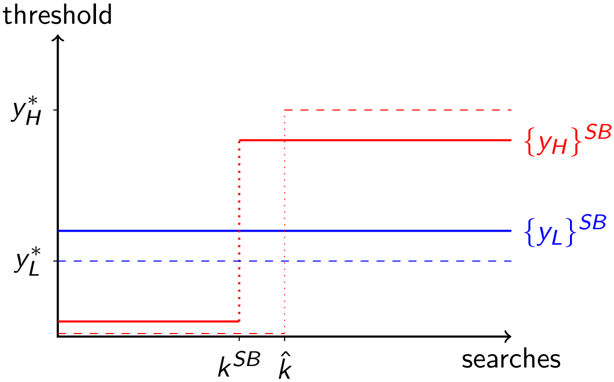
Separating rule with first best thresholds



Optimal Separating Menu

- Optimal increasing threshold rule
- Relax: thresholds fixed to first best & first threshold at zero
- Shorter wait for final thresholds: $k^{SB} \leq \hat{k}$
- All thresholds move towards pooling, but still constant for low/ one step for high state
  - $y_L^{SB} \geq y_L^*$
  - $y_{H1}^{SB} \geq 0$
  - $y_{H2}^{SB} \leq y_H^*$

Optimal Separating Menu



Conclusion

- Search delegated to informed expert
- Robust search rule without transfers and minimal monitoring: increasing thresholds.
- Optimal Pooling: increasing threshold as principal gets more pessimistic
- Separating: extract expert's knowledge by imposing minimum number of searches in unfavourable state.
- Rationalisation for common procurement rule in addition to collusion concerns

Thank you!

Literature

Bikhchandani, S. and Sharma, S. (1996). Optimal search with learning, *Journal of Economic Dynamics and Control* **20**(1-3): 333–359.

McCall, J. J. (1970). Economics of information and job search, *The Quarterly Journal of Economics* pp. 113–126.

Rothschild, M. (1978). Searching for the lowest price when the distribution of prices is unknown, *Uncertainty in Economics*, Elsevier, pp. 425–454.