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Pricing an Unknown Climate

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Intro

Equilibrium climate sensitivity

- Warming response to a doubling of preindustrial CO₂
- IPCC's AR6 guesstimate:
 - *Likely* range 2.5-4C
 - translate into hugely different scenarios for life on planet
 - Refers to 66% *subjective* probability
 - Different methods imply quite different results; paleoclimate data, instrumental records, physical process understanding, "emergent constraints",...

Any temperature level in the 'likely' range (and beyond...)

- far above any recent historic record
- huge extrapolation

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What is our *attitude* to the *subjectivity* of the uncertainty?

- Don't care: Expected Utility
- Do care: Ambiguity Aversion
 - Capacities abandon probability distributions
 - Maximin expected utility set of probability distributions, pick worst (Gilboa & Schmeidler, 1989)
 - Robust Control (Hansen and Sargent, 2001)
 - Used by most of previous literature on climate ambiguity
 - Smooth Ambiguity Aversion subjective second order distribution governed by more aversion(KMM 2005)
 - Recursive smooth ambiguity aversion (KMM 2009) permits anticipated Bayesian learning and time-consistent dynamics
- Should we care?
 - Tomorrow 16:00 room N29 session "Decision Theory": von Neumann-Morgenstern-based 'normative' axiomatization

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Contribution

Social cost of carbon under ambiguity aversion

- Formula: General climate-economy model
- Quantitative premium: DICE-based IAM
- 2 Framework
 - Recursive dynamic programming model
 - Social planner framework with
 - rational foresight and
 - anticipated learning
 - \hookrightarrow Decision maker updates subjective climate sensitivity prior based on temperature observations (Kelly & Kolstad 1999)
 - Klibanoff et al. (2009)'s recursive smooth ambiguity aversion to distinguishes
 - subjective climate sensitivity prior from
 - (objective) temperature stochasticity

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Climate-Economy model







Temperature dynamics:

$$\tilde{T}_{t+1} = \chi_t(M_{t+1})\tilde{s} + \xi_t(T_t) + \tilde{\epsilon}_t$$

 \tilde{s} : subjectively uncertain climate sensitivity (epistemological) $\tilde{\epsilon}$: objective temperature shocks (stochastic)



Climate-Economy model - Numerics



Bayesian learning dynamics:

 $\tilde{s}_t \sim N(\mu_{s,t}, \sigma_{s,t})$ & $\hat{T}_t \Rightarrow \tilde{s}_{t+1} \sim N(\mu_{s,t+1}, \sigma_{s,t+1})$

- Conjugate normal prior
- Update to $\mu_{s,t+1}, \sigma_{s,t+1}$ after observing stochastic temperature realization

KMM's (2009) smooth ambiguity aversion

- Risk averse to temperature stochasticity,
- Ambiguity averse to climate sensitivity uncertainty,
- Ambiguity and ambiguity aversion disentangled,
- Ambiguity aversion is 'moderate':
 - Limit of infinite ambiguity aversion returns
 - $\rightarrow\,$ maximin expected utility (Gilboa & Schmeidler 1989),
 - $\rightarrow\,$ and, "thus", robust control (Hansen & Sargent 2001)
- Time consistent
- Decision maker learns (and anticipates to do so!)
- Has normative axiomatic motivation (tomorrow! :-)

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Bellman equation

with stochasticity, uncertainty, and smooth ambiguity aversion:

$$V_t = \max_{c_t,\mu_t} L_t u(c_t) + \beta \times$$
$$f^{-1} \left\{ \int_S f\left(\mathbb{E}_{c_t} \left[V_{t+1} \right] \right) \mathrm{d} \Pi(s,t) \right\}$$

value fct today = current utility + disc future value fct

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KMM's smooth ambiguity aversion

Bellman equation with stochasticity, uncertainty, and smooth ambiguity aversion:

$$V_t = \max_{c_t,\mu_t} L_t u(c_t) + \beta \times f^{-1} \left\{ \int_S f\left(\mathbf{E}_{\epsilon_t} \left[V_{t+1}\right]\right) \mathrm{d}\,\Pi(s,t) \right\}$$

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value fct today = current utility + disc future value fct

How is the SCC affected by climate sensitivity ambiguity?

Without ambiguity aversion:

$$SCC_{0} = -\frac{1}{u'(c_{0})} \mathbf{E}_{0}^{s} \mathbf{E}_{0}^{T} \left[\sum_{t=1}^{\infty} \sum_{\tau=1}^{t} \prod_{j=1}^{t} \mathcal{R}_{j} \mathbf{E}_{j}^{s} \mathcal{P}_{j} \mathbf{E}_{j}^{T} u_{t}'(c_{t}) \frac{\partial F_{t}}{\partial T_{t}} \frac{\partial T_{t}}{\partial CO_{2,\tau}} \frac{\partial CO_{2,\tau}}{\partial E_{0}} \right]$$

Ambiguity aversion introduces 2 weights:

- Pessimism and
- Prudence

How is the SCC affected by climate sensitivity ambiguity?

With ambiguity aversion:

$$SCC_{0} = -\frac{1}{u'(c_{0})} \mathbf{E}_{0}^{s} \mathbf{E}_{0}^{T} \left[\sum_{t=1}^{\infty} \sum_{\tau=1}^{t} \prod_{j=1}^{t} \mathcal{R}_{j} \mathbf{E}_{j}^{s} \mathcal{P}_{j} \mathbf{E}_{j}^{T} u_{t}'(c_{t}) \frac{\partial F_{t}}{\partial T_{t}} \frac{\partial T_{t}}{\partial CO_{2,\tau}} \frac{\partial CO_{2,\tau}}{\partial E_{0}} \right]$$

Ambiguity aversion introduces 2 weights:

- Pessimism and
- Prudence

How is the SCC affected by climate sensitivity ambiguity?

Pessimism bias:

$$\mathcal{P}_t = \frac{f'(\cdot)}{\mathbf{E}_t^s \left(f'(\cdot)\right)}$$

- More weight on bad outcomes.
- All else equal, pessimism increases SCC.

 $f'(\cdot) = f'\left(\mathbf{E}^s\left[V_{t+1}(\cdot)\right]\right)$

How is the SCC affected by climate sensitivity ambiguity?

Ambiguity prudence:

$$\mathcal{R}_t = \frac{\mathbf{E}_t^s \left(f'(\cdot) \right)}{f' \left(f^{-1}(\cdot) \right)}$$

- Prudence = ambiguity aversion decreases in *welfare*.
- Mean-zero shocks to welfare: ambiguity prudence increases SCC.
- But: climate sensitivity uncertainty impact on welfare (highly) non-linear.

How is the SCC affected by climate sensitivity ambiguity?

- Optimal emission trajectory
- Expected-draws $(\epsilon_t = 0 \forall t)$
- Stochastic temperature, + Uncertain CS ,
 - + Ambiguity aversion
- Ambiguity aversion: RAA=10



How is the SCC affected by climate sensitivity ambiguity?

- Optimal emission trajectory
- Expected-draws $(\epsilon_t = 0 \forall t)$
- Stochastic temperature, + Uncertain CS ,
 - + Ambiguity aversion
- Extreme ambiguity aversion: RAA=100



How is the SCC affected by climate sensitivity ambiguity?

Impact *off* optimal trajectory:

- % difference: RAA=10 – No AA
- Year 2040
- Subsection of state-space
- Largest effect: high CO₂ & CS prior variance



Note: We show optimal policy after wandering off.

How is the SCC affected by climate sensitivity ambiguity?

Impact off optimal trajectory:

- % difference: RAA=100 – No AA
- Year 2040
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Related literature

- Lange and Treich (2008)
- Berger et al. (2016), Berger and Marinacci (2020)
- Millner et al. (2013)
- Lemoine and Traeger (2016)
- Xepapadeas & Yannacopoulos (2018), Brock & Xepapadeas (2021)
- Rudik (2020)
- Barnett et al. (2020)

Summing up:

Evaluate climate sensitivity ambiguity in setting that is

- time consistent (possibly rationally/normatively attractive)
- permits moderate ambiguity aversion
- includes anticipated Bayesian learning

Theory:

• introduces two weighting terms into SCC: *pessimism* weighting & *prudence*

Empirics:

• implies only a small positive policy premium of ambiguity Conclusion for practitioner:

- Error small simply using
 - regular Bayesian expected utility model
 - using best guess prior and neglecting ambiguity

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Appendix 1: Epstein-Zin comparison

How is the SCC affected by climate sensitivity ambiguity?

Comparison to Epstein-Zin preferences & according risk aversion:

- Optimal emission trajectory
- Moderate ambiguity aversion: **RAA=10**
- Epstein-Zin: RRA=10



Appendix 2: Ambiguity premium over Ambiguity Aversion

How is the SCC affected by climate sensitivity ambiguity?



- (Very) moderate effect in optimum $\approx < 1 - 8\%$
- At least initially almost linear increase

Appendix: IPCC Assessment of Climate Sensitivity

IPCC (AR6): Uncertainty in warming projections *dominated* by climate's sensitivity to additional emissions

Dots show central estimates (when available). Bars show likely (66% chance) range. Whiskers show very likely (5% to 90%) range.

