Deliberate Choice under a Lack of Confidence

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Premise

- Not all probabilities are created equal

Question:

- How can or should a policy maker/adviser take such differences into account

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Motivation: Background

Behavioral:

- Decision makers seem to respond differently to
  - known/objective probabilities versus
  - subjective guesstimates
- Vast literature on ambiguity, ambiguity attitude & source uncertainty
  - focused on descriptive **behavior**
  - relaxes 'normatively'/'rationally' desirable axioms
  - shows that decisions are 'as if' there exists ...
    (does not actually have to be a unique disentanglement of taste and uncertainty desciption)

↪ great for behavioral purpose

↪ difficult for policy applications

Motivation: Question

Question:

- How should a decision maker deal with a lack of confidence into probability estimates?

Possible answers:

- A lack of confidence is already expressed by merely assigning probabilities to outcomes
- Ignore it, e.g., von Neumann-Morgenstern suggest EU

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↪ No: it's about probability $\frac{1}{2}$ because of
  - a fair coin toss or
  - Laplace's principle of insufficient reason

  The lack of confidence governs the probability itself
  Different uncertainty generating processes

- Ignore it, e.g., von Neumann-Morgenstern suggest EU

↪ Let's see...

Motivation: Objective

Looking for decision support framework for policy making:

- *probabilities given* (derived by scientists)
- *seek evaluation* of probabilistic scenarios (policy/society)
- main desiderata:
  - stay close to von Neumann-Morgenstern framework (often considered 'normative benchmark')
  - impose time consistency of decisions (failed by most amgiguity models)

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Simple idea:

- Distinguish in model what is different in real world *(& leave it to axioms whether evaluated the same or not)*

Example: distinct probabilistic characterizations

Guidance notes for lead authors of Intergovernmental Panel on Climate Change (*AR4*). All scenarios are described probabilistically, but authors are asked to distinguish between

Table 1. A simple typology of uncertainties

Type	Indicative examples of sources	Typical approaches or considerations
Unpredictability	Projections of human behaviour not easily amenable to prediction (e.g. evolution of political systems). Chaotic components of complex systems.	Use of scenarios spanning a plausible range, clearly stating assumptions, limits considered, and subjective judgments. Ranges from ensembles of model runs.
Structural uncertainty	Inadequate models, incomplete or competing conceptual frameworks, lack of agreement on model structure, ambiguous system boundaries or definitions, significant processes or relationships wrongly specified or not considered.	Specify assumptions and system definitions clearly, compare models with observations for a range of conditions, assess maturity of the underlying science and degree to which understanding is based on fundamental concepts tested in other areas.
Value uncertainty	Missing, inaccurate or non-representative data, inappropriate spatial or temporal resolution, poorly known or changing model parameters.	Analysis of statistical properties of sets of values (observations, model ensemble results, etc); bootstrap and hierarchical statistical tests; comparison of models with observations.

Basic idea

In the model (part 1)

- distinguish different probalistic characterizations of the future
- labeling them with ‘index’ $s \in S \subset \mathbb{N}$
- *apply standard axioms* but respecting differences

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Model (part 2)

- characterize a possible order structure on s
- capturing the idea of confidence
- for agents averse to a lack of confidence

Relation to Ambiguity Literature

Models of ambiguity

- Rank dependent utility
- Choquet expected utility
- Variational preferences
- Multiple prior models
- Second order probabilities

Maybe closest to Klibanoff, Marinacci & Mukerji's (2005,2009) 'smooth ambiguity aversion' ($\equiv$ KMM)

Preview

The simple idea:

- Index probability measures by type or degree of confidence s
- Reduction of compound probabilities *only if* they are of same type/ same degree of confidence
- Otherwise standard axioms (von Neumann-Morgenstern, certainty separability, time consistency)

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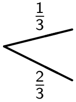
Results in:

- Decision Support Framework taking account of Confidence
- A concept of Aversion to the Lack of Confidence
- Generalization of a unified framework of
  - Epstein-Zin preferences (disentangle int subst and risk aversion)
  - KMM model (smooth ambiguity aversion)
- can nest common criteria such as *EU*, maximin, maximin EU, smooth ambiguity aversion *as functions of confidence*

Uncertainty structure

Representing 1 layer of uncertainty



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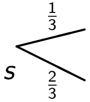

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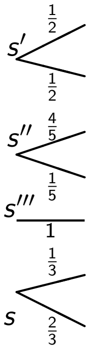
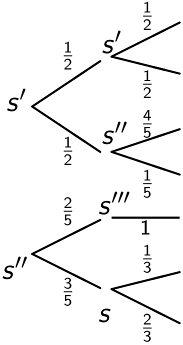


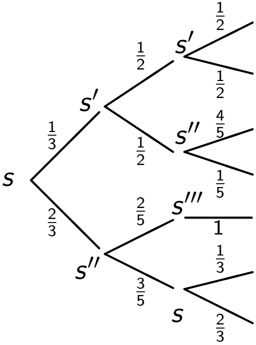
Uncertainty structure

Representing 2 layers of uncertainty



Uncertainty structure

Representing 3 layers of uncertainty



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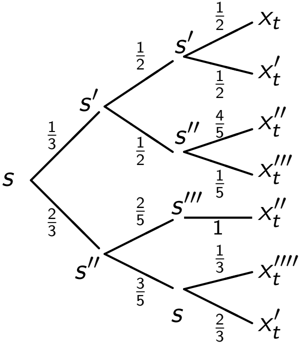
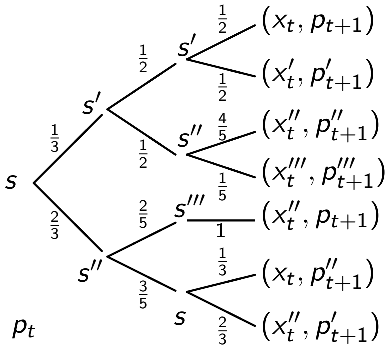


One period future

Uncertainty structure

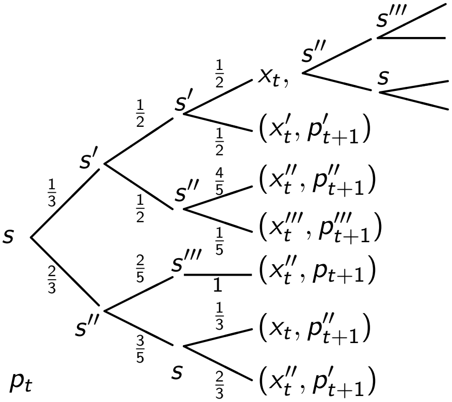
Representing 3 layers of uncertainty



Multi-period setting

Uncertainty structure

Representing 3 layers of uncertainty

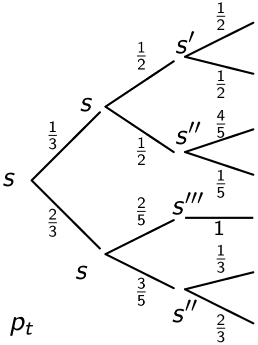


Multi-period setting

Reduction of compound lotteries (Definition)

Denote

- P_t^s: Subset of P_t with first node of type s (e.g. confidence level)
- $\hat{s}(p_t) = s$ iff $p_t \in P_t^s$



p_t

$p_t \in P_t^s$

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Reduction of compound lotteries (Definition)

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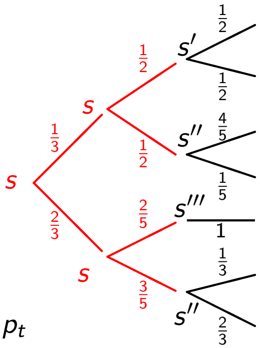

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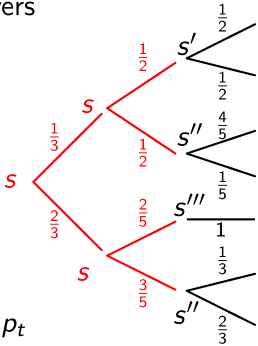
Reduction of compound lotteries (Definition)

Denote

- P_t^{ss}: Subset of P_t with first two uncertainty layers of type s
- p_t^r: Reduction of $p_t \in P_t^{ss}$ obtained by collapsing first two layers



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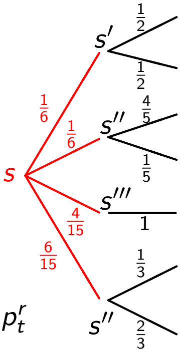


Mixing (Definition)

Mixing of lotteries: (here: same type lotteries but not crucial)

For $p_t, p'_t \in P^s_t$ define for $\alpha \in [0,1]$ and $s \in S$ the mixture

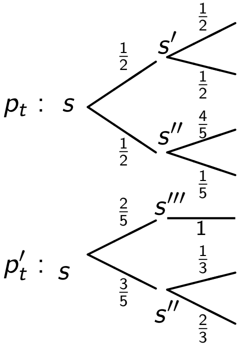
$p_t \oplus^{\alpha}_{s} p'_t$ as lottery in P^s_t yielding

- p_t with probability α and
- p'_t with probability $1 - \alpha$ with
- where lottery is of type s

Same idea as std von Neumann-Morgenstern, but index s representing type of lottery

Mixing

Example: Mixing of lotteries:



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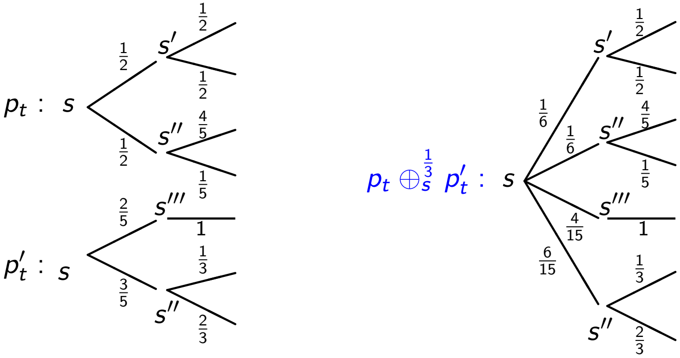

Axioms

Axioms

- Indifference to reduction of *same* degree of subjectivity lotteries:

  For all $t \in \{0, ..., T\}, s \in S$, and $p_t \in \cup_{s \in S} P_t^{ss}$: $\quad p_t^r \sim_t p_t$

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- Independence:

  For all $t \in \{0, ..., T\}, s \in S, \alpha \in [0, 1]$ and $p_t, p_t', p_t'' \in P_t^s$

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- Standard axioms: weak order, continuity, certainty separability, time consistency

shortcut

Representation

Notation for Representation:

- Uncertainty aggregator (generalized mean):
  - For f strictly increasing define: $\mathcal{M}_p^f\, z \equiv f^{-1}\left[\mathrm{E}_p\, f(z)\right]$
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Define generalized uncertainty aggregator:

Let p_t be lottery p_t^1 over lotteries p_t^2 over ... p_t^N over (x_t^*, p_{t+1}):

$$\mathcal{M}_{p_t}^{\hat{f}_t}\, W_t(x_t^*, p_{t+1}) \equiv \mathcal{M}_{p_t^1}^{f^{\hat{s}(p_t^1)}} \cdots \mathcal{M}_{p_t^N}^{f^{\hat{s}(p_t^N)}}\, W_t(x_t^*, p_{t+1})$$

Representation

The Representation:

The sequence of preference relations $(\succeq_t)_{t \in T}$ satisfies the axioms if, and only if, for all $t \in \{0, ..., T\}$ there exist

- a set of strictly increasing and continuous functions $\hat{f}_t = (f_t^s)_{s \in S}$, $f_t^s : \mathbb{R} \to \mathbb{R}$
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such that by defining recursively the functions $W_T = u_T$ and

- $W_{t-1} : X^* \times P_t \to \mathbb{R}$ by

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it holds for all $t \in T$ and all $p_t, p_t' \in P_t$

$$p_t \ \succeq_t \ p_t' \quad \Leftrightarrow \quad \mathcal{M}_{p_t}^{\hat{f}_t} W_t(x_t, p_{t+1}) \ \geq \ \mathcal{M}_{p_t'}^{\hat{f}_t} W_t(x_t, p_{t+1})$$

Example $\hookrightarrow$

Main Feature of Representation

The representation uses:

- A generalized mean for uncertainty aggregation
  - For f strictly increasing define: $\mathcal{M}_p^f\, z \equiv f^{-1}\left[\mathrm{E}_p\, f(z)\right]$
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The representation shows:

- Any uncertainty node of type s is evaluated using $\mathcal{M}_{p^s}^{f^s}$ (characterized by aversion function f_t^s depending on s)
- Evaluation is recursive (in both time and probabilitiy tree within a given period)

Example

Interpretation

$$W_{t-1}(x_{t-1}, p_t) = u_{t-1}(x_{t-1}) \; + \; \mathcal{M}_{p_t}^{\hat{f}_t} \, W_t(x_t, p_{t+1})$$

Function u measures aversion to intertemporal subst.

There are 2 effects of risk:

i) Generates fluctuations over time
   $\rightarrow$ Disliked by agents who prefer smooth consumption paths
   $\rightarrow$ Measured by u

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More about

- intrinsic risk aversion
- how to characterize concavity of individual f_t^s axiomatically

in the paper

Lack of confidence

Now let us associate the label s with level of confidence

- suggests an *order relation* $s' \triangleright s$:
  More confident about lottery labeled s' than lottery labeled s.

Definition 2:

A decision maker is (strictly) averse to the lack of confidence in belief iff for all $x, x' \in X^t$ and $s, s' \in S$:

$$s' \triangleright s \quad \Rightarrow \quad x \oplus_{s'}^{\frac{1}{2}} x' \ \succeq_t \ (\ \succ_t\ ) \ x \oplus_{s}^{\frac{1}{2}} x' \ .$$

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Proposition 2: Characterization

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which is equivalent to

$$s' \triangleright s \quad \Leftrightarrow \quad f_t^s \circ (f_t^{s'})^{-1} \text{ (strictly) concave} \quad \forall s, s' \in S \; .$$

Lack of confidence

Note: Behavioral decision theorists would probably prefer formulation:

- if there exists order on S such that...
- ... then representation satisfies...

Lack of confidence

Note: Behavioral decision theorists would probably prefer formulation:

- if there exists order on S such that...
- ... then representation satisfies...

Yet, I seek a framework that

- takes order on S characterizing lack of confidence as given:
  - Decision maker obtains it from scientists
  - or assigns it based on her judgement of advising panels
- explores how to incorporate such a statement meaningfully into evaluation

Conclusions

- Standard expected utility model suggests that differences in types of probabilities do not matter for evaluation
- this ‘finding’ is based on implicit ignoring of differences
- *von Neumann-Morgenstern* setting is easily extended to respect differences keeping main, normatively desirable axioms

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Then

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- *von Neumann-Morgenstern* setting is easily extended to respect differences keeping main, normatively desirable axioms

Then

- Confidence in probabilistic description of future matters (just as much as risk aversion does)
- Define and characterize aversion to the lack of confidence
- The evaluation model can nest decision criteria arising in
  - standard expected utility
  - decision making under ignorance by Arrow Hurwitz
  - maxi-min expected utility by Gilboa Schmeidler
  - smooth ambiguity aversion by KMM

  depending on the level of confidence

Appendix

Relation to Risk Literature

The 2 Effects of risk:

i) Generates fluctuations over time
$\rightarrow$ Disliked by agents who prefer smooth consumption over time

ii) Makes agent unsure about their future
$\rightarrow$ Disliked by agents with intrinsic aversion to risk

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- Intertemporally additive std model dismisses second effect (NOT a consequence of von Neumann-Morgenstern!)

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- Kreps & Porteus (1978), Epstein & Zin (1989,1991) and Weil (1990) developed model that incorporates both risk effects
- In Epstein-Zin-Weil model:
  Arrow Pratt risk aversion coefficient measures *i* and *ii* jointly.

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- In Epstein-Zin-Weil model:
  Arrow Pratt risk aversion coefficient measures *i* and *ii* jointly.
- Alternative (Traeger 2010):
  Intertemporal risk aversion characterizes *ii* directly
  (a convenient multi-commodity risk measure)

Intertemporal Risk Aversion (characterizing f)

Let x, x' be two consumption paths of length T.

Example, $T = 4$:

$$x = (\quad , \quad , \quad , \quad)$$

$$x' = (\quad , \quad , \quad , \quad)$$

Let $x \succ x'$ denote a strict preference for x over x'.
Let $\sim$ denote indifference.

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x =(☺,☹,☺,☹)

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Define for x and x' the consumption paths

- x^{high}: collects better outcomes of every period
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A Question of Preference

Assume you'd be indifferent between

$$(☺,☹,☺,☹) \sim (☹,☺,☹,☺)$$

A Question of Preference

Assume you'd be indifferent between

$$(☺,☹,☺,☹) \sim (☹,☺,☹,☺)$$

If not, please mentally adjust the corners of the mouth of the red frowny ☹ to reach indifference.

A Question of Preference

Assume you'd be indifferent between

$$(☺,☹,☺,☹) \sim (☹,☺,☹,☺)$$

If not, please mentally adjust the corners of the mouth of the red frowny ☹ to reach indifference.

What preference do you have in the following choice?

(☺,☹,☺,☹) vs. s: with probability $\frac{1}{2}$ (☺,☺,☺,☺), with probability $\frac{1}{2}$ (☹,☹,☹,☹)

certain path — coin toss if s=obj

A Question of Preference

Assume you'd be indifferent between

(☺,☹,☺,☹) $\sim$ (☹,☺,☹,☺)

If not, please mentally adjust the corners of the mouth of the red frowny ☹ to reach indifference.

What preference do you have in the following choice?

(☺,☹,☺,☹) $\sim$ s — $\frac{1}{2}$ → (☺,☺,☺,☺) ; $\frac{1}{2}$ → (☹,☹,☹,☹)

certain path — coin toss if s=obj

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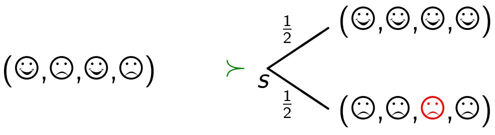

certain path

coin toss if s=obj

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Assume you'd be indifferent between

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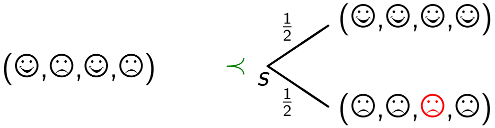


certain path

coin toss if s=obj

A Question of Preference

Assume you'd be indifferent between

(☺,☹,☺,☹) $\sim$ (☹,☺,☹ (red),☺)

If not, please mentally adjust the corners of the mouth of the red frowny ☹ to reach indifference.

What preference do you have in the following choice?

(☺,☹,☺,☹) $\sim_s$ [$\frac{1}{2}$: (☺,☺,☺,☺); $\frac{1}{2}$: (☹,☹,☹ (red),☹)]

certain path — coin toss if s=obj

$\forall\, s \Rightarrow$ STANDARD MODEL $\mathrm{E}\sum_t \beta^t u(x_t)$

A Question of Preference

Assume you'd be indifferent between

(☺,☹,☺,☹) $\sim$ (☹,☺,☹,☺)

If not, please mentally adjust the corners of the mouth of the red frowny ☹ to reach indifference.

What preference do you have in the following choice?

(☺,☹,☺,☹) $\succ$ s — $\frac{1}{2}$ → (☺,☺,☺,☺) ; $\frac{1}{2}$ → (☹,☹,☹,☹)

certain path — coin toss if s=obj

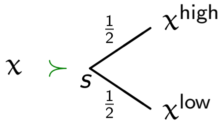
INTERTEMPORAL RISK AVERSE
with respect to degree of subjectivity s

A Question of Preference

Assume you'd be indifferent between

$$x \sim x' \quad \wedge \quad \exists \text{ period } \tau \in \{1, \ldots, T\} \text{ in which}$$
$$\text{consumption of } x \text{ and } x' \text{ are nonindifferent}$$

What preference do you have in the following choice?



certain path   coin toss if s=obj

INTERTEMPORAL RISK AVERSE
with respect to degree of subjectivity *s*

Intertemporal Risk Aversion

For any two consumption paths x, x' define composed paths

- $x^{\text{high}}(x, x')$ collecting better outcomes of every period
- $x^{\text{low}}(x, x')$ collecting inferior outcomes of every period

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Definition 1:

A decision maker is intertemporal risk averse w.r.t. to lotteries of degree of subjectivity s in period t

- iff for all certain consumption paths x and x'

$$x \sim_t x' \quad \Rightarrow \quad x \;\succeq_t\; x^{\text{high}}(x, x') \oplus_s^{\frac{1}{2}} x^{\text{low}}(x, x')$$

Intertemporal Risk Aversion

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Characterization of f:

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- iff f_t^s in the representation is concave

Intertemporal Risk Aversion

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- iff f_t^s in the representation is concave

Note relation to one-commodity Epstein Zin (1989):

- f_t^{obj} measures the difference between Arrow Pratt aversion to objective risk an aversion to intertemporal substitution

Subjectivity of Belief and Ambiguity

Three restrictions make representation a von Neumann-Morgenstern version of KMM's model of smooth ambiguity aversion:

- only 2 layers of uncertainty (in every period)
- only subjective (*subj*) over objective (*obj*) lotteries
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Another way of phrasing the restriction:

- KMM disentangle subjective Arrow Pratt risk aversion from aversion to intertemporal substitution
- But KMM set Arrow Pratt aversion to objective risk equal to aversion to intertemporal substitution

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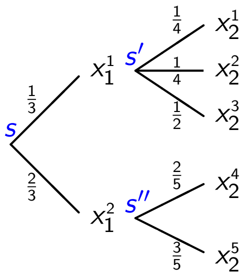
Then:
Aversion to subjectivity collapses to KMM's smooth ambiguity aversion

Representation - Illustration

back

Example ($T = 2$):

Evaluate 

Representation - Illustration

back

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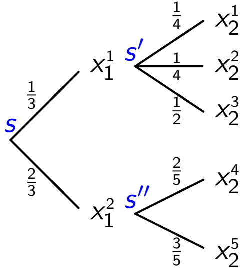
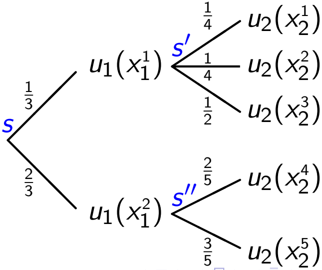
Evaluate



by



Representation - Illustration

back

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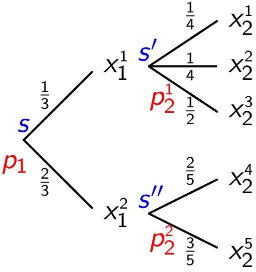

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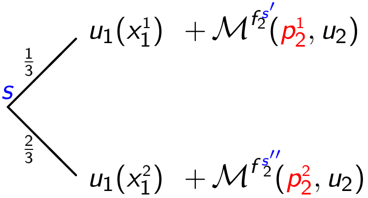

Representation - Illustration

back

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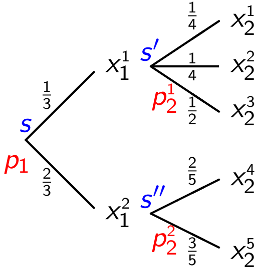
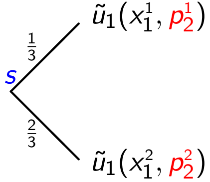


by



Representation - Illustration

back

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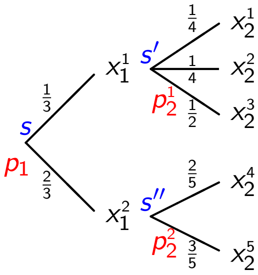
Evaluate 

by $\mathcal{M}^{f_1^s}(p_1, \tilde{u}_1)$

Setting - Details

$\rightarrow$ back

The choice space (within a period):

- X compact metric space, for example:
- X^* outcomes and observations within a period

Setting - Details

$\rightarrow$ back

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- $s \in S$ degree of subjectivity, where S is finite set
- $\Delta_s(X)$ space of Borel probability measures on X,
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Setting - Details

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- $Z^0(X) \equiv X$,
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Setting - Details

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What for?

- Static choice object: Lottery $z \in Z^N(X^*)$ for some $N \in \mathbb{N}$
- Arbitrary concatenation of lotteries with differing degrees of subjectivity

Setting - Details

→ back

The actual choice space (generalized temporal lotteries):

Assume a finite time horizon

- Last period: choices $p_T \in P_T = Z^N(X^*)$
- Period before: choices $p_{T-1} \in P_{T-1} = Z^N(X^* \times Z^N(X^*))$
- Recursively: choices in t: $p_t \in P_t = Z^N(X^* \times P_{t+1})$

Setting - Details

$\rightarrow$ back

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Interpretation P_t:

- At beginning of a period uncertainty over
  - outcome in that period
  - lottery describing the future at the end of that period
- uncertainty composed of different risks with differing degrees of subjectivity

Setting - Details

→ back

Definitions (preferences, subjectivity of lottery, reduction)

- Preferences $\succeq_t$ on P_t

Setting - Details

→ back

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- Preferences $\succeq_t$ on P_t
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- $\hat{s}(p_t) = s$ iff $p_t \in P_t^s$ (degree of subjectivity of root lottery)

Setting - Details

→ back

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  $p_t^r(A) = \int \tilde{p}(A) dp_t(\tilde{p})$ for all A in Borel algebra

Setting - Details

$\rightarrow$ back

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What's the point?

- (Only) lotteries of same subjectivity can be pulled together
- Only then the independence axioms will have power

Relation to Epstein Zin (1989)

Relation to Arrow Pratt risk aversion:

For the one-commodity setting (with utility str. increasing)

- u_t characterize aversion to intertemporal substitution

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Define:

- $g_t^{obj} \equiv f_t^{obj} \circ u_t^{-1}$: measures Arrow Pratt risk aversion with respect to objective risk
- $g_t^{subj} \equiv f_t^{subj} \circ u_t^{-1}$: measures Arrow Pratt risk aversion with respect to subjective risk
- Then $f_t^{amb} = g_t^{subj} \circ (g_t^{obj})^{-1}$

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- Then $f_t^{amb} = g_t^{subj} \circ (g_t^{obj})^{-1}$

Here, my suggested refinement of smooth ambiguity aversion is equivalent to being

- more Arrrow Pratt risk averse to subjective than to objective risk.