The Welfare of Nations : Do social preferences matter for the macroeconomy?

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Countries differ widely in terms of their fiscal system

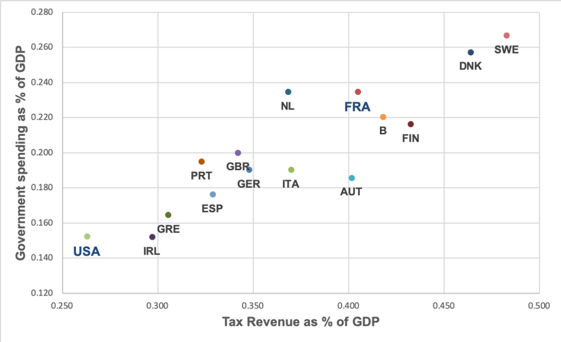

Figure – Government spending and tax revenues average from 1995 to 2007 (as a share of GDP). Source Trabandt & Uhlig (2011).

Fiscal systems and inequalities in the US and in France

	Tot	τ^K	τ^L	τ^c	B	G	Gini bef.	Gini aft.	Gini wealth
France	40	35	46	18	60	24	.48	.28	.68
United States	26	36	28	5	63	15	.48	.40	.77

Table – Summary of fiscal systems and inequalities in the US and and in France.

How can we rationalize those differences ?

1. Different **technologies**.
2. Different **preferences**, i.e., the **Social Welfare Function(SWF)** through which the government aggregates the welfare of heterogeneous agents might differ across countries.
3. The **political system** that implements the policy can differ.

What we do

Provide a methodology to identify the **Social Welfare Function (SWF)** which is compatible with the **actual tax structure** using the following fiscal instruments : **capital and consumption taxes, non-linear labor tax, and public debt**.

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Contribution

1. What is the SWF which rationalizes these fiscal choices ?
2. What is the role of SWFs for the macroeconomy ? In other words, how does the fiscal system change in case the countries alter their SWFs ?
3. How do the business cycle properties are altered if the SWF of countries differ ?

How are we able to do that ?

Our identification strategy relies on **two main identifying assumptions** :

1. Fiscal choices result from the **optimal choices** of a benevolent planner. The planner is endowed with her own SWFs, and understands distortions and the general equilibrium effects - **Inverse Optimal Taxation Problem**.
2. Select among possible SWFs the one which is the closest to the **Utilitarian SWF**.

Preview of the Results

- The SWFs in France and in the US are very **different** from each other :
  1. US SWF is **increasing**.
  2. France SWF is **U-shaped**.
  3. For US has the SWF of France : **debt and progressivity of the labor tax have to increase**.
  4. **Dynamic response** of the US economy is surprisingly similar when we **change the SWF**.

Literature Review

1. Inverse Optimal Taxation Problem, which estimate Social Welfare Functions (SWFs) from actual fiscal policies : Bargain & Keane (2010) ; Bourguignon & Amadeo (2015) ; Heathcote & Tsujiyama (2021) ; Chang et al. (2018) (use one fiscal instrument).
2. Optimal policies in heterogeneous-agent model : Heathcote (2005) ; Kaplan & Violante (2014), Heathcote & Perri (2017) , Aiyagari (1995) , Aiyagari & McGrattan (1998) ; Dávila et al. (2012) .
3. Ramsey problem to obtain the steady-state fiscal policy and level of public debt : Dyrda & Pedroni (2018) ; Açikgözet al. (2018).
4. Estimation of social welfare function : Laroque & Choné 2005, Bourguignon & Spadaro, 2012, Bargain et al., 2014.

Households preferences and program

- Y **idiosyncratic productivity states** $y_t \in \mathcal{Y} := \{1, \ldots, Y\}$, with history $y^t = \{y_0, \ldots, y_t\} \in \mathcal{Y}^{t+1}$.
- Markovian **aggregate state** $s_t \in \mathcal{S} \subset \mathbb{R}^+$, with history $s^t = \{s_0, \ldots., s_t\} \in \mathcal{S}^{t+1}$.
- **Tax on labor income :** $\mathcal{T}_t(\tilde{w}_t y_t^i l_t^i) := \tilde{w}_t y_t^i l_t^i - \kappa_t(\tilde{w}_t y_t^i l_t^i)^{1-\tau_t}$.

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$$\max_{\{c_t^i, l_t^i, a_t^i\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t^i) - v(l_t^i) + u_G(G_t) \right),$$

$$(1 + \tau_t^c) c_t^i + a_t^i \leq \underbrace{\kappa_t (\tilde{w}_t)^{1-\tau_t}}_{:=w_t} (y_t^i l_t^i)^{1-\tau_t} + (1 + \underbrace{(1 - \tau_t^K) \tilde{r}_t}_{:=r_t}) a_{t-1}^i,$$

$$a_t^i \geq -\overline{a}, c_t^i > 0, l_t^i > 0.$$

Production and government

- **Production Function :** CRS net of depreciation, $F(K_{t-1}, L_t, s_t)$.
- **Wage rate and capital interest rate :** $\tilde{r}_t = F_K(K_{t-1}, L_t, s_t)$ and $\tilde{w}_t = F_L(K_{t-1}, L_t, s_t)$.

Production and government

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- **Government budget constraint :** $G_t + (1+\tilde{r}_t)B_{t-1} = \tau_t^c C_t + \int_i \mathcal{T}_t(\tilde{w}_t y_t^i l_t^i)\ell(di) + \tau_t^K \tilde{r}_t A_{t-1} + B_t$.

Market clearing and resources constraints

- **Market clearing condition** :

$$\int_i a_t^i \ell(di) = A_t = B_t + K_t, \quad \int_i y_t^i l_t^i \ell\,(di) = L_t.$$

- **Resource constraint** :

$$G_t + C_t + K_t - K_{t-1} = F(K_{t-1}, L_t, s_t).$$

▸ Definition

▸▸ Social Welfare Function

The Social Welfare Function

- We need to compute for an **arbitrary aggregate welfare function** the fiscal policy that maximizes aggregate welfare.
- The Welfare an arbitrary agent having initial state (y_0, a_{-1}) can be expressed as follows :

$$V_0(y_0) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{y^t \in \mathcal{Y}^{t+1} | y_0} \theta_t(y^t \mid y_0) \left(u(c_t(y^t)) - v(l_t(y^t)) + u_G(G_t) \right).$$

The Social Welfare Function

- The planner considers to put some weight $\omega\left(y_t\right)$:

$$V_0^P(y_0) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{y^t \in \mathcal{Y}^{t+1}|y_0} \omega(y_t)\theta_t(y^t \mid y_0)\Big(u(c_t(y^t)) - v(l_t(y^t)) + u_G(G_t)\Big).$$

- **The SWF at date 0** then becomes :

$$W_0 = \sum_{y_0 \in \mathcal{Y}} \theta_0\left(y_0\right) V_0^P\left(y_0\right).$$

The Social Welfare Function

- The SWF can be written as : ▸ More details

$$W_0 := \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) \Big(u(c_t^i) - v(l_t^i) + u_G(G_t) \Big) \, \ell(di) \right].$$

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The goal is to estimate $\omega(y_t^i)$ that replicate

1. the actual fiscal policies (in France or in the US).
2. some key empirical steady-state moments.

The Ramsey Program

Redundancy

$$\max_{\left(w_t, r_t, \tilde{w}_t, \tilde{r}_t, \tau_t^c, \tau_t^K, \tau_t, \kappa_t, B_t, G_t, K_t, L_t, (c_t^i, l_t^i, a_t^i, \nu_t^i)_i\right)_{t \geq 0}} W_0,$$

$$G_t + (1 + r_t)B_{t-1} + w_t \int_i (y_t^i l_t^i)^{1-\tau_t} \ell(di) + r_t K_{t-1} = \tau_t^c C_t + F(K_{t-1}, L_t, s_t) + B_t,$$

for all $i \in \mathcal{I}$: $(1 + \tau_t^c)c_t^i + a_t^i = (1 + r_t)a_{t-1}^i + w_t(y_t^i l_t^i)^{1-\tau_t},$

$$a_t^i \geq -\bar{a}, \ \nu_t^i(a_t^i + \bar{a}) = 0, \ \nu_t^i \geq 0,$$

$$u'(c_t^i) = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} u'(c_{t+1}^i) \right] + \nu_t^i,$$

$$v'(l_t^i) = \frac{(1 - \tau_t)}{1 + \tau_t^c} w_t y_t^i (y_t^i l_t^i)^{-\tau_t} u'(c_t^i),$$

$$K_t + B_t = \int_i a_t^i \ell(di), \ L_t = \int_i y_t^i l_t^i \ell(di).$$

Identification of Pareto weights

Our paper focuses here on an **inverse optimal taxation problem**.

Definition (Consistent SW)

Consider a steady-state fiscal policy $(\tau^c, \tau^K, \tau, \kappa, B, G)$. **Social Pareto weights** implied by the mapping $\omega : \mathcal{Y} \to \mathbb{R}$ are said to be **consistent** when the **fiscal policy** $(\tau^c, \tau^K, \tau, \kappa, B, G)$ is the **steady-state optimal fiscal policy** of the Ramsey program with the planner's objective W_0 defined using the weights implied by ω.

Identification of Pareto weights

1. Estimate social weights that are the closest to the standard **utilitarian social welfare function :**

$$(\omega(y))_{y\in\mathcal{Y}} = \arg\min_{\{\tilde{\omega}(y))\}} \sum_{y\in\mathcal{Y}} \theta(y)(\tilde{\omega}(y) - 1)^2.$$

2. Estimate a **parametric functional** form – similarly to Heathcote & Tsujiyama (2021) :

$$\log\omega(y) := \omega_0 + \omega_1 \log(y) + \omega_2(\log(y))^2.$$

Truncated Model

Solving the Ramsey Problem

Proposition

In the Ramsey problem, where the planner can levy linear taxes on consumption and capital, progressive tax on labor, and raise one-period public debt, the ***consumption tax is redundant with other fiscal instruments****.*

Furthermore, the credit constraint $\overline{a}$ can be set to zero without loss of generality.

Redundancy Result

Solving the Ramsey Problem

The first Lagrange multiplier we introduce is denoted by $\beta^t\lambda^i_{c,t}$ and is associated to the Euler equation of agent i at date t. We also introduce $\beta^t\lambda^i_{l,t}$:

- **Social value of liquidity :**

$$\psi^i_t := \omega^i_t u'(c^i_t) - (\lambda^i_{c,t} - R_t\lambda^i_{c,t-1} - (1-\tau_t)W_t y^i_t (y^i_t l^i_t)^{-\tau_t}\lambda^i_{l,t})u''(c^i_t).$$

- **Net social value of liquidity :**

$$\hat{\psi}^i_t = \psi^i_t - \mu_t.$$

Solving the Ramsey Problem

- The FOC with respect to **individual savings** $(\tilde{a}_t^i)$:

$$\hat{\psi}_t^i = \beta \mathbb{E}_t \left[R_{t+1} \hat{\psi}_{t+1}^i \right].$$

- The FOC with respect to **labor** (l_t^i) :

$$\begin{aligned}\psi_{l,t}^i = {} & (1-\tau_t) W_t (y_t^i)^{1-\tau_t} (l_t^i)^{-\tau_t} \hat{\psi}_t^i \\ & + \mu_t F_{L,t} y_t^i - (1-\tau_t) W_t (y_t^i)^{1-\tau_t} (l_t^i)^{-\tau_t} \lambda_{l,t}^i \tau_t \frac{u'(c_t^i)}{l_t^i}.\end{aligned}$$

- The FOC with respect to the **interest rate** (R_t) :

$$\int_j \hat{\psi}_t^j \tilde{a}_{t-1}^j \ell(dj) = -\int_j \lambda_{c,t-1}^j u'(c_t^j) \ell(dj).$$

Solving the Ramsey Problem

- The FOC with respect to the **wage rate** (W_t) :

$$\int_j \hat{\psi}_t^j (y_t^j l_t^j)^{1-\tau_t} \ell(dj) = -\int_j \lambda_{l,t}^j (y_t^j l_t^j)^{1-\tau_t} (1-\tau_t) u'(c_t^j)/l_t^j \ell(dj).$$

- The FOC with respect to **progressivity** (τ_t) :

$$\begin{aligned} 0 = & \int_j (y_t^j l_t^j)^{1-\tau_t} (\hat{\psi}_t^j + \lambda_{l,t}^j (1-\tau_t)(u'(c_t^j)/l_t^j)) \ln(y_t^j l_t^j) \ell(dj) \\ & + \int_j \lambda_{l,t}^j \left((y_t^j l_t^j)^{1-\tau_t} \right) (u'(c_t^j)/l_t^j) \ell(dj). \end{aligned}$$

- The FOC with respect to the **public debt** $(\hat{B}_t)$:

$$\mu_t = \beta \mathbb{E}_t \left[(1 + \tilde{r}_{t+1}) \mu_{t+1} \right].$$

The Truncated Model

- Let $N \geq 0$ be a truncation length.
- Let

$$y^t = \{\underbrace{y_0, \ldots, y_{t-N-2}, y_{t-N-1}, y_{t-N}}_{\xi_{y^N}}, \underbrace{y_{t-N+1}, \ldots, y_{t-1}, y_t}_{=y^N}\}$$

$$:= \{\underbrace{\ldots, y^t_{-N-2}, y^t_{-N-1}, y^t_{-N}}_{\xi_{y^N}}, \underbrace{y^t_{-N+1}, \ldots, y^t_{-1}, y^t_0}_{=y^N}\}.$$

Results Ramsey Projected

- There are S_{t,y^N} agents with history y^N , with $S_{t,y^N} = \sum_{\tilde{y}^N \in \mathcal{Y}^N} S_{t-1,\tilde{y}^N} \Pi_{t,\tilde{y}^N y^N}$.
- The **model aggregation** then assigns to **each truncated history** the **average choices** (be it for consumption, savings, or and labor supply) of the **group of agents sharing the same truncated history**.

The Truncated Model

- Consider a generic variable, denoted by $X_t(y^t, s^t)$ and we denote by X_{t,y^N} the **average quantity of X assigned to truncated history** y^N. Formally :

$$X_{t,y^N} = \frac{1}{S_{t,y^N}} \sum_{y^t \in \mathcal{Y}^{t+1} \big| (y^t_{-N+1}, \dots, y^t_{-1}, y^t_0) = y^N} X_t(y^t, s^t)\theta_t(y^t),$$

  where $\theta_t(y^t)$ is the measure of agents with history y^t.

- The **beginning-of-period wealth** $\tilde{\tilde{a}}_{t,y^N}$ for truncated history y^N is :

$$\tilde{\tilde{a}}_{t,y^N} = \sum_{\tilde{y}^N \in \mathcal{Y}^N} \frac{S_{t-1,\tilde{y}^N}}{S_{t,y^N}} \Pi_{t,\tilde{y}^N y^N} \tilde{a}_{t-1,\tilde{y}^N}.$$

The Ramsey Program on the projected Model

$$\max_{\left(W_t, R_t, \tilde{w}_t, \tilde{r}_t, \tau_t^c, \tau_t^K, \tau_t, \kappa_t, \hat{B}_t, G_t, K_t, L_t, (c_{t,y^N}, l_{t,y^N}, \tilde{a}_{t,y^N}, \nu_{t,y^N})_{y^N}\right)_{t\geq 0}} W_0,$$

$$G_t + W_t \sum_{y^N \in \mathcal{Y}^N} \xi^y_{y^N} S_{t,y^N} (l_{t,y^N} y_{y^N})^{1-\tau_t} + (R_t - 1)\tilde{A}_{t-1} + \hat{B}_{t-1} = F(K_{t-1}, L_t, s_t) + \hat{B}_t.$$

Finite state-space representation :

for all $y^N \in \mathcal{Y}^N$: $c_{t,y^N} + \tilde{a}_{t,y^N} = W_t \xi^y_{y^N} (l_{t,y^N} y_0^N)^{1-\tau_t} + R_t \tilde{\tilde{a}}_{t,y^N},$

$$\tilde{a}_{t,y^N} \geq -\bar{\tilde{a}},\ \nu_{t,y^N}(\tilde{a}_{t,y^N} + \bar{\tilde{a}}) = 0,\ \nu_{t,y^N} \geq 0,$$

$$\xi^{u,E}_{y^N} u'(c_{t,y^N}) = \beta \mathbb{E}_t \left[R_{t+1} \sum_{\tilde{y}^N \in \mathcal{Y}^N} \Pi_{t+1,y^N \tilde{y}^N} \xi^{u,E}_{\tilde{y}^N} u'(c_{t+1,\tilde{y}^N}) \right] + \nu_{t,y^N},$$

$$\xi^{v,1}_{y^N} v'(l_{t,y^N}) := (1-\tau_t) W_t \xi^y_{y^N} (l_{t,y^N} y_0^N)^{1-\tau_t} \xi^{u,1}_{y^N} (u'(c_{t,y^N})/l_{t,y^N}),$$

$$K_t + \hat{B}_t = \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} \tilde{a}_{t,y^N}, \quad L_t = \sum_{y^N \in \mathcal{Y}^N} S_{t,y^N} y_{y^N} l_{t,y^N}.$$

The Ramsey Program on the projected Model

- **Computing the Pareto weights** $(\omega_y)_{y\in\mathcal{Y}}$ is the key contribution of our paper involves estimating the Pareto weights that corresponds to different fiscal systems.
-

$$\omega_y = \arg\min_{(\tilde{\omega}_y)} \theta(y) \left\| (\tilde{\omega}_y)_y - \mathbf{1} \right\|_2$$

subject to $\sum_y \theta(y)\tilde{\omega}_y = 1$ and such that planner's first-order conditions hold.

▸ Pareto weights for each history

Reproduce a realistic allocation

1. US and French Fiscal system from **1995-2007**.
2. Realistic **income process**.
3. **Wealth and Income inequality**.

Parameter values

	$\hat{\tau}$	SE	Obs	R^2
France	**0.23**	0.0056	5289	0.855
United States	**0.16**	0.0019	38111	0.942

Table – Estimate of the progressivity of the labor income tax in the US and and in France for 2005 using the LIS database.

		US		France	
Parameter	Description	Value	Target or ref.	Value	Target or ref.
Preference parameters					
β	discount factor	0.992	$K/Y = 2.7$	0.996	$K/Y = 3.1$
u	utility function	·	$\gamma = 1.8$	·	$\gamma = 1.8$
φ	Frish elasticity	0.5	Chetty et al. (2011)	0.5	Chetty et al. (2011)
χ	hours worked	0.33	Penn World Table	0.29	Penn World Table
α	capital share	36%	Profit Share, NIPA	36%	Profit Share, INSEE
δ	depreciation rate	2.5%	Krueger et al. (2018)	2.5%	Own calculations, INSEE
Productivity parameters					
σ^y	std. err. productivity	0.10	Gini for income	0.06	Fonseca et al. (2020)
ρ^y	autocorr. productivity	0.99	Gini for income	0.99	Fonseca et al. (2020)

Table – Parameter values.

Model implications for key variables

		US		France	
Parameter	Description	Model	Data	Model	Data
Public finance aspects					
B/Y	Public debt-to-GDP ratio	63%	63%	60%	60%
G/Y	Public spending-to-GDP ratio	15%	15%	25%	24%
	Total tax revenues	16%	26%	25%	40%
Aggregate quantities					
C/Y	Aggregate consumption (share of GDP)	58%	60%	44%	45%
I/Y	Aggregate investment (share of GDP)	27%	25%	31%	31%
Inequality measures					
	Gini for post-tax income	40%	40%	28%	28%
	Gini for wealth	78%	77%	68%	68%

Table – Model implications for key variables.

Estimation of Pareto weights

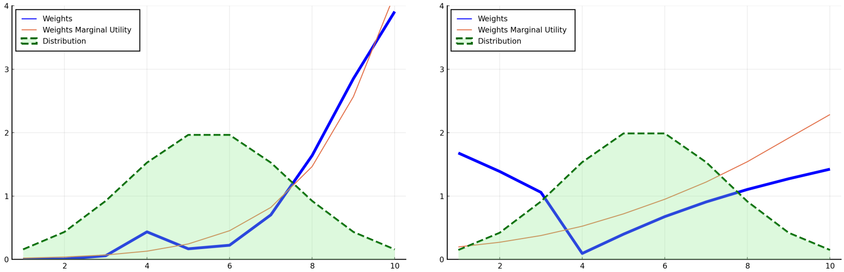
- Recall that we can represent the truncated history of an agent i whose idiosyncratic history is y^t as :

$$y^t = \{\underbrace{y_0, \ldots, y_{t-N-2}, y_{t-N-1}, y_{t-N}}_{\xi_{y^N}}, \underbrace{y^N_{t-N+1}, \ldots, y^N_{t-1}, y^N_t}_{=y^N}\},$$

- We use a truncation length of $N = 5$. We select 10 idiosyncratic productivity levels, which implies $10^5 = 100000$ different truncated histories.
- Pareto weights are estimated such that **histories with the same productivity level in the beginning of the truncation** will be assigned the **same weight** (i.e., if $y^N_t = \tilde{y}^N_t$ such that $y^N_t \in y^N$ and $\tilde{y}^N_t \in \tilde{y}^N$ with $y^N \neq \tilde{y}^N$ then $\omega(y^N) = \omega(\tilde{y}^N)$).

Estimation of Pareto weights

▸ Pareto weights by history ▸ Robustness ▸▸ Weights wealth



(a) United States (b) France

Figure – Pareto weights as a function of productivity for the US and France.

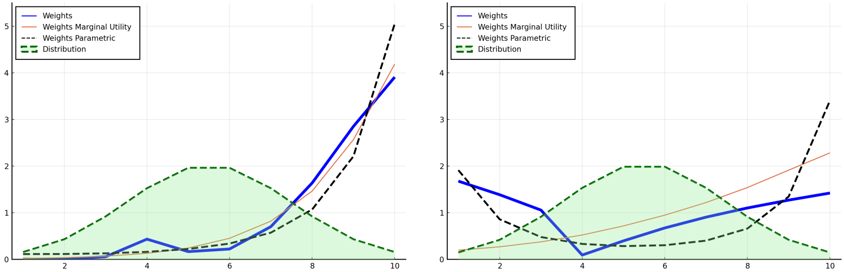
Estimation of Pareto weights

	US	France
Mean	1.00	1.00
St. deviation	1.37	0.49
Min.	0.006	0.095
Max.	3.91	1.68
Bottom 10 %	0.006	0.37
Median	0.33	1.08
Top 10%	2.96	1.45

Table – Summary statistics for the Pareto Weights of the US and France.

Estimation of Pareto weights

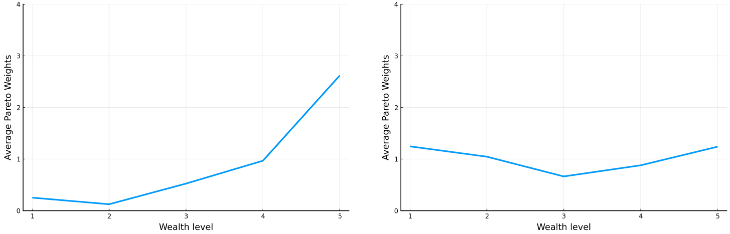
▸ Functional form



(a) United States (b) France

Figure – Parametric Pareto weights as a function of productivity for the US and France.

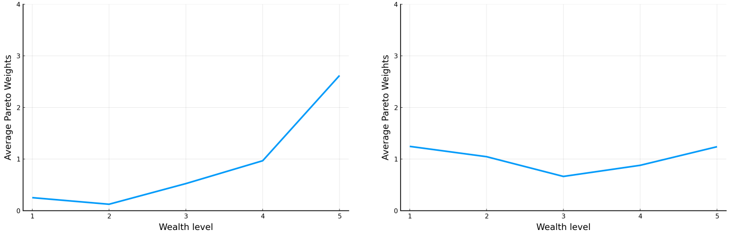
Estimation of Pareto weights



(a) United States (b) France

Figure – Average Pareto weights as a function of wealth for the US and France.

Estimation of Pareto weights



(a) United States

(b) France

Figure – Average Pareto weights as a function of wealth for the US and France.

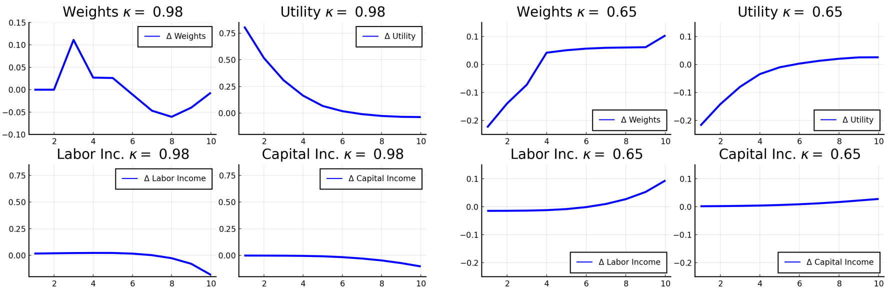
- The French shape is consistent with a **inequality or inequity aversion**.
- The US shape is consistent with a **redistribution component for low-wealth agents**, but other favors **high-income and high-wealth agents**.

A world where United States has the French tax system

- Evaluate **whether the fiscal system in France, with a higher progressivity** in labor tax can explain the **higher weights that France gives for low productivity agents**.
- Analyse whether the **business cycle properties are altered if the SWF of the US is different**, and in which dimension those properties are altered.

A world where United States has the French tax system

How the experiment runs | Skip

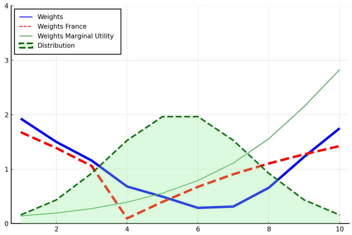


(a) United States (b) France

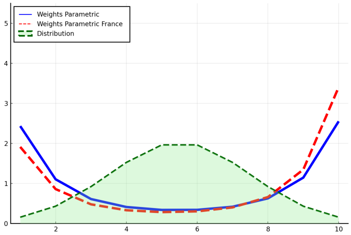
Figure – Difference between the Pareto weights between US with the French fiscal system.

A world where United States has the Pareto weights of France

Below we have the Pareto weights as a function of productivity for the **US with the new fiscal system and France**.



(a) Pareto weights



(b) Pareto weights parametric

Figure – Pareto weights for the United States with the French Pareto weights and France.

A world where United States has the Pareto weights of France

	US					
	Public debt (%GDP)	τ_k (%)	τ (%)	κ (%)	Gini a.t.	Gini wealth
Benchmark economy USA	63	36	16	85	40	78
Benchmark economy France	60	35	23	72.8	28	68
USA with the French PWs	**299**	**9**	**57**	71	27	63

Table – Comparison between the benchmark economies and the USA economy with the French Pareto weights.

- The **progressivity of US increases**, since before the progressivity was favoring the high-income agents.
- The **debt-to-GDP** ratio also **increased**.
- The **tax on capital** now was **reduced**, since savings need to absorb the additional debt.

Dynamics of the fiscal system

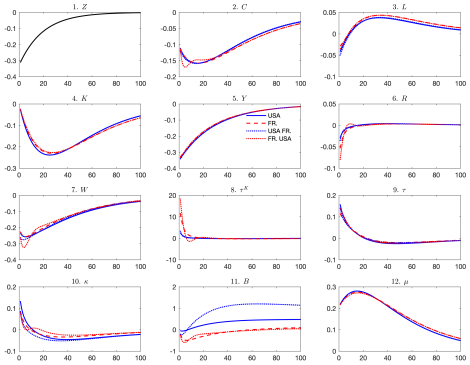


Figure – Dynamics and comparisons.

Conclusion

- **Methodology to identify the Social Welfare Function (SWF)** of a government, which is compatible with the empirical wealth and income distributions given the actual tax structure.
- Estimate the **Social Welfare Function from the data**.
- We used the estimated SWFs to assess the role of the latter to the **macroeconomy**.
- Finally, by analysing the dynamics of the economy we showed that the main business cycle **alterations occur in the fiscal policy parameters**.

The Social Welfare Function

- Using the **law of large number**, the planner maximizes at date 0 :

$$W_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{y^t \in \mathcal{Y}^{t+1}} \theta_t(y^t)\omega(y_t) \left(u(c_t(y^t)) - v(l_t(y^t)) + u_G(G_t) \right).$$

- Compared to the **(unweighted) utilitarian Social Welfare Function**, the only difference is the set of weights $\omega\left(y_t\right)$.

Notation

Definition (Simplifying Notation)

If an agent has an idiosyncratic history y_i^t, and initial wealth a_{-1}^i at period t, where the aggregate history is s^t, we will then denote the realization in state (y_i^t, a_{-1}^i, s^t) of any random variable $X_t : \mathcal{Y}^{t+1} \times \mathbb{R} \times \mathcal{S}^{t+1} \to \mathbb{R}$ simply by X_t^i.

The aggregation of the variable X_t at period t over all agents is $\int_i X_t^i \ell(di)$ instead of :

$$\int_{a_{-1}} \sum_{y^t \in \mathcal{Y}^{t+1}} \theta_t \left(y^t\right) X_t \left(y^t, a_{-1}, s^t\right) \mathbf{\Lambda} \left(y_0, da_{-1}\right).$$

◂ Return Market clearing ◂ Return SWF

A redundancy result

Using the **new definitions** :

$$
\begin{aligned}
\tilde{a}_t^i &:= \frac{a_t^i}{1+\tau_t^c}, \\
W_t &:= \frac{w_t}{1+\tau_t^c}, \\
R_t &:= \frac{(1+r_t)(1+\tau_{t-1}^c)}{1+\tau_t^c}, \\
\tilde{B}_t &:= \frac{B_t}{(1+\tau_t^c)}, \\
\tilde{A}_t &:= \frac{A_t}{1+\tau_t^c}, \\
\hat{B}_t &:= (1+\tau_t^c)\tilde{B}_t - \tau_t^c \tilde{A}_t.
\end{aligned}
$$

A redundancy result

Using the **new definitions** :

$$\tilde{a}_t^i := \frac{a_t^i}{1+\tau_t^c},$$

$$W_t := \frac{w_t}{1+\tau_t^c},$$

$$R_t := \frac{(1+r_t)(1+\tau_{t-1}^c)}{1+\tau_t^c},$$

$$\tilde{B}_t := \frac{B_t}{(1+\tau_t^c)},$$

$$\tilde{A}_t := \frac{A_t}{1+\tau_t^c},$$

$$\hat{B}_t := (1+\tau_t^c)\tilde{B}_t - \tau_t^c \tilde{A}_t.$$

The **Ramsey Problem** becomes :

$$\max_{\left(W_t, R_t, \tilde{w}_t, \tilde{r}_t, \tau_t^c, \tau_t^K, \tau_t, \kappa_t, \hat{B}_t, G_t, K_t, L_t, (c_t^i, l_t^i, \tilde{a}_t^i, \nu_t^i)_i\right)_{t\geq 0}} W_0,$$

$$G_t + W_t \int_i (y_t^i l_t^i)^{1-\tau_t} \ell(di) + (R_t - 1)\tilde{A}_{t-1} + \hat{B}_{t-1} = F(K_{t-1}, L_t, s_t) + \hat{B}_t,$$

for all $i \in \mathcal{I}$: $c_t^i + \tilde{a}_t^i = W_t (y_t^i l_t^i)^{1-\tau_t} + R_t \tilde{a}_{t-1}^i,$

$$\tilde{a}_t^i \geq -\tilde{\bar{a}},\ \nu_t^i(\tilde{a}_t^i + \tilde{\bar{a}}) = 0,\ \nu_t^i \geq 0,$$

$$u'(c_t^i) = \beta \mathbb{E}_t \left[R_{t+1} u'(c_{t+1}^i) \right] + \nu_t^i,$$

$$v'(l_t^i) = (1-\tau_t) W_t y_t^i (y_t^i l_t^i)^{-\tau_t} u'(c_t^i),$$

$$K_t + \hat{B}_t = \tilde{A}_t = \int_i \tilde{a}_t^i \ell(di),\ L_t = \int_i y_t^i l_t^i \ell(di).$$

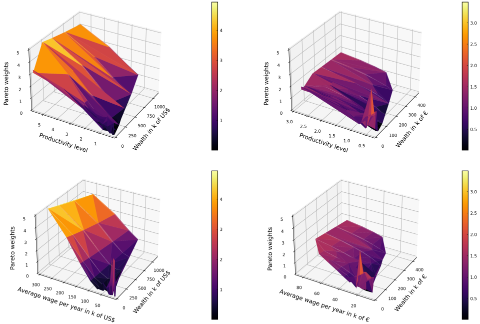
◂ Original problem ◂ Proposition

Computing Pareto weights for each history

- Solve $\omega_{y^N} = \arg\min_{(\tilde{\omega}_{y^N})} S_{y^N} \left\|(\tilde{\omega}_{y^N})_{y^N} - \mathbf{1}\right\|_2$ subject to $\sum_y S_{y^N}\tilde{\omega}_{y^N} = 1$.
- Notice that the only difference between the two approaches is that in the second one we will have $(\omega_{y^N})_{y^N}$ different for each $y^N \in \mathcal{Y}^N$, whereas in the first approach $(\omega_{y^N})_{y^N} = (\omega_{\tilde{y}^N})_{\tilde{y}^N}$ whenever $y_0^N = \tilde{y}_0^N$, i.e., everytime the productivity level in the first period of the truncation associated with the history y^N is the same as the productivity level associated with the history $\tilde{y}^N$ with $y^N \neq \tilde{y}^N$.

◂ Pareto weights by productivity

Estimation of Pareto weights



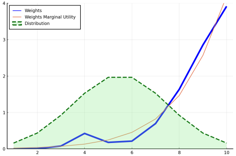
(a) United States (b) France

Figure – Pareto weights as a function of productivity (income per capita) and wealth.

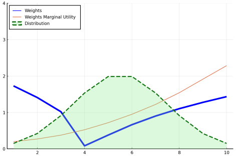
Weights productivity

Robustness check

◂ Weights productivity

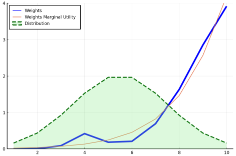


(a) United States

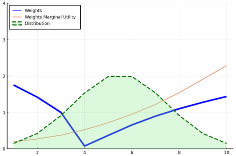


(b) France

Figure – Pareto weights as a function of productivity for the US and France N=7.



(a) United States



(b) France

Figure – Pareto weights as a function of productivity for the US and France N=8.

Robustness check

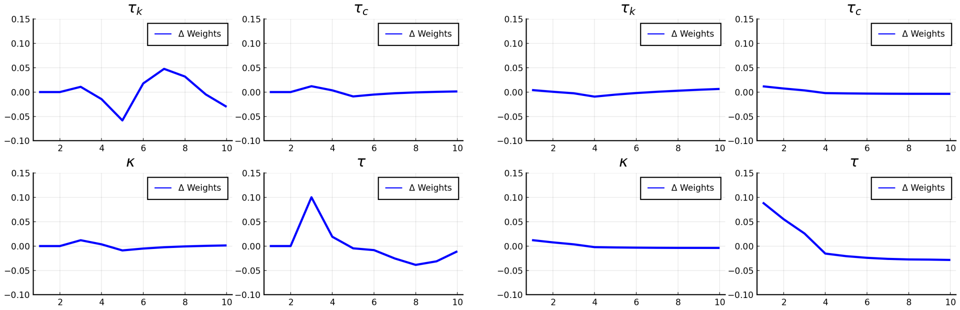
- Increase $\boldsymbol{G/Y}$.
- Finance this increase by each one of the instruments (i.e., τ_k, τ_c, κ, and τ) such that the **budget constraint of the state** is still satisfied.

	US Steady state	US Increase in G/Y	France Steady state	France Increase in G/Y
τ_k	0.36	**0.387**	0.35	**0.361**
τ_c	0.05	**0.076**	0.18	**0.19**
κ	0.85	**0.83**	0.728	**0.72**
τ	0.16	**0.22**	0.23	**0.25**

Table – Changes in the fiscal instruments after an increase in G/Y for United States and France.

Robustness check

◂ Weights productivity



(a) United States (b) France

Figure – Change in weights by increasing G/Y for United States and France.

Estimation of Pareto weights

- $\log \omega(y) := \omega_0 + \omega_1 \log(y) + \omega_2 (\log(y))^2.$
- $\log \omega(y)^{us} = -0.25 + 1.06 \log(y) + 0.22 (\log(y))^2.$
- $\log \omega(y)^{fr} = -0.51 + 0.62 \log(y) + 1.44 (\log(y))^2.$

◂ Weights parametric

How the experiment runs

1. Once the **capital-to-output ratio** is set to the value in the **steady state**, we iterate in the value for κ such that the value of **government spending to output ratio** is kept the same.
2. The model parameters keep unchanged but also the main macro ratios. In this exercise the only vector we are changing is the vector that represents the **fiscal system** $(\tau_K, \tau_c, \tau, \kappa, B)$.

	United States	France
τ_k	0.35	0.36
τ_c	0.18	0.05
κ	0.98	0.65
τ	0.23	0.16
B/Y	0.21	0.91

Table – Fiscal system for US and France.

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