Perfect Altruism Breeds Time Consistency

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EEA-ESEM

August 22, 2022

Social Dynamic Choices

The roots of most social economic decisions are linked to a choice of social lifetime utility and associated social parameters.

- Household choices (Chiappori and Mazzocco 2017)
- Fiscal policy (Barro 1974)
- Climate policy (Nordhaus 2007)

Paternalism?

Classic Treatment:

Exponential discounted utility: Ramsey (1928) and Samuelson (1937)

Near-one discounting factor: Ramsey's ethical critique

Utilitarian value: Bergerson-Samuelson welfare function

Classic Concerns:

Exponential discounted utility: Difficult to derive (Marglin, 1963 and Feldstein, 1964)

Near-one discounting factor: Violate 'everyone has a say' principle (Arrow, 1997)

Classic Assumption: Homogeneity

Heterogeity Challenge

Constant discount rate selection: Opinions about discount factor vary among experts (Weitzman 2001, Drupp et al 2018)

TABLE 1—DISTRIBUTION OF RESPONSES

Discount rate (Rounded to nearest whole percentage)	Number of responses
−3	1
−2	1
−1	1
0	46
1	236
2	454
3	427
4	362
5	227
6	136
7	71
8	44
9	28
10	44
11	15
12	25
13	12
14	5
15	8
16	3
17	2
18	3
19	1
20	4
25	2
26	1
27	1
	Total responses = 2,160

Descriptive Challenge

Time Inconsistency: Widely observed

- UK and France adopt time-dependent discounting schemes
- Paris Agreement: US enter and exit and re-enter...
- Commitment device:
  - Too costly (Laibson 2015)
  - Political power rotation (Harstad 2020)

This paper

considers: non-Paternalism way to derive social lifetime utility

proposes: separate aggregation rule

Utilitarian social utility: Weighted average of individual instantaneous utilities
Social discounting function: Weighted average of individual ones

more importantly identifies: principles that characterize the above rules

compares: various degree of inconsistency

Methodology

Preference Aggregation

- Respect individualism
- Identifiable and testable
- Adopted by Zuber 2011, Jackson and Yariv 2014, and many others

Other methodology

- Weitzman 2001: gamma discounting
- Adams et al 2014: revealed preference
- Galperti and Strulovici 2017: Axiomtization

Why we care?

- Provide foundations for constant social discounting and utilitarian social utility
  - Zuber and Jackson-Yariv confirm the difficulty of Marglin and Feldstein
  - Feng and Ke (2018) and Chambers and Echenique (2018) suggest different rules without social utility concern
- Provide foundations for quasi-hyperbolic social discounting and utilitarian social utility
  - Amador (2003) and Chatterjee and Eyigungor (2016) found that quasi-hyperbolic social discounting explains promise to invest and reverse it once in power
- Provide foundations for various degree of time inconsistency
  - Halac and Yared (2018) show that government bias relates to coordinated fiscal rules

The Model

- a society of finite individuals: $i \in \mathcal{I} = \{1, \ldots, n\}$
- discrete time horizon: $t \in \mathbb{N} = \{1, 2, \ldots\}$
- consumption space $\mathcal{L} = \Delta(X)$: a simplex on finite set X
- a stream of consumption: $\mathbf{z} \in \mathcal{L}^\infty$
- individual lifetime utility $U_i : \mathcal{L}^\infty \to \mathbb{R}$
- social lifetime utility $U : \mathcal{L}^\infty \to \mathbb{R}$

Time-separable Utility

Assumption 1: Time-separable utility for both individuals and society

Definition

$U : \mathcal{L}^\infty \to \mathbb{R}$ is *time-separable* if there exist a *discount function* $\eta : \mathbb{N} \to (0, 1)$ and a nonconstant and continuous utility function $u : \mathcal{L} \to \mathbb{R}$ such that a consumption stream $\mathbf{z} = (z_1, z_2, \ldots) \in \mathcal{L}^\infty$ is evaluated as

$$U(\mathbf{z}) = \sum_{t=1}^{\infty} \eta_t u(z_t), \tag{1}$$

Exponential discounted utlity (EDU): $\eta_t = \delta^{t-1}$

Hyperbolic discounting utility : $\eta_t = (1 + \gamma t)^{-\frac{\alpha}{\gamma}}$

Quasi-hyperbolic discounting utility : $\eta_t = \beta\delta^{t-1}$

Minimum Agreement

Assumption 2: *minimum agreement over consumption*; i.e. there are $z^{*}, z_{*} \in \mathcal{L}$ such that for $z \in \mathcal{L}$, $u_i(z^{*}) \geq u_i(z) \geq u_i(z_{*})$, for all $i \in \mathcal{I}$.

Impossibility

Classic Pareto Condition: For any $\mathbf{z}, \hat{\mathbf{z}} \in \mathcal{L}^{\infty}$, if, $U_i(\mathbf{z}) \geq U_i(\hat{\mathbf{z}})$ for all $i \in \mathcal{I}$, then $U(\mathbf{z}) \geq U(\hat{\mathbf{z}})$.

Proposition

Classic Pareto condition is satisfied if and only if U is dictatorial.

Example

A household with two individuals, *Ana* with (u_a, η_{at}) and *Bob* with (u_b, η_{bt}). If no dictator, then there is $0 < \lambda < 1$ such that $u = \lambda u_a + (1-\lambda) u_b$.

$\mathcal{L}$	x	y	z
u_a	0	$\frac{0.98}{\lambda}$	$-\frac{1}{\lambda}$
u_b	0	$-\frac{0.95}{1-\lambda}$	$\frac{9}{1-\lambda}$
u	0	0.03	8

t	1	2	3
η_a	1	0.99	0.99^2
η_b	1	0.1	0.1^2
η	1	η_2	η_3

- $U_a(y, z, x, x, \cdots) = \frac{0.98}{\lambda} - \frac{1}{\lambda} \times 0.99 < 0 = U_a(x, x, \cdots)$
- $U_b(y, z, x, x, \cdots) = -\frac{0.95}{1-\lambda} + \frac{9}{1-\lambda} \times 0.1 < 0 = U_b(x, x, \cdots)$

For all η,

$$U(y, z, x, x, \cdots) = 0.03 + 8\eta_2 > 0 = U(x, x, \cdots)$$

Intuition

- With kid (y, z) or Without kid (x, x)
- Neither Ana nor Bob wants a kid, but for different reasons
- Ana likes baby and is patient, but worries much about the future of baby
- Bob hates taking care of baby and is impatient, but enjoys the future family happiness
- Non-dictatorial household utility prefers a kid
- Classic Pareto condition, which leads to spurious unanimity, is not plausible to follow

Impartiality

- Ana and Bob should give sympathetic consideration to each other
- Change positions by switching discount factors
- Unanimity is impartial if changing positions does not change unanimity

$\mathcal{L}$	x	y	z
u_a	0	$\frac{0.98}{\lambda}$	$-\frac{1}{\lambda}$
u_b	0	$-\frac{0.95}{1-\lambda}$	$\frac{9}{1-\lambda}$
u	0	0.03	8

t	1	2	3
η_a	1	0.99	0.99^2
η_b	1	0.1	0.1^2
η	1	η_2	η_3

(y, z) is preferred to (x, x) is NOT impartially unanimous

$$u_a(y) + \eta_{b2} \cdot u_a(z) = \frac{0.98}{\lambda} - \frac{0.1}{\lambda} > 0.$$

Impartial Pareto Condition

- A *impartial* society is a product set $\mathcal{I} \times \mathcal{I}$
- A *virtual* individual, $ij \in \mathcal{I} \times \mathcal{I}$ has utility: $U_{ij}(\mathbf{z}) = \sum_{t=1}^{\infty} \eta_{it} u_j(z_t)$

Impartial Pareto Condition (IPC): For any $\mathbf{z}, \hat{\mathbf{z}} \in \mathcal{L}^{\infty}$, if, $U_{ij}(\mathbf{z}) \geq U_{ij}(\hat{\mathbf{z}})$ for all $ij \in \mathcal{I} \times \mathcal{I}$, then $U(\mathbf{z}) \geq U(\hat{\mathbf{z}})$.

Theorem

A social lifetime utility U satisfies IPC if and only if there exist nonnegative $\{\alpha_i\}_{i \in \mathcal{I}}$ and $\{\gamma_i\}_{i \in \mathcal{I}}$ with $\sum_i \alpha_i = \sum_i \gamma_i = 1$ such that

$$u = \sum_i \alpha_i u_i \quad \text{and} \quad \eta_t = \sum_i \gamma_i \eta_{it} \tag{2}$$

for all $t \in \mathbb{N}$.

Impatience

Definition

A lifetime utility $V : \mathcal{L} \to \mathbb{R}$ satisfies *decreasing impatience* if for any $t > s$ and $k \geq 1$, $V(x, \bar{z}_{*-t}) = V(y, \bar{z}_{*-s})$ implies $V(x, \bar{z}_{*-(t+k)}) \geq V(y, \bar{z}_{*-(s+k)})$ (*constant impatience* if $V(x, \bar{z}_{*-(t+k)}) = V(y, \bar{z}_{*-(s+k)})$).

Proposition

Suppose that a social lifetime utility U admits a separate aggregation as in eq (3). If each individual satisfies either constant or decreasing impatience, then a non-dictatorial social utility U exhibits decreasing impatience.

'Ought' or 'Is' Proposition

For two consumption streams $\mathbf{z}, \mathbf{z}' \in \mathcal{L}^\infty$, if, (i) $\mathbf{z}$ and $\mathbf{z}'$ are constant streams; or (ii) $u_i(z) = u_j(z)$ for all $z \in \text{conv}(\{z_t : t \in \mathbb{N}\} \cup \{z'_t : t \in \mathbb{N}\})$ and all $i, j \in \mathcal{I}$, then $\mathbf{z}$ and $\mathbf{z}'$ are common-value streams.

Common-value Pareto Condition (CV-PC). For any pair of common-value streams $\mathbf{z}, \hat{\mathbf{z}} \in \mathcal{L}^\infty$, if $U_i(\mathbf{z}) \geq U_i(\hat{\mathbf{z}})$, for all $i \in \mathcal{I}$, then $U(\mathbf{z}) \geq U(\hat{\mathbf{z}})$.

Theorem

A social lifetime utility U satisfies the CV-PC *if and only if there exist nonnegative $\{\alpha_i\}_{i\in\mathcal{I}}$ and $\{\gamma_i\}_{i\in\mathcal{I}}$ with $\sum_i \alpha_i = \sum_i \gamma_i = 1$ such that*

$$u = \sum_i \alpha_i u_i \quad \textit{and} \quad \eta_t = \sum_i \gamma_i \eta_{it} \tag{3}$$

for all $t \in \mathbb{N}$.

Constant Social Discounting

Assumption: All individuals are EDU

IPC: Decreasingly impatient social planner

Question: What principle would lead to constant impatient social planner?

Koopman Condition

Stationarity: A lifetime utility function U is *stationary* if, for all $x \in \mathcal{L}$ and all $\mathbf{z}, \mathbf{z}' \in \mathcal{L}^\infty$,

$$U(\mathbf{z}) \geq U(\mathbf{z}') \text{ if and only if } U(x, \mathbf{z}) \geq U(x, \mathbf{z}').$$

Necessary, but not sufficient

Pareto Condition is not Stationary

t	1	2	3	4
η_a	1	0.4	0.4^2	0.4^3
η_b	1	0.6	0.6^2	0.6^3
η	1	0.5	$\frac{0.4^2+0.6^2}{2}$	$\frac{0.4^3+0.6^3}{2}$

But, restriction to first 2-period consumption is compatible with stationarity

Altruism should not spill over beyond next generation (Barro, 1974; Phelps and Pollak, 1964)

Perfectly Altruistism

Example

$\mathcal{L}$	x	y	x'	y'	z
u_a	2	1	1	3	0
u_b	4	-3	-1	8	0

t	1	2	3
η_a	1	0.99	0.99^2
η_b	1	0.1	0.1^2

$(x, y, z, z \cdots)$ and $(x', y', z, z \cdots)$ only differ in first two periods and coincide for the rest

Time Consistent Planner

We say two consumption streams $\mathbf{z}$ and $\mathbf{z}'$ are *diperiodic* if $z_t = z'_t$ for $t > 2$.

Perfectly Altruistic Impartial Pareto Condition (PAI-PC). For any diperiodic consumption streams $\mathbf{z}$ and $\mathbf{z}'$, if $U_{ij}(\mathbf{z}) \geq U_{ij}(\mathbf{z}')$, for all $ij \in \mathcal{I} \times \mathcal{I}$, then $U(\mathbf{z}) \geq U(\mathbf{z})$.

Theorem

A society satisfies PAI-PC and Stationarity if and only if social lifetime utility is EDU, in which u is a convex combination of $\{u_i\}_{i \in \mathcal{I}}$ and δ is a convex combination of $\{\delta_i\}_{i \in \mathcal{I}}$.

Perfect Altruism

Is the skipped generation altruism also *perfect* in the sense of time consistence?

Example

$\mathcal{L}$	x	y	x'	y'	z
u_a	2	1	1	3	0
u_b	4	-3	-1	8	0

t	1	2	3
η_a	1	0.99	0.99^2
η_b	1	0.1	0.1^2

$(x, z, y, z \cdots)$ and $(x', z, y', z \cdots)$ only differ in two identical periods

k-PAI-PC

Let $k \geq 2$, we say two consumption streams $\mathbf{z}, \mathbf{z}'$ are k-*diperiodic* if $z_t = z'_t$ for $t \in \mathbb{N} \setminus \{1, k\}$.

k-PAI-PC Let $k \in \mathbb{N}$. For any k-diperiodic streams $\mathbf{z}, \mathbf{z}'$, if $U_{ij}(\mathbf{z}) \geq U_{ij}(\mathbf{z}')$ for all $ij \in \mathcal{I} \times \mathcal{I}$, then $U(\mathbf{z}) \geq U(\mathbf{z}')$.

Proposition

The k-PAI-PC *and* stationarity *are satisfied if and only if social lifetime utility is a EDU, in which u is a convex combination of $\{u_i\}_{i \in \mathcal{I}}$ and $\eta_t = \delta^{t-1}$ for all $t \in \mathbb{N}$, with δ being a convex combination of $\{\delta_i\}_{i \in \mathcal{I}}$.*

Quasi-hyperbolic Social Discounting

Definition

A lifetime utility $V : \mathcal{L}^\infty \to \mathbb{R}$ admits a quasi-hyperbolic discounting form if there exists a continuous function u on $\mathcal{L}$ and parameters $\beta \in (0, 1]$ and $\delta \in (0, 1)$ such that for $z \in \mathcal{L}^\infty$,

$$V(\mathbf{z}) = u(x_1) + \beta \sum_{t=2}^{\infty} \delta^{t-1} u(z_t). \tag{4}$$

Quasi-Stationarity: A lifetime utility function U is *stationary* if, for all $x, y \in \mathcal{L}$ and all $\mathbf{z}, \mathbf{z}' \in \mathcal{L}^\infty$,

$$U(x, \mathbf{z}) \geq U(x, \mathbf{z}') \text{ if and only if } U(x, y, \mathbf{z}) \geq U(x, y, \mathbf{z}').$$

We say that two consumption streams $\mathbf{z}, \mathbf{z}'$ are triperiodic if $z_t = z'_t$ for $t > 3$.

Quasi-Altruism Impartial Pareto Condition (QAI-PC). For any pair of triperiodic consumption streams $\mathbf{z}, \mathbf{z}'$, if $U_{ij}(\mathbf{z}) \geq U_{ij}(\mathbf{z}')$, for all $ij \in \mathcal{I} \times \mathcal{I}$, then $U(\mathbf{z}) \geq U(\mathbf{z})$.

Theorem

A society satisfies QAI-PC and quasi-stationarity if and only if there exists positive $\{\alpha_i\}_{i\in\mathcal{I}}$ *and* $\{\lambda_i\}_{i\in\mathcal{I}}$ *with* $\sum_i \alpha_i = \sum_i \gamma_i = 1$ *such that society has a quasi-hyperbolic discounting form as in eq (4), in which*

$$u = \sum_{i\in\mathcal{I}} \alpha_i u_i \quad \text{and} \quad \delta = \frac{\sum_{i\in\mathcal{I}} \lambda_i \delta_i^2}{\sum_{i\in\mathcal{I}} \lambda_i \delta_i} \quad \text{and} \quad \beta = \frac{(\sum_{i\in\mathcal{I}} \lambda_i \delta_i)^2}{\sum_{i\in\mathcal{I}} \lambda_i \delta_i^2}.$$

Furthermore, $\delta \in (\min_{i\in\mathcal{I}} \delta_i, \max_{i\in\mathcal{I}} \delta_i)$ *and* $\beta \in (\frac{\min_i \delta_i}{\max_i \delta_i}, 1)$.

Generalization

We say that two consumption streams $\mathbf{z}$ and $\mathbf{z}'$ are k-*periodic* if $z_t = z'_t$ for $t > k$.

k-Imperfect Altruism Impartial Pareto Condition (k-IAI-PC): Let $k \in \mathbb{N}$. For any pair of k-periodic consumption streams $\mathbf{z}, \mathbf{z}'$, if $U_{ij}(\mathbf{z}) \geq U_{ij}(\mathbf{z}')$ for all $ij \in \mathcal{I} \times \mathcal{I}$, then $U(\mathbf{z}) \geq U(\mathbf{z}')$.

Generalization

Definition

A lifetime utility $V : \mathcal{L}^\infty \to \mathbb{R}$ admits a *level k hyperbolic* form if there exists $0 < \beta_1 \leq \ldots \leq \beta_k \leq 1$ and $\delta \in (0,1)$ such that for $\mathbf{z} \in \mathcal{L}^\infty$,

$$V(\mathbf{z}) = u(z_1) + \beta_1 \delta u(z_2) + \beta_1 \beta_2 \delta^2 u(z_3) + \cdots + \prod_{\ell=1}^{k} \beta_s \sum_{t=S+1}^{\infty} \delta^{t-1} u(z_t). \tag{5}$$

Definition

A lifetime utility U is k*-delayed stationary* if, for all $x \in \mathcal{L}$ and all $\mathbf{z}, \mathbf{c}, \hat{\mathbf{c}} \in \mathcal{L}^\infty$,

$$U(\mathbf{z}_k\mathbf{c}) \geq U(\mathbf{z}_k\hat{\mathbf{c}}) \text{ if and only if } U(x, \mathbf{z}_k\mathbf{c}) \geq U(x, \mathbf{z}_k\hat{\mathbf{c}}).$$

Theorem

A social TSU function U is k-delayed stationary and satisfies k-IAI-PC if and only if there exists nonnegative numbers α_i and γ_j such that U has the form as in eq (5), in which

$$u = \sum_i \alpha_i u_i \tag{6}$$

$$\delta = \frac{\sum_j \gamma_j \delta_j^{k+1}}{\sum_j \gamma_j \delta_j^k} \tag{7}$$

$$\beta_\ell = \frac{\sum_j \gamma_j \delta_j^\ell}{\sum_j \gamma_j \delta_j^{\ell-1}} \cdot \frac{\sum_j \gamma_j \delta_j^k}{\sum_j \gamma_j \delta_j^{k+1}} \qquad \textit{for all } 1 \leq \ell \leq k. \tag{8}$$

Comparative Analysis

Definition

A utility U exhibits *more decreasing impatience* than utility V if, for any t, s in $\mathbb{N}$ and $x, y, x', y' \in \mathcal{L}$, $U(x, \bar{z}_*) = U(y_t, \bar{z}_{*-t})$, $V(x', \bar{z}_*) = V(y'_t, \bar{z}_{*-t})$, and $V(x'_s, \bar{z}_{*-s}) \leq V(y'_{t+s}, \bar{z}_{*-\{t+s\}})$ implies $U(x_s, \bar{z}_*) \leq U(y_{t+s}, \bar{z}_{*-\{t+s\}})$.

Example

V	1	2	$\cdots$
$\sim$	100	0	
	0	105	

U	1	2	$\cdots$
$\sim$	100	0	
	0	110	

V	$\cdots$	61	62	$\cdots$
$\curlywedge$	$\cdots$	100	0	
	$\cdots$	0	105	

U	$\cdots$	61	62	$\cdots$
$\curlywedge$	$\cdots$	100	0	
	$\cdots$	0	110	

Proposition

Fix nonnegative numbers α_i *and* γ_j *such that* $\sum_{i\in\mathcal{I}}\alpha_i = \sum_{j\in\mathcal{I}}\gamma_j = 1$*. If* $k \geq \hat{k}$*, then a society characterized by* $(\hat{u}, \hat{\delta}, \{\hat{\beta}_\ell\}_{\ell=1}^{\hat{k}})$*, defined as in eqs (6,7,8), is more decreasing impatience than a society characterized by* $(u, \delta, \{\beta_\ell\}_{\ell=1}^{k})$*, defined as in eqs (6,7,8).*

Perfect or Imperfect Altruism?

Consider a society has to choose between two consumption streams

$$\mathbf{z} = (\underbrace{1, 0, \ldots, 0, -100}_{11 \text{ periods}}, 0, \ldots) \quad \text{and} \quad \mathbf{z}' = (1.1, -0.4, 0, \ldots).$$

Let $\delta = 0.5$. A society prefers $\mathbf{z}$ to $\mathbf{z}'$:

$$1 - 100 \times 0.5^{10} = 0.9032 > 0.9 = 1.1 - 0.4 \times 0.5.$$

Let $\beta = 0.8$. A society would prefer $\mathbf{z}'$ to $\mathbf{z}$:

$$1 + 0.8 \times 0.5^{10} \times (-100) = 0.9219 < 0.94 = 1.1 + 0.8 \times 0.5 \times (-0.4)$$

Conclusion

- Social decisions rely on the selection of social parameters.
- We suggest various way to determine those parameters.
- The very principles are identified to characterize those methods.
- Our methods provide a solid foundation to apply various utility forms in either time consistent or inconsistent fashion.