Disclosure Services and Welfare Gains in Takeover Markets

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Abstract

We present a model that captures three features of takeover markets: (i) firms are heterogeneous in their prospects as bidders, targets, and stand-alone firms, (ii) bidders do not know true values of targets, (iii) targets pay fees to disclose their values. Despite its complexity, the model admits closed form solutions and yields predictions supported by empirical evidence. Two main results emerge. First, if full disclosure is facilitated by a monopolist, it captures a large fraction of the welfare gains. Second, adding the option of minimum disclosure, when combined with a cap regulation on price-dependent fees, significantly weakens the monopolist's power. (*JEL* L1, G3, D8)

Key words: Asymmetric information, Disclosure, Market segmentation, Takeovers.

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1 Introduction

Recent studies show that a significant amount of resources are reallocated through takeover activities.[1] While the literature on takeovers is extensive, we know little about the *reallocational efficiency* of takeover markets. On the one hand, standard macroeconomic models of resource reallocation assume away a market for corporate control.[2] On the other hand, models of takeovers designed to study the managerial efficiency of public firms are not well-suited to study takeovers at the aggregate level.[3] We aim to fill this gap by presenting a model of takeovers among a continuum of heterogenous firms.

While a large and increasing number of firms engage in takeovers, still only a small fraction of firms participate in this market. Why is the participation so limited? We argue that because a unit of trading is an entire firm (or at least its large indivisible component not tradeable via factor markets), *credibly disclosing its quality* is critical but difficult. For many firms, such disclosure is too costly to do on their own and they need to rely on professional service providers. In practice, large intermediaries (e.g. investment banks) offer such services, and except occasional "mega" deals, most takeovers involve firms with little bargaining power against these intermediaries. In fact, billions of dollars are paid to intermediaries, and such fees have been subject to criticism.[4] Yet, we lack a framework to understand how these fees affect the reallocational efficiency.[5]

We present a model that captures three salient features of takeover markets. First, there are many firms that are heterogeneous in their prospects as bidders and targets, and also in their stand alone values. Second, bidder firms do not know the true value of target firms and voluntary disclosure is costly. Finally, firms pay sizable fees. We model these fees as charged by a monopoly intermediary who provides disclosure services to target firms. In our model, each firm owns two indivisible factors: a tradeable "project" and non-tradeable "skill" to manage a project. Firms are heterogeneous in both dimensions, and complementarity between the two factors creates potential gains from trade.[6] Firms are privately informed about

[1] For example, see Eckbo (2014) and David (2021).

[2] For example, see Restuccia and Rogerson (2008). Hopenhayn (2013) surveys this literature. A recent work in this line of research, David (2021), is an exception.

[3] Studies in this area of research do not model firms' production side and/or analyze only firms directly involved in takeovers. For example, see Grossman and Hart (1980) and the literature that followed.

[4] Golubov et al. (2012) report that in 2007, \$4.2 trillion was spent on M&A deals and investment banks advised on over 85% of the deals by transaction values. Fees are estimated to be \$39.7 billion (about 1% of the transaction values). See also Hunter and Jagtiani (2003) and Calomiris and Hitscherich (2005).

[5] Previous works on merger fees typically viewed them as a device to control the incentive of intermediaries. See McLaughlin (1992) and also the papers cited in the previous footnote.

[6] To focus on the information friction as a source of inefficiency, we abstract from strategic motives of takeovers. While such motives may be more important in some industries, we believe that our modelling choice is a reasonable first step given our interest in welfare gains at the aggregate level.

their project quality and skill. We use this model to answer the following questions: how does the nature of disclosure technologies and fees affect the efficiency of resource reallocation? Should intermediaries be regulated? If so, how should such regulations be designed?

In Section 3, we establish our welfare benchmark. *If firms cannot credibly disclose their project quality at all*, matching is random but their decision to sell can reveal some information. We call this equilibrium *a no disclosure equilibrium*. Because it sets a natural lower bound on the amount of information revelation, we use a welfare gain in this equilibrium as our welfare benchmark. We also study an equilibrium with a minimum disclosure technology. We use this simple disclosure technology as a part of policy proposal in Section 5.

Section 4 contains our main technical contribution. We analyze the full disclosure equilibrium where target firms must pay fees to disclose their project quality. To our knowledge, we are the first to formulate and solve an equilibrium matching model with two dimensional heterogeneity and the transaction costs. We first derive a market-clearing condition in the form of a differential equation, and use it to derive predictions about matched pairs of firms. We then solve this equation in a closed form under a fixed fee and a fee proportional to prices. We investigate two properties of matched pairs of firms. The first property is the *relative value* – a value of a target firm relative to its matched bidder firm. We show that the relative value is less than one, and decreases in the proportional fee. The second property is the *skill gap*, defined by the skill of the bidder minus that of the matched target. We show that the skill gap is positive, increases in the project quality across deals, and increases in both types of fees. We discuss the existing empirical evidence consistent with our model, and also how to test other predictions from our model.

Finally, we endogenize the fees. We show that the monopoly intermediary uses both fees. Importantly, firms' gain net of the intermediary's profit is smaller than the benchmark welfare gain, i.e., *firms are made worse off* with the full disclosure service than without any disclosure service. While an orthodox remedy to this problem may be to promote entries, it may not be the best solution in the current context. First, a problem of the M&A advisory market is that top-tier (so called "bulge bracket") investment banks are *collectively* extracting large surplus.[7] If the major players have long established a collusive business practice, it might be difficult to promote entries and/or competition. Second, more entries may not lead to a socially better outcome.[8] Without a proper understanding of *potential* entrants, a

[7] Rau (2000) finds that top 5 investment banks' share is high and stable. Although the valuation of target firms can be done by other entities, typically investment bank advisors provide this service. For the institutional details, see Kisgen et al. (2009), Davidoff et al. (2011), and Cain and Denis (2013).

[8] For example, see Mankiw and Winston (1986), Amir et al. (2014), and von Negenborn (2019). In the current context, increasing returns due to fixed costs of information production and the heterogeneity of intermediaries seem to be relevant.

policy promoting entries of intermediaries may adversely affect firms.

In Section 5, we make a novel policy proposal to improve the efficiency of takeover markets *in the presence of the monopoly intermediary*. We construct *a hybrid market equilibrium*, where the intermediary offers the full disclosure technology (with fees) in "the upper market", and the public service provider offers the minimum disclosure technology (for free) in "the lower market". Firms endogenously select into the two markets. In equilibrium, targets in the upper market have better projects than those in the lower market, and bidders in the upper market have better skills than those in the lower market. A price function in the upper market ensures that a project of better quality is transferred to a firm with a better skill, while matching is random in the lower market.

We first identify conditions on the fees such that both markets can attract firms. In particular, a fixed fee must be positive to make a marginal target indifferent between full disclosure and pooling with lower types. We then show that, without any restriction, the intermediary would only use a proportional fee so that no firm trades in the lower market. As a result, the welfare gain and firms' gain change little from the case where the free minimum disclosure service is not offered to firms. Thus, simply offering the minimum disclosure service for free has a limited welfare impact. However, we show that regulating *either one of the two fees* significantly improves the welfare. Specifically, we show that with *a cap on the proportional fee* a small but active lower market emerges.

With the optimal cap, the welfare gain is close to the level achieved by the free full disclosure technology and, importantly, most of the welfare gain accrues to firms. We also assess the welfare contribution of the cap regulation *alone*. We show that if the same cap is imposed in the absence of the free minimum disclosure technology, the welfare gain is much smaller and firms do not gain much. Thus, it is *the cap regulation activating the lower market* that significantly improves the welfare and firms' gain. What is crucial for the success of the cap regulation is that, even though firms in the lower market do not *directly* contribute to the overall welfare, the hybrid market structure allows them to contribute to it *indirectly*. While the inefficiency due to random matching remains, its magnitude is minor because the firms in the lower market have small gains from trade. Yet, their presence makes the demand for the intermediary's full disclosure service more elastic, significantly reducing its profit.

Our policy proposal is novel and has three advantages over standard policies promoting more entries. First, implementing the cap on the proportional fee is straightforward, compared to the difficulty and the uncertainty associated with encouraging entries. Second, a public service provider is not required to have a disclosure technology comparable to that of private service providers. The most simple technology – the minium disclosure – is sufficient. Finally and most importantly, this policy *benefits the most productive firms*, because the expertise of the incumbent service provider is utilized in the upper market, but its profit is significantly reduced. While the cost of public service provision must be bourne by someone, perhaps by all publicly traded firms, the suggested magnitude of the welfare improvement indicates that it may be justified *on the basis of the reallocational efficiency.*

The rest of the paper is organized as follows. After the literature review, Section 2 describes a model. Section 3 introduces a welfare benchmark, a no disclosure equilibrium, and also studies a minimum disclosure technology. Section 4 studies the full disclosure equilibrium with fees and derive model predictions for matched pairs of firms. We also endogenize these fees by introducing a monopoly intermediary. In Section 5 we study a hybrid market equilibrium and make a policy proposal. Section 6 concludes. Supplementary proofs are gathered in Section 7.[9]

1.1 Related literature

The literature on takeovers is extensive, spanning many fields from finance, industrial organization, to macroeconomics. Lambrecht and Myers (2007) study a model of consolidating takeovers in declining industries in which inefficiency arises due to an agency friction. Legros and Newman (2013) study a model of firms' integration decision in which the shared ownership may cause inefficiency. We focus on takeovers that exploit synergies. Also, a source of inefficiency is different because we abstract from agency or ownership frictions. David (2021) embeds a takeover process in a dynamic model to study its aggregate implication. The key friction in his model is a search friction. None of these works studies information frictions and the role of disclosure. At a technical level, our model is closest to Jovanovic and Braguinsky (2004) and Nocke and Yeaple (2008). Relative to the former work, our model features a non-degenerate distribution of project quality and multiple modes of disclosures.[10] Nocke and Yeaple (2008) also model takeovers driven by two-dimensional heterogeneity. They build a two-country trade model, but abstract from trading costs and information friction in takeover markets. As a result, all the firms engage in takeovers in their model. Beyond the literature on takeovers, Fernandez and Gali (1999) is a related work on distorted matching. They study distortions due to a borrowing constraint, while we focus on the information friction and transaction costs. Moreover, they make standard assumptions of two exogenous sides (e.g. schools and students) and zero outside options for all agents. In our model, firms have heterogenous outside options (stand-alone values) and choose sides of the market (targets or bidders), both of which are crucial features of takeover markets.

[9] Long algebras omitted in the proofs are available from the author upon request.

[10] In their model a project quality takes either zero or one. It simplifies their analysis but limits its scope.

2 Model

After describing a model environment in Section 2.1, we explain our welfare measure and a notion of competitive equilibrium in Section 2.2.

2.1 Environment

Firms use two factors of production. The first factor is tradeable but indivisible, while the second factor is non-tradeable. Firms are heterogeneous in the quality of both factors. We call the tradeable factor *a project*, and the non-tradeable factor *skill*. We interpret a project as a collection of *tangible* assets, which remain productive when their ownership changes. Examples include a large plant (manufacturing), a customer base (retail), and an access to specific locations (services). We interpret skill as a collection of *intangible* assets, which are productive only within the current firm boundary.[11] Examples include organization-specific knowledge and team capital embodied in a network of people. These are useful "assets", but various coordination issues make sustaining their productivity after takeovers difficult.[12]

We assume that each firm can manage at most one project, and that each firm cannot operate on both sides of takeover markets simultaneously. These assumptions capture an aspect of small firms, consistent with a notion of competitive takeovers. Hence, options available for each firm are: (i) sell its project (*target*), or (ii) buy a new project (*bidder*), or (iii) produce with its project (*stand-alone firms*).[13] Finally, we assume that a continuum measure one of firms draw A and X independently from a uniform distribution on $[0, 1]$. A firm with a project of quality A and the skill level X can produce output using a production technology $F(A, X) = AX$.

Remark. Li et al. (2018) show that bidder firms' organization capital, measured by an accounting data, creates values in takeovers, while that of target firms does not. This is consistent with an interpretation of X in our model as non-tradeable organization capital. We discuss further evidence presented in their paper in Section 4.

[11] Some intangible assets, such as intellectual properties and human capital of workers, may be tradeable via takeovers. However, to be productive, these assets may require a firm-specific factor such as local reputation and history. If the latter is not tradeable, neither is the former.

[12] A simple example is the skill of the single owner-manager selling his firm to do other business. Generally, a bidder firm does not pay for anything it cannot utilize, no matter how valuable it is for a target firm.

[13] If firms can buy *and* sell projects without transaction costs, then all firms do both, as in Nocke and Yeaple (2008). With small transaction costs, it can be shown that some firms choose one side, and others two sides. To keep firms' decision as simple as possible, we simply assume that doing both is too costly.

2.2 Welfare measure and equilibrium

We define a welfare gain by the aggregate production after takeovers minus the aggregate production without takeovers. The aggregate production without takeovers is

$$\int_0^1 \int_0^1 (AX)\, dAdX = \left(\int_0^1 AdA\right)\left(\int_0^1 XdX\right) = \frac{1}{4}.$$

Let NP be bidders' new production, BL be their lost production, TL be targets' lost production, and Y_N be production by stand-alone firms. The welfare gain is measured by

$$WG \equiv NP + Y_N - \frac{1}{4} = NP - (BL + TL).$$

We characterize NP, BL, TL in different types of equilibria and compare WG across them.

Equilibrium. Our solution concept is a competitive equilibrium, in which firms act taking prices as given, and prices clear markets for projects given some form of disclosure technology available to target firms. More specifically, firms face the following problem.

$$\Pi(A, X) \equiv \max\{\Pi_T(A),\ \Pi_B(X),\ AX\}, \tag{1}$$

where $\Pi_T(A)$ is the payoff as a target firm, $\Pi_B(X)$ is the payoff as a bidder firm, and AX is the payoff as a stand-alone firm. A choice of prices (to sell) and projects (to buy) if any, as well as fees for disclosure, are all subsumed in $\Pi_T(A)$ and $\Pi_B(X)$. A solution to (1) endogenously determines the supply and demand of projects as a function of prices. A competitive equilibrium is a pair of firms' strategies and prices such that (i) the firms' strategies solve (1) taking prices as given, and (ii) prices clear the markets for projects. Obviously, forms of $\Pi_T(A)$, $\Pi_B(X)$, and prices depend on the nature of disclosure technology and fees. We defer further discussion of equilibrium in the corresponding sections.

Remark. Our notion of equilibrium takes disclosure technologies as given. Lizzeri (1999) studies an intermediary's *disclosure design* in a model with one seller and two buyers. In our matching model with many firms heterogeneous in two dimensions, a disclosure design interacts with both matching and market-clearing. To keep our analysis tractable, we focus on simple disclosure technologies and leave a general investigation of *the disclosure design in a heterogeneous matching environment* for a future work.

3 Welfare benchmark

In this section, we study a minimum disclosure technology. A minimum disclosure technology only verifies that a project quality is above a prespecified threshold value, which we call *a minimum standard*. This disclosure technology is important for two reasons. First, it is the most simple disclosure technology. As such, we use it as a policy tool in Section 5. Second, when the minimum standard is set low enough, it results in an equilibrium in which only target firms' decision to sell reveals some information. Because this sets a natural lower bound on the amount of information revelation, we use a welfare gain in this equilibrium as our welfare benchmark. We call an equilibrium with the minimum disclosure technology *a minimum disclosure equilibrium* (henceforth MD equilibrium), and call an equilibrium without any disclosure *a no disclosure equilibrium* (ND equilibrium).

We denote a minimum standard by $A_{\min} \in [0,1]$. The disclosure "$A \geq A_{\min}$" is common for all targets, so projects must be traded at a single price P. The expected quality of projects for sale, $E\left[A|A \text{ is for sale}\right] \equiv a$, is endogenous. Because firms' problem is $\Pi^{MD}(A,X) = \max\{P,\ aX - P,\ AX\}$, participation constraints as a target and as a bidder are

$$AX \leq P, \tag{2}$$

$$AX \leq aX - P. \tag{3}$$

In **Figure 1**, we plot (2) as a blue line decreasing in X, and (3) as a dashed red line increasing in X, assuming $A_{\min} = 0$. The intersection of the two lines defines a point $(X^*, A^*) = \left(\frac{2P}{a}, \frac{a}{2}\right)$. A vertical line below this point represents firms that are indifferent between being targets and bidders, but strictly prefer trading to not trading.

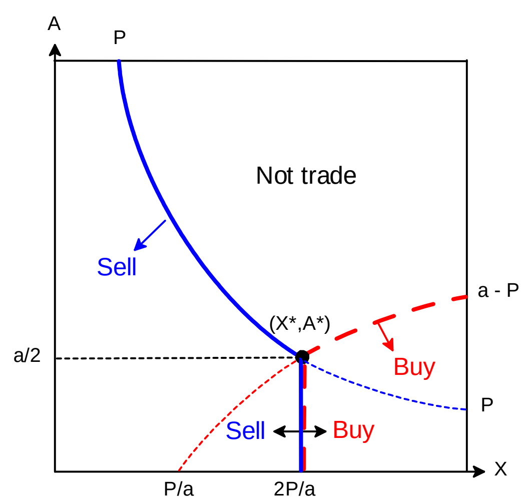


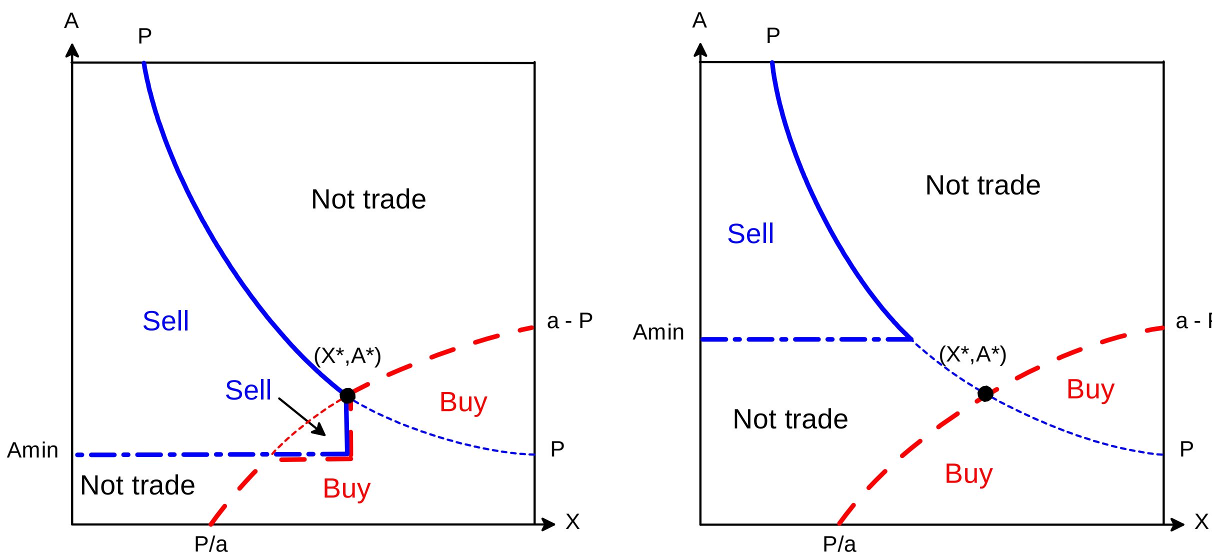
Figure 1. Sorting with $A_{\min} = 0$.

For a given P, this sorting pattern implies a supply $S(P) = A^* X^* + \int_{A^*}^{1} \frac{P}{A} dA$ and a demand $D(P) = \int_{X^*}^{1} \left(a - \frac{P}{X}\right) dX$. We show that a market-clearing condition $S(P) = D(P)$ defines a unique market-clearing price $P(a) \in \left(0, \frac{a}{3}\right)$ for any $a \in (0,1)$. Finally, given any conjectured a, the expected quality of projects for sale is

$$\frac{\int_0^{A^*} AX^* dA + \int_{A^*}^{1} A\frac{P(a)}{A} dA}{S(P(a))} \equiv \Gamma(a).$$

We show that $\Gamma(a) = a$ has a unique solution $a^* \in (0,1)$. Using a^* and $P^* \equiv P(a^*)$, we compute various equilibrium objects.

If we raise $A_{\min}$, there are two possible cases as illustrated in **Figure 2**.



(a) $0 < A_{\min} < A^*$. (b) $A^* < A_{\min} < 1$.

Figure 2. Sorting in the MD equilibrium.

In the panel (a), targets and bidders are "connected", while in the panel (b) they are "separated. The level of $A_{\min}$ determines which case occurs in equilibrium.

Proposition 1 (MD equilibrium).

(a) *A unique equilibrium exists for all* $A_{\min} \in [0,1]$, *and* $A_{\min} \lesseqgtr A^* \Leftrightarrow A_{\min} \lesseqgtr A_+$, *where* $A_+ \approx 0.285$ *is a smaller solution to* $1 = A(1 - 2\ln A)$.

(b) *The expected welfare gain is* $\left(\frac{a^* - P^*}{2}\right)^2$.

Proof. **Proposition 1 (a)** follows from **Lemma A** shown below.

Lemma A

(a) *For* $A_{\min} \in [0, A_+)$, $a^* \in (0, 2A_+)$ *is a unique solution to* $a = \Gamma(a; A_{\min})$, *where*

$$\Gamma(a; A_{\min}) \equiv \frac{1 - \frac{a}{4} - \frac{1}{a}A_{\min}^2}{1 - \ln\frac{a}{2} - \frac{2}{a}A_{\min}}, \tag{4}$$

and $A_{\min} < A^* = \frac{a^*}{2}$ *holds. A market-clearing price is a unique solution to*

$$a^* = P\left\{3 - \ln P - \left(4\frac{A_{\min}}{a^*} + \ln\left(1 - \frac{A_{\min}}{a^*}\right)\right)\right\}. \tag{5}$$

(b) *For* $A_{\min} \in [A_+, 1)$, $a^* = -\frac{1-A_{\min}}{\ln A_{\min}} \in [2A_+, 1)$ *and* $A_{\min} \geq A^* = \frac{a^*}{2}$ *holds. A market-clearing price is a unique solution to*

$$a^* = P\left(1 - \ln P + \ln\frac{a^*}{A_{\min}}\right). \tag{6}$$

We prove **Lemma A** in Section 7. To prove **Proposition 1 (b)**, we drop "$*$" from (a^*, P^*) and proceed in two steps. First, we derive the welfare gain as a function of $(a, P, A_{\min})$. Second, we use a market-clearing condition to derive the expression $\left(\frac{a-P}{2}\right)^2$.

Step 1. The expected welfare gain as a function of $(a, P, A_{\min})$.

For $A_{\min} < A_+$, the expected value of new production is given by

$$\begin{aligned} NP^{MD} &= \underbrace{a\int_{X^*}^{1} X\left(a - \frac{P}{X}\right)dX}_{a\left(\frac{a}{2}-P\right)} + \underbrace{a\int_{\frac{P}{a}}^{\frac{P}{a-A_{\min}}} X\left(a - \frac{P}{X}\right)dX}_{\frac{P^2}{2}\left(\frac{A_{\min}}{a-A_{\min}}\right)^2} + \underbrace{a\int_{\frac{P}{a-A_{\min}}}^{X^*} (XA_{\min})\,dX}_{\frac{P^2}{2}\frac{A_{\min}}{a}\left(4-\left(\frac{a}{a-A_{\min}}\right)^2\right)} \\ &= a\left(\frac{a}{2} - P\right) + \frac{P^2}{2}\frac{A_{\min}}{a}\left(\frac{3 - 4\frac{A_{\min}}{a}}{1 - \frac{A_{\min}}{a}}\right). \end{aligned}$$

Targets' lost production is

$$TL^{MD} = \underbrace{\int_{A_{\min}}^{A^*}\int_{0}^{X^*} (AX)\,dAdX}_{\frac{1}{2}\left(A^{*2}-A_{\min}^2\right)\frac{X^{*2}}{2}} + \underbrace{\int_{A^*}^{1}\left(A\int_{0}^{\frac{P}{A}} XdX\right)dA}_{-\frac{P^2}{2}\ln\frac{a}{2}} = \frac{P^2}{2}\left(\frac{1}{2} - \ln\frac{a}{2}\right) - P^2\left(\frac{A_{\min}}{a}\right)^2.$$

Bidders' lost production is

$$
\begin{aligned}
BL^{MD} &= \underbrace{\int_{X^*}^{1}\left(X\int_0^{a-\frac{P}{X}}AdA\right)dX}_{\left(\frac{a}{2}-P\right)^2+\frac{P^2}{2}\left(\ln\frac{a}{2}-\ln P\right)} + \underbrace{\int_{\frac{P}{a}}^{X^*}\left(X\int_0^{a-\frac{P}{X}}AdA\right)dX - \int_{\frac{P}{a-A_{\min}}}^{X^*}\left(X\int_{A_{\min}}^{a-\frac{P}{X}}AdA\right)dX}_{\frac{P^2}{2}\left[2\left(\frac{A_{\min}}{a}\right)^2-\frac{A_{\min}}{a-A_{\min}}+\ln\left(1+\frac{A_{\min}}{a-A_{\min}}\right)\right]} \\
&= \left(\frac{a}{2}-P\right)^2+\frac{P^2}{2}\left(\ln\frac{a}{2}-\ln P\right)+\frac{P^2}{2}\left[2\left(\frac{A_{\min}}{a}\right)^2-\frac{A_{\min}}{a-A_{\min}}+\ln\left(1+\frac{A_{\min}}{a-A_{\min}}\right)\right].
\end{aligned}
$$

The welfare gain is

$$
\begin{aligned}
WG^{MD}\left(A_{\min}<A_+\right) &= NP^{MD}-\left(TL^{MD}+BL^{MD}\right) \\
&= a\left(\frac{a}{2}-P\right)-\left\{\left(\frac{a}{2}-P\right)^2+\frac{P^2}{2}\left(\frac{1}{2}-\ln P\right)\right\} \\
&\quad +\frac{P^2}{2}\frac{A_{\min}}{a}\left(\frac{3-4\frac{A_{\min}}{a}}{1-\frac{A_{\min}}{a}}\right)-\frac{P^2}{2}\left[\ln\left(1+\frac{A_{\min}}{a-A_{\min}}\right)-\frac{A_{\min}}{a-A_{\min}}\right] \\
&= G\left(a,P\right)+\frac{P^2}{2}\left\{4\frac{A_{\min}}{a}+\ln\left(1-\frac{A_{\min}}{a}\right)\right\},
\end{aligned}
$$

where we used $G(a,P)$ defined by

$$
\left(\frac{a}{2}-P\right)\left(\frac{a}{2}+P\right)+\frac{P^2}{2}\left(\ln P-\frac{1}{2}\right)\equiv G\left(a,P\right). \tag{7}
$$

For $A_{\min}\geq A_+$, the expected value of new production is given by $NP^{MD}=a\int_{\frac{P}{a}}^{1}X\left(a-\frac{P}{X}\right)dX=\frac{(a-P)^2}{2}$. Targets' lost production is $TL^{MD}=\int_{A_{\min}}^{1}\left(A\int_0^{\frac{P}{A}}XdX\right)dA=-\frac{P^2}{2}\ln A_{\min}$. Bidders' lost production is

$$
BL^{MD}=\int_{\frac{P}{a}}^{1}\left(X\int_0^{a-\frac{P}{X}}AdA\right)dX=\frac{1}{4}\left(a-P\right)\left(a-3P\right)-\frac{P^2}{2}\ln\frac{P}{a}.
$$

Therefore, the welfare gain is

$$
\begin{aligned}
WG^{MD}\left(A_{\min}\geq A_+\right) &= NP^{MD}-\left(TL^{MD}+BL^{MD}\right) \\
&= \frac{1}{4}\left(a-P\right)\left(a+P\right)+\frac{P^2}{2}\ln\left(A_{\min}\frac{P}{a}\right).
\end{aligned}
$$

Step 2. Use a market-clearing condition to show that the welfare gain is always $\left(\frac{a-P}{2}\right)^2$.

For $A_{\min} < A_{+}$, from the market-clearing condition (5) in **Lemma A (a)**,

$$4\frac{A_{\min}}{a} + \ln\left(1 - \frac{A_{\min}}{a}\right) = 3 - \ln P - \frac{a}{P}. \tag{8}$$

Recall that $WG^{MD}(A_{\min} < A_{+}) = G(a,P) + \frac{P^2}{2}\left\{4\frac{A_{\min}}{a} + \ln\left(1 - \frac{A_{\min}}{a}\right)\right\}$, where the expression of $G(a,P)$ is given in (7). Eliminating $A_{\min}$ using (8) yields

$$WG^{MD}(A_{\min} < A_{+}) = \left(\frac{a}{2}\right)^2 - P^2 + \frac{P^2}{2}\left(\ln P - \frac{1}{2} + 3 - \ln P - \frac{a}{P}\right) = \left(\frac{a-P}{2}\right)^2.$$

For $A_{\min} \geq A_{+}$, from the market-clearing condition (6) in **Lemma A (b)**,

$$\ln\left(A_{\min}\frac{P}{a}\right) = -\frac{a-P}{P}. \tag{9}$$

Using (9) in $WG^{MD}(A_{\min} \geq A_{+}) = \frac{1}{4}(a-P)(a+P) + \frac{P^2}{2}\ln\left(A_{\min}\frac{P}{a}\right)$,

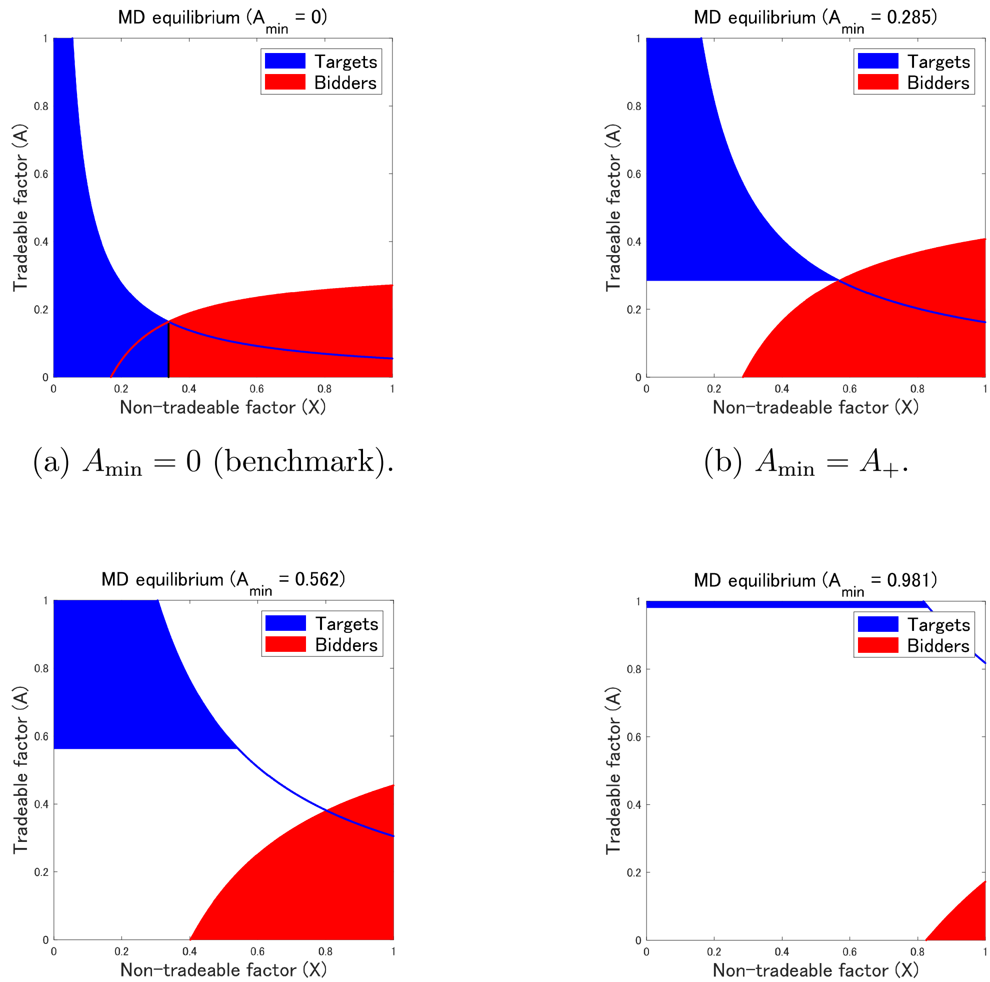
$$WG^{MD}(A_{\min} \geq A_{+}) = \frac{1}{4}\left(a^2 - P^2\right) - \frac{P}{2}(a-P) = \left(\frac{a-P}{2}\right)^2. \qquad \blacksquare$$

Proposition 1 allows us to compute the expected welfare gain as a function of $A_{\min} \in [0,1]$. Let $WG^{MD}(A_{\min})$ denote the welfare gain in the MD equilibrium with $A_{\min} \in [0,1]$. We numerically obtained the following result.[14]

Claim 1 *The benchmark welfare gain with $A_{\min} = 0$ is $WG^{ND} \equiv WG^{MD}(0) \approx 0.0184$. There is $A^*_{\min} \in (A_{+}, 1)$ that maximizes the expected welfare gain in the MD equilibrium. The maximized welfare gain $WG^{MD}(A^*_{\min})$ is 281% of the benchmark welfare gain WG^{ND}.*

[14] Throughout the paper, we use **Claim** tags for results obtained by numerical evaluations.

Figure 3 shows how the level of minimum standard $A_{\min}$ affects the behavior of firms.



(a) $A_{\min} = 0$ (benchmark). (b) $A_{\min} = A_+$.

(c) $A_{\min} = A^*_{\min}$. (d) $A_{\min} \approx 1$.

Figure 3. MD equilibria for different values of $A_{\min}$.

Targets are in the area $A \in \left[A_{\min}, \frac{P^*}{X}\right]$ (colored blue), while bidders are in the area $A \leq a^* - \frac{P^*}{X}$ (colored red). The panel (a) is the case with $A_{\min} = 0$ (the ND equilibrium). The minimum standard $A_{\min}$ increases from the panel (a) to (d). The MD equilibrium is inefficient for two reasons. First, bidders with initial projects of quality $A \in [A_{\min}, a^* - P^*]$ may end up with worse projects (panel (a)(b)).[15] By raising $A_{\min}$, this problem disappears (panel (c)). However, too high $A_{\min}$ leaves much welfare gains unrealized (panel (d)). This trade-off determines $A^*_{\min} \in (A_+, 1)$. A key for the efficiency in the MD equilibrium is to make $A_{\min}$ high enough that *firms that qualify as targets do not become bidders*, but not too

[15] Notice that the height of the red area $(a^* - P^*)$ exceeds the lowest point of the blue area $(A_{\min})$.

high such that sufficient trading occurs.

The ND equilibrium is a conceptual benchmark. In reality, there are many ways in which target firms can disclose the quality of what they sell. In the next section, we study the opposite extreme – a case where target firms can disclose their project quality perfectly.

4 Fill disclosure equilibrium subject to fees

In this section, we assume that project quality can be perfectly disclosed if target firms pay fees.[16] In Section 4.1, we formally define an equilibrium with general fees, and discuss model predictions. In Section 4.2, we study a fee structure commonly used in practice – a fixed fee and a fee proportional to target prices.[17] Finally, in Section 4.3 we endogenize fees.

4.1 General fees and model predictions

Consider a firm with (A, X). Let $\{P(A)\}_{A\in(0,1]}$ be a price function for projects. Suppose that a fee $f(A, P)$ is charged for target firms who disclose A and sell it at a price P. In general, f affects the price function, but we suppress the notation. A payoff as a target is $\Pi_T(A) \equiv P(A) - f(A, P(A))$. A payoff as a bidder is $\Pi_B(X) \equiv \max_{a\in[0,1]} \{aX - P(a)\}$. For now, we assume that $P(a)$ is twice differentiable, $P'(a) > 0$ and $P''(a) > 0$, which we verify in equilibrium. This ensures that a bidder's problem has an interior solution characterized by $X = P'(a)$. We denote this *matching function* by $m(a) \equiv P'(a)$. With these $\Pi_T(A)$ and $\Pi_B(X)$, the firm's problem is

$$\Pi^{FD}(A, X) \equiv \max\{\Pi_T(A),\ \Pi_B(X),\ AX\}. \tag{10}$$

Due to the two-dimensional heterogeneity, matching and market-clearing are distinct concepts in our model. First, the first order condition of the bidders' problem, $X = P'(a)$, determines the matching between targets' project quality and bidders' skill. Second, a supply and a demand for projects at each quality level are determined as follows. A *supply density* for projects of quality a is $S(a) \equiv \int \mathbf{1}_{\mathcal{S}(a)} dX$, where $\mathbf{1}_{\mathcal{S}(a)}$ is an indicator function for a set of *skills of target firms* with a project $A = a$:

$$\mathcal{S}(a) \equiv \left\{X \in [0,1] \,|\Pi^{FD}(a, X) = \Pi_T(a)\right\} \qquad \subset [0,1].$$

[16] Fees for bidders can be added, but we do not include them here for brevity.

[17] McLaughlin (1992) reports that most target fees are based on acquisition values (71.4% in his sample). He also reports that, while only 8.1% of target firms use fixed fees, they are used often when the investment bank only provides an opinion letter. This indicates the relevance of fixed fees to pay for information.

Similarly, a *demand density* for projects of quality a is $D(a) \equiv \int \mathbf{1}_{\mathcal{D}(a)} dA$, where

$$\mathcal{D}(a) \equiv \left\{ A \in [0,1] \,|\Pi^{FD}(A, m(a)) = \Pi_B(m(a)) \right\} \qquad \subset [0,1]$$

is a set of *project qualities initially held by bidders* with skill $X = m(a)$. A price function $\{P(A)\}_{A \in (0,1]}$ clears a market if $P'(1) = 1$ and

$$\int_0^a S(A)\, dA = \int_0^{m(a)} D\left(a^B(X)\right) dX \;\text{ for any } a \in (0,1], \tag{11}$$

where $a^B(X)$ is defined by $X = m\left(a^B(X)\right)$. The matching $X = m(A) = P'(A)$ implies $\frac{dX}{dA} = P''(A)$. Then (11) can be stated *in its density form* as

$$S(A) = D(A)\, P''(A) \;\text{ for any } A \in (0,1]. \tag{12}$$

Because $S(A)$ and $D(A)$ depend on $\{P(A)\}_{A \in (0,1]}$, (12) defines a differential equation in P.

Definition *A full disclosure equilibrium is a pair of a price function* $\{P(A)\}_{A \in (0,1]}$ *and firms' strategies such that (i) firms' strategies solve* (10) *taking* $\{P(A)\}_{A \in (0,1]}$ *as given, (ii)* $\{P(A)\}_{A \in (0,1]}$ *satisfies* $P'(1) = 1$ *and* (12).

Welfare gain. In equilibrium, targets are in the set $\mathcal{S} \equiv \underset{A \in (0,1]}{\cup} \mathcal{S}(A)$, while bidders are in the set $\mathcal{D} \equiv \underset{A \in (0,1]}{\cup} \mathcal{D}(A)$. Bidders' new production is

$$NP^{FD} \equiv \int_0^1 \left(a^B(X)\, X\right) D\left(a^B(X)\right) dX.$$

Similarly, bidders' lost production and targets' lost production are

$$BL^{FD} \equiv \int \int (AX)\, \mathbf{1}_{\mathcal{D}} dA dX \qquad \text{and} \qquad TL^{FD} \equiv \int \int (AX)\, \mathbf{1}_{\mathcal{S}} dA dX.$$

The welfare gain is $WG^{FD} = NP^{FD} - \left(BL^{FD} + TL^{FD}\right)$.

General characterization. Because bidders who demand projects of quality a have skill $m(a) = P'(a)$, their participation constraint $Am(a) \leq am(a) - P(a)$ implies the demand density $D(a) = a - \frac{P(a)}{P'(a)}$. On the other side of the market, target firms' participation constraint $aX \leq P(a) - f(a, P(a))$ implies the supply density $S(a) = \frac{P(a) - f(a, P(a))}{a}$. By substituting $D(a)$ and $S(a)$ into the market-clearing condition (12),

$$\left(\frac{P'(a)}{P(a)}a-1\right)\frac{P''(a)}{P'(a)}a=1-\frac{f(a,P(a))}{P(a)}. \tag{13}$$

Note that $\frac{P'(a)}{P(a)}a\equiv\eta_p(a)$ is the elasticity of the price function $P(a)$, while $\frac{P''(a)}{P'(a)}a\equiv\eta_m(a)$ is the elasticity of the matching function $m(a)=P'(a)$. A price function is a solution to (13) with a boundary condition $P'(1)=1$.

Given the symmetry of our environment, *without fees*, a conjectured efficient matching $m(A)=P'(A)=A$ immediately yields $P(A)=\frac{1}{2}A^2$, which solves (13) with $f(a,P(a))=0$. The benefit of deriving (13) arises because in the presence of fees we do not know a priori the form of matching function $P'(A)$. Importantly, as we demonstrate it below, we can gain some economic insight from (13) even when we cannot solve it explicitly.

Positive implications. We derive three empirical measures using (13). First, by multiplying P on both sides of (13), we have $\Pi_B(m(a))\,\eta_m(a)=\Pi_T(a)$. Thus, *a relative target value* is

$$RV(a)\equiv\frac{\Pi_T(a)}{\Pi_B(m(a))}=\eta_m(a). \tag{14}$$

Second, we measure *a fee ratio* $FR(a)$ by the amount of fees as a fraction of prices.

$$FR(a)\equiv\frac{f(a,P(a))}{P(a)}=1-\eta_m(a)\left(\eta_p(a)-1\right). \tag{15}$$

Finally, targets with $A=a$ have the average skill $\frac{S(a)}{2}$,[18] while matched bidders have skill $m(a)$. Accordingly, we measure *a skill gap* $SG(a)$ by

$$SG(a)\equiv m(a)-\frac{1}{2}S(a).$$

Using $S(a)=\frac{P(a)-\ f(a,P(a))}{a}$ and (13), the skill gap can be expressed as

$$SG(a)=m(a)\left(1-\frac{\eta_m(a)}{2}\frac{\eta_p(a)-1}{\eta_p(a)}\right). \tag{16}$$

We use $RV(a)$, $FR(a)$, and $SG(a)$ as positive implications from the model. In particular, we can use $RV(a)$ and $FR(a)$ to identify *a skill premium* of bidders $\frac{SG(a)}{m(a)}$.

[18] They satisfy the participation constraint $X\leq S(a)$.

Lemma 1 (Positive properties of FD equilibrium). *The skill premium satisfies*

$$\frac{SG(a)}{m(a)} = 1 - \frac{RV(a)}{2} \frac{1 - FR(a)}{RV(a) + 1 - FR(a)}, \tag{17}$$

which is decreasing in $RV(a)$ and increasing in $FR(a)$.

Proof. Substituting $\eta_m(a) = RV(a)$ and $\eta_p(a) - 1 = \frac{1-FR(a)}{\eta_m(a)}$ into the expression of $\frac{SG(a)}{m(a)} = 1 - \frac{\eta_m(a)}{2} \frac{\eta_p(a)-1}{\eta_p(a)}$ yields (17). The rest is obvious. ■

Lemma 1 implies that any changes in the environment that make *the relative target value smaller* and/or *the fee ratio larger* would imply the higher skill premium of bidders. These endogenous relationship can be tested if data on $RV(a)$, $FR(a)$, and $SG(a)$ are available. We further discuss predictions from the model after explicitly solving for the price function for a simple but commonly used fee structure.

4.2 Fixed and proportional fees

The differential equation (13) does not admit a general solution for a general form of fees $f(A, P)$, but fortunately we can make a further progress for important classes of fees. A fixed fee and a fee proportional to target prices can be expressed as $f = \phi + \tau P$ with fixed numbers $(\phi, \tau) \in \mathbb{R}^2$.[19] By substituting this into (13),

$$\left(\frac{P'(a)}{P(a)}a - 1\right) \frac{P''(a)}{P'(a)}a = 1 - \tau - \frac{\phi}{P(a)}. \tag{18}$$

We show that (18) has a unique solution within the class of power functions. **Proposition 2** is our main technical contribution.

Proposition 2 (FD equilibrium with fees ϕ, τ).

(a) *The price function and the matching function are*

$$P(A; \phi, \tau) = \frac{1}{1 + \sqrt{1-\tau}} \left(A^{1+\sqrt{1-\tau}} + \frac{\phi}{\sqrt{1-\tau}}\right) \quad \textit{and} \quad m(A) = A^{\sqrt{1-\tau}}. \tag{19}$$

[19] A closed form solution still obtains with $f = \phi + \tau P + dAX$ and fees for bidders $f_B = \phi_B + \tau_B P + d_B AX$.

(b) *The relative target value is* $RV(A) = \sqrt{1-\tau}$. *The fee ratio and the skill gap are*

$$\begin{aligned} FR(A) &= 1 - \sqrt{1-\tau}\frac{1-\tau-\frac{\phi}{A^{1+\sqrt{1-\tau}}}}{\sqrt{1-\tau}+\frac{\phi}{A^{1+\sqrt{1-\tau}}}}, \\ SG(A) &= \frac{1}{2\left(1+\sqrt{1-\tau}\right)}\left\{\left(1+\tau+2\sqrt{1-\tau}\right)A^{\sqrt{1-\tau}} + \frac{\phi}{A}\right\}. \end{aligned}$$

$FR(A)$ *decreases in* A *given* $\phi > 0$, *and increases in* ϕ, τ.

$SG(A)$ *increases in* A, *while* $\frac{SG(A)}{m(A)}$ *decreases in* A. *Both increase in* ϕ, τ.

(c) *The welfare gain is*

$$\begin{aligned} WG^{FD}(\phi,\tau) &= \frac{1}{4}\left(\frac{\sqrt{1-\tau}}{1+\sqrt{1-\tau}}\right)^2\left(1-\frac{\phi}{1-\tau}\right)\left\{\begin{array}{l} 2-\sqrt{1-\tau} \\ +\left(3\sqrt{1-\tau}-2\right)\frac{\phi}{1-\tau} \end{array}\right\} \\ &\quad +\frac{1}{2}\frac{\frac{\phi^2}{\sqrt{1-\tau}}\ln\frac{\phi}{1-\tau}}{\left(1+\sqrt{1-\tau}\right)^2} \end{aligned}$$

for any $\phi + \tau < 1$ *and zero otherwise.*

Proof.

(a) Suppressing the argument of P, P', P'', the differential equation (18) is

$$\left(P'a - P\right)\frac{P''}{P'}a = (1-\tau)P - \phi. \tag{20}$$

We find a solution in the class of power functions $P(a;\phi,\tau) = \frac{1}{c_0}a^{c_0} + c_1$. With this,

$$P'a - P = \frac{c_0-1}{c_0}a^{c_0} - c_1 \qquad \text{and} \qquad \frac{P''}{P'}a = c_0 - 1.$$

Substituting these into (20) and equating coefficients on both sides,

$$\frac{c_0-1}{c_0}(c_0-1) = \frac{1-\tau}{c_0} \qquad \text{and} \qquad (c_0-1)c_1 = \phi - (1-\tau)c_1.$$

This has a unique solution $c_0 = 1+\sqrt{1-\tau}$ and $c_1 = \frac{\phi}{c_0-\tau} = \frac{\phi}{1-\tau+\sqrt{1-\tau}}$. With these (c_0, c_1), the price function (19) is obtained. ■

(b) Using (19) and $m(A) = A^{\sqrt{1-\tau}}$ to compute $\eta_p(A)$ and $\eta_m(A)$ in the expressions (14), (15), (16) yields the results. That $FR(A)$ is decreasing in A given $\phi > 0$, and increasing in ϕ is obvious. To show that $FR(A)$ is increasing in τ, we let $s \equiv \sqrt{1-\tau}$ and show that $FR(A) = 1 - s\frac{s^2 - \frac{\phi}{A^{1+s}}}{s + \frac{\phi}{A^{1+s}}}$ is decreasing in s. It suffices to show that $\frac{s^2 - \frac{\phi}{A^{1+s}}}{s + \frac{\phi}{A^{1+s}}}$ is increasing in s. This holds because $\frac{s^2 - \frac{\phi}{A^{1+s}}}{s + \frac{\phi}{A^{1+s}}} = s\frac{sA^{1+s} - \frac{\phi}{s}}{sA^{1+s} + \phi}$ is increasing in s. Because $RV(A) = \sqrt{1-\tau}$ is independent of A and decreasing in τ, **Lemma 1** implies the comparative statics for $\frac{SG(A)}{m(A)}$. Because $m(A)$ is increasing in A and independent of ϕ, $SG(A) = m(A)\frac{SG(A)}{m(A)}$ is increasing in A and ϕ. Finally, to show that $SG(A)$ is increasing in τ, it suffices to show that $\frac{1+\tau+2\sqrt{1-\tau}}{1+\sqrt{1-\tau}}$ is increasing in τ. Taking a derivative,

$$\frac{\left(1 - \frac{1}{\sqrt{1-\tau}}\right)\left(1 + \sqrt{1-\tau}\right) - \left(1 + \tau + 2\sqrt{1-\tau}\right)\left(-\frac{1}{2\sqrt{1-\tau}}\right)}{\left(1 + \sqrt{1-\tau}\right)^2}.$$

The numerator is $1 + \frac{\sqrt{1-\tau}}{2} > 0$. ■

(c) We work with $s \equiv \sqrt{1-\tau}$. The price function (19) is

$$P(a;\phi,s) = \frac{1}{1+s}\left(a^{1+s} + \frac{\phi}{s}\right). \tag{21}$$

From the expression of target firms' payoff

$$P(A;\phi,\tau) - \{\phi + \tau P(A;\phi,\tau)\} = \frac{1-\tau}{1+\sqrt{1-\tau}}\left(A^{1+\sqrt{1-\tau}} - \frac{\phi}{1-\tau}\right),$$

the supply density at $A = a$ is $S(a;\phi,s) = \frac{s^2}{1+s}\left(a^s + \frac{\phi}{s^2}\frac{1}{a}\right)$. From bidder firms' payoff

$$X \times X^{\frac{1}{\sqrt{1-\tau}}} - P\left(X^{\frac{1}{\sqrt{1-\tau}}};\phi,\tau\right) = \frac{\sqrt{1-\tau}}{1+\sqrt{1-\tau}}\left(X^{\frac{1+\sqrt{1-\tau}}{\sqrt{1-\tau}}} - \frac{\phi}{1-\tau}\right),$$

the demand density at $X = x$ is $D(x;\phi,s) = \frac{1}{(1+s)s}\left(s^2 x^{\frac{1}{s}} - \frac{\phi}{x}\right)$. As bidder firms with skill X are matched with projects of quality $A = X^{\frac{1}{s}}$, the new production is

$$NP(\phi,s) = \int_{\underline{X}}^{1} X^{\frac{1+s}{s}} D(X;\phi,s)\,dX, \qquad \text{where } \underline{X} \equiv \left(\frac{\phi}{s^2}\right)^{\frac{s}{1+s}} = \underline{A}^s.$$

The lost production by bidders and targets are

$$\begin{aligned} BL(\phi, s) &= \int_{\underline{X}}^{1} \left\{ X \int_{0}^{D(X;\phi,s)} A dA \right\} dX, \\ TL(\phi, s) &= \int_{\underline{A}}^{1} \left\{ A \int_{0}^{S(A;\phi,s)} X dX \right\} dA. \end{aligned}$$

Computing $NP(\phi, s)$, $BL(\phi, s)$, $TL(\phi, s)$,

$$NP(\phi, s) = \frac{s^2}{(1+s)s} \int_{\underline{X}}^{1} X^{\frac{2+s}{s}} dX - \frac{\phi}{(1+s)s} \int_{\underline{X}}^{1} X^{\frac{1}{s}} dX = \frac{1}{2} \left(\frac{s}{1+s} \right)^2 \left(1 - \underline{X}^{\frac{1+s}{s}} \right)^2,$$

$$\begin{aligned} BL(\phi, s) &= \frac{1}{2} \int_{\underline{X}}^{1} \left\{ X \frac{1}{(1+s)^2 s^2} \left(s^4 X^{\frac{2}{s}} - 2s^2 \phi X^{\frac{1-s}{s}} + \frac{\phi^2}{X^2} \right) \right\} dX \\ &= \frac{1}{4} \left(\frac{s}{1+s} \right)^3 \left(1 - \underline{X}^{\frac{1+s}{s}} \right) \left(1 - 3\underline{X}^{\frac{1+s}{s}} \right) - \frac{\phi^2 \ln \underline{X}}{2(1+s)^2 s^2}, \end{aligned}$$

$$TL(\phi, s) = \frac{1}{2} \int_{\underline{A}}^{1} \left\{ A \left(\frac{s^2}{1+s} \right)^2 \left(A^{2s} - \frac{2\phi}{s^2} A^{s-1} + \frac{\phi^2}{s^4} \frac{1}{A^2} \right) \right\} dA = sBL(\phi, s).$$

Therefore, the welfare gain is

$$\begin{aligned} WG(\phi, s) &= NP(\phi, s) - (BL(\phi, s) + TL(\phi, s)) \\ &= \frac{1}{4} \left(\frac{s}{1+s} \right)^2 \left(1 - \underline{X}^{\frac{1+s}{s}} \right) \left\{ 2 - s + (3s-2) \underline{X}^{\frac{1+s}{s}} \right\} + \frac{\frac{\phi^2}{s} \ln \underline{X}^{\frac{1+s}{s}}}{2(1+s)^2}. \end{aligned}$$

Using $\underline{X}^{\frac{1+s}{s}} = \frac{\phi}{s^2}$, we have

$$WG(\phi, s) = \frac{1}{4} \left(\frac{s}{1+s} \right)^2 \left(1 - \frac{\phi}{s^2} \right) \left\{ 2 - s + (3s-2) \frac{\phi}{s^2} \right\} + \frac{\frac{\phi^2}{s} \ln \frac{\phi}{s^2}}{2(1+s)^2}. \tag{22}$$

Substituting $s \equiv \sqrt{1-\tau}$ back, we obtain the expression of $WG^{FD}(\phi, \tau)$. ■

Proposition 2(a) allows us to derive $\Pi_T(A)$, $\Pi_B(X)$, and the participation thresholds $X \leq \frac{\Pi_T(A)}{A}$ and $A \leq \frac{\Pi_B(X)}{X}$. It is useful to see the special case without fees.

Corollary 2 (FD equilibrium with $\phi = \tau = 0$).

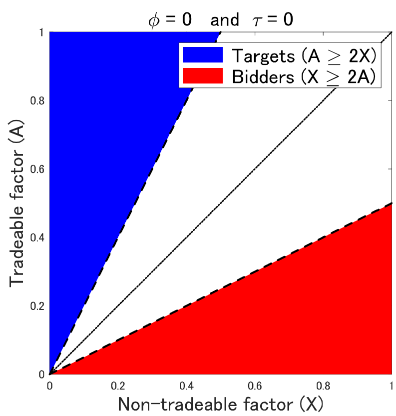
(a) $P(A) = \frac{A^2}{2}$ *and* $m(A) = A$.

(b) $RV(A) = 1$, $FR(A) = 0$, *and* $SG(A) = \frac{3}{4}A$.

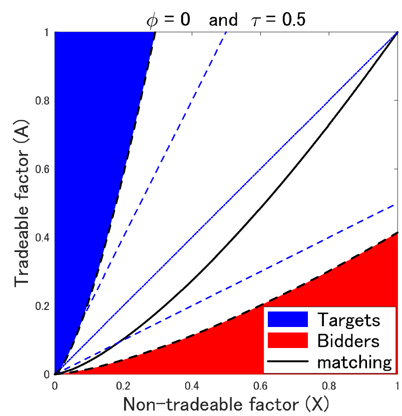
(c) $WG^{FD}(0,0) = \frac{1}{16}$ *and firms' payoff is* $\Pi^{FD}(A,X) = \begin{cases} \frac{1}{2}A^2 & \text{for } \frac{A}{X} \geq 2, & \text{(target)} \\ \frac{1}{2}X^2 & \text{for } \frac{A}{X} \leq \frac{1}{2}, & \text{(bidder)} \\ AX & \text{otherwise.} & \text{(stand-alone)} \end{cases}$

Proof. (a), (b), and $WG^{FD}(0,0) = \frac{1}{16}$ in (c) are obtained by substituting $\phi = \tau = 0$ into the results in **Proposition 2**. To derive the expression of $\Pi^{FD}(A,X)$, note that $\Pi_T(A) = P(A) = \frac{1}{2}A^2$ and $\Pi_B(X) = X^2 - \frac{1}{2}X^2 = \frac{1}{2}X^2$ imply the participation constraints $AX \leq \frac{1}{2}A^2 \Leftrightarrow \frac{A}{X} \geq 2$ and $AX \leq \frac{1}{2}X^2 \Leftrightarrow \frac{A}{X} \leq \frac{1}{2}$. ■

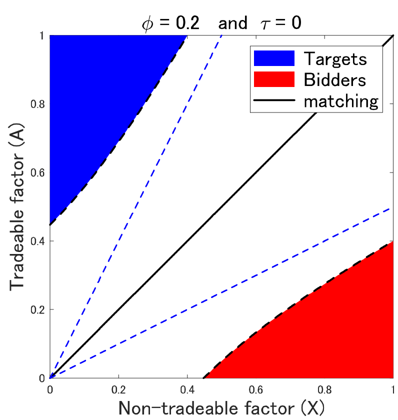
Figure 4 shows matching and sorting patterns for $(\phi, \tau) \in \{(0,0), (0,0.5), (0.2,0), (0.2,0.5)\}$.



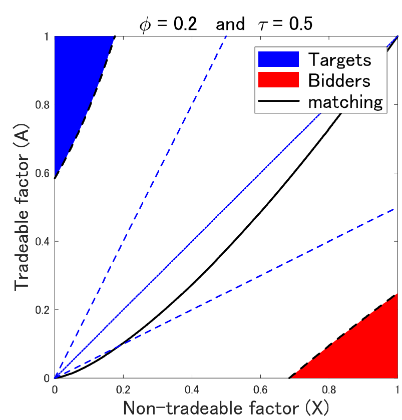
(a) No fees.



(b) Proportional fee (τ) only.



(c) Fixed fee (ϕ) only.



(d) τ and ϕ.

Figure 4. Sorting with fees.

The panel (a) shows the case without fees, where the matching function is $m(A) = A$. In the other panels, a black line is the matching function $m(A) = A^{\sqrt{1-\tau}}$ and dashed blue lines are the case without fees for comparison. Fees reduce a mass of participating firms (the colored area). The proportional fee reduces supply *by more at higher levels of* A (panel (b)), while the opposite holds for the fixed fee (panel (c)). Intuitively, the fixed fee discourages firms with small gains from trade, but it does not distort the matching of participating firms. On the other hand, the proportional fee is more taxing for better projects, and the matching exhibits asymmetry: to clear the market for higher A with a higher price, the demand must come from more skilled bidders. We also note that higher ϕ makes τ more distortionary, and vice versa. To see why this occurs, the target payoff is

$$\Pi_T(A) = \frac{1-\tau}{1+\sqrt{1-\tau}} \left(A^{1+\sqrt{1-\tau}} - \frac{\phi}{1-\tau} \right)$$

so the supply density is positive only for $\Pi_T(A) > 0$, which is equivalent to

$$A > \left(\frac{\phi}{1-\tau} \right)^{\frac{1}{1+\sqrt{1-\tau}}} \equiv \underline{A}(\phi, \tau). \tag{23}$$

This shows that, given higher ϕ, an increase in τ raises the worst quality of projects more.

Proposition 2(b) offers model predictions that can be tested against data. First, it is well documented that targets are smaller than matched bidders (see Eckbo (2014)). While there can be many reasons, our model identifies a new mechanism – a proportional fee. Because this form of fee is common in takeover markets, it could be an important determinant of the relative value of targets. Moreover, with variations in proportional fees across deals, our model predicts that, among deals where a proportional fee is higher, the relative values of targets should be smaller. One suggestive evidence in this context is that *cross-border deals* exhibit a smaller relative value of targets (see Moeller et al. (2005)). Another related evidence is that *privately held targets* have a smaller relative value (see Chang (1998)). Our model indicates the following explanation. Costs of information production for foreign and privately held targets are likely to be higher than average targets. If intermediaries pass on to firms these higher costs by raising proportional fees, then our model predicts that the relative target values in these deals are smaller.[20] It would be interesting to know if, and the extent to which, this explanation is borne out in the data.

Second, Li et al. (2018) directly measure organization capital and study its role in takeovers. They find that high organization capital bidders achieve better post-merger oper-

[20] Importantly, we can show that proportional fees charged for *bidder firms* also make the relative target value smaller, strengthening our argument.

ating performance.[21] They also find that, while target organization capital does not matter for the deal performance, *the gap between bidder and target organization capital* does. This is consistent with the equilibrium skill gap $SG(A)$ increasing in A in our model. We view Li et al. (2018)'s findings as an empirical support for our model. Additionally, our model predicts that (i) the skill premium, $\frac{SG(A)}{m(A)}$, decreases in A, and (ii) the skill gap and skill premium increase in both types of fees. The relationship between fees and the characteristics of matched pairs deserves further empirical investigation.

Proposition 2(c) allows us to compute the welfare gain as a function of fees. **Figure 5** plots $WG^{FD}(\phi, \tau)$, taking the maximum $\frac{1}{16} \approx 0.063$ at $\phi = \tau = 0$ and decreasing in (ϕ, τ).

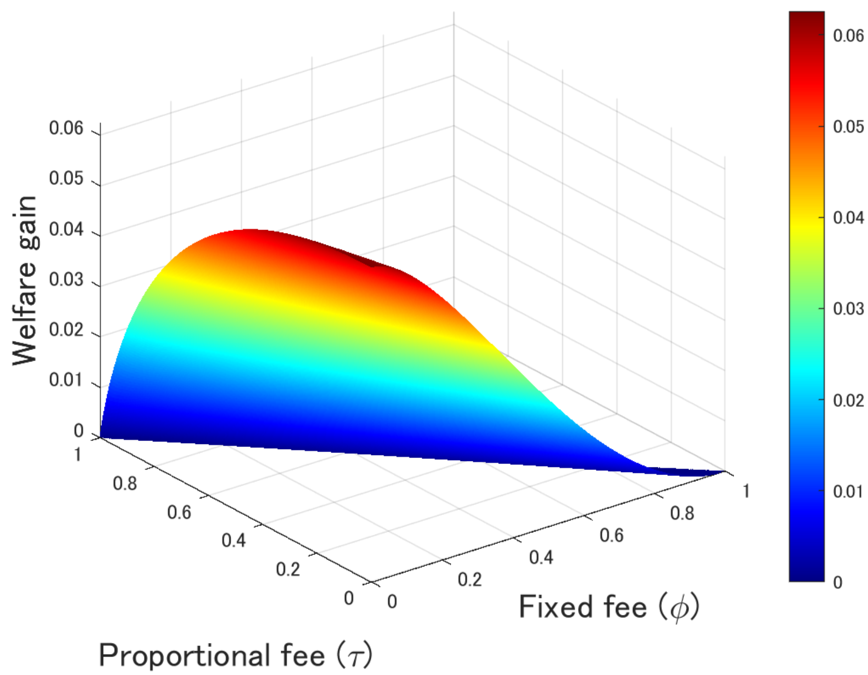


Figure 5. $WG^{FD}(\phi, \tau)$.

Using our welfare benchmark $WG^{ND} \approx 0.0184$, the welfare improvement by the *free* full disclosure technology is $\frac{WG^{FD}(0,0)}{WG^{ND}} \times 100 = 340\%$. In the next subsection, we endogenize fees and study associated welfare gains.

[21] Their measures of operating performance are (i) the decrease in the cost of goods sold, (ii) expensed related to IT and human capital, (iii) asset turnover, and (iv) innovative efficiency. See Section VI and VII in their paper for more details.

4.3 Profit-maximizing fees

We endogenize (ϕ, τ) by letting the monopoly intermediary solve

$$\max_{(\phi,\tau)\in\mathbb{R}^2} \pi(\phi,\tau), \quad \text{where } \pi(\phi,\tau) \equiv \int_{\underline{A}(\phi,\tau)}^{1} R(\phi,\tau;a)\,da, \tag{24}$$

$$R(\phi,\tau;a) \equiv \frac{(1-\tau)P(a;\phi,\tau)-\phi}{a}\left(\tau P(a;\phi,\tau)+\phi\right). \tag{25}$$

In (24), $\underline{A}(\phi,\tau)$ is the worst project quality defined in (23) and $R(\phi,\tau;a)$ is the revenue from target firms with $A=a$. Note that $\frac{(1-\tau)P(a;\phi,\tau)-\phi}{a} = \frac{\Pi_T(a)}{a}$ is the supply density of projects, which is *the demand density for the disclosure service.*[22] Integrating $R(\phi,\tau;a)$ over $a \in [\underline{A}(\phi,\tau), 1]$ yields the following result.

Lemma 2. *The intermediary's profit is*

$$\begin{aligned}\pi(\phi,\tau) &= \frac{1}{2}\left(\frac{\sqrt{1-\tau}}{1+\sqrt{1-\tau}}\right)^2\left(1-\frac{\phi}{1-\tau}\right)\left\{\begin{array}{l}1-\sqrt{1-\tau}\\ +\left(3\sqrt{1-\tau}-1\right)\frac{\phi}{1-\tau}\end{array}\right\}\\ &\quad + \frac{\frac{\phi^2}{\sqrt{1-\tau}}\ln\left(\frac{\phi}{1-\tau}\right)}{\left(1+\sqrt{1-\tau}\right)^2}\end{aligned} \tag{26}$$

for any $\phi+\tau<1$ and zero otherwise.

Proof. We work with $s \equiv \sqrt{1-\tau}$. The intermediary's profit is

$$\pi(\phi,s) = \int_{\underline{A}}^{1} \frac{s^2P(a;\phi,s)-\phi}{a}\left(\left(1-s^2\right)P(a;\phi,s)+\phi\right)da, \quad \text{where } \underline{A} \equiv \left(\frac{\phi}{s^2}\right)^{\frac{1}{1+s}}.$$

$\pi(\phi,s)>0$ for any $\phi<s^2$ and zero otherwise. Expanding the expression inside the integral,

$$s^2\left(1-s^2\right)\int_{\underline{A}}^{1}\frac{\left(P(a;\phi,s)\right)^2}{a}da + \phi\left(2s^2-1\right)\int_{\underline{A}}^{1}\frac{P(a;\phi,s)}{a}da - \phi^2\int_{\underline{A}}^{1}\frac{da}{a}. \tag{27}$$

Using the price function (21),

$$\int_{\underline{A}}^{1}\frac{\left(P(a;\phi,s)\right)^2}{a}da = \frac{1}{(1+s)^3}\left[\left(1-\frac{\phi}{s^2}\right)\left\{\frac{1}{2}\left(1+\frac{\phi}{s^2}\right)+\frac{2\phi}{s}\right\}-\left(\frac{\phi}{s}\right)^2\ln\frac{\phi}{s^2}\right],$$

$$\int_{\underline{A}}^{1}\frac{P(a;\phi,s)}{a}da = \frac{1}{(1+s)^2}\left[1-\frac{\phi}{s^2}-\frac{\phi}{s}\ln\frac{\phi}{s^2}\right].$$

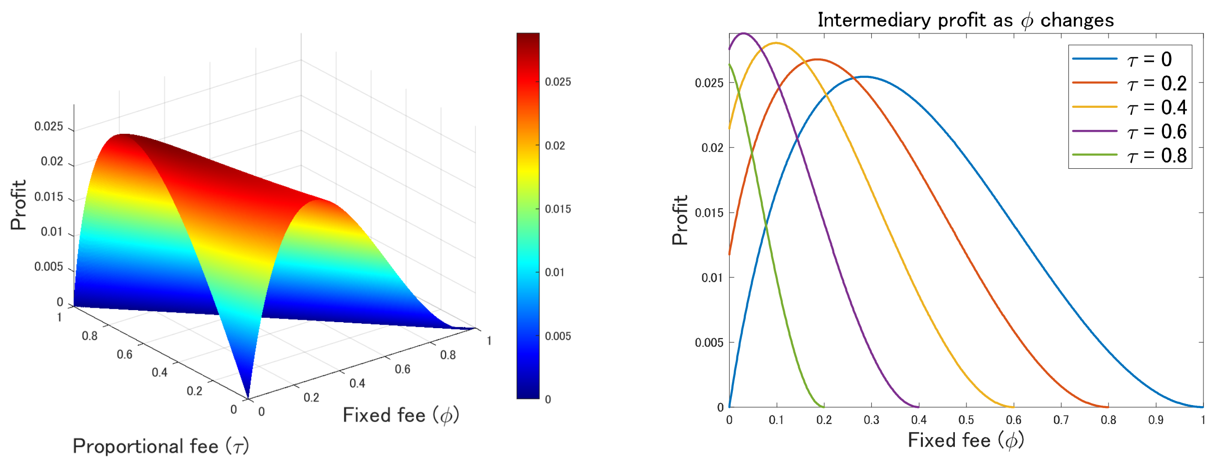
[22] The term $\tau P(a;\phi,\tau)+\phi$ in (25) is a revenue per target firm with $A=a$.

Substituting these expressions into (27), and simplifying,

$$\pi(\phi, s) = \frac{1}{2(1+s)^2}\left[\left(1-\frac{\phi}{s^2}\right)\left\{(1-s)s^2 + (3s-1)\phi\right\} + 2\frac{\phi^2}{s}\ln\frac{\phi}{s^2}\right]. \tag{28}$$

Substituting $s \equiv \sqrt{1-\tau}$ back, we obtain (26). ■

We characterize fees that maximize the intermediary's profit (26). To gain some insight, set $\tau = 0$ to obtain $\pi(\phi, 0) = \frac{\phi}{4}(1 - \phi + \phi\ln\phi)$, which is maximized at $\phi = \overline{\phi} \approx 0.285$ and the associated profit is $\frac{\overline{\phi}(1-\overline{\phi})}{8}$.[23] Similarly, setting $\phi = 0$ in (26) yields $\pi(0, \tau) = \frac{1}{2}\left(\frac{\sqrt{1-\tau}}{1+\sqrt{1-\tau}}\right)^2\left(1-\sqrt{1-\tau}\right)$, which is maximized at $\tau = \overline{\tau} \approx 0.685$. **Figure 6** plots $\pi(\phi, \tau)$.



(a) $\pi(\phi, \tau)$. (b) Profit as ϕ changes.

Figure 6. Intermediary's profit.

To characterize the profit-maximizing fees, we derive the following two functions:

$$\phi(\tau) \equiv \arg\max_{\phi}\{\pi(\phi, \tau)\} \qquad \text{and} \qquad \tau(\phi) \equiv \arg\max_{\tau}\{\pi(\phi, \tau)\},$$

and find the intersection of these two functions. From the intermediary's problem (24), the first order condition with respect to ϕ is

$$\int_{\underline{A}(\phi,\tau)}^{1} \frac{\partial R(\phi, \tau; a)}{\partial \phi} da = R(\phi, \tau; \underline{A}(\phi, \tau)) \frac{\partial \underline{A}(\phi, \tau)}{\partial \phi}. \tag{29}$$

[23] Substituting $\frac{\partial}{\partial\phi}\pi(\phi, 0) = 0 \Leftrightarrow \phi\ln\phi = -\frac{1-\phi}{2}$ into $\pi(\phi, 0)$ yields $\pi\left(\overline{\phi}, 0\right) = \frac{\overline{\phi}(1-\overline{\phi})}{8}$.

The condition (29) and $R(\phi,\tau;a)$ in (25) show how ϕ affects the intermediary's profit. First, ϕ directly affects $R(\phi,\tau;a)$ through a marginal fee revenue $\tau P+\phi$ and a mass of target firms $\frac{(1-\tau)P-\phi}{a}$. Second, ϕ indirectly affects $R(\phi,\tau;a)$ through $P(a;\phi,\tau)$. These two effects are subsumed in $\frac{\partial R(\phi,\tau;a)}{\partial \phi}$ in (29). Third, ϕ affects the participation threshold $\underline{A}(\phi,\tau)$. However, at the threshold $\underline{A}(\phi,\tau)$, $R(\phi,\tau;\underline{A}(\phi,\tau))=0$ holds. Therefore, the extent to which ϕ affects $\underline{A}(\phi,\tau)$, i.e., $\frac{\partial \underline{A}(\phi,\tau)}{\partial \phi}>0$, drops from (29), and $\phi(\tau)$ is defined by $\int_{\underline{A}(\phi,\tau)}^{1}\frac{\partial R(\phi,\tau;a)}{\partial \phi}da=0$. With this $\phi(\tau)$, we obtain **Lemma 3**.

Lemma 3 (profit-maximizing fees). *For a given proportional fee $\tau\in[0,1)$, the intermediary chooses $(\phi,\tau)=(\phi(\tau),\tau)$, where $\phi(\tau)\in(0,1-\tau)$ is a unique solution to $\frac{\frac{1-\tau}{\phi}-1}{\ln\frac{1-\tau}{\phi}}=\frac{2\sqrt{1-\tau}}{2\sqrt{1-\tau}-1}$. The intermediary's profit is*

$$\pi(\phi(\tau),\tau)=\frac{1}{2}\left(\frac{\sqrt{1-\tau}}{1+\sqrt{1-\tau}}\right)^2\left(1-\frac{\phi(\tau)}{1-\tau}\right)\left(1-\sqrt{1-\tau}+\frac{\phi(\tau)}{\sqrt{1-\tau}}\right), \tag{30}$$

and the associated welfare gain is

$$WG^{FD}(\phi(\tau),\tau)=\frac{1}{4}\left(\frac{\sqrt{1-\tau}}{1+\sqrt{1-\tau}}\right)^2\left(1-\frac{\phi(\tau)}{1-\tau}\right)\left\{1+\left(1-\sqrt{1-\tau}\right)\left(1-\frac{\phi(\tau)}{1-\tau}\right)\right\}. \tag{31}$$

Proof. We work with $s\equiv\sqrt{1-\tau}$. Taking a partial derivative $\frac{\partial \pi(\phi,s)}{\partial \phi}$,

$$\frac{\partial \pi(\phi,s)}{\partial \phi}\geq 0 \qquad \Leftrightarrow \qquad (2s-1)\left(s^2-\phi\right)\geq 2\phi s\ln\frac{s^2}{\phi}$$

Note that $\phi=s^2$ does not satisfy the second order condition. Hence, we have

$$\frac{\partial \pi(\phi,s)}{\partial \phi}\geq 0 \qquad \Leftrightarrow \qquad \frac{\frac{s^2}{\phi}-1}{\ln\frac{s^2}{\phi}}(2s-1)\geq 2s.$$

Therefore, if $s\leq\frac{1}{2}$ ($\Leftrightarrow \tau\geq\frac{3}{4}\equiv\widehat{\tau}$), it is optimal to set $\phi=0$, i.e., $\phi(\tau)=0$ for $\tau\geq\widehat{\tau}$.

For a given $s>\frac{1}{2}$ ($\Leftrightarrow \tau<\widehat{\tau}$), the profit-maximizing ϕ must satisfy

$$\frac{\frac{s^2}{\phi}-1}{\ln\frac{s^2}{\phi}}=\frac{2s}{2s-1} \quad \Leftrightarrow \quad \frac{\frac{1-\tau}{\phi}-1}{\ln\frac{1-\tau}{\phi}}=\frac{2\sqrt{1-\tau}}{2\sqrt{1-\tau}-1}. \tag{32}$$

This has a unique solution $\phi(\tau)<1-\tau$ decreasing in τ, that satisfies $\lim_{\tau\uparrow\widehat{\tau}}\phi(\tau)=0$. Also, $\phi(0)=\overline{\phi}\approx 0.285$ is defined as a smaller solution to $\phi(1-2\ln\phi)=1$.

Rearrange (32) as $\frac{\phi^2}{s}\ln\frac{\phi}{s^2}=\frac{1-2s}{2}\phi\left(1-\frac{\phi}{s^2}\right)$ and substitute it into (28) to get

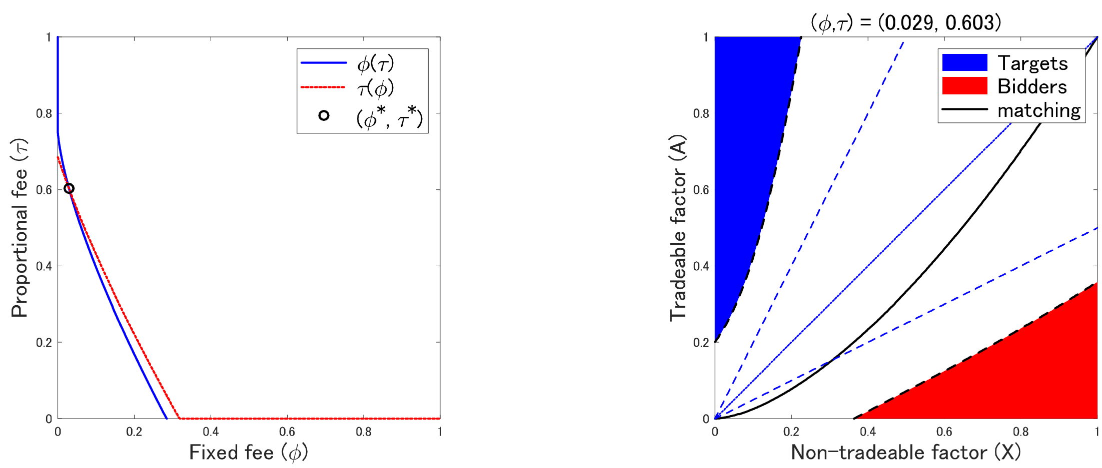
$$\pi\left(\phi\left(s\right),s\right)=\frac{1}{2}\left(\frac{s}{1+s}\right)^2\left(1-\frac{\phi\left(s\right)}{s^2}\right)\left(1-s+\frac{\phi\left(s\right)}{s}\right).$$

Similarly, by substituting $\frac{\phi^2}{s}\ln\frac{\phi}{s^2}=\frac{1-2s}{2}\phi\left(1-\frac{\phi}{s^2}\right)$ into (22),

$$WG\left(\phi\left(s\right),s\right)=\frac{1}{4}\left(\frac{s}{1+s}\right)^2\left(1-\frac{\phi}{s^2}\right)\left\{1+\left(1-s\right)\left(1-\frac{\phi}{s^2}\right)\right\}.$$

With $s\equiv\sqrt{1-\tau}$, we have $\pi\left(\phi\left(\tau\right),\tau\right)$ and $WG^{FD}\left(\phi\left(\tau\right),\tau\right)$. ■

Figure 7 plots $\{\phi\left(\tau\right),\tau\left(\phi\right)\}$ and the sorting pattern with profit-maximizing fees.



(a) Profit-maximizing fees. (b) Sorting with profit-maximizing fees.

Figure 7. Profit-maximizing fees and sorting with (ϕ^*,τ^*).

In the panel (a), a marker "∘" indicates $(\phi^*,\tau^*)=(0.029,0.603)$. The panel (b) shows the sorting pattern. By substituting (ϕ^*,τ^*) into (30) and (31), we obtain $\pi\left(\phi^*,\tau^*\right)$ and $WG^{FG}\left(\phi^*,\tau^*\right)$. We define *firms' gain* by $FG^{FG}\left(\phi^*,\tau^*\right)\equiv WG^{FG}\left(\phi^*,\tau^*\right)-\pi\left(\phi^*,\tau^*\right)$.

Claim 2 *When the intermediary chooses* $(\phi^*,\tau^*)=(0.029,0.603)$, *the welfare gain is 253% and firms' gain is 96% of the benchmark welfare gain.*

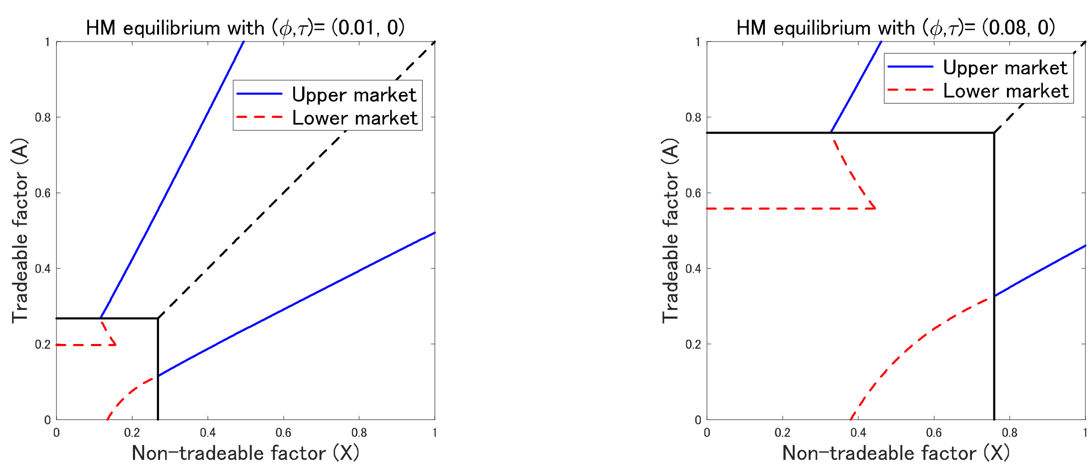
With profit-maximizing fees, despite using the full disclosure technology, firms' gain become *smaller* than the benchmark welfare gain in the ND equilibrium. Thus, a combination of fixed and proportional fees impose a heavy burden on firms. In the next section, we make a policy proposal that alleviates this problem.

5 Policy proposal : hybrid market structure

So far, we investigated the FD technology and the MD technology in isolation. Two natural questions are whether they can coexist and whether they should. A short answer is "yes and yes". In Section 5.1, we construct *a hybrid market equilibrium* (henceforth HM equilibrium) and identify conditions under which both disclosure technologies are used by firms. In Section 5.2, we study the incentive of the intermediary in this market structure. In Section 5.3, we propose a regulation to support the HM equilibrium.

5.1 Hybrid market equilibrium

Suppose that projects of quality $A \geq \overline{A} \in (0,1)$ are fully disclosed in one market (*the upper market*), while projects of quality $A \in \left[A_{\min}, \overline{A}\right)$ are pooled in the other market (*the lower market*). The marginal project quality $\overline{A}$ as well as the minimum standard $A_{\min}$ will be endogenously determined. Prices in the upper market are given by a price function $\{P(A)\}_{A \geq \overline{A}}$. We denote the price and the expected project quality in the lower market by (P_0, a_0). **Figure 8** shows two examples with $(\phi, \tau) = (0.01, 0)$ and $(\phi, \tau) = (0.08, 0)$.



(a) $(\phi, \tau) = (0.01, 0)$. (b) $(\phi, \tau) = (0.08, 0)$.

Figure 8. Hybrid market equilibrium for a given (ϕ, τ).

Lower market. The analysis of the lower market follows that of the MD equilibrium.

Given that bidders and targets are not connected (which we verify is true as shown by red dashed lines in **Figure 8**), for a given price P_0 a supply is $S(P_0) = \int_{A_{\min}}^{\overline{A}} \frac{P_0}{A} dA$ and a demand is $B(P_0) = \int_{\frac{P_0}{a_0}}^{\overline{X}} \left(a_0 - \frac{P_0}{X}\right) dX$. A market-clearing condition $S(P_0) = B(P_0)$ yields

$$\frac{a_0 \overline{X}}{P_0} - 1 = \ln \frac{a_0 \overline{X}}{P_0} + \ln \frac{\overline{A}}{A_{\min}}, \tag{33}$$

with a unique solution $P_0 \in \left(0, a_0 \overline{X}\right)$ for any $\frac{\overline{A}}{A_{\min}} > 1$. The expected quality of projects is

$$a_0 = \frac{\overline{A} - A_{\min}}{\ln \overline{A} - \ln A_{\min}}. \tag{34}$$

Upper market. In the upper market, the analysis of the FD equilibrium applies. Importantly, however, targets with $A = \overline{A}$ and bidders with $X = \overline{X} = \overline{A}^{\sqrt{1-\tau}}$ must be indifferent between the two markets. This implies the following two indifference conditions:

$$\Pi_T\left(\overline{A}\right) = P_0 \qquad \text{and} \qquad \Pi_B\left(\overline{A}^{\sqrt{1-\tau}}\right) = a_0 \overline{A}^{\sqrt{1-\tau}} - P_0. \tag{35}$$

From (33), (34), (35), we derive $\left\{P_0, a_0, A_{\min}, \overline{A}\right\}$. The marginal project quality is

$$\overline{A} = \left(\frac{\phi}{1 - \tau - \kappa(\tau)\sqrt{1-\tau}}\right)^{\frac{1}{1+\sqrt{1-\tau}}} \equiv \overline{A}(\phi, \tau), \tag{36}$$

where $\kappa(\tau) > 0$ is a function derived in the proof of **Lemma 4** below. Because $\overline{A}(\phi, \tau)$ given in (36) affects the size of the upper market, it affects the intermediary's choice of fees.

Coexistence of the two markets. For the ease of notations, we define

$$\phi_{\max}(\tau) \equiv 1 - \tau - \kappa(\tau)\sqrt{1-\tau}, \qquad \Phi(\phi, \tau) \equiv \frac{\phi}{\phi_{\max}(\tau)},$$

so that $\overline{A}(\phi, \tau) = (\Phi(\phi, \tau))^{\frac{1}{1+\sqrt{1-\tau}}}$ and $\Phi(\phi, \tau) > \frac{\phi}{1-\tau}$ whenever $\phi_{\max}(\tau)\phi > 0$. For the two markets to attract some firms, we must have $\overline{A}(\phi, \tau) \in (0, 1)$. The next result provides a condition on (ϕ, τ) to achieve this.

Lemma 4. *There is $\tau_{\max} \in (0, 1)$ such that the two markets coexist if and only if*

$$\tau < \tau_{\max} \text{ and } 0 < \phi < \phi_{\max}(\tau). \tag{37}$$

Proof. From the two indifference conditions (35) with $\Pi_T\left(\overline{A}\right) = \frac{1-\tau}{1+\sqrt{1-\tau}}\left(\overline{A}^{1+\sqrt{1-\tau}} - \frac{\phi}{1-\tau}\right)$

and $\Pi_B\left(\overline{A}^{\sqrt{1-\tau}}\right)=\frac{\sqrt{1-\tau}}{1+\sqrt{1-\tau}}\left(\overline{A}^{1+\sqrt{1-\tau}}-\frac{\phi}{1-\tau}\right)$, we obtain

$$P_0=\frac{\sqrt{1-\tau}}{1+\sqrt{1-\tau}}a_0\overline{A}^{\sqrt{1-\tau}}, \tag{38}$$

$$\overline{A}^{1+\sqrt{1-\tau}}-\frac{a_0}{\sqrt{1-\tau}}\overline{A}^{\sqrt{1-\tau}}-\frac{\phi}{1-\tau}=0. \tag{39}$$

Substituting (38) into the market-clearing condition in the lower market (33) yields

$$A_{\min}=\kappa_{\min}(\tau)\,\overline{A},\qquad \text{where } \kappa_{\min}(\tau)\equiv\frac{1+\sqrt{1-\tau}}{\sqrt{1-\tau}}\exp\left(-\frac{1}{\sqrt{1-\tau}}\right). \tag{40}$$

By substituting (40) into (34) to eliminate $A_{\min}$,

$$a_0=\kappa(\tau)\,\overline{A},\qquad \text{where } \kappa(\tau)\equiv\frac{1-\kappa_{\min}(\tau)}{-\ln\kappa_{\min}(\tau)}\qquad\in(\kappa_{\min}(\tau),1). \tag{41}$$

That $\kappa_{\min}<\kappa(\tau)<1$ implies $A_{\min}<a_0<\overline{A}$. Finally, combining (39) and (41) yields $\overline{A}=\left(\frac{\phi}{1-\tau-\kappa(\tau)\sqrt{1-\tau}}\right)^{\frac{1}{1+\sqrt{1-\tau}}}$, which is (36). We derived $(A_{\min},a_0,P_0)$ as functions of $\overline{A}(\phi,\tau)$:

$$\begin{aligned}A_{\min}&=\kappa_{\min}(\tau)\,\overline{A}(\phi,\tau)\equiv A_{\min}(\phi,\tau),\\ a_0&=\kappa(\tau)\,\overline{A}(\phi,\tau)\equiv a_0(\phi,\tau),\\ P_0&=\frac{\sqrt{1-\tau}}{1+\sqrt{1-\tau}}\kappa(\tau)\left\{\overline{A}(\phi,\tau)\right\}^{1+\sqrt{1-\tau}}\equiv P_0(\phi,\tau).\end{aligned} \tag{42}$$

From (36) and (42), both markets attract some firms if and only if $0<\overline{A}(\phi,\tau)<1\Leftrightarrow 0<\phi<\phi_{\max}(\tau)$. Because $\phi_{\max}(\tau)\equiv 1-\tau-\kappa(\tau)\sqrt{1-\tau}$, $0<\phi_{\max}(\tau)$ requires $\kappa(\tau)<\sqrt{1-\tau}$. To show that there is $\tau_{\max}\in(0,1)$ such that $\kappa(\tau)<\sqrt{1-\tau}\Leftrightarrow\tau<\tau_{\max}$, first note that $\kappa(\tau)<\sqrt{1-\tau}$ is equivalent to $1-\kappa_{\min}(\tau)<-\ln\kappa_{\min}(\tau)\sqrt{1-\tau}$, where $\kappa_{\min}(\tau)$ is defined in (41). Using $s\equiv\sqrt{1-\tau}$ and further evaluating this inequality,

$$\kappa(\tau)<\sqrt{1-\tau}\qquad\Leftrightarrow\qquad\frac{1}{s}\left(1+\frac{1}{s}\right)>\exp\left(\frac{1}{s}\right)\ln\left(1+\frac{1}{s}\right).$$

At $s=1$ ($\Leftrightarrow\tau=0$), $2>\exp(1)\ln 2\approx 1.8842$ holds, while for sufficiently small s, $\frac{1}{s}\left(1+\frac{1}{s}\right)<\exp\left(\frac{1}{s}\right)\ln\left(1+\frac{1}{s}\right)$ holds. Therefore, there is $s>1$ that solves $\frac{1}{s}\left(1+\frac{1}{s}\right)=\exp\left(\frac{1}{s}\right)\ln\left(1+\frac{1}{s}\right)$. To show uniqueness, we let $\widehat{s}\equiv\frac{1}{s}$ and show that the derivative of $\widehat{s}(1+\widehat{s})$ is smaller than that of $\exp(\widehat{s})\ln(1+\widehat{s})$, whenever $\widehat{s}(1+\widehat{s})=\exp(\widehat{s})\ln(1+\widehat{s})$ holds. This is equivalent to

$$1+2\widehat{s}<\widehat{s}(1+\widehat{s})\frac{\exp(\widehat{s})}{1+\widehat{s}}\qquad\Leftrightarrow\qquad\left(1+\widehat{s}-\widehat{s}^2\right)(1+\widehat{s})<\exp(\widehat{s}),$$

which holds for any $\widehat{s} \geq 1$. Below, we plot $\kappa(\tau)$ and $\sqrt{1-\tau}$ as functions of τ.

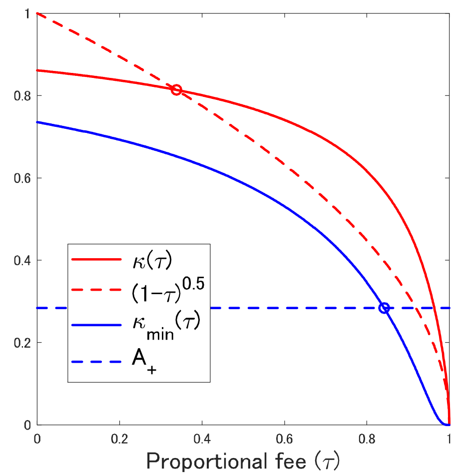


Figure L4. $\kappa(\tau) < \sqrt{1-\tau}$ implies $\kappa_{\min}(\tau) > A_{+}$.
Note. $\tau_{\max} \approx 0.338$ (a red marker $\circ$).

Finally, we show $A_{\min} > \frac{a_0}{2}$ to verify that bidders and targets are not connected in the lower market. First, $A_{\min} > \frac{a_0}{2} \Leftrightarrow 2\kappa_{\min}(\tau) > \kappa(\tau) = \frac{1-\kappa_{\min}(\tau)}{-\ln \kappa_{\min}(\tau)} \Leftrightarrow \kappa_{\min}(\tau)(1 - 2\ln \kappa_{\min}(\tau)) > 1 \Leftrightarrow \kappa_{\min}(\tau) > A_{+}$, where A_{+} was defined in **Proposition 1**. The last inequality holds given $\tau < \tau_{\max}$, as shown in **Figure L4**. $\blacksquare$

The two upper bounds on fees $(\tau_{\max}, \phi_{\max}(\tau))$ naturally arise from $\overline{A}(\phi, \tau) < 1$, i.e., the intermediary cannot set too high fees no firm is willing to pay. We obtain $\phi_{\max}(\tau) > 0 \Leftrightarrow \tau < \tau_{\max} \approx 0.338$. A more subtle condition, $0 < \phi$, follows from $\overline{A}(\phi, \tau) > 0$. Intuitively, to make targets with the marginal project quality $\overline{A}(\phi, \tau)$ indifferent between fully disclosing $\overline{A}(\phi, \tau)$ and pooling with lower quality $A \in [A_{\min}, \overline{A})$, ϕ must be positive. Otherwise, unravelling (i.e., full disclosure by paying $\tau P(A)$) occurs and pooling in the lower market cannot be sustained. This implies that, *conditional on ϕ being positive*, the intermediary would satisfy $\tau < \tau_{\max}$ and $\phi < \phi_{\max}(\tau)$. However, we show in the next subsection that the intermediary would choose $\phi = 0$. Then, $\overline{A}(0, \tau) = 0$ implies that the lower market attracts no firm.

Welfare gain. Assuming that (37) holds, we compute welfare gains. We suppress the dependence of $\kappa(\tau)$ and $\Phi(\phi, \tau)$ on ϕ and τ in the next result.

Lemma 5 *Given (37), a welfare gain in the upper market is*

$$\begin{aligned} WG^U(\phi,\tau) &= \frac{1}{4}\left(\frac{\sqrt{1-\tau}}{1+\sqrt{1-\tau}}\right)^2 (1-\Phi)\left\{2-\sqrt{1-\tau}+\left\{\begin{array}{c}\left(4-\frac{\sqrt{1-\tau}}{\sqrt{1-\tau}-\kappa}\right)\sqrt{1-\tau} \\ -2\left(2-\frac{\sqrt{1-\tau}}{\sqrt{1-\tau}-\kappa}\right)\end{array}\right\}\frac{\phi}{1-\tau}\right\} \\ &\quad + \frac{\frac{\phi^2}{\sqrt{1-\tau}}\ln\Phi}{2\left(1+\sqrt{1-\tau}\right)^2}, \end{aligned}$$

and a welfare gain in the lower market is $WG^L(\phi,\tau) = \frac{1}{4}\left(\frac{\kappa\Phi}{1+\sqrt{1-\tau}}\right)^2$.

Proof. The derivation process follows closely that used for the FD equilibrium (for the upper market) and for the MD equilibrium (for the lower market). We work with $s=\sqrt{1-\tau}$. In the upper market, the matching function is $A^s = X$ and the only difference from the FD equilibrium is that the worst pair $(\underline{X},\underline{A})$ is replaced with $\left(\overline{X},\overline{A}\right)$. Hence the new production is

$$NP^U(\phi,s) = \int_{\overline{X}}^1 X^{\frac{1+s}{s}} D(X;\phi,s)\,dX, \text{ where } \overline{X} \equiv \left(\frac{\phi}{s(s-\kappa)}\right)^{\frac{s}{1+s}} = \overline{A}^s.$$

The lost production by bidders and targets are

$$BL^U(\phi,s) = \int_{\overline{X}}^1 \left\{X\int_0^{D(X;\phi,s)} A dA\right\} dX \quad \text{and} \quad TL^U(\phi,s) = \int_{\overline{A}}^1 \left\{A\int_0^{S(A;\phi,s)} X dX\right\} dA.$$

Therefore,

$$\begin{aligned} NP^U(\phi,s) &= \frac{1}{2}\left(\frac{s}{1+s}\right)^2\left(1-\overline{X}^{\frac{1+s}{s}}\right)\left(1+\overline{X}^{\frac{1+s}{s}}-2\frac{\phi}{s^2}\right), \\ BL^U(\phi,s) &= \frac{1}{4}\left(\frac{s}{1+s}\right)^3\left(1-\overline{X}^{\frac{1+s}{s}}\right)\left(1+\overline{X}^{\frac{1+s}{s}}-\frac{4\phi}{s^2}\right)-\frac{\phi^2\ln\overline{X}}{2(1+s)^2 s^2}, \\ TL^U(\phi,s) &= sBL^U(\phi,s). \end{aligned}$$

The welfare gain in the upper market is

$$\begin{aligned} WG^U(\phi,s) &= NP^U(\phi,s) - \left(BL^U(\phi,s)+TL^U(\phi,s)\right) \\ &= \frac{1}{4}\left(\frac{s}{1+s}\right)^2\left(1-\overline{X}^{\frac{1+s}{s}}\right)\left\{2-s+\left\{\begin{array}{c}\left(4-\frac{s}{s-\kappa}\right)s \\ -2\left(2-\frac{s}{s-\kappa}\right)\end{array}\right\}\frac{\phi}{s^2}\right\}+\frac{\phi^2\ln\overline{X}^{\frac{1+s}{s}}}{2(1+s)^2 s}. \end{aligned}$$

Substituting $\overline{X}^{\frac{1+s}{s}} = \frac{\phi}{s(s-\kappa)} = \Phi$ and $s\equiv\sqrt{1-\tau}$ back, we obtain $WG^U(\phi,\tau)$ in **Lemma 5**.

Next, we compute $WG^L(\phi,\tau) = NP^L(\phi,s) - \left(BL^L(\phi,s) + TL^L(\phi,s)\right)$. These are obtained by replacing the upper bound of 1 with $\overline{X}$ and $\overline{A}$ in the corresponding expressions in the MD equilibrium. The new production is

$$NP^L(\phi,s) = a_0 \int_{\frac{P_0}{a_0}}^{\overline{X}} X\left(a_0 - \frac{P_0}{X}\right) dX = \frac{\left(a_0\overline{X} - P_0\right)^2}{2}.$$

Targets' lost production is

$$TL^L(\phi,s) = \int_{A_{\min}}^{\overline{A}} \left(A\int_0^{\frac{P_0}{A}} X dX\right) dA = \frac{P_0^2}{2}\ln\frac{\overline{A}}{A_{\min}}.$$

Bidders' lost production is

$$BL^L(\phi,s) = \int_{\frac{P_0}{a_0}}^{\overline{X}} \left(X\int_0^{a_0 - \frac{P_0}{X}} A dA\right) dX = \frac{1}{4}\left(a_0\overline{X} - P_0\right)\left(a_0\overline{X} - 3P_0\right) + \frac{P_0^2}{2}\ln\frac{a_0\overline{X}}{P_0}.$$

Therefore,

$$TL^L(\phi,s) + BL^L(\phi,s) = \frac{1}{4}\left(a_0\overline{X} - P_0\right)\left(a_0\overline{X} - 3P_0\right) + \frac{P_0^2}{2}\ln\left(\frac{\overline{A}}{A_{\min}}\frac{a_0\overline{X}}{P_0}\right).$$

Because $\overline{X} = \overline{A}^s$, from the market-clearing condition of the lower market (33),

$$\frac{a_0\overline{X}}{P_0} - 1 = \ln\frac{a_0\overline{X}}{P_0} + \ln\frac{\overline{A}}{A_{\min}}.$$

Using this, $TL^L(\phi,s) + BL^L(\phi,s) = \frac{1}{4}\left(a_0\overline{X} - P_0\right)^2$. Therefore,

$$WG^L(\phi,\tau) = \frac{\left(a_0\overline{X} - P_0\right)^2}{2} - \frac{1}{4}\left(a_0\overline{X} - P_0\right)^2 = \left(\frac{a_0\overline{X} - P_0}{2}\right)^2.$$

From (38) and (41),

$$a_0\overline{X} - P_0 = \left(1 - \frac{s}{1+s}\right)a_0\overline{X} = \frac{1}{1+s}\left(\kappa\overline{X}^{\frac{1}{s}}\right)\overline{X} = \frac{\kappa}{1+s}\overline{X}^{\frac{1+s}{s}}.$$

With $\overline{X}^{\frac{1+s}{s}} = \frac{\phi}{s(s-\kappa)} = \Phi$ and $s \equiv \sqrt{1-\tau}$, we obtain $WG^L(\phi,\tau)$ in **Lemma 5**. ■

By **Lemma 5**, we can compute $WG^{HM}(\phi,\tau) \equiv WG^U(\phi,\tau) + WG^L(\phi,\tau)$.

5.2 Profit-maximizing fees in the hybrid market structure

The intermediary solves

$$\max_{(\phi,\tau)\in\mathbb{R}^2} \pi^{HM}(\phi,\tau), \quad \text{where } \pi^{HM}(\phi,\tau) \equiv \int_{\overline{A}(\phi,\tau)}^{1} R(\phi,\tau;a)\,da, \tag{43}$$

$$R(\phi,\tau;a) \equiv \frac{(1-\tau)\,P(a;\phi,\tau)-\phi}{a}\left(\tau P(a;\phi,\tau)+\phi\right),$$

and $\overline{A}(\phi,\tau)$ is a marginal project quality defined in (36). Compared with the problem (24) without the lower market, the only difference is the higher quality threshold $\overline{A}(\phi,\tau) > \underline{A}(\phi,\tau)$.[24] The first order condition with respect to ϕ is

$$\int_{\overline{A}(\phi,\tau)}^{1} \frac{\partial R(\phi,\tau;a)}{\partial\phi}da = R\left(\phi,\tau;\overline{A}(\phi,\tau)\right)\frac{\partial\overline{A}(\phi,\tau)}{\partial\phi}.$$

Importantly, $R\left(\phi,\tau;\overline{A}(\phi,\tau)\right) > 0$ if $\overline{A}(\phi,\tau) > 0$. Therefore, in the presence of an *active* lower market, ϕ affects the intermediary's profit not only through $R(\phi,\tau;a)$ but also through $\overline{A}(\phi,\tau)$. This is the key feature of the HM equilibrium with an active lower market.

However, the problem (43) exhibits a discontinuity at $\phi = 0$: setting $\phi = 0$ makes $\overline{A}(\phi,\tau) = 0$ for any $\tau \in [0,1]$, while for a given $\phi \in (0, \phi_{\max}(0))$, $\overline{A}(\phi,\tau)$ is quickly increasing in τ. We compute the intermediary's profit as before.

Lemma 6 *The intermediary's profit in the upper market is*

$$\begin{aligned}\pi^{HM}(\phi,\tau) &= \frac{1}{2}\left(\frac{\sqrt{1-\tau}}{1+\sqrt{1-\tau}}\right)^2(1-\Phi)\left\{\begin{array}{l}1-\sqrt{1-\tau}\\ +\left(3\sqrt{1-\tau}-1+\frac{1-\sqrt{1-\tau}}{\sqrt{1-\tau}-\kappa}\kappa\right)\frac{\phi}{1-\tau}\end{array}\right\} \\ &\quad +\frac{\frac{\phi^2}{\sqrt{1-\tau}}\ln\Phi}{\left(1+\sqrt{1-\tau}\right)^2}\end{aligned} \tag{44}$$

for any (ϕ,τ) such that $\Phi \equiv \frac{\phi}{\phi_{\max}(\tau)} < 1$ and zero otherwise.

Proof. We work with $s = \sqrt{1-\tau}$. Again, we start by replacing $\underline{A}$ with $\overline{A}$ in the expression (27) derived for the FD equilibrium.

$$\int_{\overline{A}}^{1}\frac{\left(P(a;\phi,s)\right)^2}{a}da = \frac{1}{(1+s)^3}\left\{\left(1-\overline{A}^{1+s}\right)\left(\frac{1+\overline{A}^{1+s}}{2}+\frac{2\phi}{s}\right)-\left(\frac{\phi}{s}\right)^2\ln\overline{A}^{1+s}\right\},$$

[24] From (23) and (36), $\frac{\overline{A}(\phi,\tau)}{\underline{A}(\phi,\tau)} = \left(\frac{\sqrt{1-\tau}}{\sqrt{1-\tau}-\kappa(\tau)}\right)^{\frac{1}{1+\sqrt{1-\tau}}} > 1.$

$$\int_{\overline{A}}^{1} \frac{P(a;\phi,s)}{a} da = \frac{1}{1+s}\left[\frac{a^{1+s}}{1+s} + \frac{\phi}{s}\ln a\right]_{\overline{A}}^{1} = \frac{1}{(1+s)^2}\left(1-\overline{A}^{1+s} - \frac{\phi}{s}\ln \overline{A}^{1+s}\right).$$

Therefore,

$$\begin{aligned}
\pi^{HM}(\phi,s) &= \frac{1}{(1+s)^2}\left(1-\overline{A}^{1+s}\right)\left\{s^2(1-s)\left(\frac{1+\overline{A}^{1+s}}{2} + \frac{2\phi}{s}\right) + \phi\left(2s^2-1\right)\right\} \\
&\quad - \left\{\left(\frac{s}{1+s}\right)^2 (1-s)\left(\frac{\phi}{s}\right)^2 + \frac{\phi\left(2s^2-1\right)}{(1+s)^2}\frac{\phi}{s} - \frac{\phi^2}{1+s}\right\}\ln \overline{A}^{1+s}.
\end{aligned}$$

The first term is $\frac{1-s}{2}\left(\frac{s}{1+s}\right)^2\left(1-\overline{A}^{2(1+s)}\right) + \frac{(2s-1)\phi}{(1+s)^2}\left(1-\overline{A}^{1+s}\right)$, while the second term is

$$-\left\{\left(\frac{s}{1+s}\right)^2(1-s)\left(\frac{\phi}{s}\right)^2 + \frac{\phi\left(2s^2-1\right)}{(1+s)^2}\frac{\phi}{s} - \frac{\phi^2}{1+s}\right\}\ln \overline{A}^{1+s} = \frac{\phi^2}{(1+s)^2 s}\ln \overline{A}^{1+s}.$$

Therefore,

$$\begin{aligned}
\pi^{HM}(\phi,s) &= \frac{1}{s^2(1+s)^2}\left[s^4\frac{1-s}{2}\left(1-\overline{A}^{2(1+s)}\right) + s^2(2s-1)\phi\left(1-\overline{A}^{1+s}\right) + s\phi^2\ln \overline{A}^{1+s}\right] \\
&= \frac{1}{2}\left(\frac{s}{1+s}\right)^2\left(1-\overline{A}^{1+s}\right)\left\{1-s+\left\{3s-1+\frac{\kappa}{s-\kappa}(1-s)\right\}\frac{\phi}{s^2}\right\} + \frac{\frac{\phi^2}{s}}{(1+s)^2}\ln \overline{A}^{1+s}.
\end{aligned}$$

Using $\overline{A}^{1+s} = \frac{\phi}{s(s-\kappa)} = \Phi$,

$$\pi^{HM}(\phi,s) = \frac{1}{2}\left(\frac{s}{1+s}\right)^2(1-\Phi)\left\{1-s+\left\{3s-1+\frac{\kappa}{s-\kappa}(1-s)\right\}\frac{\phi}{s^2}\right\} + \frac{\frac{\phi^2}{s}\ln\Phi}{(1+s)^2}.$$

Substituting $s \equiv \sqrt{1-\tau}$ back, we obtain $\pi^{HM}(\phi,\tau)$ in **Lemma 6**. ■

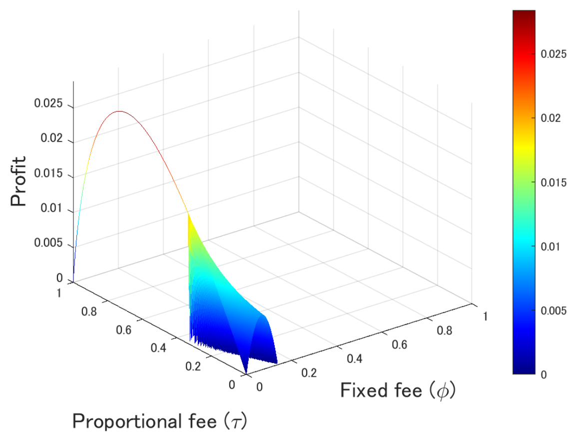
We proceed similarly as before by deriving the following two functions:

$$\phi^{HM}(\tau) \equiv \arg\max_{\phi}\left\{\pi^{HM}(\phi,\tau)\right\} \qquad \text{and} \qquad \tau^{HM}(\phi) \equiv \arg\max_{\tau}\left\{\pi^{HM}(\phi,\tau)\right\}.$$

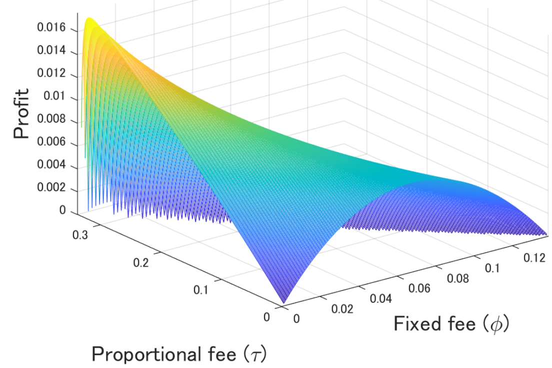
We then find the intersection of these two functions, and numerically verify that the profit function (44) is indeed maximized at this point. Similarly as before, we define firms' gain by $FG^{HM}(\phi^*,\tau^*) \equiv WG^{HM}(\phi^*,\tau^*) - \pi^{HM}(\phi^*,\tau^*)$.

Claim 3 *In the HM equilibrium without any fee regulation, the intermediary chooses* $(\phi^*_{HM},\tau^*_{HM}) = (0,\tau(0))$. *The welfare gain is 253% of the benchmark welfare gain, and firms' gain is 99%.*

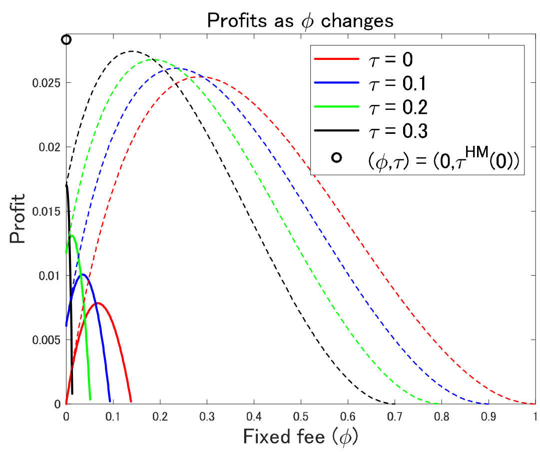
When competing with the free minimum disclosure technology, the intermediary can use a fixed fee *but chooses not to*. Comparing **Claim 3** with **Claim 2**, however, the welfare impact of this change in the intermediary's behavior is small. **Figure 9** plots the profit function (44), and the associated profit-maximizing fees.



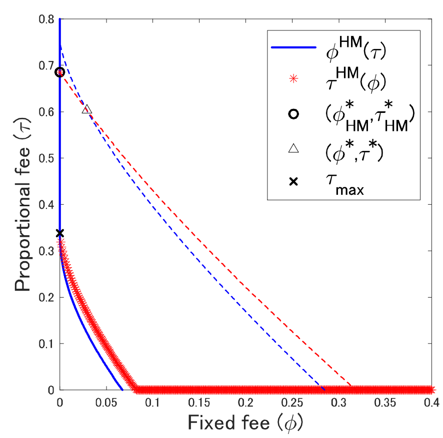
(a) $\pi^{HM}(\phi, \tau)$.



(b) Magnified $\pi^{HM}(\phi, \tau)$ for $\phi > 0$.



(c) Profit as ϕ changes.



(d) Profit-maximizing fees.

Figure 9. Profit and fees with the lower market.

The panel (a) plots $\pi^{HM}(\phi, \tau)$ with the same scale used for $\pi(\phi, \tau)$ in **Figure 6 (a)**. Importantly, the line $\pi^{HM}(0, \tau)$ along the τ-axis is identical to $\pi(0, \tau)$, suggesting the discontinuity of $\pi^{HM}(\phi, \tau)$ at $\phi = 0$. By choosing a large τ and $\phi = 0$, the intermediary enjoys its monopoly status even in the presence of the free minimum disclosure service. This no longer works with

$\phi > 0$. No matter how small it is, a positive fixed fee makes the lower market active, which makes setting high τ not a viable option for the intermediary. The panel (b) plots $\pi^{HM}(\phi, \tau)$ for $\phi > 0$ with a magnified scale. It shows that for $\phi > 0$, $\tau^{HM}(\phi) \in (0, \tau_{\max})$ is unique. The panel (c) plots $\pi^{HM}(\phi, \tau)$ as a function of ϕ for different values of τ. Dashed lines are $\pi(\phi, \tau)$, with the same color for the same value of τ. Comparing solid lines with dashed lines, the profit is significantly reduced for any $\phi > 0$. However, at $\phi = 0$, the intermediary will choose $\tau^{HM}(0) = \tau(0)$. Finally, the panel (d) plots $\phi^{HM}(\tau)$ and $\tau^{HM}(\phi)$.[25] Dashed lines are $\phi(\tau)$ and $\tau(\phi)$. This shows the optimality of $(\phi^*_{HM}, \tau^*_{HM}) = (0, \tau(0))$ (a marker $\circ$), which is close to (ϕ^*, τ^*) (a marker $\triangle$).

In sum, the welfare improvement in the HM equilibrium is small *if the intermediary can secure a large profit without the fixed fee*. Intuitively, the force of the lower market works through $\overline{A}(\phi, \tau) \equiv \left(\frac{\phi}{1-\tau-\kappa\sqrt{1-\tau}}\right)^{\frac{1}{1+\sqrt{1-\tau}}}$, i.e., by making the demand for the full disclosure technology more elastic to (ϕ, τ). The intermediary *can* eliminate this force by setting $\phi = 0$, and it *will do so* if using τ *alone* secures a high enough profit.

5.3 Regulation to support the active lower market

In the previous subsection we showed that if the intermediary can freely choose fees, a welfare improvement in the HM equilibrium is limited. However, if *either ϕ or τ is regulated*, then the lower market has a large welfare impact. More precisely, if the lower bound $\phi \geq \phi_b > 0$ (b for *bottom*) is imposed on the fixed fee, then the intermediary chooses $(\phi, \tau) = \left(\phi_b, \tau^{HM}(\phi_b)\right)$. Similarly, if the upper bound $\tau \leq \tau_c < \tau_{\max}$ (c for *cap*) is imposed on the proportional fee, then the intermediary chooses $(\phi, \tau) = \left(\tau_c, \phi^{HM}(\tau_c)\right)$. Because the former "bottom" regulation may face opposition from *firms*, we focus on the latter "cap" regulation. With a cap $\tau_c \in [0, \tau_{\max})$ imposed on τ, we have the following result.

Proposition 3 (HM equilibrium with a cap regulation on τ). *With a cap regulation $\tau \leq \tau_c \in [0, \tau_{\max})$, the intermediary chooses $(\phi, \tau) = \left(\phi^{HM}(\tau_c), \tau_c\right)$, where $\phi^{HM}(\tau) \in (0, \phi_{\max}(\tau))$ is a unique solution to $\frac{1}{\Phi} - 1 = \frac{1}{2\sqrt{1-\tau}-1}\frac{\kappa(1-\kappa)}{\sqrt{1-\tau}-\kappa} + \frac{2\left(\sqrt{1-\tau}-\kappa\right)}{2\sqrt{1-\tau}-1}\ln\frac{1}{\Phi}$ with $\Phi = \frac{\phi}{\phi_{\max}(\tau)}$. The associated welfare gain is*

$$WG^{HM}\left(\phi^{HM}(\tau_c), \tau_c\right) = \frac{1}{4}\left(\frac{\sqrt{1-\tau_c}}{1+\sqrt{1-\tau_c}}\right)^2\left(1 - \frac{\phi^{HM}(\tau_c)}{1-\tau_c}\right)\left\{1 + \left(1 - \sqrt{1-\tau_c}\right)\left(1 - \frac{\phi^{HM}(\tau_c)}{1-\tau_c}\right)\right\}.$$

[25] $\tau^{HM}(\phi)$ is plotted with asterisk ($*$) markers to show the discontinuity of $\tau^{HM}(\phi)$ at $\phi = 0$: $\lim_{\phi \downarrow 0}\tau^{HM}(\phi) = \tau_{\max} < \tau^{HM}(0) = \tau(0)$.

Proof. We work with $s=\sqrt{1-\tau}$. First, $\frac{\partial\Phi}{\partial\phi}=\frac{1}{s(s-\kappa)}$, and $\frac{\partial\pi^{HM}}{\partial\phi}\geq 0\Leftrightarrow$

$$2\left(2\phi\ln\Phi+\phi\right)\geq\frac{1}{s-\kappa}\left\{\left(1-s\right)s^{2}+\left(3s-1+\frac{1-s}{s-\kappa}\kappa\right)\phi\right\}-s\left(1-\Phi\right)\left(3s-1+\frac{1-s}{s-\kappa}\kappa\right).$$

Computing the right hand side yields $2s\left\{1-2s+\left(3s-1+\frac{1-s}{s-\kappa}\kappa\right)\Phi\right\}$, so

$$\frac{\partial\pi^{HM}}{\partial\phi}\geq 0\qquad\Leftrightarrow\qquad 2\phi\ln\Phi\geq s\left\{\left(2s-1+\frac{1-\kappa}{s-\kappa}\kappa\right)\Phi-\left(2s-1\right)\right\}.$$

At the equality, this can be written as

$$\frac{1}{\Phi}-1=\frac{1}{2s-1}\frac{\kappa\left(1-\kappa\right)}{s-\kappa}+\frac{2\left(s-\kappa\right)}{2s-1}\ln\frac{1}{\Phi}.$$

This has a unique solution $\Phi\in(0,1)$, with which we obtain $\phi^{HM}\left(s\right)=s\left(s-\kappa\right)\Phi$. Substituting $s\equiv\sqrt{1-\tau}$, we obtain the equation whose solution defines $\phi^{HM}\left(\tau\right)$.

From the derivation of $WG^{HM}\left(\phi,\tau\right)$, it can be written as

$$WG^{HM}\left(\phi,s\right)=\frac{1}{4}\frac{1}{\left(1+s\right)^{2}}\left\{s^{2}\left(2-s\right)\left(1-\Phi^{2}\right)-4\left(1-s\right)\phi\left(1-\Phi\right)+\kappa^{2}\Phi^{2}+\frac{\phi}{s}\left(2\phi\ln\Phi\right)\right\}.$$

Using $\frac{\partial\pi^{HM}}{\partial\phi}=0\Leftrightarrow 2\phi\ln\Phi=s\left\{\left(2s-1+\frac{1-\kappa}{s-\kappa}\kappa\right)\Phi-\left(2s-1\right)\right\}$ in the curly bracket above yields $s^{2}\left(2-s\right)-\left(3-2s\right)\phi+\left(1-s\right)\left(s-\kappa\right)^{2}\Phi^{2}$. Therefore,

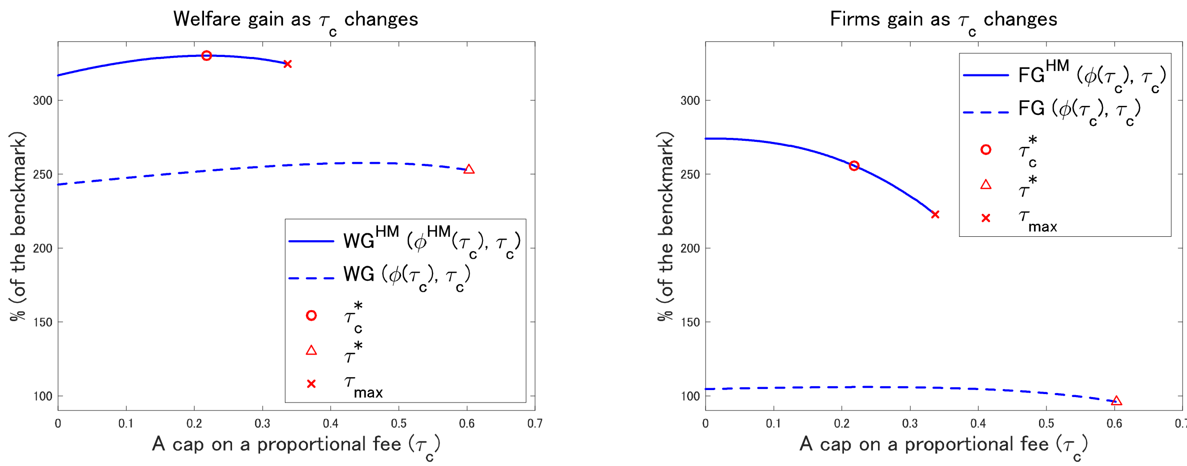
$$\begin{aligned}WG^{HM}\left(\phi^{HM}\left(s\right),s\right)&=\frac{1}{4}\frac{1}{\left(1+s\right)^{2}}\left\{s^{2}\left(2-s\right)-\left(3-2s\right)\phi^{HM}\left(s\right)+\frac{1-s}{s^{2}}\left\{\phi^{HM}\left(s\right)\right\}^{2}\right\}\\&=\frac{1}{4}\left(\frac{s}{1+s}\right)^{2}\left(1-\frac{\phi^{HM}\left(s\right)}{s^{2}}\right)\left(1+\left(1-s\right)\left(1-\frac{\phi^{HM}\left(s\right)}{s^{2}}\right)\right).\end{aligned}$$

With $s\equiv\sqrt{1-\tau}$ and $\tau=\tau_c$, we obtain $WG^{HM}\left(\phi^{HM}\left(\tau_c\right),\tau_c\right)$ in **Proposition 3**. ■

As a simple example, consider $\tau_c=0$, i.e., a complete ban on the proportional fee. The welfare gain $WG^{HM}\left(\phi^{HM}\left(0\right),0\right)=\frac{1}{16}\left(1-\phi^{HM}\left(0\right)\right)$ is 317% of the benchmark welfare gain, and the corresponding firms' gain is 274%. Let us compare these numbers with $WG^{FD}\left(\phi\left(0\right),0\right)$ and $FG^{FD}\left(\phi\left(0\right),0\right)$, i.e., the effects of *the same regulation* $\tau_c=0$ *in the absence of the lower market*. $WG^{FD}\left(\phi\left(0\right),0\right)$ is 243% of the benchmark welfare gain and $FG^{FD}\left(\phi\left(0\right),0\right)$ is only 105%. Thus, while a cap regulation without a lower market does little for firms, in the presence of the lower market it significantly improves firms' gain.

Figure 10 plots $WG^{HM}\left(\phi^{HM}\left(\tau_c\right),\tau_c\right)$ and $FG^{HM}\left(\phi^{HM}\left(\tau_c\right),\tau_c\right)$ as functions of $\tau_c\in[0,\tau_{\max})$. For comparison, dashed lines are $WG^{FD}\left(\phi\left(\tau_c\right),\tau_c\right)$ and $FG^{FD}\left(\phi\left(\tau_c\right),\tau_c\right)$ for

$\tau_c < \tau^*$.[26] These are obtained under the same cap, but in the absence of the lower market.



(a) Welfare gains.

(b) Firms' gains.

Figure 10. $WG^{HM}\left(\phi^{HM}(\tau_c), \tau_c\right)$ and $FG^{HM}\left(\phi^{HM}(\tau_c), \tau_c\right)$.

In **Figure 10**, a marker "△" is $\tau^* \approx 0.603$, and a marker "×" is $\tau_{\max} \approx 0.338$. The panel (a) plots $WG^{HM}\left(\phi^{HM}(\tau_c), \tau_c\right)$ and $WG\left(\phi(\tau_c), \tau_c\right)$. It shows that $WG^{HM}\left(\phi^{HM}(\tau_c), \tau_c\right)$ is maximized by the interior cap $\tau_c^* \approx 0.218 \in (0, \tau_{\max})$ (a marker ○). The associated fixed fee is $\phi^{HM}(\tau_c^*) \approx 0.009$. The panel (b) plots $FG^{HM}\left(\phi^{HM}(\tau_c), \tau_c\right)$ and $FG\left(\phi(\tau_c), \tau_c\right)$. The former (a solid line) shows that most of the welfare gain accrues to firms.[27] In contrast, the latter (a dashed line) shows that, without the lower market, the intermediary absorbs almost all the welfare improvement by the disclosure.

Claim 4 *There is the optimal cap regulation $\tau_c^* \approx 0.218 \in (0, \tau_{\max})$ that maximizes the welfare gain in the HM equilibrium. The maximized welfare gain is 330% and the corresponding firms' gain is 256% of the benchmark welfare gain WG^{ND}. With the same cap but without the lower market, the welfare gain is 252% and firms' gain is 106%.*

[26] $\tau_c \geq \tau^*$ is a non-binding regulation. See **Figure 9 (d)**.

[27] While it is hard to read off from **Figure 10 (b)**, firms' gain also takes the interior maximum 274% at $\tau_c \approx 0.008$. The associated fixed fee is 0.065 and the welfare gain is 318%. Because the intermediary's (constrained) profit increases in τ_c, the optimal cap $\tau_c^* \in (0.008, \tau_{\max})$ balances firms' gain and the intermediary's profit.

Figure 11 shows the sorting pattern of firms with $(\phi, \tau) = \left(\phi^{HM}\left(\tau_c^*\right), \tau_c^*\right)$.

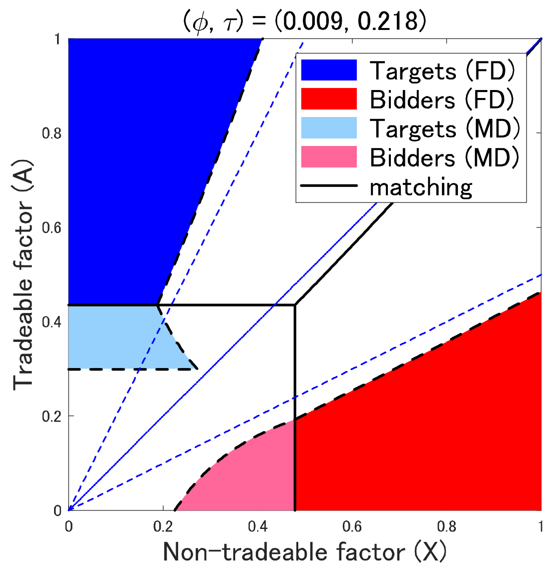


Figure 11. Sorting in the HM equilibrium with $\tau_c^* \approx 0.218$.

Figure 11 shows that with the optimal cap τ_c^*, the lower market attracts much smaller mass of firms relative to the upper market. If measured by transaction values, its relative size is even smaller. Nevertheless, these small transactions play a key role in improving the overall welfare. Trading in the lower market is inefficient due to random matching, and a *direct* welfare contribution of the lower market is small because firms in this market have small gains from trade. Yet, a hybrid market structure allows these firms to contribute to the aggregate welfare *indirectly*, and its aggregate impact is significant. The welfare analysis with the intermediary is summarized in **Table 1**.

Table 1. The welfare gain with a monopoly intermediary

	Single market		Hybrid market	
	Welfare gain	Firms' gain	Welfare gain	Firms' gain
Without regulation on fees.	253%	96%	253%	99%
The cap on a proportional fee.	252%	106%	330%	256%

In sum, the minimum disclosure technology, although inefficient on its own, might be a useful regulatory tool to control non-competitive and/or collusive behaviors of disclosure service providers. While implementing the minimum disclosure service takes real resources, the suggested magnitude of the welfare gain indicates that the benefit may exceed the cost.

6 Conclusion

We presented a competitive matching model of takeover markets. The model yields testable predictions for the characteristics of matched pair of firms, some of which are consistent with the existing empirical evidence. We showed that the full disclosure service by a monopoly intermediary and the minimum disclosure service by a regulatory body can coexist, if the intermediary's fees are appropriately regulated. A presence of the active market with the minimum disclosure service raises the demand elasticity facing the intermediary. As a result, more welfare gains are realized, and more productive firms benefit more from this policy. The result shows that a minimum disclosure technology, although it appears inefficient on its own, might be a useful regulatory tool. More generally, our analysis indicates that in a situation where a natural monopoly (or collusion) has been established but its (or their) expertise is valuable, "pseudo" competition induced by a regulatory body may be an alternative, indirect measure of regulation.

There are a number of limitations to our analysis: we remained in a static model, studied only simple disclosure technologies, focused on the monopoly intermediary, and ignored reallocation through factor markets. How much do we gain by allowing for a second round of trading? Does the intermediary have an incentive to provide a more general disclosure service? How do intermediaries with different disclosure technologies compete? Does a market for corporate control substitute or complement other factor markets? We believe that our model of takeovers is a useful first step toward answering these questions.

7 Omitted proof

7.1 Proof of Lemma A (used in the proof of Proposition 1)

(a) Suppose $A_{\min} < A^* = \frac{a}{2}$. We verify later that this occurs if and only if $A_{\min} < A_+$. Given $A_{\min} < A^*$ and (a, P) such that $0 < P < a < 1$, the sorting pattern implies that targets satisfy $X \leq \frac{P}{A}$, $X \leq X^*$, and $A \geq A_{\min}$. Hence, a supply curve is

$$S(P) = \int_{A_{\min}}^{A^*} X^* dA + \int_{A^*}^{1} \frac{P}{A} dA = X^*(A^* - A_{\min}) - P \ln A^*.$$

Substituting $A^* = \frac{a}{2}$ and $X^* = \frac{2P}{a}$,

$$S(P) = P\left(1 + \ln\frac{2}{a} - A_{\min}\frac{2}{a}\right).$$

Bidders satisfy $A \leq a - \frac{P}{X}$, and additionally, if $A \in [A_{\min}, A^*]$, $X > X^*$. Note that $a - \frac{P}{X} = 0$ defines a skill threshold $X = \frac{P}{a}$ (below which no bidder exists), while $a - \frac{P}{X} = A_{\min}$ defines another threshold $X = \frac{P}{a - A_{\min}} \in \left(\frac{P}{a}, X^*\right)$ (above which $A \geq A_{\min}$ becomes a binding constraint for some firms). Using these, a demand curve is

$$\begin{aligned} D(P) &= \int_{\frac{P}{a}}^{1} \left(a - \frac{P}{X}\right) dX - \int_{\frac{P}{a - A_{\min}}}^{X^*} \left(a - \frac{P}{X} - A_{\min}\right) dX \\ &= P\left\{\frac{a}{P} - \ln\frac{a}{P} + \ln\left(2\frac{a - A_{\min}}{a}\right) - 2\frac{a - A_{\min}}{a}\right\}. \end{aligned}$$

For $P > 0$, a market-clearing condition $S(P) = D(P)$ is equivalent to

$$a = P\left\{3 - \ln P - \left(4\frac{A_{\min}}{a} + \ln\left(1 - \frac{A_{\min}}{a}\right)\right)\right\} \equiv \Phi_1(P; a, A_{\min}).$$

This is (5). A brief inspection of Φ_1 yields

$$\frac{\partial \Phi_1}{\partial P} = 2\left(1 - 2\frac{A_{\min}}{a}\right) + \ln\frac{a}{a - A_{\min}} - \ln P.$$

This is positive for any $0 < P < a < 1$ such that $A_{\min} < A^* = \frac{a}{2}$, because

$$2\left(1 - 2\frac{A_{\min}}{a}\right) > 0 > \ln P - \ln\frac{a}{a - A_{\min}}.$$

Also, $\Phi_1(0; a, A_{\min}) = 0$ and $\Phi_1(a; a, A_{\min}) = a\left\{3 - 4\frac{A_{\min}}{a} - \ln(a - A_{\min})\right\}$. Note that $\Phi_1(a; a, A_{\min}) > a \Leftrightarrow 2 - 4\frac{A_{\min}}{a} > \ln(a - A_{\min})$ holds because $A_{\min} < A^* = \frac{a}{2}$ implies $2\left(1 - 2\frac{A_{\min}}{a}\right) > 0 > \ln(a - A_{\min})$. This establishes a unique solution $P(a) \in (0, a)$ to (5).

Given the sorting pattern, the expected quality of projects for sale is

$$\Gamma(a; A_{\min}) \equiv \frac{\int_{A_{\min}}^{A^*} (AX^*)\, dA + \int_{A^*}^{1} \left(A\frac{P}{A}\right) dA}{S(P)}.$$

The numerator can be evaluated as

$$P\left\{\frac{1}{2}\frac{2}{a}\left(A^{*2} - A_{\min}^2\right) + 1 - A^*\right\} = P\left(1 - \frac{a}{4} - \frac{1}{a}A_{\min}^2\right).$$

Combining this with $S(P) = P\left(1 + \ln\frac{2}{a} - A_{\min}\frac{2}{a}\right)$ yields (4).

We prove that, for any $A_{\min} < A_+$, $\Gamma(a; A_{\min})$ satisfies

$$\text{(i) } \Gamma(2A_{\min}; A_{\min}) > 2A_{\min}, \qquad \text{(ii) } \Gamma(1; A_{\min}) < 1, \qquad \text{(iii) } \frac{\partial\Gamma(a; A_{\min})}{\partial a}|_{a=a*} < 1.$$

(i) - (iii) imply that $\Gamma(a; A_{\min}) = a$ has a unique solution $a^* \in (2A_{\min}, 1)$.

For the property (i),

$$\Gamma(2A_{\min}; A_{\min}) = \frac{1 - \frac{A_{\min}}{2} - \frac{A_{\min}}{2}}{-\ln A_{\min}} > 2A_{\min}$$

$$\Leftrightarrow 1 > A_{\min}(1 - 2\ln A_{\min}) \qquad \Leftrightarrow \qquad A_{\min} < A_+.$$

For the property (ii),

$$\Gamma(1; A_{\min}) = \frac{\frac{3}{4} - A_{\min}^2}{1 + \ln 2 - 2A_{\min}} < 1 \qquad \Leftrightarrow \qquad \frac{3}{4} - \ln 2 < (1 - A_{\min})^2.$$

This holds because $\frac{3}{4} - \ln 2 = 0.057 < (1 - A_+)^2 = (1 - 0.285)^2 = 0.511 < (1 - A_{\min})^2$.

For the property (iii), let $N \equiv 1 - \frac{a}{4} - \frac{1}{a}A_{\min}^2$ and $D \equiv 1 - \ln\frac{a}{2} - \frac{2}{a}A_{\min}$ so that $\Gamma(a; A_{\min}) = \frac{N}{D}$. Then

$$\frac{\partial\Gamma(a; A_{\min})}{\partial a} < 1 \qquad \Leftrightarrow \qquad \frac{\partial N}{\partial a}D - N\frac{\partial D}{\partial a} < D^2 \qquad \Leftrightarrow \qquad \frac{\partial N}{\partial a} - D < \frac{N}{D}\frac{\partial D}{\partial a}$$

$$\Leftrightarrow \left(\frac{A_{\min}}{a}\right)^2 - \frac{1}{4} - D < \frac{N}{D}\left(\frac{2A_{\min}}{a} - 1\right)\frac{1}{a}.$$

Because both sides are negative for $A_{\min} < \frac{a}{2}$, this is equivalent to

$$\frac{N}{D} = \Gamma(a; A_{\min}) < a\frac{1 - \ln\frac{a}{2} - \frac{2}{a}A_{\min} + \frac{1}{4}\left\{1 - \left(\frac{2A_{\min}}{a}\right)^2\right\}}{1 - \frac{2A_{\min}}{a}}.$$

The right hand side can be written as $a + a\frac{\ln\frac{2}{a}+\frac{1}{4}\left(1-\frac{2A_{\min}}{a}\right)\left(1+\frac{2A_{\min}}{a}\right)}{1-\frac{2A_{\min}}{a}}$, so

$$\frac{\partial\Gamma\left(a;A_{\min}\right)}{\partial a}<1 \qquad \Leftrightarrow \qquad \frac{\Gamma\left(a;A_{\min}\right)-a}{a}<\frac{\ln\frac{2}{a}}{1-\frac{2A_{\min}}{a}}+\frac{1}{4}\left(1+\frac{2A_{\min}}{a}\right).$$

Because the right hand side is positive for $A_{\min}<\frac{a}{2}$ while the left hand side is zero at $a=a^*$, this implies that $\frac{\partial\Gamma(a;A_{\min})}{\partial a}<1$ holds at $a=a^*$.

To show that a^* is increasing in $A_{\min}$, it suffices to show that $\frac{\partial\Gamma(a;A_{\min})}{\partial A_{\min}}|_{a=a^*}>0$.

$$\frac{\partial\Gamma\left(a;A_{\min}\right)}{\partial A_{\min}}>0 \qquad \Leftrightarrow \qquad \frac{\partial N}{\partial A_{\min}}D>N\frac{\partial D}{\partial A_{\min}}.$$

Because $\frac{\partial N}{\partial A_{\min}}=-\frac{2A_{\min}}{a}$ and $\frac{\partial D}{\partial A_{\min}}=-\frac{2}{a}$ are both negative,

$$\frac{\partial\Gamma\left(a;A_{\min}\right)}{\partial A_{\min}}>0 \qquad \Leftrightarrow \qquad \frac{N}{D}=\Gamma\left(a;A_{\min}\right)>A_{\min}.$$

This holds at $a=a^*$, because $A_{\min}<\frac{a^*}{2}<a^*=\Gamma\left(a^*;A_{\min}\right)$.

Finally, we already know that, for $A_{\min}<1$, $\Gamma\left(2A_{\min};A_{\min}\right)=2A_{\min}\Leftrightarrow 1=A_{\min}\left(1-2\ln A_{\min}\right)\Leftrightarrow A_{\min}=A_+$. Therefore, $\lim_{A_{\min}\nearrow A_+}a^*=2A_0$. ■

(b) Suppose $A_{\min}\geq A^*=\frac{a}{2}$. We verify later that this occurs if and only if $A_{\min}\geq A_+$. A supply curve is

$$S\left(P\right)=\int_{A_{\min}}^{1}\frac{P}{A}dA=-P\ln A_{\min}.$$

A demand curve is

$$D\left(P\right)=\int_{\frac{P}{a}}^{1}\left(a-\frac{P}{X}\right)dX=a-P+P\ln\frac{P}{a}.$$

A market-clearing condition $S\left(P\right)=D\left(P\right)$ is equivalent to

$$a=P\left(1-\ln P+\ln\frac{a}{A_{\min}}\right)\equiv\Phi_2\left(P;a,A_{\min}\right).$$

This is (6). A brief inspection of Φ_2 yields

$$\frac{\partial\Phi_2}{\partial P}=\ln\frac{a}{PA_{\min}}.$$

This is positive if and only if $\frac{a}{P}>A_{\min}$, which is true for any $A_{\min}<1$ and $P<a$. Also, $\Phi_2\left(0;a,A_{\min}\right)=0$ and $\Phi_2\left(a;a,A_{\min}\right)=a\left(1-\ln A_{\min}\right)>a$ for any $A_{\min}<1$. This establishes a unique solution $P\in(0,a)$ to (6).

Given the sorting pattern, the expected quality of projects for sale is

$$\frac{\int_{A_{\min}}^{1}\left(A\frac{P}{A}\right)dA}{S\left(P\right)}=\frac{P\left(1-A_{\min}\right)}{-P\ln A_{\min}}=\frac{A_{\min}-1}{\ln A_{\min}}.$$

Therefore, given $A_{\min}\geq A^{*}=\frac{a}{2}$, $a^{*}=\frac{A_{\min}-1}{\ln A_{\min}}$. To verify the conjecture $A_{\min}\geq A^{*}=\frac{a}{2}$,

$$A_{\min}\geq\frac{1}{2}\frac{A_{\min}-1}{\ln A_{\min}}\qquad\Leftrightarrow\qquad 1\leq A_{\min}\left(1-2\ln A_{\min}\right)\Leftrightarrow A_{\min}\geq A_{+}.$$

That $a^{*}=\frac{A_{\min}-1}{\ln A_{\min}}$ is increasing in $A_{\min}$ is immediate from

$$\ln A_{\min}-\frac{A_{\min}-1}{A_{\min}}>0\qquad\Leftrightarrow\qquad 1>A_{\min}\left(1-\ln A_{\min}\right),$$

where the right hand side is increasing in $A_{\min}$ and approaches one as $A_{\min}\nearrow 1$. Note also that $\lim_{A_{\min}\nearrow 1}\frac{A_{\min}-1}{\ln A_{\min}}=1$. At $A_{\min}=A_{+}$, $a^{*}=\frac{A_{+}-1}{\ln A_{+}}=2A_{+}$ holds because this is equivalent to $1=A_{+}\left(1-2\ln A_{+}\right)$. This means that a^{*} is continuous in $A_{\min}\in[0,1)$. ■

7.2 Analysis behind Claim 2

We establish the existence of the interior optimum (ϕ^{*},τ^{*}). In the proof of **Lemma 3**, we showed that $\phi\left(\tau\right)=0$ for $\tau\geq\widehat{\tau}=\frac{3}{4}$ and $\phi\left(0\right)=\overline{\phi}\approx 0.285$. Here, we similarly show that $\tau\left(\phi\right)=0$ for $\phi\geq\widehat{\phi}\approx 0.319$ and $\tau\left(0\right)=\overline{\tau}\approx 0.685$. Then, $\overline{\phi}<\widehat{\phi}$, $\overline{\tau}<\widehat{\tau}$, and the continuity of $\{\phi\left(\tau\right),\tau\left(\phi\right)\}$ imply the existence of the interior solution (ϕ^{*},τ^{*}). Numerically, this is the unique optimum as shown in **Figure 7 (a)**.

Taking a partial derivative of $\pi\left(\phi,s\right)$ with respect to s,

$$\frac{\partial\pi\left(\phi,s\right)}{\partial s}\geq 0\quad\Leftrightarrow\quad\begin{array}{l} s^{2}\left(1+s\right)^{2}\frac{\partial}{\partial s}\left[\left(s^{2}-\phi\right)\frac{(1-s)s^{2}+(3s-1)\phi}{2}-s\phi^{2}\ln\frac{s^{2}}{\phi}\right]\\ \geq\left[\left(s^{2}-\phi\right)\frac{(1-s)s^{2}+(3s-1)\phi}{2}-s\phi^{2}\ln\frac{s^{2}}{\phi}\right]\frac{\partial}{\partial s}\left\{s^{2}\left(1+s\right)^{2}\right\}\end{array}$$

Because $\frac{\partial}{\partial s}\left\{s^{2}\left(1+s\right)^{2}\right\}=2s\left(1+s\right)\left(1+2s\right)$, this is equivalent to

$$\frac{\frac{\partial}{\partial s}\left[\left(s^{2}-\phi\right)\frac{(1-s)s^{2}+(3s-1)\phi}{2}-s\phi^{2}\ln\frac{s^{2}}{\phi}\right]}{\left(s^{2}-\phi\right)\frac{(1-s)s^{2}+(3s-1)\phi}{2}-s\phi^{2}\ln\frac{s^{2}}{\phi}}\geq\frac{2\left(1+2s\right)}{s\left(1+s\right)}\in\left[2,3\right].$$

Evaluating the numerator of the left hand side,

$$\begin{aligned}&\frac{\partial}{\partial s}\left[\left(s^{2}-\phi\right)\frac{\left(1-s\right)s^{2}+\left(3s-1\right)\phi}{2}-s\phi^{2}\ln\frac{s^{2}}{\phi}\right]\\ =\;& s^{3}\left(2-\frac{5}{2}s\right)-\frac{7}{2}\phi^{2}+2\phi s\left(3s-1\right)-\phi^{2}\ln\frac{s^{2}}{\phi}.\end{aligned}$$

Therefore,

$$\frac{\partial \pi(\phi,s)}{\partial s} \geq 0 \quad \Leftrightarrow \quad \frac{s^4\left(2-\frac{5}{2}s\right)-\frac{7}{2}\phi^2 s+2\phi s^2(3s-1)-s\phi^2\ln\frac{s^2}{\phi}}{\left(s^2-\phi\right)\frac{(1-s)s^2+(3s-1)\phi}{2}-s\phi^2\ln\frac{s^2}{\phi}} \geq \frac{2(1+2s)}{1+s}. \tag{45}$$

For $\phi=0$, the left hand side of (45) becomes $\frac{s^4\left(2-\frac{5}{2}s\right)}{\frac{(1-s)s^4}{2}}=\frac{4-5s}{1-s}$. For this to be positive, we need $s<\frac{4}{5}$. The optimal τ when $\phi=0$, i.e. $\tau(0)$, is given by a solution to

$$(4-5s)(1+s)=2(1+2s)(1-s) \quad \Leftrightarrow \quad s^2+3s-2=0.$$

This has a unique positive solution $s=\frac{-3+\sqrt{17}}{2} \Leftrightarrow \tau=1-\frac{26-6\sqrt{17}}{4}=\frac{3\sqrt{17}-11}{2}=\overline{\tau}\approx 0.685$. Finally, we verify $s=\frac{-3+\sqrt{17}}{2}<\frac{4}{5}\Leftrightarrow\sqrt{17}<\frac{21}{5}$.

For $\phi>0$, rewrite (45) with equality as

$$\frac{\frac{s^2}{\phi}\left\{4(3s-1)-\frac{s^2}{\phi}(5s-4)\right\}-7s-2s\ln\frac{s^2}{\phi}}{\left(\frac{s^2}{\phi}-1\right)\left\{(1-s)\frac{s^2}{\phi}+3s-1\right\}-2s\ln\frac{s^2}{\phi}}=\frac{2(1+2s)}{1+s}. \tag{46}$$

Numerically, we find that this has a unique solution $\tau(\phi)$ decreasing in ϕ. It also satisfies $\lim_{\phi\uparrow\widehat{\phi}}\tau(\phi)=0$ and $\tau(0)=\overline{\tau}\approx 0.685$, where $\widehat{\phi}\approx 0.319$ is characterized as follows. By setting $s=1$ in (46),

$$\begin{aligned}\frac{\frac{1}{\phi^2}\left(-\frac{1}{2}\right)-\frac{7}{2}+\frac{4}{\phi}-\ln\frac{1}{\phi}}{\frac{1}{\phi}-1-\ln\frac{1}{\phi}} &= 3 \;\Leftrightarrow\; -\frac{1}{2\phi}-\frac{7}{2}\phi+4+\phi\ln\phi=3(1-\phi+\phi\ln\phi)\\ &\Leftrightarrow\; 1-\frac{1}{2\phi}-\frac{1}{2}\phi=2\phi\ln\phi\\ &\Leftrightarrow\; 1=\phi(2-\phi-4\phi\ln\phi).\end{aligned}$$

This has two solutions and the smaller one is $\widehat{\phi}\approx 0.319$. Therefore, $\overline{\phi}<\widehat{\phi}$ and $\overline{\tau}<\widehat{\tau}$, as shown in **Figure 7 (a)**. This establishes the existence of the interior optimum (ϕ^*,τ^*).

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