Competitive Fair Redistricting

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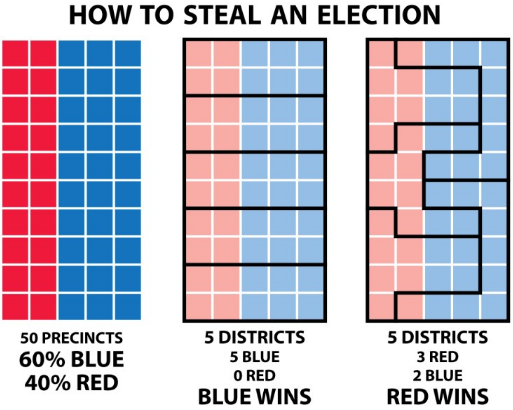
Problems with Gerrymandering in the US

- Redistricting after new census in the US, every ten years. New district map proposed by the ruling party.
- Frequent discrepancy between party winning the popular vote and the party winning the election/ a majority of districts.
- Difficulty to define a legal standard for acceptable vs. abusive redistricting.

Incentives in partisan gerrymandering: Cracking and Packing



This paper

Mechanism/ Market Design Approach: Is there a game of redistricting that gives rise to a desirable outcome: an implementation of the popular vote.

- Let both parties keep each other in check.
- Hope: Eliminate need for a standard of acceptability.
- Market design element: Work with given institutional constraints
    - There have to be many districts
    - Districts boundaries are adjusted every now and then

$\Rightarrow$ "Trivial solution" of simply having one district not available

The main result

Theorem

There is a dynamic game of redistricting in which each party has a strategy that "guarantees" winning a majority of districts conditional on winning the popular vote.

- When a party plays this strategy, the "victory in the election cannot be stolen".
- Result follows from the analysis of a fictitious zero sum game in which both parties maximizing the probability of winning a majority of seats.
- We are not predicting that parties will actually follow this strategy. They may have objectives different from maximizing the probability of winning a majority.
- By the Maximin-property for zero sum games, if one parties deviates from equilibrium, the outcome for the other party can only get better.

Related Literatures

1. Normative Approaches

   Vickrey (1961), Ely (2019)

2. Colonel Blotto / Divide-the-dollar-games

   Myerson (1993), Lizzeri and Persico (2001, 2005), Laslier and Picard (2002), Konrad (2009), Kovenock and Roberson (2020)

3. Partisan Gerrymandering

   Owen and Grofman (1988) ... Kolotilin and Wolitzky (2020).

Outlook

1. Introduction
2. Model and Main result
3. Simple example
4. Concluding remarks

Model I

- $2N$ local districts, indexed by $k \in \{1, 2, \ldots, 2N\}$, and one at-large district.
- Two parties, labeled R and D.
- Two types of voters/ precincts $t \in \{t_1, t_2\}$
- Possible states of the world $\omega \in \Omega$
- $v(t, \omega)$ the prob type t votes for R in state ω; v is increasing in both arguments.
- The mass of type t_j voters is given by

$$b_j = 2N\ \beta_j\ , \quad \text{where} \quad \beta_1 + \beta_2 = 1 \quad \text{and} \quad \beta_1 \leq \frac{1}{2}\ .$$

Model II

The popular vote:

- State $\hat{\omega} \in \Omega$ defined by $\beta_1 \; v(t_1, \hat{\omega}) + \beta_2 \; v(t_2, \hat{\omega}) = \frac{1}{2}$.
- Party R / D wins the popular vote if $\omega > \hat{\omega}$ / $\omega < \hat{\omega}$.
- Assumptions:

$$\begin{aligned} &\text{i)} \; v(t_1, \hat{\omega}) \quad < \quad \tfrac{1}{2} \quad < \quad v(t_2, \hat{\omega}) \; . \\ &\text{ii)} \; 1 - v(t_1, \hat{\omega}) \quad \geq \quad v(t_2, \hat{\omega}) \; . \end{aligned}$$

Model III

District Outcomes:

- Voter assignment to districts over several rounds.
- In this process, each party assigns every voter to one of the districts. Thus, any one voter is assigned twice, once by D and once by R.
- A voter assignment by party $P \in \{D, R\}$ is a collection $\sigma_P = (\sigma_{Pk})_{k=1}^{2N}$, where

$$\sigma_{Pk} = (\sigma_{Pk}^1, \sigma_{Pk}^2) \quad \text{with} \quad \sigma_{Pk}^1 + \sigma_{Pk}^2 = 1 \; ,$$

  is the assignment of voters to district k by party P. Party R wins district k in state ω if

$$(\sigma_{Dk}^1 + \sigma_{Rk}^1)\; v(t_1, \omega) + (\sigma_{Dk}^2 + \sigma_{Rk}^2)\; v(t_2, \omega) \quad > \quad \frac{1}{2} \; . \tag{1}$$

Model IV

The sequence of moves/ the game form:

- L rounds. In any round l, Party P assigns a mass of $\frac{1}{L}$ voters to every district. Formally, party P specifies $\sigma_{Pl} = (\sigma^1_{kPl}, \sigma^2_{kPl})^{2N}_{k=1}$ so that

$$\sigma^1_{kPl} + \sigma^2_{kPl} = \frac{1}{L} \ .$$

- For concreteness, we assume that, for l odd, R moves first and D second.
- Denote the total mass of type t_1 partisans assigned by party P to district k over the L rounds by $\sigma^1_{kP} := \sum_{l=1}^{L} \sigma^1_{kPl}$. Analogously, let $\sigma^2_{kP} := \sum_{l=1}^{L} \sigma^2_{kPl}$.
- To be consistent with the overall distribution of voters, $(\sigma_{kP})^{2N}_{k=1}$ must satisfy

$$\frac{1}{2N} \sum_{k=1}^{2N} \sigma^1_{kP} = \beta_1 \quad \text{and} \quad \frac{1}{2N} \sum_{k=1}^{2N} \sigma^2_{kP} = \beta_2 \ .$$

Model V

Winning a majority of seats:

- Recall that there are $2N$ districts and an at-large-district. Thus, the party that wins at least $N + 1$ seats wins a majority in the legislature.
- Given a pair of voter assignments (σ_D, σ_R), we denote the probability that party R wins a majority of seats, conditional on it winning the popular vote, by $\Pi_R(\sigma_D, \sigma_R \mid \omega > \hat{\omega})$.
- We define $\Pi_D(\sigma_D, \sigma_R \mid \omega < \hat{\omega})$ analogously.

The main result

Theorem

Let $N \geq 3$. For every $\varepsilon > 0$, there is $\hat{L}$, so that, for $L \geq \hat{L}$: There is a strategy σ_R so that

$$\Pi_R \left(\sigma_D, \sigma_R \mid \omega > \hat{\omega} \right) = 1 \;, \quad \text{for every} \quad \sigma_D \;,$$

and there is a strategy σ_D so that

$$\Pi_D \left(\sigma_D, \sigma_R \mid \omega < \hat{\omega} \right) = 1 \;, \quad \text{for every} \quad \sigma_R \;.$$

Illustrative example I

- Type 1 and type 2 with equal population shares, $\beta_1 = \beta_2$.
- $v(t_1, \omega) = 0.3 + 0.1\omega$, $v(t_2, \omega) = 0.6 + 0.1\omega$, for $\omega \in [0, 1]$.
- Let $L = 1$.
- R moves second. R can undue deviations from the popular vote by D.

  E.g. if D assigned 60 percent t_1 and 40 percent t_2 precincts to district k, R can respond by assigning 40 percent t_1 and 60 percent t_2 precincts.
- D moves first. Suppose that D assigns only type t_1 precincts to districts 1 to N districts, and only type t_2 precincts to districts $N + 1$ to $2N$.

  Whatever the response of R, the first N districts will have at least a 50 percent share of Democratic-leaning precincts, so will be won by D whenever $\omega < 0.5$.

Illustrative example II

Why is the general case more complicated than this?

- Let $\beta_1 = 2/3$ and $\beta_2 = 1/3$.
- $v(t_1, \omega) = 0.3 + 0.2\omega$, $v(t_2, \omega) = 0.6 + 0.2\omega$, for $\omega \in [0, 1]$.
- As before, D / R wins the the popular vote whenever $\omega < 0.5$ / $\omega > 0.5$.
- With $\beta_1 = 2/3$, not possible for D to block all t_1 precincts together in one-half of the districts: the type of move that guaranteed Democrats a victory in the previous example is no longer feasible.
- Blocking them in $2/3$ of districts is feasible, but this strategy does not work in the sense of ensuring a majority whenever $\omega > 0.5$.

Concluding remarks

- We show that it is possible to neutralize the distortions due to partisan gerrymandering by having both parties participate in the redistricting process.
- Possibility result based on a particular sequential game
- The protocol does not have be taken literally as a specific proposal for how redistricting should be done in practice.
- It is of theoretical value in that it provides an upper bound for what is achievable when the rules governing the redistricting process are well designed.
- Presumably, there are other protocols that also implement the popular vote.

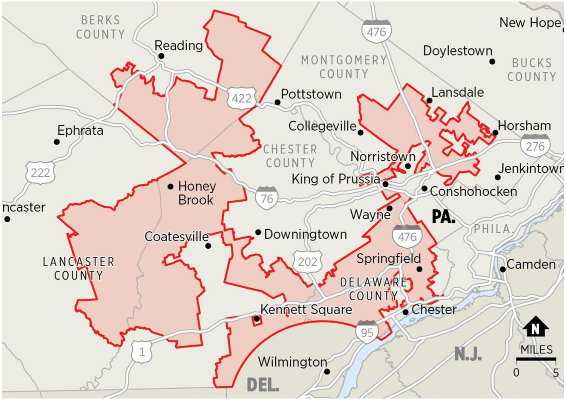
Ethymology I

From Wikipedia:

- The word gerrymander (...) was created in reaction to a redrawing of Massachusetts state senate election districts under Governor Elbridge Gerry, later Vice President of the United States.
- When mapped, one of the contorted districts in the Boston area was said to resemble a mythological salamander.
- Appearing with the term, and helping spread and sustain its popularity, was a political cartoon, printed in March 1812.

Ethymology II

A recent gerrymander: Goofy kicking Donald Duck



Pennsylvania's 7th congressional district