On the Saving Behavior of European Households

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Motivation

- Average saving rate have been trending downward.
- Saving rate declined since the 1990s in many European economies.
- Eurozone household's saving jumped from 12.8% of disposable income in 2019 to 19% in 2020.

	Average saving rate				
	1990- 1994	1995 - 2003	2004-2008	2009-2014	2015-2020
Italy	19.53	11.13	8.38	3.76	4.42
Spain	6.72	6.37	3.11	4.40	5.03
Netherlands	11.89	6.46	2.79	7.76	11.76
France	13.66	14.07	14.50	15.37	15.42
Denmark	−1.46	−2.25	−2.97	0.48	5.70
Austria	15.48	10.88	11.41	8.69	10.08
Belgium	13.06	12.96	10.93	8.44	7.31
Finland	5.65	2.03	0.16	1.48	0.71
Germany	12.92	10.34	10.68	9.92	12.06
Sweden	2.72	2.15	5.63	11.54	14.79
Switzerland	13.71	13.43	14.23	17.01	18.80
Canada	11.52	4.70	2.33	4.36	5.19
Japan	14.65	8.95	3.13	2.67	4.09
USA	8.29	5.87	4.13	7.07	9.71

This Paper

- Contribution:
  - We develop and use an overlapping generations model to identify the factors behind the saving behavior.
  - In a theoretical ground: we propose a specification of the optimal saving function.
- Research Question: What is behind the saving behavior?
  - The demographics: the rapid aging of the European population.
    Next slide Next slide Next slide
  - Wages, labor supply, capital, return on equity, and interest rate.
- To address this question, we use cross-country data to measure the effects of various factors (potentially important) on saving.
- Main Findings
  - We find strong evidence that an increase in youth labor supply leads to a rise in household savings.
  - We find an inverse relationship between corporate equity and household savings.
  - Falling interest rates may be related to the downward trend in saving.

Literature

- Household saving: Ma and Yi (2010), Mody et al. (2012), Cronqvist and Siegel (2015), Fagereng et al. (2019), Choukhmane et al. (2019), Mian et al. (2020), Nardi et al. (2021), and Ordonez and Piguillem (2021).
- Portfolio choice over the life cycle Ameriks and Zeldes (2004), Campanale et al. (2015), Fagereng et al. (2017), Gomes (2020), and Catherine (2021).
- Overlapping generations model: Chen et al. (2007), Coeurdacier et al. (2015), Mehlum et al. (2016), and Irmen (2017), Eggertsson et al. (2019a), Eggertsson et al. (2019b), and Miranda-Pinto et al. (2020).
- Demographics and saving behavior: Chamon and Prasad (2010), Curtis et al. (2015), Bairoliya and Ray (2021).
- Interest rate and saving: Constantinides et al. (2002), Eeckhoudt and Schlesinger (2008),Wang (2004), Chetty et al. (2014), Cao and Werning (2018).
- Precautionary savings and risk: Apps et al. (2014), Mehlum et al. (2016), Floden (2008), Angeletos (2007), Carroll (2009), Lachowska and Myck (2018).

Model Set up

- There are three generations: young, middle-age, and old.
- Consider an economy of overlapping generations in which individuals work and choose how much to consume, save, and invest when young, middle-aged and old.
- Equity and safe assets are substitutes.
- Representative producer.
- Capital producers face adjustment cost.
- Safe assets are issued by the government.
- Monetary authority.

Households

- Households lifetime utility function $U(c_t^j, l_t^j)$

$$E_0 \sum_{t=0}^{T} \beta^t U(c_t^j, l_t^j) = E_0 \sum_{t=0}^{T} \{U(c_t^y, l_t^y) + \beta U(c_{t+1}^m, l_{t+1}^m) + \beta^2 U(c_{t+2}^o)\},$$

- We define the budget constraint of young agent y as follow:

$$c_t^y + z^y a_t + \zeta^y k_t^i = \omega_t l_t^y,$$

- The budget of the middle-aged agent m is:

$$c_{t+1}^m + z^m a_{t+1} + \zeta^m k_{t+1}^i = \omega_{t+1} l_{t+1}^m + z^y a_t r_t + ((1-\delta) + r_t^k)\zeta^y k_t^i,$$

- The budget of the old agent o is:

$$c_{t+2}^o = z^m a_{t+1} r_{t+1} + ((1-\delta) + r_{t+1}^k)\zeta^m k_{t+1}^i.$$

- Finally, the law of motion for capital is given by:

$$k_t^i = (1-\delta) k_{t-1}^i + f(i) i_t^i,$$

- Adjustment cost function Next slide.
- Households optimality conditions. Next slide.

Capital Producers and Firms

- We assume that capital producer maximizes the expected profits subject to capital accumulation by choosing capital k_t^p and investment i_t^p

$$\text{maximize} \quad E_t[r_{t-1}^k k_{t-1}^p - i_t^p]$$

- The law of motion for capital

$$k_t^p = (1-\delta)k_{t-1}^p + f(i)i_t^p,$$

  - Adjustment cost function ▸ Next slide.
  - Optimality conditions ▸ Next slide.
- Each producer operates the following technology:

$$y_t = z_t(k_t)^\alpha (l_t^y + l_t^m)^{1-\alpha},$$

  - Producers optimality conditions. ▸ Next slide

Aggregation, Monetary Policy Rule, and Government Budget Constraint

- We define the market clearing condition:

$$y_t = c_t + i_t$$

- The market for capital and labor clear

$$k_t = k_t^i + k_t^p, \quad l_t = l_t^y + l_t^m.$$

- Monetary policy rule:

$$\ln \iota_t = \rho^r \ln \iota_{t-1} + \rho^y \ln y_t + \epsilon_t,$$

- The government budget constraint

$$a_t r_t = g_t + a_{t-1}$$

Next slide

Theoretical Framework

- We consider the optimality condition:

$$\beta^2\left(z^m a_t r_t + ((1-\delta) + r_t^k)\zeta^m k_t^i\right)^{-\sigma} = \left(\omega_t l_t^y - z^y a_t - \zeta^y k_t^i\right)^{-\sigma} \frac{\beta z^y r_t \zeta^y}{z^m},$$

- Proposition 1. *Under non-linear optimal saving function, it follows that an increases in youth labor supply l_t^y causes a rise in aggregate savings a_t, $\frac{\partial a_t}{\partial l_t^y} > 0$.*
- Proposition 2. *A relative increase in the average wage rate lead to higher aggregate saving, $\frac{\partial a_t}{\partial \omega_t} > 0$.*
- Proposition 3. *The optimal saving function implies that in response to an increase in corporate equity, aggregate saving experience a sustained decline, $\frac{\partial a_t}{\partial k_t} < 0$. A relative increase in capital rental rate leads to a drop in aggregate saving, $\frac{\partial a_t}{\partial r_t^k} < 0$.*
- Proposition 4. *Under the houshold saving optimal function, it follows that a fall in interest rate translates into a decline aggregate saving, $\frac{\partial a_t}{\partial r_t} > 0$.*

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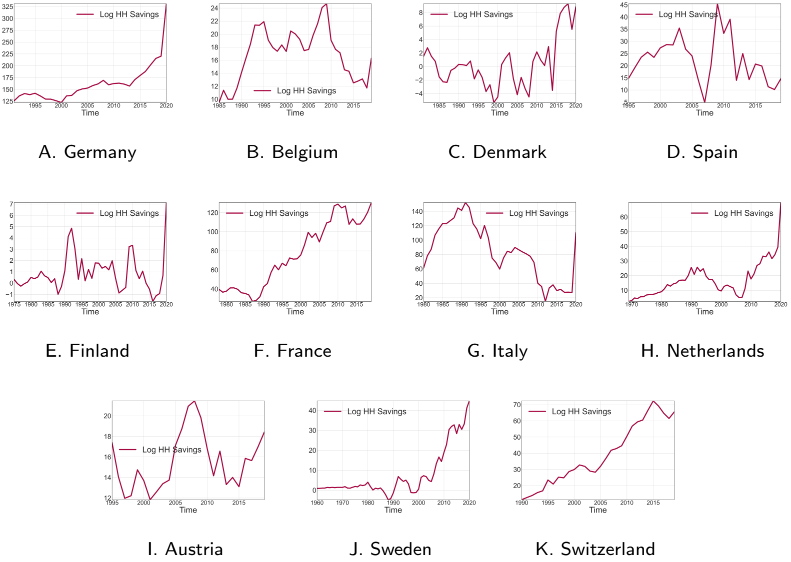
Optimal Saving Function: Specification and Econometric Estimation

- We consider the optimality condition:

$$\beta^2\left(z^m a_t r_t + ((1-\delta) + r_t^k)\zeta^m k_t^i\right)^{-\sigma} = \left(\omega_t l_t^y - z^y a_t - \zeta^y k_t^i\right)^{-\sigma} \frac{\beta z^y r_t \zeta^y}{z^m},$$

- We identify the origin of personal saving by taking the logs of terms of the optimal saving function:

$$\ln a_t = \frac{z^y \zeta^y}{z^{y2}\zeta^y + \beta z^{m2}} \ln \omega_t - \frac{\zeta^y z^y + \beta z^m \sigma}{\sigma(z^{y2}\zeta^y + \beta z^{m2})} \ln r_t + \frac{z^y \zeta^y}{z^{y2}\zeta^y + \beta z^{m2}} \ln l_t^y$$
$$- \frac{z^y \zeta^y - \beta(\delta - 2)\zeta^m z^m}{z^{y2}\zeta^y + \beta z^{m2}} \ln k_t^i - \frac{\beta z^m \zeta^m}{z^{y2}\zeta^y + \beta z^{m2}} \ln r_t^k.$$



A. Germany B. Belgium C. Denmark D. Spain

E. Finland F. France G. Italy H. Netherlands

I. Austria J. Sweden K. Switzerland

Figure 1: Dynamics of Household Saving in Europe

Notes: The data is in Log and at year level over the period 1960-2020.Source: Datastream.

Specification and Econometric Estimation

- To estimate the aggregate effect on personal saving at the country level using the following panel regression:

$$\ln a_{n,t} = \varsigma_0 + \varsigma_\omega \ln \omega_{n,t} + \varsigma_l \ln l^y_{n,t} + \varsigma_r \ln r_{n,t} + \varsigma_{r^k} \ln r^k_{n,t} + \varsigma_k \ln k_{n,t} + \varsigma_d \ln d_{n,t} + \varsigma_t + \varsigma_n + \epsilon_{n,t}, \quad (0.1)$$

- Annual data on: the net saving of households, the central bank policy rate, the average compensation per hour worked, the hours worked by young, return on equity, capital, and the demographic variables.
Next slide
- The panel covers the period from 1960 to 2020.
- The data-set includes the following European countries: Italy, Spain, Netherlands, Sweden, Switzerland, Denmark, Austria, Belgium, Finland, Germany, and France.

The Effect of Wage Rate on Aggregate Saving - Evidence in the Cross-section

	Dependent variable: Aggregate saving $\ln(a)$			
	(1)	(2)	(3)	(4)
Constant	-5.23**	1.28	0.08	-0.85
	(2.28)	(1.40)	(0.39)	(0.78)
Average wage rate $\ln(\omega)$	2.59***	0.52	0.90***	1.20***
	(0.73)	(0.45)	(0.12)	(0.25)
Country fixed effects	Yes	No	Yes	No
Time fixed effects	Yes	Yes	No	No
No. Observations	289	289	289	289
H_0: OLS model with no FE is preferred				
OLS model with Time and Country FE				
Chi-2 Statistic		4.16		
P-value		0.12		
Decision		Accept H_0		

This table shows the results from ordinary least squares regressions over the sample (1960-2020), (1) with country and time fixed effects; (2) with time fixed effects; (3) with country fixed effects; (4) with no fixed effects. Statistical significance (Std. error in parentheses): 0.1*, 0.05**, 0.01 ***.

Youth Labor Supply and Aggregate Saving

	Dependent variable: Aggregate saving $\ln(a)$			
	(1)	(2)	(3)	(4)
Constant	-25.19***	-1.29	-14.53***	-1.23
	(3.24)	(0.92)	(3.27)	(0.89)
Youth labor $\ln(l^y)$	2.04***	0.32***	1.27***	0.31***
	(0.23)	(0.07)	(0.24)	(0.06)
Country fixed effects	Yes	No	Yes	No
Time fixed effects	Yes	Yes	No	No
No. Observations:	251	251	251	251
H_0: OLS model with no FE is preferred				
OLS model with Time and Country FE				
Chi-2 Statistic			58.95	
P-value			0.00	
Decision			Reject H_0	

This table shows the results from ordinary least squares regressions over the sample (1960-2020), (1) with country and time fixed effects; (2) with time fixed effects; (3) with country fixed effects; (4) with no fixed effects. Statistical significance (Std. error in parentheses): 0.1*, 0.05**, 0.01 ***.

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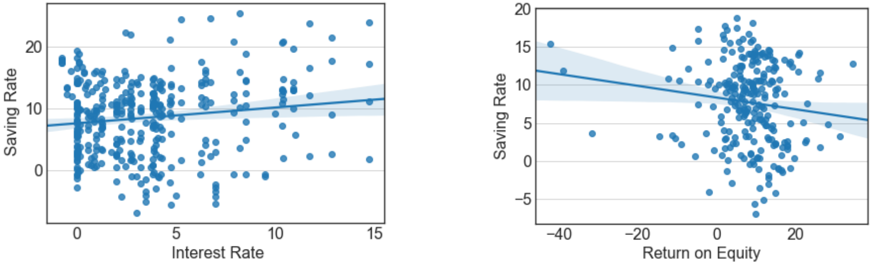
Interest Rate, Return on Equity, and Saving Rate

- An increase in interest rate is associated with an increase in saving rate.
- Saving rate is decreasing with the return on equity.



A. Saving Rate vs Interest Rate

B. Saving Rate vs Return on Equity

Figure 2: Saving Rate, Return on Equity and Interest Rate, Country Level

Optimal Saving Function

	Dependent variable: Aggregate saving $\ln(a)$				
	(1)	(2)	(3)	(4)	(5)
Constant	-3.61	-5.42	-11.49	-8.05	-1.74
	(9.56)	(8.14)	(13.17)	(18.71)	(10.34)
Average wage rate $\ln(\omega)$	0.80	0.81	0.60	0.55	0.59
	(1.06)	(1.05)	(1.05)	(1.07)	(1.06)
Capital $\ln(k)$	-0.92**	-0.85*	-1.13**	-1.13**	-1.03**
	(0.44)	(0.47)	(0.53)	(0.53)	(0.47)
Youth labor supply $\ln(l^y)$	1.36***	1.24**	1.53***	1.51***	1.44***
	(0.36)	(0.48)	(0.35)	(0.35)	(0.30)
Return on equity $\ln(r^k)$	0.01	0.01	0.01	0.01	0.01
	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)
Interest rate $\ln(r)$	0.53***	0.53***	0.55***	0.55***	0.54***
	(0.20)	(0.20)	(0.20)	(0.20)	(0.20)
% of pop. ages $0-14$	-0.46				
	(1.54)				
% of pop. ages $15-24$		0.45			
		(0.93)			
% of pop. ages $25-64$			2.03	1.54	
			(3.30)	(3.80)	
% of pop. ages over 65				-0.37	-0.65
				(1.41)	(1.22)
No. Observations:	177	177	177	177	177
Country fixed effects	Yes	Yes	Yes	Yes	Yes
Time fixed effects	Yes	Yes	Yes	Yes	Yes

This table shows the results from ordinary least squares regressions over the sample (1960-2020). Independent variables are: average wage rate, capital, youth labor supply, return on equity, interest rate, the share of population between 0-14 years, the share of population between 15-24 years, the share of population between 25-64 years, the share of population over 64 years, time and country fixed effects. Statistical significance (Std. error in parentheses): 0.1*, 0.05**, 0.01 ***.

Optimal Saving Function: Additional Results

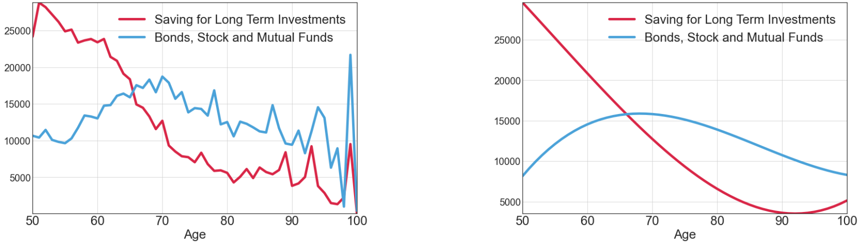
- An inverse relationship between corporate equity and saving at country level.
- Households are willing to substitute savings for corporate equity (offer high returns).

	Dependent variable: Aggregate saving $\ln(a)$		
	(1)	(2)	(3)
Constant	-5.12	2.91***	17.05***
	(8.10)	(0.17)	(5.81)
Return on equity $\ln(r^k)$	0.01	0.08	
	(0.08)	(0.08)	
Capital $\ln(k)$	-0.94**		-0.91**
	(0.43)		(0.38)
Youth labor supply $\ln(l^y)$	1.42***		
	(0.30)		
Interest rate $\ln(r)$	0.53***		
	(0.20)		
Average wage rate $\ln(\omega)$	0.72		
	(1.03)		
No. Observations:	177	186	223
Country fixed effects	Yes	Yes	Yes
Time fixed effects	Yes	Yes	Yes

This table shows the results from ordinary least squares regressions over the sample (1960-2020). Independent variables are: average wage rate, capital, youth labor supply, return on equity, interest rate, country and time dummies. Statistical significance (Std. error in parentheses): 0.1*, 0.05**, 0.01***.

Savings by age

- How European households over 50 years alternate their saving decisions?
    - Saving in long term investment is not sustainable after the age of 50.
    - Agents appear to prefer long term saving before retirement. When agents reach the retirement age, they are willing to substitute long term savings for corporate bonds.



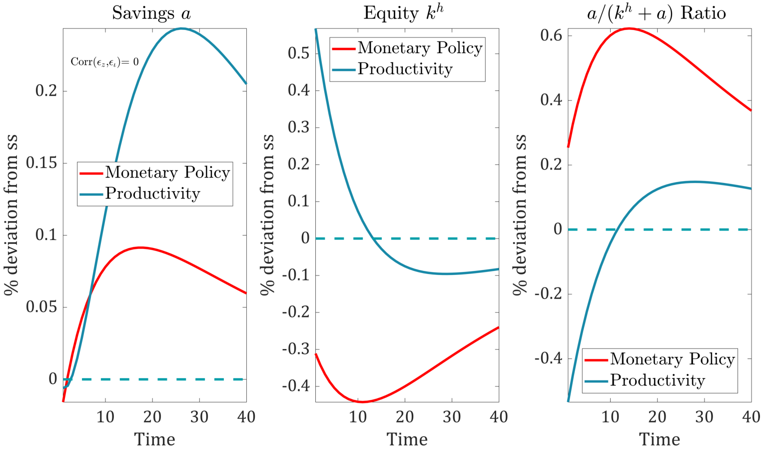
A. Savings by Age

B. Filtered Data

Figure 3: Household Savings by Age

Notes: We take the mean value of saving for long term investment and for bonds, stocks and mutual funds by age. The data is fitted to polynomial of order three. Source: The Survey of Health, Ageing and Retirement.

The Response of Saving to Productivity and Monetary Policy Shocks



Calibration: Next slide. Model Fit: Next slide. Model vs Data: Next slide. Correlation Matrix: Next slide.

Conclusion

We propose an overlapping generations model to understand the factors behind the saving behavior in Europe.

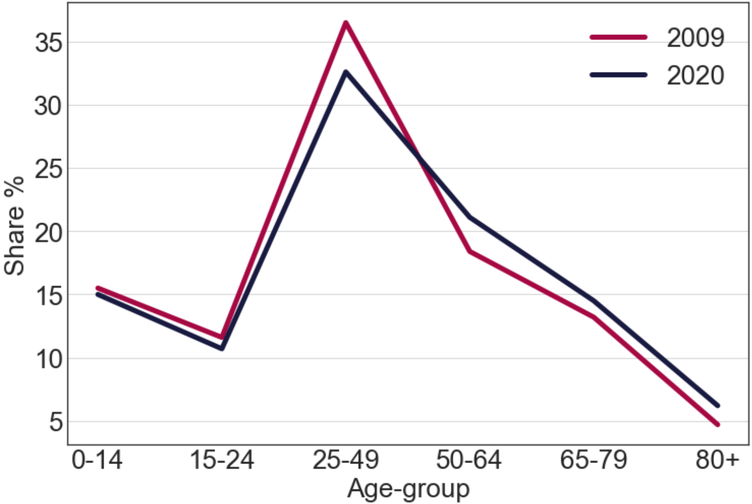
- We find strong evidence that an increase of youth labor supply leads to a rise in aggregate saving.
  - An increase of 1 percent of the hours worked by young people lead on average to a higher aggregate saving by 2.04 percent.
- Our empirical analysis reveals a positive relationship between interest rate and saving behavior.
- A negative relationship between capital and aggregate saving is confirmed by our empirical investigation.

Thank You . . .

Share of Age Group between 2009 and 2020

The demographic structure can potentially affect saving.

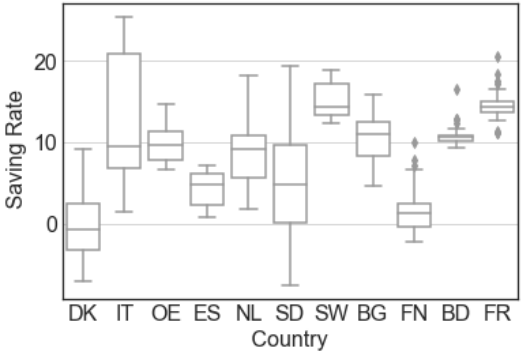
Figure 4: Share of Age Group between 2009 and 2020



Divergence in saving rate

- The distribution of saving rate is more volatile in Sweden and Italy. There is low variability over time for France and Germany. Saving rate in Denmark and Finland tend to concentrate around negative values.

Figure 5: Saving Rate across Europe



Note: Annual saving rate for the period going from 1960 to 2020 across European countries. Source: OECD

Share of Age Group between 2009 and 2020

Table 1: Population demographics in Europe

Economy	Proportion of Population							
	0-14		15-24		25-64		65+	
	2009	2019	2009	2019	2009	2019	2009	2019
Belgium	16.9	16.9	12.1	11.4	53.9	52.8	17.1	18.9
Denmark	18.3	16.5	12.0	12.6	53.8	51.3	15.9	19.5
Germany	13.6	13.6	11.4	10.4	54.6	54.4	20.4	21.6
Spain	14.8	14.8	10.9	9.8	57.7	56.0	16.6	19.4
France	18.5	18.0	12.6	11.8	52.4	50.2	16.5	20.1
Italy	14.1	13.2	10.1	9.8	55.6	54.3	20.3	22.9
Netherlands	17.7	15.9	12.2	12.3	55.1	52.7	15.0	19.2
Austria	15.1	14.4	12.3	10.9	55.2	55.7	17.4	18.8
Finland	16.7	16.0	12.4	11.2	54.1	51.1	16.8	21.8
Sweden	16.7	17.8	13.2	11.3	52.3	51.0	17.7	19.9
Switzerland	15.3	15.0	11.9	10.6	56.1	55.9	16.6	18.5

Source: Eurostat.

Back

Model Equilibrium

- The household optimality conditions:

$$\begin{aligned}
\lambda_t &= (c_t^y)^{-\sigma} \\
(l_t^y)^\eta &= \lambda_t \omega_t \\
\frac{\lambda_t}{\beta} &= (c_{t+1}^m)^{-\sigma} \\
\beta (l_{t+1}^m)^\eta &= \lambda_t \omega_{t+1} \\
\lambda_t z^m &= E_t \beta \lambda_{t+1} z^y r_{t+1} \\
\lambda_t \zeta^m &= E_t \beta \lambda_{t+1} ((1-\delta) + r_{t+1}^k) \zeta^y \\
\lambda_t &= \beta^2 (c_{t+2}^o)^{-\sigma}
\end{aligned}$$

Household Euler equation

$$\beta^2 \left(z^m a_{t+1} r_{t+1} + ((1-\delta) + r_{t+1}^k) \zeta^m k_{t+1}^i \right)^{-\sigma}$$
$$= \left(\omega_{t+1} l_{t+1}^y - z^y a_{t+1} - \zeta^y k_{t+1}^i \right)^{-\sigma} \frac{\beta z^y r_{t+1} \zeta^y}{z^m}$$

Back

Model Equilibrium

- Capital producer:
  the first order condition with respect to capital is

$$E_t \beta \chi_{t+1}(r_t^k + (1-\delta)) = \chi_t$$

  The first order condition with respect to investment is given by

$$\chi_t = \chi_t \left[1 - \kappa \frac{i_t^p}{i_{t-1}^p}\left(\frac{i_t^p}{i_{t-1}^p} - 1\right) - \frac{\kappa}{2}\left(\frac{i_t^p}{i_{t-1}^p} - 1\right)^2\right] + \beta \chi_{t+1}\left[\kappa \left(\frac{i_{t+1}^p}{i_t^p}\right)^2 \left(\frac{i_{t+1}^p}{i_t^p} - 1\right)\right] \tag{0.2}$$

Back

- Producer first order conditions

$$\mu_t = \frac{\omega_t}{(1-\alpha) k_t^{\alpha} (l_t^y + l_t^m)^{-\alpha}},$$

$$\mu_t = \frac{r_t^k}{\alpha (l_t^y + l_t^m)^{1-\alpha} k_t^{\alpha - 1}}.$$

Back

Calibration

Table 2: Model Parameters

Variable	Symbol	Value	Target / Source
Panel A: Calibrated parameters			
Curvature on the disutility of labor	η	0.54	Ziliak and Kniesner (2005)
Coef. of relative risk aversion	σ	1	Standard in literature
Depreciation rate (Annual)	δ	0.025	Krusell and Smith (2015)
Cobb Douglas parameter	α	0.3	Standard in literature
Discount factor (Annual)	β	0.985	The ECB policy rate ι
Capital to labor ratio	$\frac{k}{l}$	0.62	Datastream
Investment adjustment cost	κ	2.48	Christiano et al. (2014)
Panel B: Exogenous shock parameters			
Coefficient on lagged interest rate	ρ	0.845	Albonico et al. (2017)
Coefficient on output	ρ^y	0.0592	Albonico et al. (2017)
Standard deviation	σ^r	0.520	Albonico et al. (2017)
Coefficient on gov. revenues	ρ^g	0.89	Albonico et al. (2017)
Standard deviation	σ^g	0.0012	Albonico et al. (2017)
Coefficient on productivity	ρ^z	0.87	Uhlig (2007)
Standard deviation	σ^z	0.0069	Uhlig (2007)

Back

Model Fit

Table 3: Data

	Ratios				
	$\frac{c}{y}$	$\frac{k}{y}$	$\frac{k}{l}$	$\frac{l^y}{l}$	$\frac{l^m}{l}$
Italy	0.79	6.72	0.23	0.07	0.93
Spain	0.77	6.05	0.17	0.08	0.91
Netherlands	0.68	4.81	0.22	0.15	0.85
France	0.77	5.45	0.24	0.09	0.91
Denmark	0.70	5.05	2.05	0.15	0.86
Austria	0.71	9.13	0.36	0.13	0.87
Belgium	0.75	5.75	0.27	0.08	0.92
Finland	0.75	5.49	0.23	0.11	0.89
Germany	0.73	5.18	0.22	0.11	0.89
Sweden	0.71	5.40	2.31	0.10	0.90
Switzerland	0.63	5.57	0.44	0.14	0.86

Source: Datastream and Eurostat.
The consumption to output ratio is given c/y, the capital to output ratio is k/y, and the capital to labor ratio is k/l.
The share of youth labor in labor force is given by l^y/l, and the share of middle-age workers in labor force is l^m/l.

Back

Model Fit

Table 4: Model vs Data

Ratio	Description	Model	Data
k/l	Capital to labor ratio	0.62	0.42
k/y	Capital to output ratio	4.13	5.78
c/y	Consumption to output ratio	0.82	0.72
l^y/l	Share of young people in labor force	0.10	0.10
l^m/l	Share of middle aged people in labor force	0.90	0.90

▸ Back

Correlation Matrix

Table 5: Correlation Matrix

Variables	a	r	r^k	ω	l^y	l^m	k^i	i^i
Savings a	1.00							
Policy rate r	-0.07	1.00						
Return on capitalr^k	-0.07	1.00	1.00					
Wage rate ω	0.19	0.59	0.59	1.00				
Hours worked (young) l^y	-0.46	0.54	0.54	0.79	1.00			
Hours worked (middle-aged) l^m	-0.45	0.56	0.56	0.79	1.00	1.00		
Household capital k^i	-0.45	0.55	0.55	0.79	1.00	1.00	1.00	
Household investment i^i	-0.06	-0.13	-0.13	0.27	0.30	0.30	0.30	1.00

Table shows the correlation matrix of the calibrated model.

Back

Capital Accumulation

The law of motion for capital is given by:

$$k_t^i = (1-\delta)k_{t-1}^i + f(i)i_t^i, \tag{0.3}$$

The quantity of investment at period t is proportional to the adjustment cost function $f(i) = \left[1 - \frac{\kappa}{2}\left(\frac{i_t^i}{i_{t-1}^i} - 1\right)^2\right]$.

Back

Capital Accumulation

The law of motion for capital is given by:

$$k_t^p = (1-\delta)k_{t-1}^p + f(i)i_t^p, \qquad (0.4)$$

The quantity of investment at period t is proportional to the adjustment cost function $f(i) = \left[1 - \frac{\kappa}{2}\left(\frac{i_t^p}{i_{t-1}^p} - 1\right)^2\right]$.

Back

Government Revenues

We further assume that government does not optimize and government policies are assumed to be exogenous. Let assume that net government revenues shock is given by

$$\ln g_t = \rho^g \ln g_{t-1} + \epsilon_t^g \tag{0.5}$$

where ρ^g is a smoothing parameter and ϵ_t^g is a shock to government revenues.

Back

Data

- Cross-country data.
- Covers the period from 1960 to 2020.

Variable	Source
Household savings	OECD
Gross Domestic Product	Oesterreichische Nationalbank, Deutsche Bundesbank, OECD, Eurostat.
Purchasing power parities	OECD
Capital Stock	University of Groningen
Hours Worked	Eurostat, INE - National Statistics Institute, DG ECFIN AMECO, Statistics Sweden
Employment by age	Eurostat
Demographic structure	Eurostat

Back

Proof

$$\frac{\partial a_t}{\partial r_t} = \frac{\frac{\beta z^y \zeta^y}{z^m\left(-k_t\zeta^y - a_t z^y + l_t^y \omega_t\right)^{\sigma}} + a_t \beta^2 \sigma z^m \left(k_t \left(r_t^k - \delta + 1\right)\zeta^m + a_t r_t z^m\right)^{-\sigma-1}}{-\frac{\beta r_t \sigma z^{y2} \zeta^y \left(-k_t\zeta^y - a_t z^y + l_t^y \omega_t\right)^{-\sigma-1}}{z^m} - \beta^2 r_t \sigma z^m \left(k_t \left(r_t^k - \delta + 1\right)\zeta^m + a_t r_t z^m\right)^{-\sigma-1}}$$

The effects on saving is straightforward:

$$\frac{\partial a_t}{\partial r_t} > 0$$

Back

Proof

$$\frac{\partial a_t}{\partial \omega_t} =$$

$$-\frac{\beta l_t^y r_t \sigma z^y \zeta^y \left(-k_t \zeta^y - a_t z^y + l_t^y \omega_t\right)^{-\sigma-1}}{-\beta r_t \sigma z^{y2} \zeta^y \left(-k_t \zeta^y - a_t z^y + l_t^y \omega_t\right)^{-\sigma-1} - \beta^2 r_t \sigma z^{m2} \left(k_t \left(r_t^k - \delta + 1\right) \zeta^m + a_t r_t z^m$$

The effects on saving is straightforward:

$$\frac{\partial a_t}{\partial \omega_t} > 0$$

Proof

$$\frac{\partial a_t}{\partial l_t^y} =$$

$$-\frac{\beta \omega_t r_t \sigma z^y \zeta^y \left(-k_t \zeta^y - a_t z^y + l_t^y \omega_t\right)^{-\sigma-1}}{-\beta r_t \sigma z^{y2} \zeta^y \left(-k_t \zeta^y - a_t z^y + l_t^y \omega_t\right)^{-\sigma-1} - \beta^2 r_t \sigma z^{m2} \left(k_t \left(r_t^k - \delta + 1\right) \zeta^m + a_t r_t z^m\right.}$$

The effects on saving is straightforward:

$$\frac{\partial a_t}{\partial l_t^y} > 0$$

Proof

$$\frac{\partial a_t}{\partial r_t^k} = \frac{\beta^2 k_t \sigma \zeta^m \left(k_t \left(r_t^k - \delta + 1\right) \zeta^m + a_t r_t z^m\right)^{-\sigma-1}}{-\frac{\beta r_t \sigma z^{y2} \zeta^y \left(-k_t \zeta^y - a_t z^y + l_t^y \omega_t\right)^{-\sigma-1}}{z^m} - \beta^2 r_t \sigma z^m \left(k_t \left(r_t^k - \delta + 1\right) \zeta^m + a_t r_t z^m\right)^{-\sigma-1}}$$

The effects on saving is straightforward:

$$\frac{\partial a_t}{\partial r_t^k} < 0$$

Proof

$$\frac{\partial a_t}{\partial k_t} = \frac{\frac{\beta r_t \sigma z^y \zeta^{y2}\left(-k_t \zeta^y - a_t z^y + l_t^y \omega_t\right)^{-\sigma-1}}{z^m} + \beta^2 \left(r_t^k - \delta + 1\right) \sigma \zeta^m \left(k_t \left(r_t^k - \delta + 1\right) \zeta^m + a_t r_t z\right.}{-\frac{\beta r_t \sigma z^{y2} \zeta^y \left(-k_t \zeta^y - a_t z^y + l_t^y \omega_t\right)^{-\sigma-1}}{z^m} - \beta^2 r_t \sigma z^m \left(k_t \left(r_t^k - \delta + 1\right) \zeta^m + a_t r_t z^m\right)^{-}}$$

The effects on saving is straightforward:

$$\frac{\partial a_t}{\partial k_t} < 0$$

Savings and Interest Rate

	Dependent variable: Aggregate saving $\ln(a)$				
	(1)	(2)	(3)	(4)	(5)
Constant	3.27***	-21.18***	-19.31***	-1.07	175.33
	(0.15)	(3.77)	(3.27)	(1.21)	(157.37)
Interest rate $\ln(r)$	-0.16**	0.33*	0.32*	-0.08	0.07
	(0.07)	(0.18)	(0.18)	(0.09)	(0.08)
Average wage rate $\ln(\omega)$		0.79		1.32***	1.49***
		(0.80)		(0.36)	(0.46)
Capital $\ln(k)$					0.74***
					(0.12)
Youth labor supply $\ln(l^y)$		1.54***	1.59***		0.19**
		(0.24)	(0.24)		(0.08)
Return on equity $\ln(r^k)$					-0.09
					(0.14)
% of pop. ages $0-14$					-14.40*
					(7.67)
% of pop. ages $15-24$					-3.37
					(5.57)
% of pop. ages $25-64$					-27.87
					(25.24)
% of pop. ages over 65					-10.90
					(7.81)
Country fixed effects	No	Yes	Yes	No	No
Time fixed effects	No	Yes	Yes	No	No
No. Observations:	288	207	208	243	177

This table shows the results from panel regressions over the sample (1960-2020). Independent variables are: interest rate, youth labor, average wage rate, capital, return on equity, the share of population between 0 and 14 years, the share of population between 15 and 24 years, the share of population between 25 and 64 years, the share of population over 65 years, country and time dummies. Statistical significance (Std. error in parentheses): 0.1*, 0.05**, 0.01 ***.

What is the Effect of Aging on Aggregate Saving?

	Dependent variable: Aggregate saving $\ln(a)$			
	(1)	(2)	(3)	(4)
const	-4.79**	-7.98***	36.55***	12.56***
	(2.05)	(1.19)	(6.42)	(1.95)
% of pop. ages 0-14	2.63***			
	(0.72)			
% of pop. ages 15-24		4.20***		
		(0.47)		
% of pop. ages 25-64			-8.51***	
			(1.62)	
% of pop. ages over 65				-3.55***
				(0.70)
Country fixed effects	Yes	Yes	Yes	Yes
Time fixed effects	Yes	Yes	Yes	Yes
No. Observations:	388	388	388	388

This table shows the results from panel regressions over the sample (1960-2020). Independent variables are: the share of population between 0 and 14 years, the share of population between 15 and 24 years, the share of population between 25 and 64 years, the share of population between over 65 years, country fixed effects, and time fixed effects. Statistical significance (Std. error in parenthese): 0.1*, 0.05**, 0.01 ***.

▸ Back