

The Tipping Point: Interest Rates and Financial Stability^{a,b}

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^aLink to the *paper's latest version* and *slides' latest version* on www.dporcellacchia.com.

^bThis paper represents my own views, not necessarily those of the European Central Bank or Eurosystem.

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Very quick answer:

- Theoretically, it depends.
→ Valuation effect vs margin compression.

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→ Valuation effect vs margin compression.
- Quantitatively, sufficiently *low* rates are destabilizing.

Literature

1. *Effect of interest rates on bank value.*

Duration-gap view: Kaufman (1984), Gertler and Kiyotaki (2010), Gertler and Karadi (2011), English, van den Heuvel, and Zakrajšek (2018), and Akinci et al. (2021).

Deposit-franchise view: Borio, Gambacorta, and Hofmann (2017), Drechsler, Savov, and Schnabl (2017), Di Tella and Kurlat (2021), and Drechsler, Savov, and Schnabl (2021).

2. *Bank stability:* Allen and Gale (1998), Gertler and Kiyotaki (2015), and Segura and Suárez (2017).

3. *Low rates.*

Credit supply: Brunnermeier and Koby (2019), Ulate (2021), and Altavilla et al. (2022).

Risk taking: Maddaloni and Peydró (2011), Jiménez et al. (2014), Dell’Ariccia, Laeven, and Marquez (2014), Di Maggio and Kacperczyk (2017), Martinez-Miera and Repullo (2017), Heider, Saidi, and Schepens (2019), and Whited, Wu, and Xiao (2021).

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Result #1: Condition for dominant effect.

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Result #2: Tipping point.

- Simple analytical solution.
- Quantitatively, interest rate below 0.32% generates bank instability.

- Unit measure of infinitely-lived households with
- unit endowment at time 0.

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Preferences:

- Households uncertain about timing of consumption $\theta \in \{1, 2, \dots\}$ with $\theta \sim \text{Geo}(\phi)$.

$$E_0(\mathcal{U}) = \phi \cdot u(C_1) + (1 - \phi) \cdot \phi \cdot u(C_2) + (1 - \phi)^2 \cdot \phi \cdot u(C_3) + \dots \quad (1)$$

- Flow utility u has constant relative risk aversion $1/\alpha > 1$.

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Investment:

1. Productive technology K :
 - one-period net return $\rho > 0$,
2. Storage technology S :
 - one-period net return 0,

$\rightarrow K \succ S$.

1. Households

- hold deposits or storage.
- ZLB on deposit rate.

2. Banks

- lends to firms via long-term bonds and
- borrows via deposits.

3. Firms

- operate the productive technology.

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- Bond issued at time $t - 1$ is equivalent to δ new bonds issued at t .

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By **arbitrage**, $1 + \rho = (1 + \delta \cdot q_{t+1}^*)/q_t^*$. With no-bubble condition,

$$q_t^* = \frac{1}{1 + \rho - \delta}.$$

Households

- At time 0, uses unit endowment to purchase deposit contract $D_0 = 1$.
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- Deposit contract specifies deposit rates $\{r_t\}_{t=0}^{+\infty}$.
- At a given time t ,
 - Impatient households (i.e., $\theta = t$) withdraw their deposits.
 - Patient households (i.e., $\theta \neq t$) do not withdraw $\iff r_t \geq 0$.
 - Households' outside option is storage.

Bank

At time 0, competitive banks choose $\{B_{t+1}, D_{t+1}, r_t\}_{t=0}^{+\infty}$ to maximize

$$\sum_{t=1}^{+\infty} (1 - \phi)^{t-1} \cdot \phi \cdot u(D_t) \quad (3)$$

subject to budget constraints

$$q_0 \cdot B_1 = D_0 = 1, \quad (4)$$

$$q_t \cdot B_{t+1} + \phi \cdot (1 - \phi)^{t-1} \cdot D_t = (1 + \delta \cdot q_t) \cdot B_t \quad \text{for all } t \geq 1, \quad (5)$$

$$D_{t+1} = (1 + r_t) \cdot D_t, \quad (6)$$

a boundary condition, and incentive-compatibility constraints

$$r_t \geq 0 \quad \text{for all } t \geq 1. \quad (7)$$

Bank failure

The bank fails at time $s \iff$ there exists no $\{r_t\}_{t=s}^{\infty} \geq 0$ that is feasible.

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Proposition 1 (Solvency condition)

At time $t \geq 1$, the bank does not fail



$$\underbrace{(1 + \delta \cdot q_t) \cdot B_t}_{\text{Bank-asset value}} \geq \frac{\phi + \rho}{\phi \cdot (1 + \rho)} \cdot \underbrace{(1 - \phi)^{t-1} \cdot D_t}_{\text{Outstanding deposits}} \quad (8)$$

Proposition 2 (PF equilibrium conditions)

PF equilibrium implies

$$1 + r_t^* = (1 + \rho)^\alpha \quad \text{for all } t \geq 1, \quad (9)$$

$$(1 + \delta \cdot q_t^*) \cdot B_t^* = \frac{\phi \cdot (1 + \rho)^{1-\alpha}}{(1 + \rho)^{1-\alpha} - (1 - \phi)} \cdot (1 - \phi)^{t-1} \cdot D_t^* \quad \text{for all } t \geq 1, \quad (10)$$

and q_t^* given by no-arbitrage condition (2).

→ With infinite risk aversion (i.e., $\alpha \rightarrow 0$), $r_t^* = 0$.

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→ With infinite risk aversion (i.e., $\alpha \rightarrow 0$), $r_t^* = 0$.

In PF equilibrium, IC never binding and no bank failure.

Deposit-franchise interpretation

- Interest margin.

$$1 + m_t \stackrel{\text{def}}{=} \frac{1 + \rho}{1 + r_t}. \quad (11)$$

→ In PF equilibrium, $m_t^* > 0$.

- Per-unit deposit franchise.

$$f(\{m_t\}) \stackrel{\text{def}}{=} \underbrace{-\phi - \phi \cdot (1 - \phi) \cdot \frac{1 + r_t}{1 + \rho} - \dots}_{\text{Value of cashflows}} - \underbrace{(-1)}_{\text{Face value}}. \quad (12)$$

→ $\{m_t\} > 0 \implies f(\{m_t\}) > 0$.

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Corollary 1 (Solvency condition with deposit-franchise interpretation)

At time $t \geq 1$, the bank does not fail

$$\underbrace{(1 + \delta \cdot q_t) \cdot B_t}_{\text{Bank asset value}} + \underbrace{f(\{\rho\}) \cdot (1 - \phi)^{t-1} \cdot D_t}_{\text{Deposit franchise}} \overset{\updownarrow}{=} \underbrace{-(1 - \phi)^{t-1} \cdot D_t}_{\text{Deposit face value}} \geq 0.$$

Interest-rate shock

Consider an unanticipated and persistent shock $\rho \rightarrow \hat{\rho}$ at time $t \geq 1$.

$$\underbrace{(1 + \delta \cdot \hat{q}_t)}_{\text{Valuation} \ominus} \cdot B_t^* + \underbrace{f(\{\hat{\rho}\})}_{\text{Margin} \oplus} \cdot (1 - \phi)^{t-1} \cdot D_t^* - (1 - \phi)^{t-1} \cdot D_t^* \geq 0? \quad (13)$$

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Proposition 4a (Dominant effect)

Three parametric regions:

1. Given $\delta < 1 - \phi$, then bank fails $\iff \hat{\rho} < \rho^{\text{TP}}$.
2. Given $\delta > (1 - \phi) \cdot (1 + \rho)^\alpha$, then bank fails $\iff \hat{\rho} > \rho^{\text{TP}}$.
3. In intermediate parameter region, bank is fully resilient to the interest-rate shock.

- US data from 1997-2007.

Model	Empirical counterpart	Value	Source
$\delta/(1 + \rho - \delta)$	Average bank-asset repricing time (years)	4.5	English, van den Heuvel, and Zakrajšek (2018)
f^*	Average per-unit deposit franchise	20%	Sheehan (2013)

Parameter	Description	Value
δ	Common ratio of coupons' progression	85%
$1 - \phi$	Probability of staying patient	95%

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$\delta < 1 - \phi$. Quantitatively, margin effect dominates. Beware low rates!

- For valuation effect to dominate, we need bank-asset duration of 18 years.

Tipping point

Proposition 4b (Tipping point)

The critical tipping point is given by

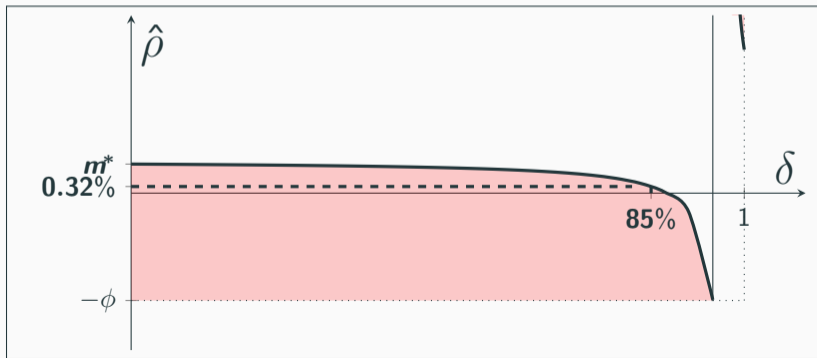
$$\rho^{\text{TP}} = m_t^* - \delta \cdot \frac{(\rho - m_t^*) \cdot (\phi + m_t^*)}{(1 - \phi) \cdot (1 + \rho) - \delta \cdot (1 + m_t^*)}. \quad (14)$$

Tipping point

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Effect of interest-rate shock on bank stability?

Method. Diamond-Dybvig model with fundamental runs plus:

- infinite-horizon and
- long-term assets.

Theory results. Margin effect vs revaluation effect.

- Condition for dominance.
- Tipping point.

Quantitative results. Margin effect dominates.

→ The threat to bank stability are low rates.

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Implications.

- Bank's maturity mismatch *alone* bad measure for interest-rate risk exposure.
- Effective lower bound on policy rates.


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Appendix

A **social planner** chooses $\{C_t, K_{t+1}\}_{t=1}^{+\infty}$ to maximise aggregate welfare

$$\sum_{t=1}^{+\infty} (1 - \phi)^{t-1} \cdot \phi \cdot u(C_t) \quad (15)$$

subject to resource constraints

$$K_{t+1} + (1 - \phi)^{t-1} \cdot \phi \cdot C_t = (1 + \rho) \cdot K_t \quad \text{for all } t \geq 1 \quad (16)$$

and initial condition $K_1 = 1$.

Efficiency requires

$$\frac{C_{t+1}}{C_t} = (1 + \rho)^\alpha \quad \text{for all } t \geq 1. \quad (17)$$

- $1/\alpha > 1 \implies$ relatively smooth consumption pattern.

1. Privately-observed type θ .
2. No fully-contingent deposit contract.
3. Households can only deposit at their bank or store.
 - Cost of direct finance (Diamond 1997) and switching cost.
4. Bank loans have fixed duration.

Bond selling

$B_t < \phi \cdot (1 - \phi)^{t-1} \cdot D_t \iff$ the bank sells bonds at time $t \geq 1$.

Proposition 5 (Failure and asset liquidation)

Consider $\delta \leq 1 - \phi$.

A bank does not fail at time $t \geq 1 \implies$ it does not sell bonds at any time $s \geq t$.

- For low enough bond duration, the coupon is always enough to pay off withdrawals as long as the bank is solvent.
- Hence, asset-liquidation costs are not relevant.

Equilibrium

Equilibrium is a sequence $\{B_t^f, B_t, D_t, K_t, q_t, r_t, \Pi_t\}_{t=0}^{+\infty}$ such that:

1. The firm chooses $\{B_t^f, K_t, \Pi_t\}_{t=0}^{+\infty}$ to solve its maximization problem, taking $\{q_t\}_{t=0}^{+\infty}$ as given.
2. The bank chooses $\{B_t, D_t, r_t\}_{t=0}^{+\infty}$ to solve its maximization problem, taking $\{q_t\}_{t=0}^{+\infty}$ as given.
3. If and only if there exists no $\{r_t\}_{t=s}^{+\infty}$ that is feasible and IC, then the bank fails at time s and its assets are paid out on a pro-rata basis.
4. Prices $\{q_t\}_{t=0}^{+\infty}$ ensure $B_{t+1}^f = B_{t+1}$ for all $t \geq 0$ subject to $\lim_{T \rightarrow +\infty} q_t^* \neq \pm\infty$.

Model	Empirical counterpart	Value	Source
ρ	Average fed funds rate	3.81%	FRED
r^*	Average interest rate on core deposits	2.39%	US Call Reports
$\delta/(1 + \rho - \delta)$	Average bank-asset repricing time (years)	4.46	English, van den Heuvel, and Zakrajšek (2018)
f^*	Average per-unit deposit franchise	20.2%	Sheehan (2013)

Parameter	Description	Value
ρ	Short-term interest rate	3.81%
$1/\alpha$	Coefficient of relative risk aversion	1.58
δ	Common ratio of coupons' progression	84.8%
ϕ	Household's probability of turning impatient	5.13%