Public Persuasion in Elections Single-Crossing Property and the Optimality of Censorship

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Motivation and Research Question

Motivation

- In modern democracies, many important choices are made through elections.
- Many actors (e.g., governments, politicians, media outlets, interest groups) may try to influence election outcomes by manipulating public information.
- From the normative perspective, what is the socially optimal way to provide public information?

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Research Question:

Given an objective, what is the optimal public information policy?

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We study public Bayesian persuasion in elections with following novel features:

- a wide class of designer preferences: any convex combination of *self-interest* and *social welfare*.
- **2** both monopolistic and competitive persuasion.

For today's talk: monopolistic persuasion by a single persuader.

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2 Single-crossing property and the optimality of censorship policy

3 Concluding remarks

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State and voter preferences

• Two alternatives R and SQ, with unknown state $k \sim F$ on [-1, 1].

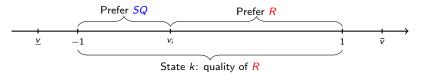


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State and voter preferences

• Two alternatives *R* and *SQ*, with unknown state $k \sim F$ on [-1, 1].



• There are n + 1 ex-ante identical voters.

- Each voter *i* has private type ("threshold-of-acceptance") $v_i \sim G$ drawn i.i.d.
- Voter *i*'s payoff is given by

$$\begin{cases} k - v_i, & \text{if } R \text{ wins} \\ 0, & \text{if } SQ \text{ wins} \end{cases}$$

 \implies Voter *i* strictly prefers *R* iff $k > v_i$.

• Voters do not directly observe k.

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An information designer can provide public information about k, but **does not** observe voters' type realizations (v_1, \dots, v_{n+1}) .

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Example 1 (Self-interested): Prefer R iff $k \ge \chi$ for some $\chi \in \mathbb{R}$.

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Example 1 (Self-interested): Prefer R iff $k \ge \chi$ for some $\chi \in \mathbb{R}$.

Example 2 (Utilitarian planner): Prefer R iff the average voter's payoff is higher under R.

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Example 3 ('Pro-Reform' planner): Prefer R iff the average payoffs of the "top 50%" of voters are higher under R.

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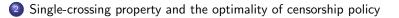
In general, the objective function can be any convex combination of 'self-interest' and any (rank-dependent) weighted average of voters' payoffs.
Details

Timing

- The designer sets an *information policy* π = (S, σ) that generates public signal s about the realized state k.
 - S is a sufficiently large signal space.
 - $\sigma(k): [-1,1] \mapsto \Delta(S)$ produces public signal s.
- Observing the public signal s and her private type v_i, each voter i simultaneously makes her voting decision (vote for R or SQ).
- Solution outcome is decided by *q*-rule: R wins iff it receives $\geq nq + 1$ votes.
 - For today's talk: simple majority rule with q = 1/2.
- All players' payoffs then realize.

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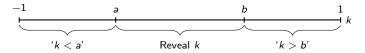
3 Concluding remarks

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Censorship policy

Censorship policy: an interval revelation strategy that 'censors' extreme states



• If $k \in [a, b]$, voters' posterior belief degenerates to k due to full revelation.

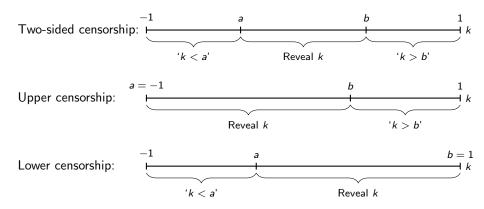
- If k > b, voters' posterior expectation of k is $\mathbb{E}_{F}[k|k > b]$.
- If k < a, voters' posterior expectation of k is $\mathbb{E}_{F}[k|k < a]$.

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Censorship policy

Three types of censorship policies (with $-1 \le a \le b \le 1$):



All these include full disclosure ([a, b] = [-1, 1]) and no disclosure (a = b $\in \{-1, 1\}$) as special cases.

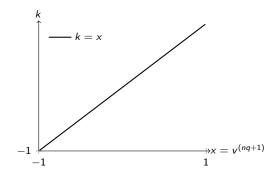
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The indifference curves

Let $x = v^{(nq+1)}$ be the pivotal voter's 'threshold of acceptance' for *R*.

• The pivotal voter's 'indifference curve' is simply k = x.



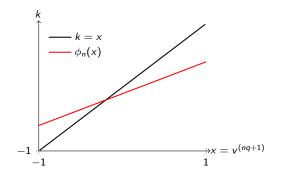
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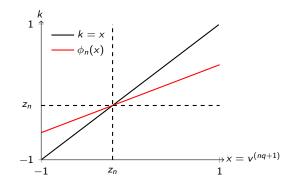
- The pivotal voter's 'indifference curve' is simply k = x.
- Conditional on $v^{(nq+1)} = x$, the designer prefers R iff $k \ge \phi_n(x)$. Examples



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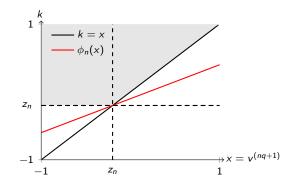
Single-crossing property: $x - \phi_n(x)$ cross 0 at most once (if so, from below).



Implication of the single-crossing property:
Temptation to manipulate voters' beliefs

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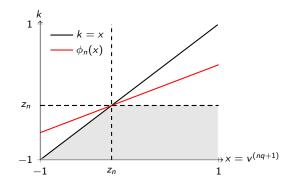


Implication of the single-crossing property:
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- If $k = z_n$, designer prefers *R* iff pivotal voter prefers *R*;
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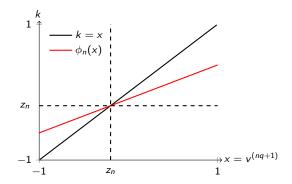
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- If $k = z_n$, designer prefers *R* iff pivotal voter prefers *R*;
- If $k > z_n$, pivotal voter prefers $R \implies$ designer strictly prefers R;
- If $k < z_n$, pivotal voter prefers $SQ \implies$ designer strictly prefers SQ;

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Single-crossing property: $x - \phi_n(x)$ cross 0 at most once (if so, from below).



If $\phi_n(x)$ satisfies single-crossing property, exactly one of three cases will apply:

- **1** $z_n \in (-1,1)$: $x > (<)\phi_n(x)$ for $x > (<)z_n$.
- **③** Uniformly biased towards alternative $R: x > \phi_n(x)$ for all $x \in (-1, 1)$.

• Uniformly biased towards alternative SQ: $x < \phi_n(x)$ for all $x \in (-1, 1)$.

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Main result

Theorem 1: Single-crossing property \implies optimality of censorship policy

Suppose $\phi_n(x)$ satisfies the single-crossing property. There exists $N \ge 0$ such that for all $n \ge N$, the following holds: • Sketch of proof

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- **2** Uniformly biased towards $R \implies upper \ censorship$ is uniquely optimal.
- **(3)** Uniformly biased towards $SQ \implies$ *lower censorship* is uniquely optimal.

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When does the single-crossing property (SCP) hold?

- **(**) If the designer is purely self-interested, then SCP holds for all G and q.
- If both G and 1 G are strictly log-concave, then SCP holds for all q and generic designer objectives. Sketch of proof
 - This condition holds if G admits a strictly log-concave density function, which is often assumed in applied theories.

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Return to Examples (1)

Self-interested designer: $\phi_n(x) = \chi$ for all x. Illustration

- If *x* ∈ (−1, 1), then some *two-sided censorship* policy with *a_n* < *x* < *b_n* is optimal in large elections.
- **2** If $\chi < -1$, then *upper censorship* policy is optimal in large elections.
- § If $\chi > 1$, then *lower censorship* policy is optimal in large elections.

Cases 2 and 3 replicate Kolotilin, Mylovanov and Zapechelnyuk (2021TE).

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Non-utilitarian social planner: • Illustration

- "Pro-Reform" planner:
 - If q = 1/2, then $x > \phi_n(x)$ on (-1, 1) so that *upper censorship* is optimal.
- (2) "Anti-Reform" planner:
 - If q = 1/2, then $x < \phi_n(x)$ on (-1, 1) so that *lower censorship* is optimal.

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Return to Examples (3)

Utilitarian planner: • Illustration

Let z^* denote the unique solution to

$$x = q\mathbb{E}_G[v_i|v_i \leq x] + (1-q)\mathbb{E}_G[v_i|v_i \geq x]$$

(Uniqueness holds if both G and 1 - G are strictly log-concave)

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- **2** If $z^* \leq -1$, then *upper censorship* policy is optimal in large elections.
- § If $z^* \ge 1$, then *lower censorship* policy is optimal in large elections.

Implication: Full information disclosure is generically NOT socially optimal.

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Single-crossing property and the optimality of censorship policy

3 Concluding remarks

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Summary

We study public Bayesian persuasion in elections in a general framework:

- (i) a wide class of objective functions
- (ii) both monopolistic and competitive persuasion

Main result: Single-crossing property \implies the optimality of censorship policies in sufficiently large elections.

More in paper:

- Comparative statics: How does the optimal information policy vary with a designer's preference and the voting rule?
- Extension to competition in persuasion and an analysis of the welfare impact of media competition. Details

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Related literature

Persuasion in elections

- Alonso and Camara (2016a,b), Schnakenberg (2015,17), Bardhi and Guo (2018), Liu (2019), Chan et al (2019), Heese and Lauermann (2021), Sun et al (2021a,b), Van der Straeten and Yamashita (2021), etc.
- We contribute by analyzing a wide class of objective functions under both monopolistic and competitive persuasion in a unified framework.

Bayesian persuasion and information design

- Kamenica and Gentzkow (2011,2017), Kolotilin (2018), Kolotilin et al (2017,2021), Mathevet et al (2018), Bergermann and Morris (2016,2019), Taneva (2019), Dworczak and Martini (2019), Ariel et al (2021), etc.
- We exploit the duality methods for linear persuasion problems, and we deliver technical contributions that apply for general linear persuasion problems.

Welfare impact of media competition on voters.

- Besley and Prat (2006), Gentzkow and Shapiro (2006), Chan and Suen (2008,2009), Cage (2019), Chen and Suen (2018), Perego and Yuksel (2020), etc.
- In contrast to all these papers, we show that even if media competition improves information disclosure, this might be socially optimal.

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Thank you!

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Designer's objective function (1) • Back

Given profile $v = (v_1, \dots, v_{n+1})$ and state k, the designer's payoff when R is implemented equals

$$u(k, v) = \rho \sum_{j=1}^{n+1} w_j(k - v^{(j)}) + (1 - \rho)(k - \chi)$$

where $v^{(1)} \leq \cdots \leq v^{(n+1)}$ is an ascending permutation of profile v.

- $\rho \in [0,1]$: designer's weight on "social welfare";
- $\chi \in \mathbb{R}$ reflects the designer's "self-interest".
- $(w_1, \cdots, w_{n+1}) \in \Delta^n$: rank-dependent welfare weighting vector; \bullet more

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Designer's objective function (2) • Back

Rewrite the designer's payoff function:

$$u(k, v) = \rho \sum_{j=1}^{n+1} w_j(k - v^{(j)}) + (1 - \rho)(k - \chi)$$
$$= k - \left(\rho \sum_{j=1}^{n+1} w_j v^{(j)} + (1 - \rho)\chi\right)$$

Given profile v, the designer's "threshold-of-acceptance" for R equals

$$\varphi_n(\mathbf{v}) := \rho \sum_{j=1}^{n+1} w_j \mathbf{v}^{(j)} + (1-\rho)\chi$$

• If $\rho = 0$, then $\varphi_n(v) = \chi$ is independent of v.

• If $\rho = 1$, then $\varphi_n(v) = \sum_{j=1}^{n+1} w_j v^{(j)}$ is a weighted average of voters' realized "threshold-of-acceptance".

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More on welfare weighting function • Back

Generate weighting vector (w_1, \dots, w_{n+1}) from weighting function $w(\cdot)$:

- $w(\cdot)$ can be any (absolutely continuous) CDF on [0, 1].
- For all $j \in \{1, \cdots, n+1\}$, set $w_j = w\left(\frac{j}{n+1}\right) w\left(\frac{j-1}{n+1}\right)$.
- Well-defined for all n: $w_j \ge 0$, and $\sum_{j=1}^{n+1} w_j = 1$.

Examples

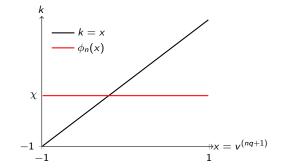
- Utilitarian planner: w(x) = x for $x \in [0, 1]$ so that $w_1 = \cdots = w_n = \frac{1}{n}$.
- "Pro-Reform" planner: $w(\cdot)$ is the CDF of a uniform distribution on [0, 0.5].

• "Anti-Reform" planner: $w(\cdot)$ is the CDF of a uniform distribution on [0.5, 1].

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Examples of $\phi_n(\cdot)$ \bullet Indifference Curves \bullet Optimal Censorship Policy



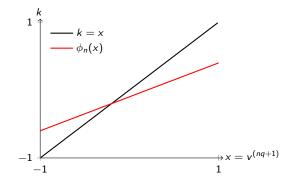
Self-interested designer: $\phi_n(x) = \chi$ is flat.

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Examples of $\phi_n(\cdot)$ Indifference Curves Optimal Censorship Policy



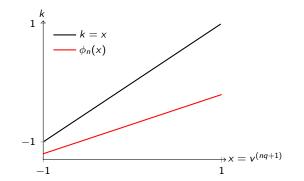
Utilitarian planner:

$$\phi_n(x) = \frac{nq}{n+1} \mathbb{E}_G[v_i | v_i \le x] + \frac{n(1-q)}{n+1} \mathbb{E}_G[v_i | v_i \ge x] + \frac{x}{n+1}$$

If $v^{(nq+1)} = x$, there are nq voters with $v_i \le x$, and n(1-q) others with $v_i \ge x$.

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Examples of $\phi_n(\cdot)$ Indifference Curves Optimal Censorship Policy



"**Pro-Reform**" planner: Suppose q = 1/2, then

$$\phi_n(x) = \mathbb{E}_G[v_i | v_i \leq x] < x$$

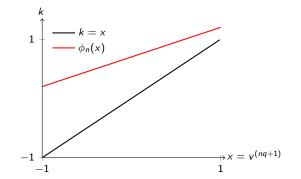
Intuition: If $v^{(n/2+1)} = x$, then $v^{(i)} \le x$ must hold for all $i \le n/2$.

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Examples of $\phi_n(\cdot)$ Indifference Curves Optimal Censorship Policy



"Anti-Reform" planner: Suppose q = 1/2, then

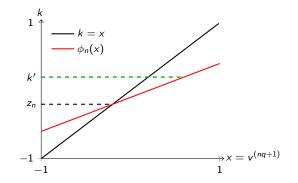
$$\phi_n(x) = \mathbb{E}_G[v_i | v_i \ge x] > x$$

Intuition: If $v^{(n/2+1)} = x$, then $v^{(i)} \ge x$ must hold for all $i \ge n/2 + 2$.

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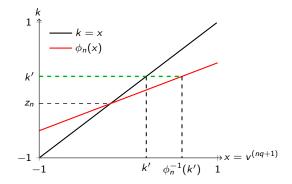
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Temptation to manipulate voters' belief • Single-Crossing Property



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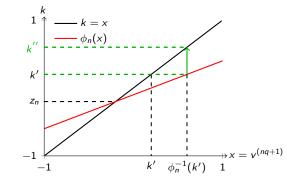
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Suppose the realized state is $k' > z_n$.

Conflict of interest: designer wants *R* to win if x < φ_n⁻¹(k'), while the pivotal voter wants it only when x < k'.

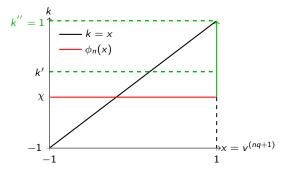
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- The designer is tempted to manipulate and let voters believe that the state is $k^{''}$ such that $k^{''} = \phi_n^{-1}(k')$.

Temptation to manipulate voters' belief Single-Crossing Property



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Conflict of interest: designer wants *R* to win if x < φ_n⁻¹(k'), while the pivotal voter wants it only when x < k'.

• The designer is tempted to *manipulate* and let voters believe that the state is k'' such that $k'' = \phi_n^{-1}(k')$.

The persuasion problem • Main Result

Notations:

- $\hat{G}_n(\cdot; q)$: the distribution of $v^{(nq+1)}$, the pivotal voter's type.
- $\theta \in [-1,1]$: posterior expectation about state k.

Designer's expected payoff under common posterior mean $\boldsymbol{\theta}$ equals

$$W_n(\theta) = \int_{\underline{v}}^{\theta} (\theta - \phi_n(x)) d\hat{G}_n(x;q)$$

The persuasion problem

$$\max_{H \in \Delta([-1,1])} \int_{-1}^{1} W_n(\theta) dH(\theta), \text{s.t. } F \succeq_{MPS} H$$

• H is the distribution of posterior means induced by an information policy.

• *H* is feasible iff prior *F* is a mean-preserving spread of *H* (i.e., $F \succeq_{MPS} H$).

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Step 1: The increasing slope property and its implication for optimal information policy Main Result

Lemma 1: The increasing slope property

Suppose that SCP holds with an interior switching point $z_n \in (-1, 1)$. Then $W_n(\cdot)$ satisfies the 'increasing-slope property' at point z_n , that is,

$$\frac{W_n(x) - W_n(z_n)}{x - z_n} \le \frac{W_n(y) - W_n(z_n)}{y - z_n}, \forall y > x$$

and strict inequality holds if $x < z_n < y$. • Graphical illustration

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and strict inequality holds if $x < z_n < y$. \bigcirc Graphical illustration

Moreover, any solution H to the persuasion problem must be (weakly) more informative than a 'cutoff' policy that precisely reveals whether the realized k is above, equal or below z_n .

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Moreover, any solution H to the persuasion problem must be (weakly) more informative than a 'cutoff' policy that precisely reveals whether the realized k is above, equal or below z_n .

Implication: We can divide the original persuasion problem to two auxiliary problems on sub-intervals $[z_n, 1]$ and $[-1, z_n]$, and then solve them separately.

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Step 2: The curvature properties and their implications for optimal information policy Main Result

Lemma 2: The curvature properties of $W_n(\cdot)$

Suppose that the SCP holds. Then there exists an $N \ge 0$ such that for all $n \ge N$ there are ℓ_n and r_n with $-1 \le \ell_n \le z_n \le r_n \le 1$ such that \bigcirc Graphical illustration

- $W_n(\cdot)$ is strictly S-shaped on $[z_n, 1]$ with inflection point r_n .
- **2** $W_n(\cdot)$ is strictly inverse-S-shaped on $[-1, z_n]$ with inflection point ℓ_n .

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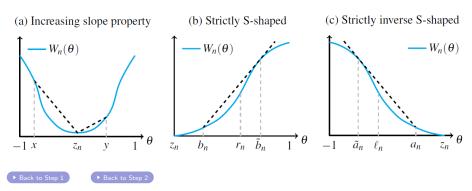
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Implication: By Kolotilin, Mylovanov and Zapechelnyuk (2021TE),

- Strictly S-shaped on $[z_n, 1] \Longrightarrow$ optimality of upper censorship policy with threshold $b_n \ge z_n$.
- Strictly inverse-S-shaped on $[-1, z_n] \implies$ optimality of lower censorship policy with threshold $a_n \le z_n$.
- Together \implies censorship policy with revelation interval $[a_n, b_n]$ is optimal.

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Graphical illustrations of mentioned curvature properties

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A sufficient condition for SCP Main Result

Proposition: Designer's interim "threshold-of-acceptance"

Conditional on $v^{(nq+1)} = x$, the designer strictly prefers R iff $k > \phi_n(x)$. Specifically, for all $x \in [\underline{v}, \overline{v}]$,

$$\phi_n(x) := \mathbb{E}[\varphi_n(v)|v^{(nq+1)} = x] = \rho \sum_{j=1}^{n+1} w_j \varphi_j(x;q,n) + (1-\rho)\chi$$

where

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Moreover, $\varphi_i(x; q, n)$ satisfies

(i) If G is strictly log-concave, then $\varphi'_j(x; q, n) < 1$ for j < nq + 1.

(ii) If 1 - G is strictly log-concave, then $\varphi'_j(x; q, n) < 1$ for j > nq + 1.

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By (i) and (ii), if both G and 1 - G are strictly log-concave, then

$$\phi'_n(x) = \rho \sum_{j=1}^{n+1} w_j \varphi'_j(x; q, n) < \rho \le 1 \quad \Longrightarrow \quad SCP$$

Sun, Schram, Sloof (EUI, UvA)

Setup: Two designers, with interim "threshold-of-acceptance" $\phi'_n(\cdot)$ and $\phi''_n(\cdot)$.

- Designers simultaneously choose information policies π_I and π_{II} .
- Equilibrium outcome $\pi = \langle \pi_I, \pi_{II} \rangle$: public signals from both policies.
- Information environment is Blackwell-connected.1

¹See Gentzkow and Kamenica (2017).

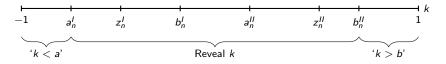
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Equilibria under competition in persuasion • Back

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Figure: Equilibrium outcome with two competing designers I and II



Note: a_n^m and b_n^m are cutoffs of the optimal censorship policy under monopolistic persuasion for $m \in \{I, II\}$. *n* is sufficiently large to ensure optimality of censorship policy.

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Equilibria under competition in persuasion • Back

Setup: Two designers, with interim "threshold-of-acceptance" $\phi_n^{\prime}(\cdot)$ and $\phi_n^{\prime\prime}(\cdot)$.

- Designers simultaneously choose information policies π_I and π_{II} .
- Equilibrium outcome $\pi = \langle \pi_I, \pi_H \rangle$: public signals from both policies.
- Information environment is Blackwell-connected.1

Theorem 2: Equilibrium outcome under competitive persuasion

Suppose

- i. both $\phi_n^{\prime}(\cdot)$ and $\phi_n^{\prime\prime}(\cdot)$ satisfy single-crossing property, and
- ii. *n* is sufficiently large such that $\mathcal{P}(a_n^m, b_n^m)$ is the unique optimal censorship policy under monopolistic persuasion for $m \in \{I, II\}$.

Then the *least informative* equilibrium outcome is given by $\mathcal{P}(a_n^*, b_n^*)$ with

$$a_n^* = \min\{a_n^l, a_n^{ll}\}, \text{ and } b_n^* = \max\{b_n^l, b_n^{ll}\}$$

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¹See Gentzkow and Kamenica (2017).

When is full disclosure the unique equilibrium? • Back

Corollary:

Full information disclosure is the unique equilibrium outcome if

$$\phi'_n(x) < x < \phi''_n(x), \forall x \in (-1,1)$$

That is, designer I is uniformly biased towards R, and designer II is uniformly biased towards SQ.

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Examples:

- **(**) Two opposite-minded self-interested designers: $\chi_I = -1$ and $\chi_{II} = 1$.
- (2) "Pro-Reform" versus "Anti-Reform" planners, under simple majority rule.

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Application to media competition:

- Model competing media outlets as in the two examples above.
- Implication 1: media competition leads to full information disclosure.
- Implication 2: media competition is not Utilitarian optimal because it induces excessive information revelation.

Sun, Schram, Sloof (EUI, UvA)

Public Persuasion in Elections