Strong Rationalizability, Learning, and Equilibrium in Repeated Games with Imperfect Feedback

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Overview

- We study limits of learning dynamics in the infinite repetition of a one-period game where
 - there may be imperfect monitoring,
 - players are strategically sophisticated,
 - the one-period game may be sequential or simultaneous move.
- As particular case, we focus on impatient players who maximize their one-period payoffs
 - to relate the implications we find to one-period solution concepts.
- We provide a sort of foundation for a strategically sophisticated refinement of Self-Confirming Equilibrium (SCE) that arises as limit play of learning dynamics.

SCE and motivation, I

- In SCE (also called "conjectural equilibrium"), players best reply to confirmed conjectures (=1st-order beliefs) about co-players' behavior,
 - confirmation refers to beliefs correctly predicting what a player observes about the play (determined by her feedback).
- SCE has been shown to characterize limits of learning dynamics of infinitely repeated games, with possibly imperfect monitoring, where players are impatient and (possibly) naive:
 - not only they do not necessarily engage in any type of strategic reasoning,
 - but, additionally, they believe they are facing a fixed time-invariant distribution of co-players' strategy profiles.
- It is thus natural to ask: What characterizes the limits of learning dynamics when players realize they are repeatedly facing each other and engage in sophisticated reasoning (a form of common belief in rationality)?

SCE and motivation, II

The literature has provided two kinds of answers, that neglect an explicit analysis of learning dynamics:

- SCE in rationalizable conjectures/1st-order beliefs (Battigalli 1987, Battigalli & Guaitoli 1988)
 - adds to confirmation of conjectures the requirement that a player assign probability 1 to co-players' rationalizable strategies,
 - a condition that (in static one-period games) follows from common belief in rationality (CBR).
- Rationalizable SCE (Rubinstein & Wolinsky 1994)
 - implied by assuming (on top of CBR) common belief in confirmation of conjectures.

Results, I

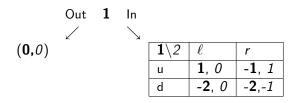
- SCE in rationalizable conjectures within the infinitely repeated game characterizes limits of learning dynamics in our setting:
 - strategically sophisticated players reason about the whole infinite interaction.
 - under precise conditions, players asymptotically learn to correctly predict what they will observe.
- We model **strategic sophistication** assuming that:
 - behavior and interactive beliefs satisfy Rationality (conditional SEU maximization)
 - and Common Strong Belief in Rationality (best rationalization) principle: always ascribe to co-players' the highest degree of strategic sophistication consistent with what one observes).
 - Behavioral implication: players implement strongly rationalizable strategies (see Battigalli & Tebaldi 2019).
- When players are *impatient*, strong rationalizability in the supergame implies one-period strong rationalizability (in every period) along the path of play.

Results, II

There are no chance moves and players do not randomize.

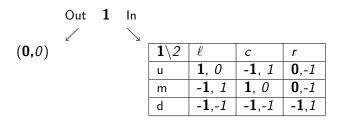
- RCSBR=Rationality and Common Strong Belief in Rationality.
- IMP=Impatience.
- **OGT**=Observational Grain of Truth:
 - eventually, each player assigns positive probability to her actual infinite sequence of observations (given the true state);
 - it ensures asymptotic learning, similar to Kalai & Lehrer (1993, 1995).
- Under RCSBR, IMP, and OGT, players end up playing a sequence of one-period SCEs in strongly rationalizable conjectures for the one-period game.
 - yet, if there are multiple SCEs in rationalizable conjectures, the play can alternate among them.
- Conversely, one can show that any sequence of one-period SCEs with strongly rationalizable conjectures can be played by rational impatient players under common strong belief in rationality.

Example 1: Feedback = own realized payoff



- Strongly rationalizable strategies: $s_1 = \text{Out.u}$ and $s_2 = r$ (if In). With this,
 - the only outcome of SCE in rationalizable conjectures is Out;
 - (In.u, \(\ell \)) is an SCE (supported by a non-rationalizable conjecture of pl. 2); pl. 1 holds a correct conjecture, pl. 2 assigns prob. 1 to In (observed), and at least 1/2 to In.d;
 - this profile of strategies can be played in the limit if and only if we remove common strong belief in rationality, and maintain rationality and observational grain of truth.

Example 2: Feedback = own realized payoff



- All strategies except In.d (pl. 1) and r (pl. 2) are strongly rationalizable.
 - The only outcome of SCE in rationalizable conjectures is Out.
 - Under strong rationalizability, disequilibrium pairs in $\{{\rm In.u,In.m}\} \times \{\ell,c\}$ can be played infinitely often if and only if observational grain of truth does not hold.
 - Yet, In.d and r are never played under rationality and common strong belief in rationality.

Discussion

- If marginal one-period beliefs converge, then—generically—we obtain convergence to one fixed SCE in rationalizable conjectures.
 - But we do not have interesting sufficient conditions for convergence of one-period beliefs.
- Why no randomization? Because SEU maximizer have no incentive to randomize. This buys a deterministic path, which simplifies the analysis.
- Harder problem: analyze recurrent play within a (large) population game with random matching in each period, using RCSBR and OGT, but not IPM.

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