Strong Rationalizability, Learning, and Equilibrium in Repeated Games with Imperfect Feedback

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Overview

- We study limits of learning dynamics in the infinite repetition of a one-period game where
  - there may be *imperfect monitoring,*
  - players are *strategically sophisticated,*
  - the *one-period game may be sequential* or simultaneous move.
- As particular case, we focus on *impatient* players who maximize their one-period payoffs
  - to relate the implications we find to one-period solution concepts.
- We provide a sort of foundation for a strategically sophisticated refinement of *Self-Confirming Equilibrium (SCE)* that arises as limit play of learning dynamics.

SCE and motivation, I

- In **SCE** (also called "**conjectural equilibrium**"), players *best reply to confirmed conjectures* (=1^{st}-order beliefs) about co-players' behavior,
  - **confirmation** refers to beliefs correctly predicting what a player observes about the play (determined by her feedback).
- SCE has been shown to characterize limits of learning dynamics of infinitely repeated games, with possibly imperfect monitoring, where players are impatient and (possibly) naive:
  - not only they do not necessarily engage in any type of strategic reasoning,
  - but, additionally, they believe they are facing a fixed time-invariant distribution of co-players' strategy profiles.
- It is thus natural to ask: *What characterizes the limits of learning dynamics when players realize they are repeatedly facing each other and engage in sophisticated reasoning* (a form of common belief in rationality)?

The literature has provided two kinds of answers, that neglect an explicit analysis of learning dynamics:

- *SCE in rationalizable conjectures/1*st-order beliefs (Battigalli 1987, Battigalli & Guaitoli 1988)
  - adds to confirmation of conjectures the requirement that a player assign probability 1 to co-players' rationalizable strategies,
  - a condition that (in static one-period games) follows from *common belief in rationality (CBR).*
- *Rationalizable SCE* (Rubinstein & Wolinsky 1994)
  - implied by assuming (on top of CBR) *common belief in confirmation of conjectures.*

Results, I

- SCE in rationalizable conjectures within the infinitely repeated game characterizes limits of learning dynamics in our setting:
  - *strategically sophisticated* players reason about the whole infinite interaction,
  - under precise conditions, players asymptotically learn to correctly predict what they will observe.
- We model **strategic sophistication** assuming that:
  - behavior and interactive beliefs satisfy *Rationality* (conditional SEU maximization)
  - and *Common Strong Belief in Rationality* (**best rationalization principle**: always ascribe to co-players' the highest degree of strategic sophistication consistent with what one observes).
  - Behavioral implication: players implement *strongly rationalizable strategies* (see Battigalli & Tebaldi 2019).
- When players are *impatient*, strong rationalizability in the supergame implies one-period strong rationalizability (in every period) along the path of play.

Results, II

There are *no chance moves* and players *do not randomize*.

- **RCSBR=***Rationality and Common Strong Belief in Rationality*.
- **IMP=***Impatience*.
- **OGT**=*Observational Grain of Truth***:**
  - eventually, each player assigns positive probability to her actual infinite sequence of observations (given the true state);
  - it ensures asymptotic learning, similar to Kalai & Lehrer (1993, 1995).

- *Under RCSBR, IMP, and OGT, players end up playing a sequence of one-period SCEs in strongly rationalizable conjectures for the one-period game.*
  - yet, *if* there are *multiple* SCEs in rationalizable conjectures, the *play can alternate* among them.
- *Conversely*, one can show that any sequence of one-period SCEs with strongly rationalizable conjectures can be played by rational impatient players under common strong belief in rationality.

Example 1: Feedback = own realized payoff

Out **1** In

↙ ↘

(0,*0*)

1*2*	ℓ	r
u	**1**, *0*	**-1**, *1*
d	**-2**, *0*	**-2**,*-1*

- Strongly rationalizable strategies: s_1 =Out.u and $s_2 = r$ (if In). With this,
  - the *only outcome of SCE in rationalizable conjectures is* Out;
  - (In.u, ℓ) is an SCE (supported by a non-rationalizable conjecture of pl. *2*); pl. **1** holds a correct conjecture, pl. *2* assigns prob. 1 to In (observed), and at least 1/2 to In.d;
  - this profile of strategies can be played in the limit if and only if we remove common strong belief in rationality, and maintain rationality and observational grain of truth.

Example 2: Feedback = own realized payoff

Out **1** In

↙ ↘

(**0**,*0*)

1*2*	*ℓ*	*c*	*r*
u	**1**, *0*	-**1**, *1*	**0**,-*1*
m	-**1**, *1*	**1**, *0*	**0**,-*1*
d	-**1**,-*1*	-**1**,-*1*	-**1**,*1*

- *All* strategies *except* In.d (pl. **1**) and *r* (pl. *2*) are strongly rationalizable.
  - The *only outcome of SCE in rationalizable conjectures is Out*.
  - Under strong rationalizability, disequilibrium pairs in $\{\mathrm{In.u}, \mathrm{In.m}\} \times \{\ell, c\}$ can be played infinitely often if and only if observational grain of truth does not hold.
  - Yet, In.d and *r* are never played under rationality and common strong belief in rationality.

Discussion

- If marginal one-period beliefs converge, then—generically—we obtain convergence to one fixed SCE in rationalizable conjectures.
  - But we do not have interesting sufficient conditions for convergence of one-period beliefs.
- Why no randomization? Because SEU maximizer have no incentive to randomize. This buys a deterministic path, which simplifies the analysis.
- Harder problem: analyze recurrent play within a (large) population game with random matching in each period, using RCSBR and OGT, but *not* IPM.

Battigalli P. (1987): *Comportamento razionale ed equilibrio nei giochi e nelle situazioni sociali*, unpublished thesis, U. Bocconi.

Battigalli, P. and D. Guaitoli (1988): "Conjectural equilibria and rationalizability in a macroeconomic game with incomplete information," Quad. Ricerca 1988-6, I.E.P., U. Bocconi (also in *Decisions, Games and Markets*, Kluwer, 97-124, 1997).

Battigalli, P., and P. Tebaldi (2019): "Interactive Epistemology in Simple Dynamic Games with a Continuum of Strategies," *Economic Theory*, 68, 737-763.

Kalai, E. and E. Lehrer (1993): "Rational learning leads to Nash equilibrium," *Econometrica*, 61, 1019-1045.

Kalai, E. and E. Lehrer (1993): "Subjective equilibrium," *Econometrica*, 61, 1231-1240.

Rubinstein, A. and A. Wolinsky (1994): "Rationalizable Conjectural Equilibrium: Between Nash and Rationalizability," *Games and Economic Behavior*, 6, 299-311.