Signaling in Dynamic Markets with Adverse Selection

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Introduction

- Adverse selection is a feature of many dynamic decentralized markets.
  - When sellers in such markets have both price setting ability and private information about the quality of the products they sell, prices can be used to signal quality.
- Despite this, the literature on dynamic trading with adverse selection has focused on screening.
- This focus is restrictive, though.
  - We know since at least Wilson (1980) that the price setting mechanism affects market outcomes.
- **This paper:** Study signaling through prices in dynamic decentralized markets with adverse selection.

Introduction

- As in screening models of trade, delay in trade occurs if adverse selection is severe enough to prevent pooling.
- Standard Intuition:
  - Sellers of higher-quality goods endogenously more patient, so delay in trade restores trade by ensuring owners of lower-quality goods do want not to pool with owners of higher-quality goods.
- In screening models of trade, by lowering opportunity cost of not trading, reducing market frictions reduces ability of buyers to screen, which in turn hurts market efficiency.
- **Key result:** Market efficiency *does not depend on* trading frictions.

Related Literature

- Static models in which prices signal quality:
    - Wilson (1980), Wolinsky (1983), Bagwell and Riordan (1991), and Ellingsen (1997).
- Trade of lemons in dynamic decentralized markets with adverse selection:
    - Blouin (2003), Moreno and Wooders (2010), Camargo and Lester (2014), Chiu and Koeppl (2016), and Kim (2017).
- Dynamic centralized trading with adverse selection and competitive search models with adverse selection:
    - Janssen and Roy (2002) and Fuchs and Skrzypacz (2015).
    - Guerrieri and Shimer (2014) and Chang (2018).

Environment

- Discrete time, infinite horizon.
- Single indivisible good, which can be of finitely many types.
  - $\mathcal{I} = \{1, \ldots, N\}$ with $N \geq 2$ is set of possible types of the good.
- In each period, a mass one of anonymous infinitely-lived sellers and an equal mass of anonymous infinitely-lived buyers enter the market.
  - Sellers can produce one unit of the good and are privately informed about its type.
  - $f_i > 0$ = fraction of type-i sellers (sellers who produce type-i good) in the population.

Environment

- Payoffs
  - $v_i - p =$ payoff to buyer who buys type-i good at price p.
  - $p - c_i =$ payoff to type-i seller who sells the good at price p.
  - v_i and c_i nonnegative and strictly increasing with i (quality increases with type) and $v_i > c_i$ for all i.
- Trade
  - In each period, buyers and sellers in the market are randomly and anonymously matched in pairs.
  - Seller in a match posts price, which buyer either accepts or rejects.
  - Agents in the match trade and exit market if buyer accepts, otherwise match is dissolved and agents remain in the market.

Environment

- Trading Frictions
  - Agents have a common discount factor $\delta \in (0, 1)$.
  - Discount factor δ captures opportunity cost of not trading.
- Remarks
  - Gains from Trade: can extend analysis to allow nonpositive gains from trade for some i.
  - Timing: similar results if agents who enter the market do not get an immediate trading opportunity.
  - Trading Frictions: similar results if δ is an exit prob. or if buyers and sellers in the market are matched with prob. $\alpha \in (0, 1)$ in each period.

Strategies and Beliefs

- Anonymity of agents implies we can consider distributional strategies.

  1. Strategy profile for sellers is list $\mu = (\mu_1, \ldots, \mu_N)$ of prob. measures on $\mathbb{R}_+$ such that $\mu_i(P)$ = mass of type-i sellers who post price $p \in P$.
  2. Strategy profile for buyers is map $\sigma : \mathbb{R}_+ \to [0,1]$ with $\sigma(p)$ = prob. price p is accepted or, equivalently, a map $\theta : \mathbb{R}_+ \to [0,1]$ such that

  $$\theta(p) = \frac{\sigma(p)}{1 - \delta(1 - \sigma(p))} = \text{ discounted prob. of trade at price } p.$$

     - Remark: $\theta(p) = \mathbb{E}[\delta^{\tau(p)}]$, where $\tau(p)$ = random time of trade for seller who posts price p while in the market.

- Belief system (for buyers) is map $\pi : \mathbb{R}_+ \to \Delta^N$ with $\pi_i(p)$ = prob. buyers assign to buying type-i good should they trade at price p.

Equilibrium: Definition

Definition (Informal)

An equilibrium is a list consisting of strategy profiles, belief system, payoffs for sellers and buyers, and seller masses with the following properties.

1. *Seller Optimality*. Sellers' offers are optimal given buyers' acceptance behavior and sellers' continuation payoffs should they not trade.
2. *Buyer Optimality*. Buyers' behavior given a price offer is optimal given their beliefs and continuation payoffs should they not trade.
3. *Rational Beliefs*. Beliefs satisfy Bayes' rule for prices on path of play.
4. *Payoff Consistency.* Payoffs consistent with behavior.
5. *Stationarity.* Mass of each type of seller in the market is such that their outflow equals their inflow.

Equilibrium: Remarks and Gains From Trade

- Remarks

  1. Stationary $\Rightarrow$ all goods trade in equilibrium.

  2. **No equilibrium refinement**: agnostic about the belief formation process for off-equilibrium prices (additional comments at the end).

- Gains from Trade

  - $\mathbb{E}_{\mu_i}[\sigma] =$ probability that type-i good trades in a given period.

  - Gains from trade are

$$G = \sum_{i=1}^{N} f_i \underbrace{\frac{\mathbb{E}_{\mu_i}[\sigma]}{1 - \delta\big(1 - \mathbb{E}_{\mu_i}[\sigma]\big)}}_{\text{Discounted Prob. Trade } i} (v_i - c_i).$$

Basic Properties of Equilibria

Let S_i = set of prices posted by type-i sellers and $S_i^* \subseteq S_i$ be the set of prices at which type-i sellers trade.

Moreover, let U_i be type-i sellers' payoff and V be buyers' payoff.

1. An equilibrium with $p' \in S_1^* \setminus \bigcup_{j=1}^N S_j^*$ (only type-1 sellers trade at p') is such that $p' = v_1$, $\theta(v_1) = 1$ and $V = 0$.
2. For all $i, j \in \mathcal{I}$, $j > i \Rightarrow p \leq p'$ for all $p \in S_i$ and $p' \in S_j$. So, for all $i, j \in \mathcal{I}$, at most one price that both types of seller offer.
3. Set S^* of prices at which trade takes place in equilibrium is finite.
4. $\sum_{i=1}^N f_i v_i < c_N$ (severe adverse selection) $\Rightarrow$ there exists $p \in S_1^*$ such that $p \notin S_N^*$. So, severe selection leads to delay in trade.
5. Equilibria with $V = 0$ always exist.

Basic Properties of Equilibria

Proposition

Set of equilibrium payoff vectors for equilibria with $V = 0$ is invariant to δ.

- Buyers' payoff $= 0 \Rightarrow$ if $p \in S^*$, then $p = \sum_{i=1}^{N} \pi(p) v_i$.
- Fix $\delta \in (0,1)$ and σ. For each $\delta' \in (0,1)$ there exists σ' such that

$$\sigma/(1-\delta(1-\sigma)) = \sigma'/(1-\delta'(1-\sigma')).$$

- Keeping prices the same, can adjust buyer behavior to keep discounted probabilities of trade, and thus seller payoffs, the same.
- Challenge: adjusting buyer behavior changes eq. masses of sellers in the market, affecting buyer beliefs.
- Can adjust seller behavior to keep buyer beliefs the same.

Two-Type Case with Severe Adverse Selection

Suppose $\mathcal{I} = \{1, 2\}$ and $f_1 v_1 + f_2 v_2 < c_2$.

- Severe adverse selection $\Rightarrow$ there exists $p_1 \in S_1^*$ such that $p_1 \notin S_2^*$.
- So, $V = 0$, $p_1 = v_1$, and $\theta(v_1) = 1$. In particular, set of equilibrium payoff vectors is invariant to δ (can compute it).
- Monotonicity in prices $\Rightarrow$ two cases to consider:
  - (i) $S_1^* \cap S_2^* = \emptyset$;
  - (ii) $S_1^* \cap S_2^*$ a singleton.

Two-Type Case with Severe Adverse Selection

- Suppose $S_1^* \cap S_2^* = \emptyset$. Monotonicity in prices $\Rightarrow S_1^*$, S_2^* singletons.
- Since $V = 0$, $S_i^* = \{v_i\}$ for each i (Bayes' rule).
- Since $U_i > 0$ for each i, $S_i = S_i^*$ for each i (separating equilibria).
- Seller IC:

$$\theta(v_2) \leq \overline{\theta}(v_2) = \frac{v_1 - c_1}{v_2 - c_1} \text{ (necessary and sufficient)}$$

- Maximum gains from trade:

$$\overline{G} = f_1(v_1 - c_1) + f_2\overline{\theta}(v_2)(v_2 - c_2) = \left(f_1 + f_2\frac{v_2 - c_2}{v_2 - c_1}\right)(v_1 - c_1).$$

Two-Type Case with Severe Adverse Selection

- It turns out that equilibria with $S_1^* \cap S_2^*$ a singleton realize less gains from trade than the most efficient separating equilibrium.

Proposition

All equilibria in the two-case with severe adverse selection are such that $V = 0$. Most efficient equilibria are separating and maximum gains from trade are invariant to δ.

- Remarks:
  - Comparison with screening: Gains from trade $\downarrow \delta$ and smaller than $\overline{G}$ in the frictionless limit ($\delta \to 1$).
  - $N \geq 3$: can have equilibria with $V > 0$ and most efficient equilibrium in the presence of adverse selection need not be separating.

General Case

Proposition

Maximum equilibrium welfare is invariant to δ.

Step 1: Gains from trade when $V = 0$ bounded above by $\sum_{i=1}^{N} f_i U_i$, with equality iff sellers do no randomize.

- Randomization by sellers hurts gains from trade without reducing seller payoffs (as sellers are indifferent between all prices they offer).

Step 2: For any equilibrium with $V > 0$, there exists a more efficient equilibrium with $V = 0$.

- $V > 0 \Rightarrow$ can increase prices at which trade takes place, relaxing seller ICs and allowing greater gains from trade.

General Case

Step 3: For any equilibrium with $V = 0$, there exists a more efficient one in which sellers do not randomize.

- Randomization by sellers only possible if a given type of seller mixes with higher-type sellers.
- $V = 0 \Rightarrow$ randomization lowers the higher-type sellers' payoff by $\downarrow$ expected quality of the good to buyers without benefiting the sellers who randomize.
- Eliminating randomization then increases average seller payoffs, which increases gains from trade by Step 1.

- Steps 1 to 3 $\Rightarrow$ gains from trade maximized when $V = 0$ and sellers play pure strategies. Invariance of equilibrium payoff vectors to δ when $V = 0$ establishes desired result.

General Case: Equilibrium Refinements

- Agnostic about belief formation process for off-equilibrium prices: no equilibrium refinement.
- Possible to extend the Intuitive Criterion and the D1 refinement to our dynamic setting.
- Intuitive Criterion: does not refine the equilibrium set (every eq. satisfies IC).
- D1: the set of equilibrium payoff vector is the set of equilibrium payoff vectors for separating equilibria.
    - Most efficient equilibrium need not be separating.
    - But separating equilibria are such that $V = 0 \Rightarrow$ maximum gains from trade still invariant to trading frictions.

Final Remarks

- Characterize equilibria in dynamic decentralized markets with adverse selection when sellers make the offers (signaling through prices).
- Signaling through prices can lead to greater gains from trade.
- Unlike the screening case, market efficiency (i.e., maximum gains from trade) is invariant to trading frictions.
- Agnostic about belief formation process for off-equilibrium process: no equilibrium refinements.
  - Results survive with standard equilibrium refinements (Intuitive Criterion and D1).