Setting Interim Deadlines to Persuade

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Research questions:

- how does startup choose terms of self-reporting?
- necessary and sufficient conditions for interim reporting deadline to emerge?

This paper

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- Promises of future provision of information on progress of project serve as "carrot" incentivizing investor to start funding
- When project is sufficiently attractive to investor ex ante, startup promises **provision of only good news** (project completion)
- However, when project is not attractive ex ante, startup provides **both good news** (project completion) **and bad news** (not reaching milestone) that are released at interim date,

  i.e., startup sets an **interim reporting deadline** to persuade the investor

Literature

Dynamic Bayesian persuasion: Ely and Szydlowski (2020), Orlov et al. (2020), Ely (2017), Smolin (2021), Liu (2021), Renault et al. (2017), Ball (2019)

- ES (2020): dynamic info provision regarding static state (difficulty of task), this paper: dynamic info provision regarding state that endogenously evolves over time (progress toward completion)

Design of incentives for experimentation: Bergemann and Hege (1998), Green and Taylor (2016), Wolf (2017), Madsen (2020)

Model: investor

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c - investment cost incurred at each t until stopping

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$x_t \in \{0, 1, 2\}$ - number of stages completed by t

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- completion of stage occurs at Poisson rate λ (conditional on continuation of funding)
- $\tau_n \in \mathbb{R}_+$ - random time at which nth stage is completed

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Information about x_t **controlled by startup**

Model: payoffs

Investor (receiver)

- gets project completion payoff v iff **2nd stage completed** by moment of stopping

Startup (sender)

- gets c at each t until investor stops funding $\Rightarrow$ wants to postpone stopping

Startup's problem

At $t = 0$ startup **commits** to information policy σ,
σ_t maps from history up to t to $\Delta(M)$, $\forall t$

Timing within t: x_t draw $\rightarrow$ m_t draw $\rightarrow$ a_t choice

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$$\max_{\sigma} \left[\mathrm{E} \left[\tau \right] \right],$$

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Example 1. No information: $\sigma^{NI} = m, \forall t, h_t$

Example 2. Full information: $\sigma^{FI} \in \{m_0, m_1, m_2\}$, m_n sent at all t such that $x_t = n$

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- **Zero stages** completed by t:

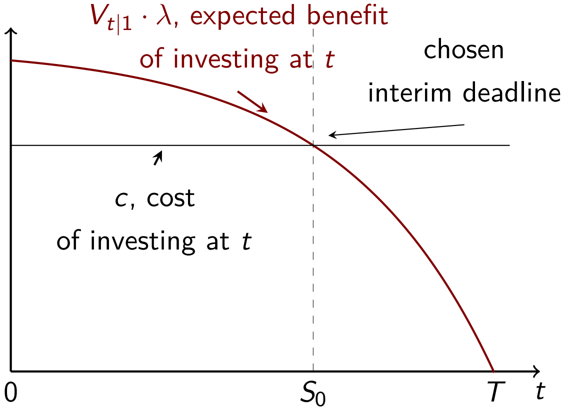
$$\mathrm{E}\left[u_0\right]_{a_t=1} = V_{t|1} \cdot h\Delta t - c \cdot \Delta t,$$

  where $V_{t|1}$ - state 1 continuation value at t

Continuation value

Investor-preferred interim deadline

$$V_{t|1} \cdot h\Delta t - c \cdot \Delta t = 0$$



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- Properties of optimal info policy:
  if policy is optimal for startup then it implements investment schedule that is **efficient**
  - **efficient** – no room for improvement without harming investor
  - necessary for efficiency: **feasible** – investor is willing to start at $t = 0$

Towards optimal info policy

- Investment schedule is feasible iff investor gets at least reservation value (**IR constraint** satisfied at $t = 0$),

$$\text{reservation value} = \max\left(0, V^{NI}\right),$$

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- Info policy is **optimal for startup** iff implemented investment schedule is (1) **efficient** and (2) promises investor **precisely its reservation value**

Optimal policy when project is promising (low κ)

$$\text{total surplus} = \underbrace{v \cdot \mathrm{P}\left(x_\tau = 2\right) - c \cdot \mathrm{E}\left[\tau\right]}_{\text{investor's expected payoff}} + \underbrace{c \cdot \mathrm{E}\left[\tau\right]}_{\text{startup's expected payoff}} = v \cdot \mathrm{P}\left(x_\tau = 2\right)$$

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Implementation: WLOG, use **direct recommendation mechanism** (DRM)

- $|M| = 2$, $m = 1$ received at t - recommendation to continue at t, $m = 0$ - to stop.

Obedience

Optimal policy when project is promising (low κ)

Proposition 1: Assume $\kappa \in (0, \tilde{\kappa}(T, \lambda)]$. If in no-information benchmark investor invests until T, then startup chooses not to provide any information. Otherwise, optimal information policy is direct recommendation mechanism that has following properties:

1. whenever stopping is recommended by mechanism, second stage of project is already completed;
2. recommendation to stop is postponed so that investor's IR constraint is binding, i.e. $V(\tau) = \max\left(V^{NI}, 0\right)$, where V^{NI} - investor's expected payoff under no info provision. Example

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Key takeaway: when project is good ex ante, it is better to promise no reports on reaching the milestone of project

Optimal policy when project is not promising (high κ)

- When $\kappa > \tilde{\kappa}(T, \lambda)$, disclosure only of 2nd stage completion does not motivate investor to start $\Rightarrow$ it needs to provide at least some **information on 1st stage completion** to satisfy IR constraint
- Startup prefers disclosure of 2nd stage to disclosure of 1st stage as incentive device (efficiency)

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Thus, startup

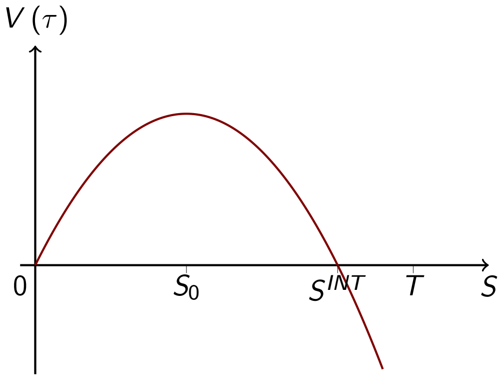
1. **immediately** discloses 2nd stage completion (using preferred instrument fully);
2. chooses deterministic date at which it reports if 1st stage completed or not (**interim reporting deadline**)

Proposition 3

Interim deadline optimal for startup

Startup **postpones date of interim reporting** so that investor's IR binds: $V(\tau) = 0$

$V(\tau)$, as a function of interim reporting deadline chosen by startup, S:



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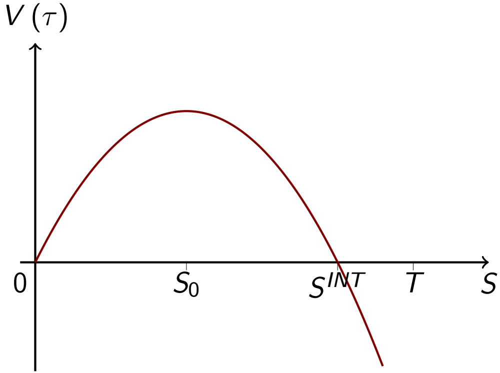

At **interim deadline** S^{INT}, stopping is recommended with certainty if 1st stage is not yet completed!

Conclusion

- Startup designs information provision to investor with goal of postponing stopping of funding

- Ex ante promising project $\Rightarrow$ startup stays silent at interim stages and discloses only completion of project with delay
- Ex ante unattractive project $\Rightarrow$ startup both immediately discloses completion of project and provides progress reports at the interim date

- Interim self-reporting deadline emerges when:
  (i) there is hard project completion deadline for investor and
  (ii) project has sufficiently high cost-benefit ratio for investor

Thank you for your attention!

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Full information benchmark

Continuation value of investor at time t under full information and conditional on completion of 1st stage of project:

$$V_{t|1} = \left(v - \frac{c}{\lambda}\right)\left(1 - e^{-\lambda(T-t)}\right)$$

Inv-preferred deadline

Investment schedule τ

Formally: stopping time τ with respect to filtration $F = (\mathcal{F}_t)_{t \geq 0}$ generated by stochastic process x_t.

Informally: τ is random variable with support $[0, T]$ induced by rule specifying when to stop based on history of x_t, e.g.,

- "stop 1 minute after x_t first reaches 2"
- "stop at $t = S$ if $x_S = 1$"

Inv. schedule

Implementability of investment schedule

Lemma: investment schedule τ is implementable using DRM if

$$V_t(\tau) \geq 0, \forall t \geq 0,$$

and, given recommendation to stop at t, investor's continuation value at t in absence of any future information from startup is negative for all $t \geq 0$.

Interpretation:

1. Given recommendation not to stop, continuation value stays non-negative $\Rightarrow$ optimal to continue
2. Given recommendation to stop, continuation value is negative $\Rightarrow$ optimal to stop

DRM

Obedient DRM when project is promising (low κ)

Consider **candidate mechanism**: (only) at $t = S^*$, stop if 2nd stage is already completed.

Note that for $t > S^*$, belief that state is 2 drifts up $\Rightarrow$ at some date recommendation to continue **can cease to be obedient**!

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Example of optimal mechanism: no recommendation to stop during $t \in [0, S^*)$. At $t = S^*$, stop if second stage is already completed. If 2nd stage is not yet completed, then stop at moment of its completion. Formally,

$$\tau = \begin{cases} S^*, & \text{if } x_{S^*} = 2 \\ \min(\tau_2, T), & \text{otherwise,} \end{cases}$$

where S^* is chosen s.t. $V(\tau) = \max\left(0, V^{NI}\right)$. Proposition 1

Interim deadline chosen by startup

Proposition 3: assume $\kappa \in \left(\tilde{\kappa}\left(T,\lambda\right), \kappa^{FI}\left(T,\lambda\right)\right)$. Optimal information policy is DRM that generates

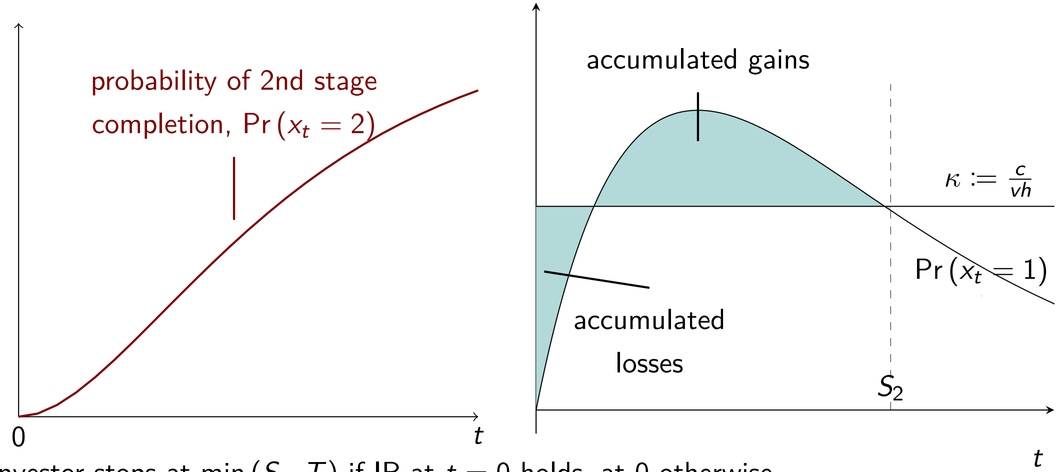
a. recommendation to stop at moment of 2nd stage completion, $t = \tau_2$ and
b. conditional recommendation to stop at interim deadline $t = S^{INT}$.

At $t = S^{INT}$, stopping is recommended with certainty if 1st stage is not yet completed. S^{INT} is chosen so that IR constraint is binding, $V\left(\tau\right) = 0$.

Startup-optimal deadline

Investor's choice under no information

Investor solves $\max_{S \in [0,T]} v \cdot \Pr(x_S = 2) - c \cdot S$



Investor stops at $\min(S_2, T)$ if IR at $t = 0$ holds, at 0 otherwise