Measuring Utility

An Application to Higher Order Risk Preferences

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- Theoretically, higher order risk preferences are related to a wide range of domains, e.g.,
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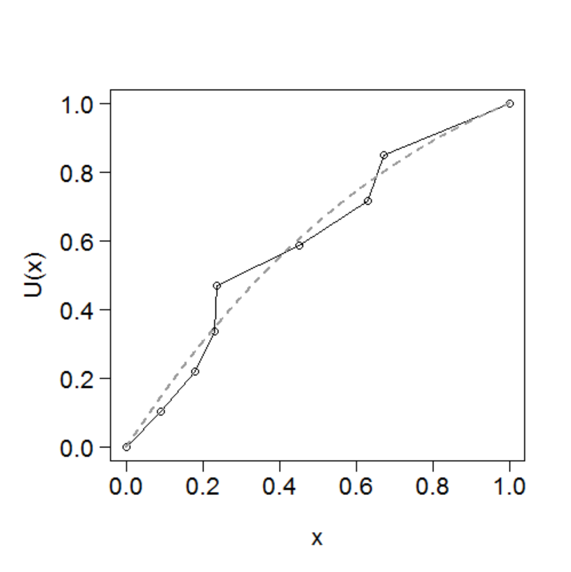

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Need for a sophisticated alternative!

Contribution

We propose a method for direct (non-parametric) elicitation of utility-based intensity coefficients of HORPs

- The method (a supervised machine learning approach)
    - Data
    - Model
    - Objective function
    - Algorithm/optimization
- Simulation and validation
- (Application to savings in Bogota w./ theoretical model)

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  - Model
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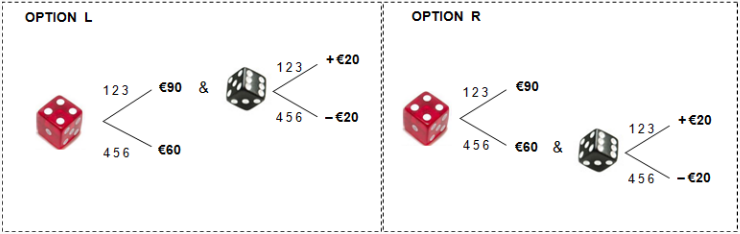
Preview on the performance of the method:

- Correlation with DGP in simulation .96 (lin. interpol.: .37)

Third Order Risk Preference: Prudence

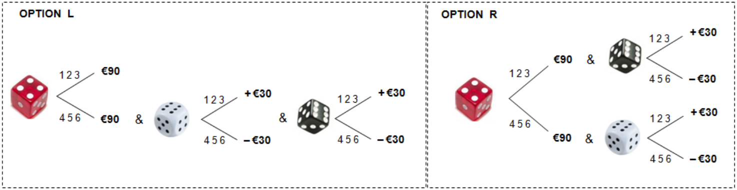


Prudence: Preference for the left lottery

- Prefer allocation of risk in states of higher wealth
- Downside Risk Aversion (Menezes et al., 1980)
- Preference for (right) skewness
  (Modica and Scarsini, 2005; Ebert, 2013)

Distribution of Options

Fourth Order Risk Preference: Temperance



Temperance: Preference for the right lottery

- Prefer disaggregation of risks over states of the world
- Dislike of Kurtosis, i.e., fat tails
  (Denuit and Eeckhoudt, 2010; Ebert, 2013)

▸ Distribution of Options

Higher Order Risk Preferences

Utility-based definitions

Risk Aversion $u'' < 0$, i.e., concave utility function

Prudence $u''' > 0$, i.e., convex marginal utility function

Temperance $u^{(iv)} < 0$, i.e., u'' concave

$$\overset{\text{Eeckhoudt \& Schlesinger (2006)}}{\Leftrightarrow}$$

"Behavioral definitions" (previous slides)

Allocation/avoidance of mean zero risks

Higher Order Risk Preferences

Intensity measures of (higher order) risk preferences

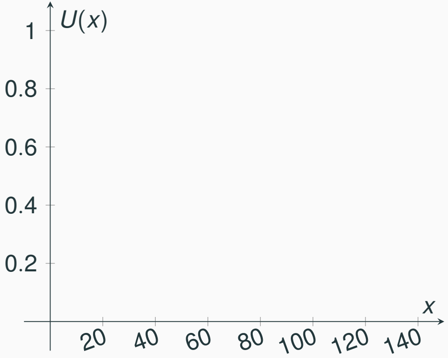
- Risk Aversion: $r^{AP} = -\frac{u''}{u'}$ (Arrow-Pratt)
- Prudence: $m = \frac{u'''}{u'}$ (Crainich and Eeckhoudt)
- Temperance: $t = -\frac{u^{(iv)}}{u'}$ (Denuit and Eeckhoudt)

$$\overset{?}{\Leftrightarrow}$$

"Behavioral definitions" (previous slides)

Allocation/avoidance of mean zero risks

Data: Utility Points via Certainty Equivalents



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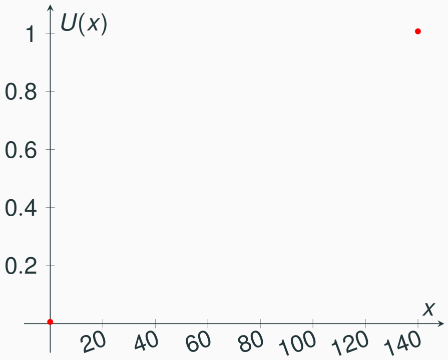
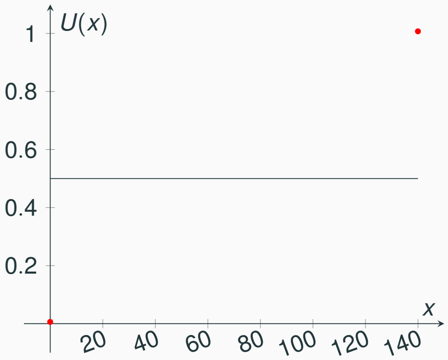


Data: Utility Points via Certainty Equivalents

$$EU_L = 0.5 \cdot U(140) + 0.5 \cdot U(0) = 0.5$$



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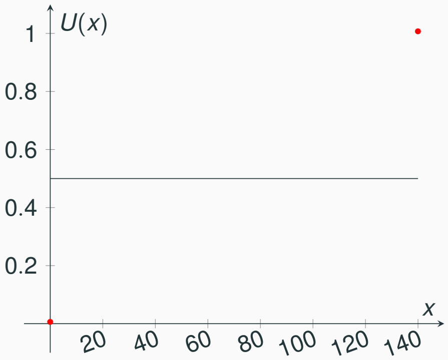

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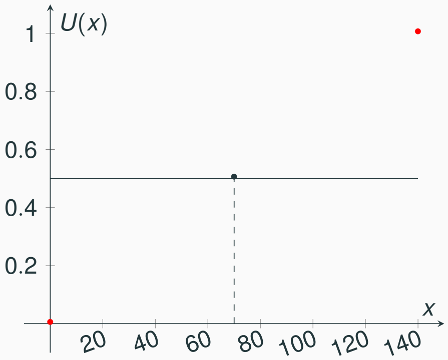

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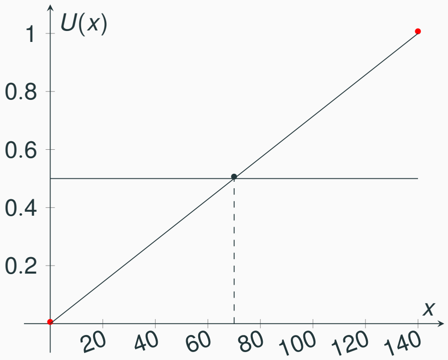

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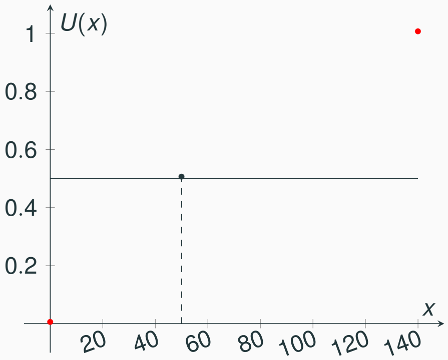

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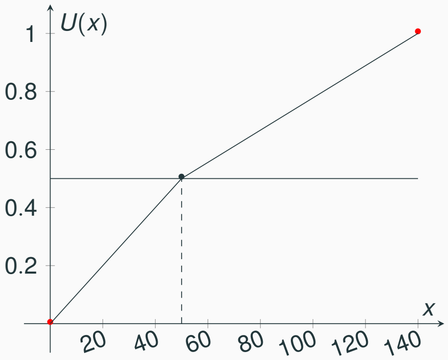

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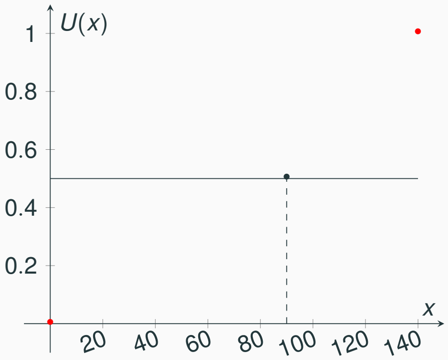

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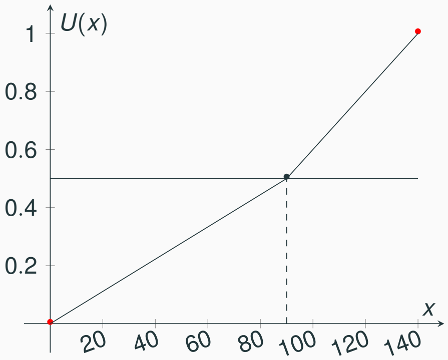

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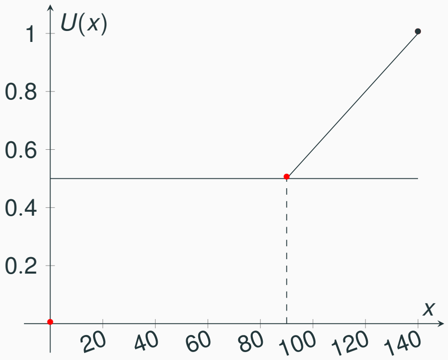

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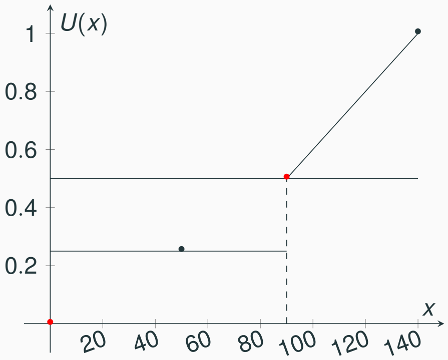

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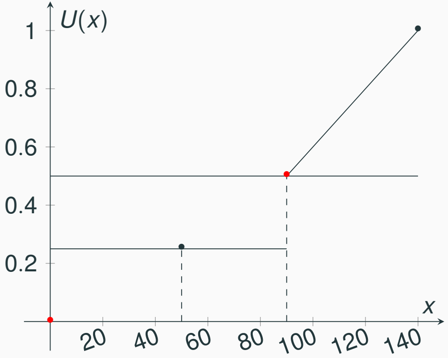

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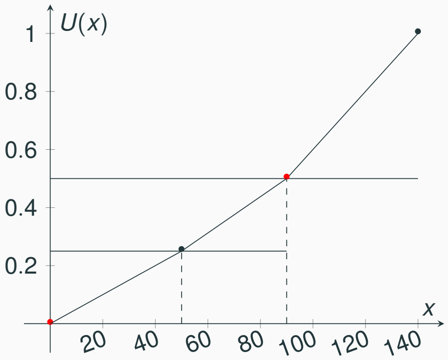

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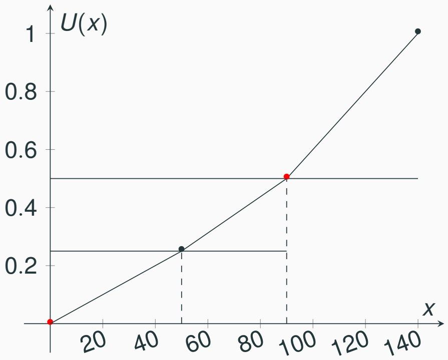
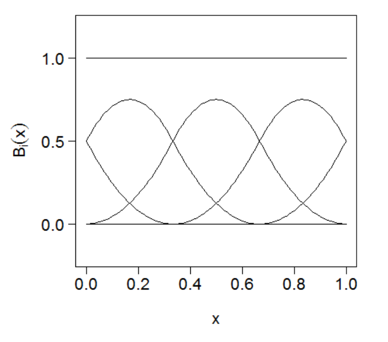


- Iterative
- Over small and large stakes
- Free of second-best assumptions about the shape of the utility curve and sign of its derivatives
- Challenges
  - Utility between elicited points?
  - Error correction?

Model: Lin. Model Using an Adapted P-Splines Approach



(a) B-spline basis

- Fit the following equation: $y_i = \beta_1 x_i + \varepsilon_i$
- Matrix notation

$$\boldsymbol{X} = \begin{bmatrix} x_0 & x_1 & \dots & x_6 & x_7 \end{bmatrix}^\top = \begin{bmatrix} 0 & 0.125 & \dots & 0.875 & 1 \end{bmatrix}^\top$$

- Objective: $\operatorname{argmin}_{\boldsymbol{\beta}} Q(\boldsymbol{\beta}) = \|\boldsymbol{y} - \boldsymbol{X}\beta\|^2$
- Solution: $\hat{\beta} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y}$

Polynomial Regression

- Fit the following equation: $y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 + \varepsilon_i$
- Matrix notation

$$\boldsymbol{X} = \begin{bmatrix} 1 & x_1 & x_1^2 & x_1^3 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_7 & x_7^2 & x_7^3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0.125 & 0.125^2 & 0.125^3 \\ \vdots & & & \\ 1 & 0.375 & 0.375^2 & 0.375^3 \\ 1 & 0.5 & 0.5^2 & 0.5^3 \end{bmatrix}; \ \boldsymbol{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix}$$

- Objective: $\operatorname{argmin}_{\boldsymbol{\beta}} Q(\boldsymbol{\beta}) = \|\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\|^2$
- Solution: $\hat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y}$

B-Spline Regression

- Generate basis functions $B_1, \ldots, B_d$: De Boor or differences of truncated power functions (TPF) ▸ Example
- Fit the following equation:
  $y_i = \beta_1 B_1(x_i) + \beta_2 B_2(x_i) + \cdots + \beta_d B_d(x_i) + \varepsilon_i$
- Matrix notation

$$\boldsymbol{X} = \begin{bmatrix} B_1(x_1) & B_2(x_1) & \ldots & B_d(x_1) \\ \vdots & \vdots & \vdots & \vdots \\ B_1(x_n) & B_2(x_n) & \ldots & B_d(x_n) \end{bmatrix} ; \ \boldsymbol{\beta} = \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_d \end{bmatrix}$$

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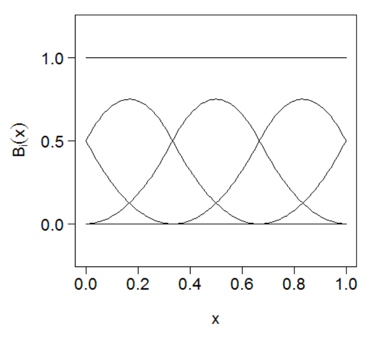
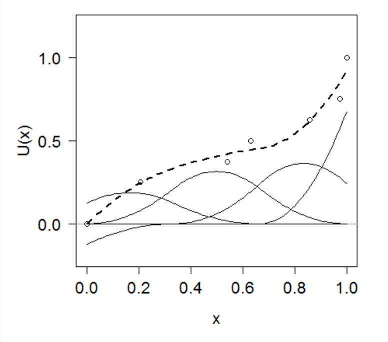


(a) B-spline basis



(b) Regression on B-spline basis

Analytical expression for derivative!

P-spline regression

Objective function of the P-spline regression

$$\text{argmin}_{\beta}\, Q(\beta) = \|\boldsymbol{y} - \boldsymbol{X}\beta\|^2 + \omega\|\boldsymbol{D_d}\beta\|^2,$$

where

- $\boldsymbol{D_d}$ the matrix representation of the dth diff. operator $\triangle^d$
- $\triangle^d a_j := \triangle(\triangle^{d-1} a_j)$ with $\triangle a_j := (a_j - a_{j-1})$

Model and Objective: Adapting the P-Spline Approach

Minimize MSPE by choice of penalty (i.e., choice of smoothness parameter)

Model and Objective: Adapting the P-Spline Approach

Minimize MSPE by choice of penalty (i.e., choice of smoothness parameter) s. th.

- Monotonicity fulfilled
- Value conditions $U(0) = 0$ and $U(1) = 1$ fulfilled
- Roughness penalty jointly smoothes several derivatives
- Roughness penalty “suits” the data, i.e., choose a data-driven minimum

Model and Objective: Adapting the P-Spline Approach

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Objective and Algorithm: Optimization via Cross Validation

How do we optimize the MSPE? Using the following algorithm:

For every iteration:

- Pick penalty weight
- Perform exhaustive K-fold cross validation where $K = \binom{n}{j}$
  In our validation study: $n = 6$, $j = 2$, and thus $K = 15$
- Compute MSPE as

$$\frac{1}{K}\sum_{k=1}^{K}\sum_{i\in I^{(k)}}\{y_i - \hat{y}_{(-k)}(x_i)\}^2,$$

  with $I^{(k)}$ the set containing the kth choice of j points for validation and $\hat{y}_{-k}(x_i)$ the prediction of y_i, obtained by estimating the model using all points but those in $I^{(k)}$
- Stop when MSPE minimized (or stopping criteria reached)

Simulation & Validation

To illustrate the suitability of our method, we conduct

- Simulation Study:

- Validation Study:

Simulation & Validation

To illustrate the suitability of our method, we conduct

- Simulation Study:
  - Generate data according to parametric function
  - Elicit utility points using certainty equivalents
  - Fit parametric function & our model
  - Derive intensity coefficient for risk aversion
  - Introduce decision error
- Validation Study:

Simulation & Validation

To illustrate the suitability of our method, we conduct

- Simulation Study: Comparable or superior to alternatives – yet with the required flexibility! ▸ Show results

- Validation Study:

Simulation & Validation

To illustrate the suitability of our method, we conduct

- Simulation Study: Comparable or superior to alternatives – yet with the required flexibility! ▸ Show results

- Validation Study:
  - Compare our method with elicitation using the "behavioral defintion"
  - Several variants, among them one for elicitation of premia
  - Test-Retest-Reliability
  - Online and in the lab, $N_1 = 585 = 527 + 58$, ($N_2 = 523 = 465 + 58$)

Simulation & Validation

To illustrate the suitability of our method, we conduct

- Simulation Study: Comparable or superior to alternatives – yet with the required flexibility! ▸ Show results

- Validation Study: Significantly related to standard method – yet yielding proper intensity measures! ▸ Show results

Conclusion

- Tailor-made solution for predicting utility functions
- Develop a new method for elicitation of (utility-based) intensity measures of higher order risk preferences
- For higher order risk preferences, method yields very favorable results
- Method has been used to uncover novel results on the relation between field behavior and risk preferences and to see previous results in a new light (Schneider & Sutter, 2021)
- Method not restricted to higher order risk preferences - possible to study other fields!

Thank You

Thank you very much for your attention!

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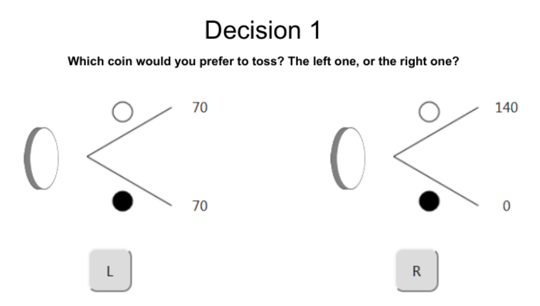
Appendix

Appendix

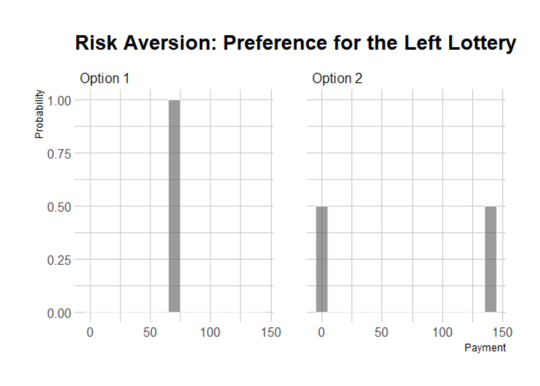
Second Order Risk Preference: Risk Aversion



Risk Aversion: Preference for the left lottery

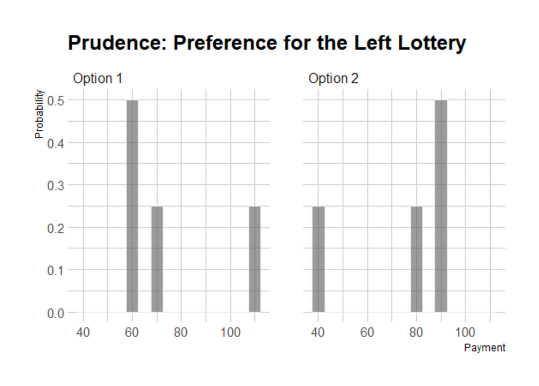
Distribution of Options

Second Order Risk Preference: Risk Aversion



▸ Definition: Risk Aversion

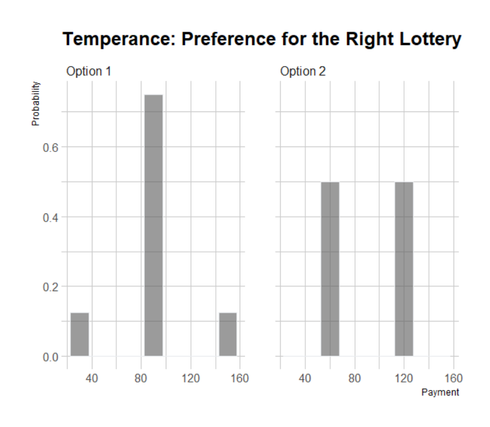
Third Order Risk Preference: Prudence



- Downside Risk Aversion (Menezes et al., 1980)
- Preference for (right) skewness
  (Modica and Scarsini, 2005; Ebert, 2013)

Definition: Prudence

Fourth Order Risk Preference: Temperance



- Dislike of Kurtosis, i.e., fat tails
  (Denuit and Eeckhoudt, 2010; Ebert, 2013)

Definition: Temperance

Example of Local Basis Function: Truncated Power Function of Degree 1 (Linear)

For a given knot κ_i, the truncated power function of degree 1 is defined as

$$(x - \kappa_i)_+ = \begin{cases} x - \kappa_i & \text{if } x - \kappa_i > 0 \\ 0 & \text{if } x - \kappa_i \leq 0 \end{cases} \tag{1}$$

Basis of the spline model is

$$\begin{aligned} \mathbf{B} &= \begin{bmatrix} B_1(x) & \dots & B_n(x) \end{bmatrix} \\ &= \begin{bmatrix} x & (x - \kappa_1)_+ & (x - \kappa_2)_+ & \dots & (x - \kappa_n)_+ \end{bmatrix} \end{aligned}$$

Back to B-Spline Regression

Simulation Results: Measurement Error, Error Propagation and Error Correction

	$AR_{\text{Expo-Power}}$	AR_{CRRA}	AR_{Linear}	$AR_{\text{Schneider et al.}}$
Correlation ρ with AR_{DGP}	0.95	0.97	0.37	0.96
Δ in ρ caused by error	0.04	0.00	0.38	0.00

Notes: This table shows in the first row the Pearson correlation coefficients between the "true" Arrow-Pratt measure of risk aversion according to the assumed data generating process, AR_{DGP}, and the Arrow-Pratt measures obtained via elicitation of certainty equivalents and subsequent estimation of a utility curve according to the respective methods. The entries in the second row denote the difference between the correlation coefficients in the first row and correlation coefficients obtained when (simulated) measurement error is introduced.

▸ Simulation & Validation

Test-Retest Reliability: Prudence

	Risk App.	EW	Schneider et al.
Mean Squared Prediction Error	1.63	1.55	**1.19**
Mean Absolute Prediction Error	0.96	0.89	**0.76**
Correlation Coeff. (Spearman)	0.18	0.14	**0.25**
p-value Correlation Coeff.	0.02	0.41	0.00
Outliers Wave 1 or 2 in % (Excluded)	0.00	0.00	4.27
N	175.00	36.00	359.00

▸ Simulation & Validation

Test-Retest Reliability: Temperance

	Risk App.	EW	Schneider et al.
Mean Squared Prediction Error	1.62	1.41	**1.10**
Mean Absolute Prediction Error	0.96	0.83	**0.61**
Correlation Coeff. (Spearman)	**0.18**	0.14	0.15
p-value Correlation Coeff.	0.02	0.41	0.00
Outliers Wave 1 or 2 in % (Excluded)	0.00	0.00	5.87
N	175.00	36.00	353.00

Simulation & Validation