

# On the Relevance of Irrelevant Strategies

Ayala Arad<sup>1</sup>, Benjamin Bachi<sup>2</sup>, Amnon Maltz<sup>2</sup>

<sup>1</sup>Tel Aviv University, <sup>2</sup>University of Haifa

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# Example

- Two public transport companies are considering opening a line between two cities
- They choose one of two possibilities
  - Express - direct between central stations
  - Local-town - travels through small towns along the way
- Payoffs

	<i>Local – town</i>	<i>Express</i>
<i>Local – town</i>	40, 40	60, 80
<i>Express</i>	80, 60	50, 50

## Example - Continued

- Suppose one company is considering another local line:  
Local-village
- Payoffs are identical to Local-town

	<i>Local – town</i>	<i>Express</i>
<i>Local – town</i>	40, 40	60, 80
<i>Local – village</i>	40, 40	60, 80
<i>Express</i>	80, 60	50, 50

## Example - Continued

- Suppose one company is considering another local line:  
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	<i>Local – town</i>	<i>Express</i>
<i>Local – town</i>	40, 40	60, 80
<i>Local – village</i>	40, 40	60, 80
<i>Express</i>	80, 60	50, 50

- The game is strategically identical

## Example - Continued

- Suppose one company is considering another local line:  
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	<i>Local – town</i>	<i>Express</i>
<i>Local – town</i>	40, 40	60, 80
<i>Local – village</i>	40, 40	60, 80
<i>Express</i>	80, 60	50, 50

- The game is strategically identical
- Local-Village is “irrelevant” in terms of the game’s outcome

## Example - Continued

- Suppose one company is considering another local line:  
Local-village
- Payoffs are identical to Local-town

	<i>Local – town</i>	<i>Express</i>
<i>Local – town</i>	40, 40	60, 80
<i>Local – village</i>	40, 40	60, 80
<i>Express</i>	80, 60	50, 50

- The game is strategically identical
- Local-Village is “irrelevant” in terms of the game’s outcome
- Will behavior change?

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- In individual choice **irrelevant alternatives** may indirectly impact agents' choices

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# Motivation

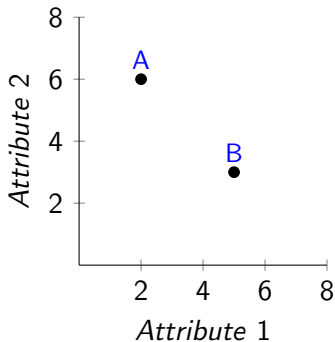
- In individual choice **irrelevant alternatives** may indirectly impact agents' choices
- Extensively studied in individual choice problems
- Our goal - explore whether **irrelevant strategies** impact behavior in strategic environments, i.e., games

# Goal Breakdown

- Direct effect: how do the players who are introduced with an “irrelevant” strategy react?
- Indirect effect: do the other players respond?
- Outcome: in coordination games, is equilibrium reached more often in the presence of the irrelevant strategy?

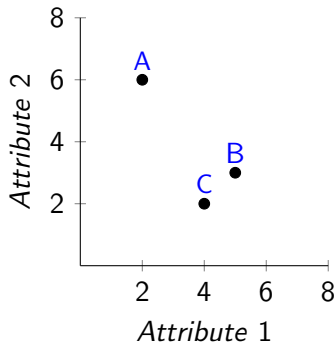
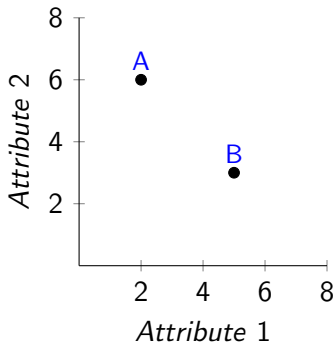
# Irrelevant Alternatives in Individual Choice

# Asymmetrically Dominated Alternative

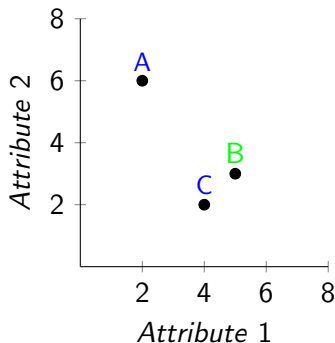
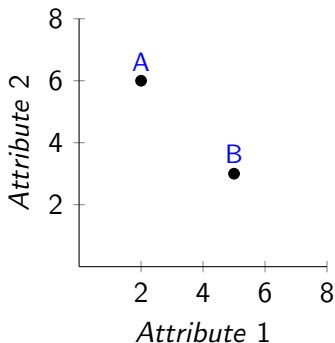


- Choice with trade-off/dilemma

# Asymmetrically Dominated Alternative

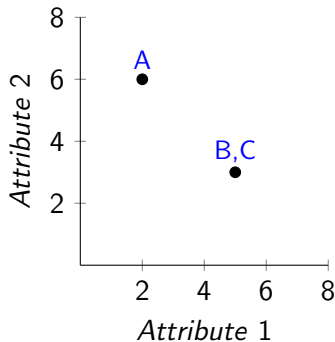
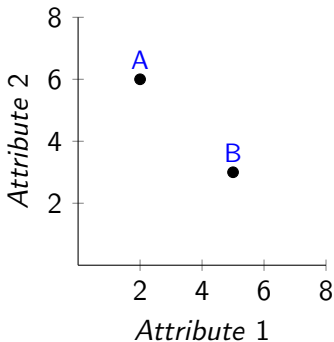


# Asymmetrically Dominated Alternative

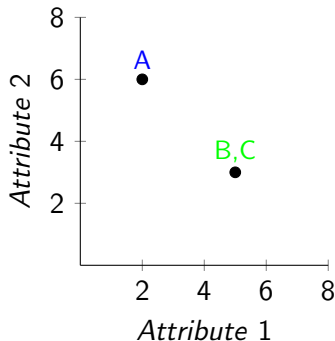
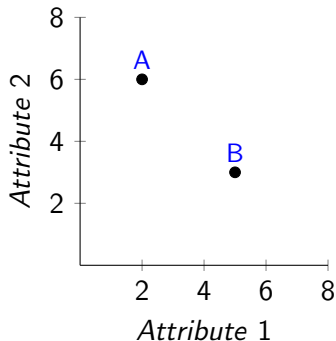


- Also known as the attraction effect (Huber et al., 1982)
- Finding of Interest: Choice share of *B* increases in the presence of *C*

# Duplicated Alternative



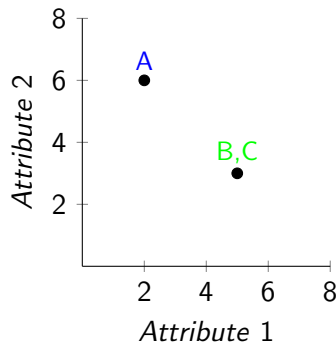
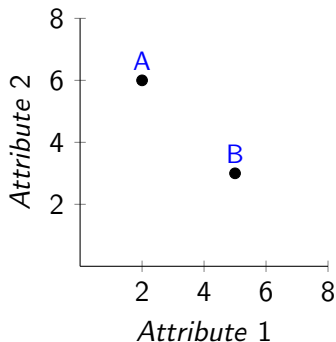
# Duplicates Alternative



- Luce (1959), Becker et al. (1963)

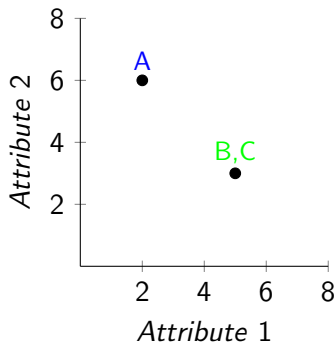
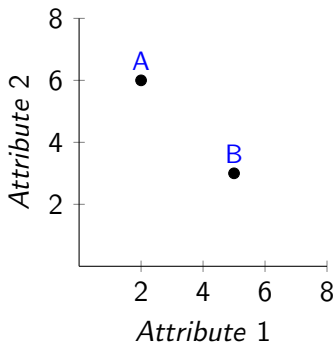


# Duplicates Alternative



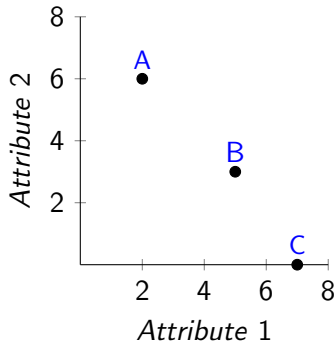
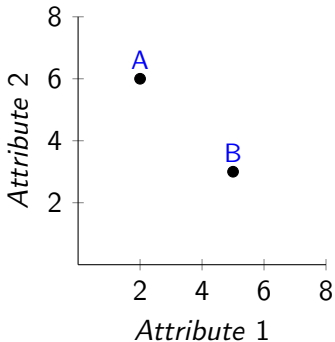
- Luce (1959), Becker et al. (1963)
- Related to the *similarity effect*

# Duplicates Alternative

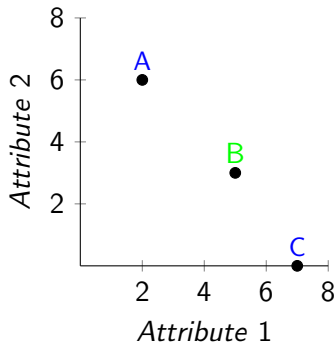
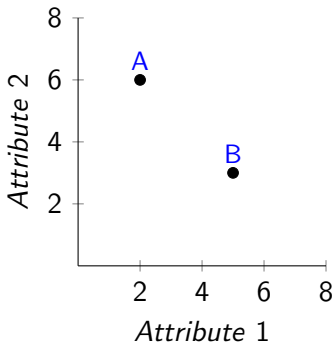


- Luce (1959), Becker et al. (1963)
- Related to the *similarity effect*
- Finding of interest: Choice share of *B* and *C* is higher than choice share of *B* in the absence of *C*

# Extreme Alternative

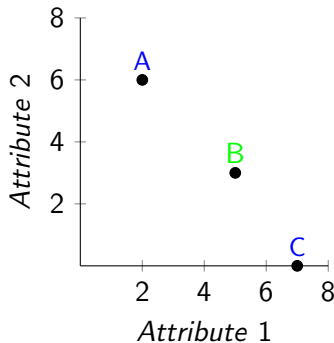
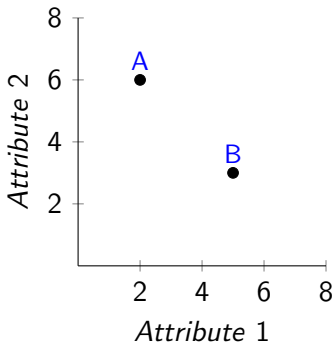


# Extreme Alternative



- Compromise effect (Simonson, 1989)
- Finding of Interest: Share of  $B/A$  increases when  $C$  is added
- Note: Added alternative **is relevant**

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- Finding of Interest: Share of  $B/A$  increases when  $C$  is added
- Note: Added alternative **is relevant**
- Won't mention today

# Irrelevant Strategies in Games

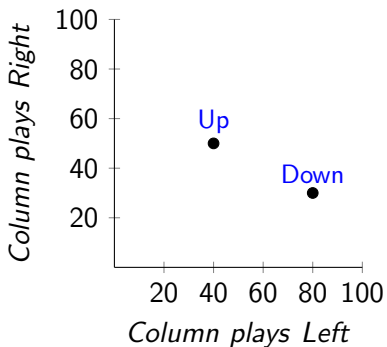
# Attributes in Games

	<i>L</i>	<i>R</i>
<i>U</i>	40, 40	50, 80
<i>D</i>	80, 50	30, 30

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	<i>L</i>	<i>R</i>
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- What are the attributes for the row player?





# Dominated Strategies

- Base game

	<i>L</i>	<i>R</i>
<i>U</i>	40, 40	50, 80
<i>D</i>	80, 50	30, 30

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- Dominance extension:

	<i>L</i>	<i>R</i>
<i>U</i>	40, 40	50, 80
<i>M</i>	35, 20	45, 20
<i>D</i>	80, 50	30, 30

# Dominated Strategies

- Base game

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- Dominance extension:

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<i>U</i>	40, 40	50, 80
<i>M</i>	35, 20	45, 20
<i>D</i>	80, 50	30, 30

- Prediction:  $U_p$  chosen more frequently in extension

# Duplicated Strategies

	<i>L</i>	<i>R</i>
<i>U</i>	40, 40	50, 80
<i>D</i>	80, 50	30, 30

- Duplicates extension:

	<i>L</i>	<i>R</i>
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# Duplicated Strategies

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- Duplicates extension:

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<i>U</i>	40, 40	50, 80
<i>M</i>	40, 40	50, 80
<i>D</i>	80, 50	30, 30

- Prediction: *Middle* and *Up* chosen more frequently in extension than *Up* in base game

## Related Literature

- Attraction effect in games
  - Dominated strategy for both players (Colman et al., 2007)
  - Dominated strategy for row player in coordination games (Amaldoss et al., 2008)

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  - Irrelevant strategies: dominated and duplicated
  - Two types of strategic interactions: coordination games and single equilibrium games
  - Allows an insight into considerations leading to effects

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- Our contribution
  - Irrelevant strategies: dominated and duplicated
  - Two types of strategic interactions: coordination games and single equilibrium games
  - Allows an insight into considerations leading to effects
- Attraction and compromise in bargaining (Galeotti et al., 2021)
  - Cooperative games
  - Dominated/extreme **equilibrium**



# Experimental Design

# Experimental Design

- 8 base games: 4 coordination, 4 single equilibrium
- 4 versions of each game: base + 3 extensions
- 4 non-matrix form “refreshment” games
- Total of 36 games
- Between subjects
- Feedback only at the end

# Experimental Design - Continued

- 2 experimental versions - players' roles flipped
- For each base game, players play **base as Row** and **extensions as Column**, or vice versa
- Players randomly matched with another (anonymous) player each game
- One game randomly selected for payment
- 2 orders

# Data Collection

- Interactive Decision Making Lab, Coller School of Management, TAU
- Computer Based
- 238 subjects - TAU undergraduates
- Average payment - 75 ILS ( $\approx$  20 USD)
- Duration - 45 minutes (including training)
- Pre-registered on AEA RCT Registry
- ISF grant

# Results

# Direct Effects

- Percent of target choices
- Definition of target strategy
  - Base games - Up
  - Dominance extensions - Up
  - Duplicates extensions - Up and Middle

# Direct Effects in Coordination Games

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	Game 1	Game 2	Game 3	Game 4
<b>Base</b>	<b>59</b>	<b>51</b>	<b>59</b>	<b>56</b>
Dominance Extension	62	62	62	66
Duplicates Extension	73	76	75	66



# Direct Effects in Coordination Games

	Game 1	Game 2	Game 3	Game 4
<b>Base</b>	<b>59</b>	<b>51</b>	<b>59</b>	<b>56</b>
Dominance Extension	62	62	62	66
Duplicates Extension	73	76	75	66

- Direct effects of dominated and duplicated strategies

# Direct Effect in Coordination Games: Regressions

	<i>Dependent variable: Target Choice</i>					
	Dominance Extension			Duplicates Extension		
	(1)	(2)	(3)	(4)	(5)	(6)
Extension	0.28** (0.13)	0.28** (0.12)	0.45** (0.20)	0.71*** (0.14)	0.71*** (0.12)	1.19*** (0.22)
Order	-0.05 (0.14)	-0.05 (0.17)		-0.06 (0.14)	-0.06 (0.18)	
Gender (male=1)	-0.10 (0.13)	-0.10 (0.17)		0.01 (0.14)	0.01 (0.18)	
correct	0.12 (0.09)	0.12 (0.14)		0.07 (0.09)	0.07 (0.13)	
game <sub>2</sub>	-0.15 (0.19)	-0.15 (0.17)	-0.26 (0.28)	-0.11 (0.20)	-0.11 (0.18)	-0.19 (0.30)
game <sub>3</sub>	0.01 (0.19)	0.01 (0.18)	0.01 (0.30)	0.04 (0.20)	0.04 (0.18)	0.06 (0.30)
game <sub>4</sub>	0.04 (0.19)	0.04 (0.17)	0.06 (0.28)	-0.23 (0.19)	-0.23 (0.18)	0.39 (0.31)
Constant	-0.49 (0.75)	-0.49 (0.75)	-0.18 (0.21)	-0.16 (0.75)	-0.16 (1.04)	-0.46** (0.20)
Observations	935	935	639	952	952	644

Notes: Numbers represent coefficients ( $\beta$ ), std. errors in parentheses.

\*  $p < 0.1$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$ .

# Direct Effects in Single Equilibrium Games

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	Game 1	Game 2	Game 3	Game 4
<b>Base</b>	<b>46</b>	<b>44</b>	<b>54</b>	<b>49</b>
Dominance Extension	52	53	54	53
Duplicates Extension	49	49	54	51

- No effects

# Direct Effect in Single Equilibrium Games: Regressions

	<i>Dependent variable: Target Choice</i>					
	Dominance Extension			Duplicates Extension		
	(1)	(2)	(3)	(4)	(5)	(6)
Extension	0.19 (0.13)	0.19* (0.10)	0.45** (0.22)	0.10 (0.13)	0.10 (0.09)	0.24 (0.21)
Order	0.16 (0.13)	0.16 (0.19)		0.12 (0.13)	0.12 (0.19)	
Gender (male=1)	0.23* (0.13)	0.23 (0.19)		0.16 (0.13)	0.16 (0.19)	
correct	0.14 (0.09)	0.14 (0.11)		0.06 (0.09)	0.06 (0.11)	
game <sub>6</sub>	-0.04 (0.19)	-0.04 (0.15)	-0.10 (0.32)	-0.05 (0.18)	-0.05 (0.14)	-0.13 (0.31)
game <sub>7</sub>	0.19 (0.19)	0.19 (0.15)	0.39 (0.33)	0.25 (0.18)	0.25* (0.15)	0.56* (0.34)
game <sub>8</sub>	0.08 (0.19)	0.08 (0.13)	0.15 (0.28)	0.10 (0.18)	0.10 (0.15)	0.21 (0.33)
Constant	-1.54** (0.76)	-1.54* (0.90)	0.796*** (0.20)	-0.90 (0.73)	-0.90 (0.93)	0.85*** (0.21)
Observations	939	939	510	952	952	528

Notes: Numbers represent coefficients ( $\beta$ ), std. errors in parentheses.

\*  $p < 0.1$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$ .

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	Game 1	Game 2	Game 3	Game 4
<b>Base</b>	<b>41</b>	<b>48</b>	<b>48</b>	<b>46</b>
Dominance Extension	50	61	61	65
Duplicates Extension	68	76	62	78



# Indirect Effects in Coordination Games

- Again, we look at % of target choices
- target for the column player is the best response to the row player's target

	Game 1	Game 2	Game 3	Game 4
<b>Base</b>	<b>41</b>	<b>48</b>	<b>48</b>	<b>46</b>
Dominance Extension	50	61	61	65
Duplicates Extension	68	76	62	78

- Column players seem to respond to direct effects

# Indirect Effect in Coordination Games: Regressions

	<i>Dependent variable: Target Choice</i>					
	Dominance Extension			Duplicates Extension		
	(1)	(2)	(3)	(4)	(5)	(6)
Extension	0.56*** (0.13)	0.56** (0.11)	0.981** (0.20)	1.07*** (0.14)	1.07*** (0.12)	1.78*** (0.23)
Order	0.08 (0.13)	0.08 (0.17)		0.06 (0.14)	0.06 (0.17)	
Gender (male=1)	0.19 (0.13)	0.19 (0.17)		0.06 (0.14)	0.06 (0.17)	
correct	-0.03 (0.09)	-0.03 (0.09)		0.01 (0.09)	0.01 (0.08)	
game <sub>2</sub>	0.36* (0.19)	0.36** (0.16)	0.68** (0.28)	0.32 (0.19)	0.32* (0.17)	0.49 (0.30)
game <sub>3</sub>	0.36* (0.19)	0.36** (0.17)	0.62** (0.30)	0.02 (0.19)	0.02 (0.18)	0.03 (0.30)
game <sub>4</sub>	0.40** (0.19)	0.40** (0.17)	0.74*** (0.28)	0.34* (0.19)	0.34* (0.17)	0.52* (0.30)
Constant	-0.44 (0.74)	-0.44 (0.76)	0.13 (0.20)	-0.54 (0.76)	-0.54 (0.70)	-2.58*** (0.30)
Observations	952	952	680	952	952	704

Notes: Numbers represent coefficients ( $\beta$ ), std. errors in parentheses.

\*  $p < 0.1$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$ .

# Indirect Effects in Single Equilibrium Games

	Game 1	Game 2	Game 3	Game 4
<b>Base</b>	<b>53</b>	<b>55</b>	<b>46</b>	<b>50</b>
Dominance Extension	46	58	49	55
Duplicates Extension	63	57	46	51

- No indirect effect

# Indirect Effect in Single Equilibrium Games: Regressions

	<i>Dependent variable: Target Choice</i>					
	Dominance Extension			Duplicates Extension		
	(1)	(2)	(3)	(4)	(5)	(6)
Extension	0.05 (0.13)	0.05 (0.10)	0.09 (0.23)	0.15 (0.13)	0.15 (0.10)	0.33 (0.22)
Order	-0.39*** (0.13)	-0.39** (0.19)		-0.37*** (0.13)	-0.37* (0.20)	
Gender (male=1)	0.16 (0.13)	0.16 (0.19)		0.16 (0.13)	0.16 (0.20)	
correct	-0.34*** (0.10)	-0.34** (0.15)		-0.28*** (0.10)	-0.28** (0.13)	
game <sub>6</sub>	0.28 (0.19)	0.28* (0.14)	0.60* (0.31)	-0.09 (0.19)	-0.09 (0.13)	-0.17 (0.29)
game <sub>7</sub>	-0.09 (0.19)	-0.09 (0.14)	-0.19 (0.31)	-0.48*** (0.19)	-0.48*** (0.15)	-1.07*** (0.34)
game <sub>8</sub>	0.12 (0.19)	0.12 (0.13)	0.26 (0.29)	-0.31* (0.19)	-0.31* (0.16)	-0.67* (0.36)
Constant	3.09*** (0.82)	3.09*** (1.19)	0.91*** (0.19)	2.85*** (0.80)	2.85*** (1.07)	1.46*** (0.24)
Observations	952	952	524	952	952	504

Notes: Numbers represent coefficients ( $\beta$ ), std. errors in parentheses.

\*  $p < 0.1$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$ .

# Coordination Rates

	Base		Dominance		Duplicates	
Game 1	33	<b>26*</b>	33	<b>28*</b>	26	<b>47*</b>
	<b>26</b>	15	<b>17</b>	21	<b>6</b>	21
Game 2	<b>24*</b>	28	<b>37*</b>	24	<b>55*</b>	21
	24	<b>24</b>	24	<b>13</b>	21	<b>3</b>
Game 3	<b>30*</b>	29	<b>36*</b>	24	<b>50*</b>	25
	18	<b>24</b>	24	<b>13</b>	13	<b>13</b>
Game 4	33	<b>24*</b>	21	<b>40*</b>	12	<b>54*</b>
	<b>21</b>	23	<b>12</b>	19	<b>10</b>	24

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	Base		Dominance		Duplicates	
Game 1	33	<b>26*</b>	33	<b>28*</b>	26	<b>47*</b>
	<b>26</b>	15	<b>17</b>	21	<b>6</b>	21
Game 2	<b>24*</b>	28	<b>37*</b>	24	<b>55*</b>	21
	24	<b>24</b>	24	<b>13</b>	21	<b>3</b>
Game 3	<b>30*</b>	29	<b>36*</b>	24	<b>50*</b>	25
	18	<b>24</b>	24	<b>13</b>	13	<b>13</b>
Game 4	33	<b>24*</b>	21	<b>40*</b>	12	<b>54*</b>
	<b>21</b>	23	<b>12</b>	19	<b>10</b>	24

- Coordination increases with irrelevant strategies

# Coordination Rates: Regressions

<i>Dependent variable: Target Equilibrium</i>				
	Dominance Extension		Duplicates Extension	
	(1)	(2)	(3)	(4)
Extension	0.45*** (0.14)	0.64*** (0.17)	1.11*** (0.14)	1.62*** (0.18)
game <sub>2</sub>	0.167 (0.20)	0.347 (0.25)	0.12 (0.20)	0.10 (0.24)
game <sub>3</sub>	0.30 (0.20)	0.46* (0.24)	0.15 (0.20)	0.15 (0.24)
game <sub>4</sub>	0.246 (0.20)	0.362 (0.25)	0.10 (0.20)	0.11 (0.24)
Constant	-1.237*** (0.167)	-0.651*** (1.03)	-1.15*** (0.16)	-1.36*** (1.16)
Observations	952	851	952	920

*Notes:* Numbers represent coefficients ( $\beta$ ), Std. errors in parentheses.  
\*  $p < 0.1$ ; \*\*  $p < 0.05$ ; \*\*\*  $p < 0.01$ .

# Discussion



# Main Findings

- Dominance and duplicates effects in coordination games
- Lead to higher coordination rates
- No effects in single equilibrium games

# Psychological Mechanism

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- Not an individual, strategy-free response. Rather,
- Serves a strategic purpose
- Facilitates coordination

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- Alternative approaches
  - Quantal Response Equilibrium (McKelvey and Palfrey, 1995)
  - Sampling Equilibrium (Osborne and Rubinstein, 1998)
  - Level- $k$
  - Cognitive Hierarchy (Camerer et al., 2004)
  - Generalized Cognitive Hierarchy (Chong et al., 2016)



# Adapted Level- $k$

- General idea: level  $k$  best responds to level  $k - 1$
- Normally: level-0 plays uniformly
- Adaptation: “context-effected level-0” who is attracted to dominant/duplicated strategies
- Allow for heterogeneous risk attitudes (at least some are moderately risk averse)
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  - No behavioral level-0 types
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- With these adjustment, our findings may be explained
  - No behavioral level-0 types
  - Level-1 and level-2 who anchor beliefs on such a type
  - Sometimes level-0 is only in the minds of higher types (Crawford and Iriberry, 2007)

Thank You!

# Types of Base Games

	<i>L</i>	<i>R</i>
<i>U</i>	40, 40	<b>50, 80</b>
<i>D</i>	<b>80, 50</b>	30, 30

*Coordination*

	<i>L</i>	<i>R</i>
<i>U</i>	40, 40	<b>50, 50</b>
<i>D</i>	80, 80	30, 90

*Single Equilibrium*

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