Reforms meet fairness concerns in school and college admissions

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Introduction

Abstract

Did reforms in matching systems around the world make them more fair?

Which matching systems and what reforms?

- School choice in US, Ghana, UK (Pathak and Sönmez, 2013)
- College admission in Chinese provinces (Chen and Kesten, 2017)

We compare mechanisms before and after reforms by two fairness criteria. Criteria are based on stability.

We find (partial) evidence that reforms made mechanisms more fair.

Motivation

Why study the reforms?

- mechanisms were criticized for vulnerability to gaming and unfairness

"high-scoring kids were being rejected simply because of the order in which they listed their college prep preferences"

Chicago authorities (Pathak and Sönmez, 2013)

"My child has been among the best students in his school and school district. [...] Unfortunately, he was not accepted by his first choice. [...] his second and third choices were already full. My child had no choice but to repeat his senior year"

Chinese parent (Chen and Kesten, 2017)

- the reforms responded to this criticism
- BUT mechanisms remained unfair and manipulable even after reforms

- did the mechanisms become BETTER?

Motivation: why fairness?

Fairness is the reported aim of the reforms:

"This Code and the related legislation will ensure that admission authorities – whether local authorities or schools – operate in a fair way"
Alan Johnson (School Admission Code, 2007)

Much is clear about vulnerability...

After the reforms the mechanisms became:
- less manipulable by profile inclusion (Pathak and Sönmez, 2013 AER; Chen and Kesten, 2017 JPE)
- more truthful by preference inclusion (Decerf and Linden, 2021 JET)
- less strategically accessible by environment inclusion (B&N, 2021 TE) - reduced the number of manipulating agents (B&N, forthcoming TE)

... but not much is known about fairness.

Fairness criteria

Metric of unfairness – **blocking student**
- the student desires and deserves a better school

Matching without blocking students is **stable**

What does it mean to deserve a seat?
- Chicago and China: priorities based on grades
- England: over-subscription criteria and appeals

We do counterfactual analysis instance by instance
We compare sets of blocking students before and after each reform

Related literature

Most stable among non-manipulable and efficient is Top Trading Cycles:

- instance by instance (Abdulkadiroglu, Che, Pathak, Tercieux, 2020)
- by set inclusion of blocking students (Dogan and Ehlers, 2020)
- by set inclusion of blocking triples (Kwon and Shorrer, 2019)
- by any comparison satisfying few properties (Dogan&Ehlers, 2020)

Most stable among efficient is Kesten's (2010) Efficiency Adjusted DA:

- by set inclusion of blocking pairs (Dogan&Ehlers, 2020; Tang&Zhang, 2020)
- by set inclusion of blocking triples (Kwon and Shorrer, 2019)
- by counting blocking students – but only for acyclic priorities, (Dogan and Ehlers, 2020)

Related literature: comparing mechanisms in use

Comparing Chinese mechanisms instance by instance (Chen and Kesten, 2017):

- symmetric Chinese parallel is more stable than Boston

Comparing constrained DA and Boston using game-theoretic solution concepts (Decerf and Van der Linden, 2016):

- in undominated strategies

  - constrained DA is more stable than Boston
  - longer list makes DA more stable

- in Nash equilibrium

  - constrained DA is less stable than Boston
  - longer list makes DA less stable

Model

School choice model (Balinski and Sönmez, 1999; Abdulkadiroglu and Sönmez, 2003)

- set of students I
- set of schools S
- each student i has a **preference** P_i over $S \cup \{\emptyset\}$
- if $s \; P_i \; \emptyset$, then s is **acceptable** to i
- collection of preferences of all students P is preference **profile**
- each school s has a **priority** $\succ_s$ over I and a **capacity** q_s
- $(I, S, P, \succ, q)$ is school choice **problem**
- mapping $\mu : I \cup S \to I \cup S \cup \{\emptyset\}$ is **matching** satisfying
  - correspondence: $\mu(i) = s \iff i \in \mu(s)$
  - capacity is not exceeded: $\mu(s) \leq q_s$
  - if i is unmatched, then $\mu(i) = \emptyset$
- mapping φ from each problem to a matching is **mechanism**

Fairness Criteria

- i is a **blocking student** at μ if there exists $s \in S \cup \emptyset$ such that
  1. i prefers s over his matching: $sP_i\mu(s)$
  2. at least one seat at s is empty: $|\mu(s)| < q_s$
     or given to some student j with lower priority: $i \succ_s j$
- matching without blocking students is **stable**

- Mechanism φ' is **more stable** than φ if
  1. at each problem where φ is stable, φ' is also stable and
  2. at some problem φ' is stable but φ is not

- Mechanism φ' is **more stable by counting** than φ if
  1. at each problem φ has at least as many blocking students as φ'
  2. at some problem φ' has fewer blocking students than φ

Mechanisms and Reforms

- consider $(P, \succ, q)$
- Constrained mechanism φ^k
  - constrained profile P^k includes only top k rows of P:
  - $\varphi^k(P) = \varphi(P^k)$

- Gale and Shapley (1962) (GS, aka Deferred Acceptance)
  - each student proposes to the best school that didn't reject him yet
  - each school tentatively accepts up to capacity according to priority
  - others are rejected and proceed to the next step
- Boston Mechanism (BM, aka Immediate Acceptance)
  - each school immediately accepts up to capacity according to priority
- First Preference First (FPF)
  - schools are partitioned into FPF and equal priority schools
  - each FPF school admits according to BM
  - each equal priority school admits according to GS
- Chinese parallel ($Ch^{(e)}$, Chen and Kesten, 2017)
  - each student proposes to the best school that didn't reject him yet
  - after each student proposed e times, acceptance is finalized
  - unassigned students continue in the same way

Results

Preview of results

- Most mechanisms became more stable
  Except 50 districts in England
- Only few mechanisms became more stable by counting
  (Ghana (2007, 2008), Chicago (2010), Newcastle (2010), Surrey (2010))
- Stability and vulnerability are strongly related

Reforms and Stability

From	To	more stable?		more stable by counting?	
		Arbitrary $\succ$	Common $\succ$	Arbitrary $\succ$	Common $\succ$
FPF^k	GS^k	not comparable	more	not comparable	not comparable
β^k	GS^k	more	more	not comparable	not comparable
GS^k	GS^{k+1}	more	more	more	more
β	$Ch^{(e)}$	more	more	not comparable	not comparable

Notes: Each row compares the mechanism in the second column to the mechanism in the first column with respect to stability and stability by counting. Common priority is a special case of arbitrary priority.

$\underline{GS^k}$ is not more stable than $\boxed{FPF^k}$

$I = \{i_1, \ldots, i_7\}$ and $S = \{s_1, \ldots, s_5\}$, unit capacity

s_3 is the only FPF school in the old system

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}	P_{i_6}	P_{i_7}	$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$	$\succ_{s_5}$
s_1	s_1	$\underline{s_4}$	$\boxed{\underline{s_1}}$	$\boxed{\underline{s_2}}$	s_1	$\boxed{\underline{s_5}}$	i_4	i_5	i_3	i_1	i_7
s_2	s_3	$\boxed{s_3}$	s_2	s_1	s_2	s_1	$\vdots$	$\vdots$	i_1	i_6	$\vdots$
$\underline{s_3}$	$\boxed{\underline{\emptyset}}$	$\emptyset$	s_3	s_3	s_5	s_2			i_2	i_3	
$\boxed{s_4}$			$\emptyset$	$\emptyset$	s_3	$\emptyset$			$\vdots$	$\vdots$	
$\emptyset$					s_4						
					$\boxed{\underline{\emptyset}}$						

i_6 blocks with s_4 under $\underline{GS^4}$ but not under $\boxed{FPF^4}$

$\underline{GS^k}$ is not more stable than $\boxed{FPF^k}$

FPF is more stable when it shortens rejection chains leading to stability

This does not occur without rejection chains, e.g.:
- not under Boston
- not with common priority

$\underline{GS^{k+1}}$ is more stable by counting than $\boxed{GS^k}$

$I = \{i_1, \ldots, i_5\}$ and $S = \{s_1, \ldots, s_4\}$, unit capacity
Replace $\underline{GS^2}$ by $\boxed{\mathrm{GS}^1}$ only for student i_2

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}	$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$
$\boxed{s_1}$	s_1	$\boxed{s_2}$	s_3	$\boxed{\underline{s_3}}$	i_3	i_2	i_1	i_5
s_2	$\underline{s_2}$	$\underline{s_1}$	s_1	s_4	i_1	i_4	i_5	$\vdots$
s_3	s_3	s_3	s_2	$\vdots$	$\vdots$	$\vdots$	$\vdots$	

To show: the number of blocking students did not increase

Student i_2: from matched to blocking
Student i_1: from blocking to matched
Student i_4: from unmatched & not blocking to blocking

$\underline{GS^{k+1}}$ is more stable by counting than $\boxed{GS^k}$

When dropping one school in list of one student:
- nothing changes
- unless this student was (tentatively) accepted at this school
- then he is a new blocking student
- rejection chains that he did not generate “save” at most one blocking student

$\underline{GS^k}$ is not more stable by counting than $\boxed{BM^k}$

$n \geq 7$, $I = \{i_1, ..., i_n\}$ and $S = \{s_1, \ldots, s_5\}$; $q_s = 1$

Replace $\boxed{\text{BM}^3}$ with $\underline{GS^3}$

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}	$\ldots$	$P_{i_{n-1}}$	P_{i_n}	$\succ_{s,\, s \in S}$
$\boxed{\underline{s_1}}$	$\boxed{\underline{s_2}}$	$\boxed{\underline{s_3}}$	s_1	s_1	s_1	s_1	$\boxed{s_4}$	i_1
$\vdots$	$\vdots$	$\vdots$	$\underline{s_4}$	s_2	s_2	s_2	$\underline{s_5}$	i_2
			$\boxed{s_5}$	s_3	s_3	s_3	$\vdots$	i_3
			$\vdots$	s_5	s_5	s_5		i_4
				$\boxed{\underline{\emptyset}}$	$\boxed{\underline{\emptyset}}$	$\boxed{\underline{\emptyset}}$		i_4
								$\vdots$
								i_n

Similarly, we can replace $\boxed{\text{BM}^3}$ with $\underline{Ch^{(3)}}$...

Reforms and Stability

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GS^k	GS^{k+1}	more	more	more	more
β	$Ch^{(e)}$	more	more	not comparable	not comparable

Notes: Each row compares the mechanism in the second column to the mechanism in the first column with respect to stability and stability by counting. Common priority is a special case of arbitrary priority.

Stability and Manipulability

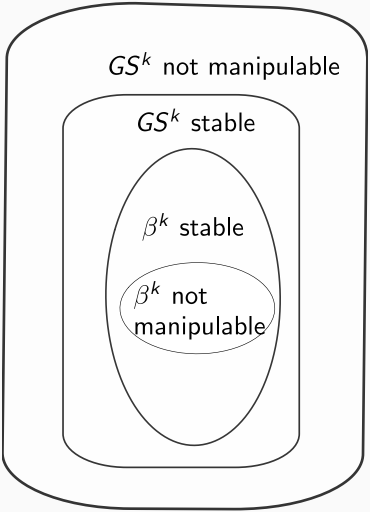
- φ is not manipulable at $(P, q, \succ)$ if no i can benefit by any P_i':

$$\varphi_i(P, \succ, q) \; R_i \; \varphi_i(P_i', P_{-i}, \succ, q)$$

- For GS^k stability is stronger than non-manipulability
  - only unmatched students can be block or manipulate
  - no student can benefit by manipulating stable matching,
  - some manipulations at unstable matchings are not beneficial
- For BM^k non-manipulability is stronger than stability
  - non-manipulable profile is always stable

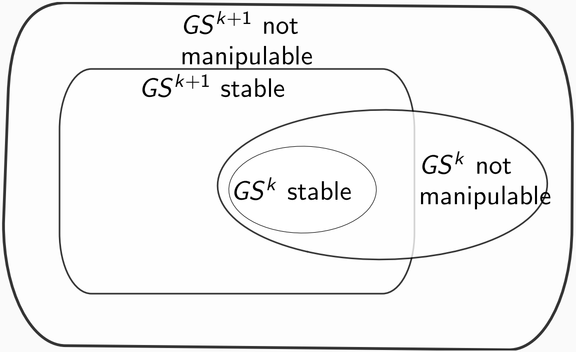
Stability and Manipulability

- For GS^k stability is stronger than non-manipulability
- For BM^k non-manipulability is stronger than stability



Stability and Manipulability

- For GS^k stability is stronger than non-manipulability



Stability and Manipulability

- For GS^k stability is stronger than non-manipulability
- For BM^k non-manipulability is stronger than stability
- For SD^k non-manipulability is equivalent to stability
  (no rejection chain can come back to blocking student)
- For FPF^k stability implies non-manipulability of GS^k
  Hence, the reforms in England either reduced manipulability or improved stability:
  - if FPF was stable, then switching to GS might lose stability
  - but then GS is non-manipulable and thus manipulability is not gained

Strategic settings

Stability and manipulability

- We compare mechanisms by stability
  - at each preference profile
  - assuming truthful reporting
- However: the mechanisms are also manipulable
- Methodological difficulty:
  - when reports are truthful, set of blocking students is clear
  - when reports are strategic, set of blocking students is empty
- We develop two partially strategic settings:

  * some students are strategic, others are truthful

  ** all students are semi-strategic:
  they avoid clearly unfeasible schools, but otherwise are truthful
  (a school is unfeasible if it is filled in the first round)

Final slide

Reforms	From	To	more fair by stability?		more fair by counting?	
			Arbitrary priority	Common priority	Arbitrary priority	Common priority
UK(54), 2007/11	FPF^k	GS^k	not comparable*,**	more	not comparable*,**	not comparable*,**
Chicago, 2009 UK(4), 2007	β^k	GS^k	more	more*	not comparable*,**	not comparable*,**
Chicago, 2010 Ghana, 2007/08 UK(2), 2010	GS^k	GS^{k+1}	more**	more*,**	more**	more*,**
China(13), 2001/12	β	$Ch^{(e)}$	more	more	not comparable*,**	not comparable*,**

Table 1: Comparison of the matching mechanisms by fairness criteria.

Notes: Each row represents a comparison of the mechanism in the third column to the mechanism in the second column according to one of the two fairness notions. Asterisk * shows the case for mixed population of strategic and truthful students; double asterisk ** shows the case for all students being semi-strategic.