Reactance: a Freedom-Based Theory of Choice*

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Abstract

A choice exhibits reactance if it is not directly welfare maximizing but represents a way to restore a threatened freedom. We provide a first axiomatic revealed preference characterization of this phenomenon, which yields necessary and sufficient conditions for deviations from rational choice to be ascribed to reactance. These conditions are shown to characterize a (unique) representation of choices consistent with reactance. We next derive the resulting preference ordering over opportunity sets for agents whose final choices are consistent with reactance. Three applications are analyzed. We first look at two social phenomena that have been (informally) associated with reactance in the psychology literature and demonstrate that reactance provides plausible explanations of the emergence of conspiracy theories and the backlash of integration policy targeted towards immigrants. We finally study a principal's problem who delegates decision to a better-informed agent that is biased and subject to reactance. We find that the effect of reactance on the agent's welfare is ambiguous.

KEYWORDS: revealed preferences, freedom, reactance.

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"Prohibitions create the desire they were intended to cure"

Lawrence Durell

Freedom is arguably an important reason for action that potentially conflicts with material welfare maximization. For instance, people that were reluctant to accept Covid 19 vaccination policy also suffered significant welfare costs, including the renunciation of previous occupations or leisure activities. Psychologists have long observed this propensity of freedom to counterbalance well-being motives—by means of natural and lab experiments—and developed the theory of *psychological reactance* to explain their findings. Despite the wide range of its applications, reactance has received little attention in economics. A possible reason is that the existing theory neither yields a formal criterion to distinguish reactance from other motivations to deviate from welfare maximization, nor clarifies whether reactance lends itself to rigorous modeling. The main contribution of this article is an axiomatic foundation of a choice procedure that captures reactance and highlights trade-offs between welfare and freedom motives.

Reactance occurs when the decision makers (DM) reverse their choice when they are deprived of what they perceive as part of their freedom. In a field experiment, Mazis, Settle and Leslie (1973) consider how consumers reacted when Miami-Dade county decided to forbid phosphate use for laundry. Despite its strong environmental rationales, this decision raised significant protests as well as unexpected reactions. For the sake of the "American freedom", some consumers that were not using phosphate-based detergent prior to the law started buying it in neighbouring counties, smuggling it at extra cost and stockpiling the (now) precious product for the 20 years to come.[1]

Such a scenario violates the standard requirement of rationality, namely, the *Weak Axiom of Revealed Preferences* (WARP), and is incompatible with a straightforward utility representation. Indeed, denoting by x the phosphate detergent in a neighbouring county, y the same product in Miami, and z a phosphate-free detergent in Miami, the following choice is observed: z is chosen when the

[1] As Mazis, Settle and Leslie (1973) showed, this astonishing effect on behavior was consistent with consumers' beliefs reversal: Miami consumers were, on average, more prone to praise phosphate detergent for its efficiency than their Tampa county neighbors.

three options are available, i.e. in the menu $\{x, y, z\}$, while x is chosen over z once y is removed, i.e. in the menu $\{x, z\}$. When this occurs, we say that x *reacts to the absence of* y. We interpret this as revealing that the DM perceives the absence of y as a threat to her freedom and considers the choice of x as a mean to "restore" this lost freedom (Section 1.2).[2] This kind of WARP violation, however, is not exclusive to reactance and need not be interpreted this way. For instance, the analysis of the attraction effect by Ok, Ortoleva and Riella (2015) is also based on similar choice patterns. The key challenge, then, is to provide choice-based axioms ensuring that an ascription of a choice reversal to reactance is valid and that distinguishes reactance from alternative psychological phenomena (Section 1.3).

Psychologists emphasize that reactance reflects an attempt to restore the loss of concrete freedoms, that is freedoms to choose diverse types of option.[3] Reactance is therefore observed between options embodying similar freedoms: if x reacts to the absence of y, then x and y are relevant to the same type of freedom. Importantly, types are not objectively observed but subjectively perceived by the DM. Hence they cannot be postulated *a priori* by an observer and must be revealed through the analysis. For instance, some DMs may perceive that buying environmentally harmful vehicles is relevant to protest against car environmental regulations,[4] but not against the phosphate ban studied by Mazis, Settle and Leslie (1973), because cars and detergents are different types of product—their consumption relates to different freedoms. In contrast, some DMs may perceive it as relevant because they take the right to pollute as a relevant type of freedom. Hence, we do not presume which type of options

[2]As emphasised by psychologists, reactance is about freedom as *subjectively perceived* by the DM: "For the theory [of reactance], freedoms are the creatures of subjective reality. If a person thinks he or she has the freedom to do X, and the ability to exercise this freedom, and perceives the conditions of the freedom's existence to be met, then this freedom exists for this person. It is theoretically irrelevant whether this freedom exists according to some more objective criteria." (Brehm and Brehm, 2013, p. 22)

[3]"Contrary to some interpretations (e.g. Dowd, 1975), the freedoms addressed by the theory are not "abstract considerations," but concrete behavioral realities. If a person knows that he or she can do X (or think X, or believe X, or feel X), then X is a specific, behavioral freedom for that person." (Brehm and Brehm, 2013, p.12)

[4]Here we are referring to the "rolling coal": in reaction to President Obama's environmental laws, some drivers modified their engine at significant costs in order to pollute more. See for instance https://www.nytimes.com/2016/09/05/business/energy-environment/rolling-coal-in-diesel-trucks-to-rebel-and-provoke.html

are relevant freedom-wise for the DM. A contribution of the paper is to reveal which of them are, under the requirement that they partition the set of options.

We show that any DM making choices consistent with our reactance axioms can be represented by the following choice procedure that we call a *reactance choice rule* (Section 2.1). First, the DM sorts the options into *types* (forming a partition), where objects of each type share features that the DM subjectively perceives as relevant for her freedom. Implicitly, x reacts to the absence of y only if x and y belong to the same type. Second, each type forms a set that is well ordered (i.e. WARP is satisfied) by a welfare criterion represented by a *utility function* u. Third, the DM determines a set that we interpret as a her *freedom requirement*. She considers these options to be vital to satisfy her freedom demands, as revealed by the observation that their absence sometimes triggers choice reversals. These options share an intuitive threshold property for each type (according to u); the threshold represents the DM's minimum welfare level to satisfy her freedom request. Choice is then made sequentially. First, the DM determines the best available options from each type according to u. Second, from these best options, she chooses the one with the highest valuation according to a criterion v that we interpret as *reactance function*. For $v(.)$ equates with $u(.)$ for all options in the freedom requirement set, but $v(.) > u(.)$ otherwise. We interpret this as v combining welfare and freedom motives, that is, the DM assigns a strictly positive freedom value only for options not in the freedom requirement set, which captures her propensity to restore a threatened freedom by choosing these options. We further study the uniqueness of our representation (Section 2.2). We show that the type partition is unique and that there exists a maximal freedom requirement set that includes all the others. We also find necessary and sufficient conditions for increasing transformations of u and v to preserve the reactance choice rule.

We then ask the following question: how does a DM whose choices can be explained by a reactance choice rule would evaluate the freedom of choice offered by the opportunity sets she is facing? Building on the series of papers by Pattanaik and Xu (1990, 1998, 2000), we axiomatize a criterion to rank menus, which simply counts the number of types from which sufficiently good options (i.e. in the freedom requirement set) are feasible (Section 3). We argue that this ordering integrates considerations about similarities between options (see Pattanaik and Xu, 2000; Nehring and Puppe, 2002) and the role of the preferences of the agent (see Pattanaik and Xu, 1998). Hence, it helps reconcile these two aspects, which have been studied separately, and it formally connects our choice rule to the existing literature on freedom of choice.

We finally study three applications of our choice model (Section 4). Two social phenomena have often been related to reactance and documented by the psychology literature, but they are not readily explained using existing (economic) models of choice. First, reactance is introduced as a possible determinant of the formation of conspiracy theories. To accommodate this phenomenon, we study how reactance impacts the DM's belief when she has to choose a biased source of information. By removing an unchosen moderately biased source, the DM might reverse his choice and choose a more biased source in the opposite direction. This can represent why, if a DM feels that some information is not accessible or hidden, he might end-up holding extreme belief or adhere to conspiracy theories. Second, reactance provides an explanation of why repressive policies towards immigrant minorities may generate backlash, as suggested by empirical evidence. Additionally, it provides an argument for the evolutionary efficiency of reactance and its persistence in the long run. Finally, we introduce reactance in a principal-agent's setting. We study a typical delegation problem: a principal can constrain the decision set of an informed but biased agent, but cannot commit to contingent monetary transfers. In addition to the standard model (e.g. Alonso and Matouschek, 2008), the agent behaves according to a reactance choice rule. We find that the presence of reactance modifies the optimal delegation strategy. Either it forces the principal to restrict even more the set of allowed actions to prevent the agent from taking worse actions because of reactance; or it forces the principal to allow the agent's preferred options. Hence the effect of reactance on the agent's material welfare is ambiguous. This depends on the principal's payoff and prior distribution over the states of the world.

Related literature. The theory of reactance originated in Brehm (1966). Since then, a huge and vivid literature has spurred in psychology (see Brehm and

Brehm, 2013; Reynolds-Tylus, 2019). In economics, few papers mention reactance, among which it is worth citing Arad and Rubinstein (2018), who use this concept to explain results of their experiment about people's perspective on libertarian-paternalistic policies (the so-called nudges). Once aware of the implementation of the policy, some subjects backfire by willingly choosing the reverse of what they are nudged to, although they would have chosen it without the policy. Nonetheless, to our knowledge, no theoretical model of reactance has been proposed so far.

Our work contributes to investigating mechanisms that can cause menu-dependence. The first to argue that menu-independence can be theoretically counterintuitive is Sen in many works (see Sen (1994, 1997) among others). Since then, many contributions have been proposed in the literature about conflicting motivations, among which Kalai, Rubinstein and Spiegler (2002), Manzini and Mariotti (2007), Cherepanov, Feddersen and Sandroni (2013), Bernheim and Rangel (2008), Borie and Jullien (2020), Dietrich and List (2016), and Ridout (2021) consider choices that can be explained (or justified) by different rationales that may depend on menus. Masatlioglu, Nakajima and Ozbay (2012) and Lleras et al. (2017) focus on the role of attention which can differ across menus. The contribution of Ok, Ortoleva and Riella (2015)—henceforth OOR—is worth emphasizing, for they propose a theory of revealed reference-dependent preferences that relies on similar choice irregularities as ours. The latter are however interpreted in the opposite direction: while they stress that choice reversals result from the addition of options (which then play the role of a reference point), we push forward the opposite interpretation that choice reversals result from the removal of some options. We interpret this as the effect of a feeling of freedom threat induced by the deprivation of some options. Consequently, the axioms and the obtained representation are very different (see Section 1 for a more detailed comparison).

Finally, our model offers a choice theoretic counterpart to the normative literature on freedom of choice—see, among others, Sen (1988), Pattanaik and Xu (1990, 1998, 2000), and Baujard (2007) for a survey. These works mainly studied the *objective* value of freedom, focusing on the opportunity aspect of freedom. While we similarly endorse this latter view, we rather study how the *subjective* perception of freedom may have an impact on agents' decisions. Following Sen (2002), we think that a theory of rational choice should encompass freedom of choice, "not merely because without [it], the idea of rational choice would be quite vacuous, but also because the concept of rationality must accommodate the diversity of reasons that may sensibly motivate choice" (p. 5).

1 Reactance-Induced Choices

1.1 Preliminaries

We work with a finite set of options X and denote by $\mathcal{X} = 2^X \setminus \emptyset$ the collection of non-empty subsets of X. Elements of $\mathcal{X}$ stand for the menus of options available to the DM and will typically be denoted $A, B, C, \ldots$ A **choice function**[5] $c : \mathcal{X} \longrightarrow X$ associates to each menu the option chosen by the DM in this menu. Namely, for any menu A, $c(A) \in A$.[6]

Let us stress that options are defined by objective features that can incorporate contextual properties—for instance, in our introductory example, we differentiated the phosphate laundry in a supermarket in Miami from the same product in a supermarket in a neighbouring county. Yet, we need not formalize these objective features, we only require the observer to be able to distinguish the different options. Some of these features may matter for the DM's subjective perception of freedom and thus will be revealed through the representation.

1.2 Revealed Reactance

Following Brehm (1966), reactance is meaningful only when freedom conflicts with another motive (e.g. welfare maximization).[7] We argue that reactance

[5]We focus on choice functions for the sake of simplicity: dealing with choice correspondences would add another layer of complexity that, we think, is not necessarily relevant in the present context. Nonetheless, we are confident that, with an additional axiom, our results would extend to choice correspondences. This axiom is a weakening of Sen's α property : If $A \subset B$ and $x, y \in C(B) \cap A$, then $x \in C(A) \iff y \in C(A)$.

[6]For simplicity, if we enumerate a set $\{x_1, \ldots, x_k\}$, we write $c\{x_1, \ldots, x_k\}$ instead of $c(\{x_1, \ldots, x_k\})$.

[7]"Reactance is conceived to be a counterforce motivating the person to reassert or restore the threatened or eliminated freedom. It exists only in the context of other forces motivating

is to be revealed through choice reversals that are inconsistent with the maximization of a single ordering, i.e. that violate WARP. As was illustrated by our introductory phosphate example, such choice reversals are induced by the removal of an option that was not chosen when it was feasible. Namely, consider a triplet of options x, y, z and suppose that z is chosen from $\{x, y, z\}$ but a reversal happens once y is no more available, i.e. $x = c\{x, z\}$. When reactance is at work, this choice reversal results from the DM being concerned by the freedom loss he suffers when being denied the access to y. The choice of x over z is an effective way for the DM to restore this threatened freedom. We say that x *reacts* to the absence of y, as capture by the following definition.

Definition 1. *Let c be a choice function on $\mathcal{X}$ and $x, y \in X$. We say that x* ***reacts to the absence of*** *y, relative to c, if there exists z such that, $z = c\{x, y, z\}$, and $x = c\{x, z\}$. We denote it $x\mathbf{R}^c y$.*

Such behaviors could well be explained by different motives. In particular, OOR base their definition of *revealed reference* on similar choice patterns. Their interpretation is however significantly different: they argue that z beats x only with the "help" of y. Hence, while they interpret these reversals as revealing a relationship between y and z, we interpret it as revealing a relationship between x and y.[8]

In addition, a definition of *potential* reactance is required. Assume we observe that *(i)* y is preferred to x, and *(ii)* for each t such that $y\mathbf{R}^c t$, we also observe that $x\mathbf{R}^c t$. In such a situation we might suspect x to be an effective way to restore the lost freedom incurred by the absence of y. Indeed, *(i)* states that y is preferred to x, while *(ii)* suggests that the choice of x is as efficient as the choice of y to restore the DM's threatened freedom. Yet, it need not be the case that $x\mathbf{R}^c y$, for there might be no third option z that allows for revealing a reversal as stated by definition 1—i.e. no z is chosen in $\{x, y, z\}$ while x is chosen in $\{x, z\}$. When this happens, we posit that x *potentially reacts* to the absence of y.

the person to give up the freedom and comply with the threat or elimination." (Brehm and Brehm, 2013, p.37).

[8]More precisely, the application of OOR's definition to these choices would identify y as a *revealed reference* of z.

Definition 2. *Let c be a choice function on $\mathcal{X}$ and $x, y \in X$. We say that x **potentially reacts to the absence of** y, relative to c, if $y = c\{x, y\}$, there exists t such that $y\mathbf{R}^c t$, and for any such option t, $x\mathbf{R}^c t$. We denote it $x\mathbf{P}^c y$.*

The next section is dedicated to providing conditions based on these relations in order to make our interpretation meaningful and to restrict violations of WARP to reactance.

1.3 Reactance Choice Properties

Our first axiom relaxes WARP by observing that the DM should feel threatened in her freedom only when some options are made unavailable. Conversely, when her menu expands, the threats should disappear. The axiom states that if an option x is chosen in two menus A and B, then no freedom concern could justify a choice reversal when these menus are gathered, i.e. in $A \cup B$. Otherwise, the reversal from $A \cup B$ to A would be triggered by the loss of an option from $B \setminus A$, which would prevent x from being chosen in B. This leads to our first axiom.[9]

EXPANSION (Exp). *For any $x \in X$, $A, B \in \mathcal{X}$, if $x = c(A) = c(B)$, then $x = c(A \cup B)$.*

Note that if x reacts to the absence of y, then **Exp** implies that $y = c\{x, y\}$. That is, for reactance to be meaningful, it must trigger a choice of an even "'worse" option than the one that is no more available, or equivalently, a sacrifice of welfare, as otherwise, reactance would hardly express a discontent. Furthermore, if z plays the same role as in definition 1, then **Exp** also implies that $z = c\{y, z\}$. Therefore, a typical pattern of reactance is revealed through a binary choice cycle—this is forbidden by OOR's axiom *No Cycle*.

We now posit axioms on the binary relations $\mathbf{R}^c, \mathbf{P}^c$. Consider our introductory example. Assume that, when only an expensive phosphate-free detergent is available in her county, the DM reacts to the prohibition by going

[9] It was already present in Sen (1971), named property γ, and was later used by Manzini and Mariotti (2007) under the name *Expansion*.

to the neighbouring county to get some phosphate detergent, while she stays in her own county when a cheap phosphate-free detergent is available. This reveals that "buying phosphate detergent in the neighbouring county" reacts to the absence of "buying phosphate detergent in Miami supermaket", though it is revealed only when the price of the phosphate-free detergent is high. Assume also that, while she prefers not to transgress the law when she can buy phosphate in the neighbouring county, the DM finally decides to go on the black market when the latter is forbidden, and that she does so whatever the price of the available phosphate-free detergent may be. This reveals that "buying phosphate detergent on the black market" reacts to the absence of "buying phosphate detergent in the neighbouring county". We would like to conclude from these two assumptions that the DM goes to the black market because she feels even more threatened in her freedom now that the detergent is prohibited in both counties. This necessarily implies that "buying phosphate detergent on the black market" also reacts to the absence of "buying phosphate detergent in Miami supermakets". Hence our first axiom requires $\mathbf{R}^c$ to be transitive—note that $\mathbf{P}^c$ is transitive by definition. Because $\mathbf{R}^c$ and $\mathbf{P}^c$ are typically incomplete, we also require their negative transitivity.

Reactance Transitivity (R-Tran). *For any $x, y, z \in X$,*

(*i*) *if $x\mathbf{R}^c y$ and $y\mathbf{R}^c z$, then $x\mathbf{R}^c z$,*

(*ii*) *let $y = c\{x, y\}$, $z = c\{y, z\} = c\{x, z\}$: if $\neg[x\mathbf{R}^c y]$ and $\neg[y\mathbf{R}^c z]$, then $\neg[x\mathbf{R}^c z]$; if $\neg[x\mathbf{P}^c y]$ and $\neg[y\mathbf{P}^c z]$, then $\neg[x\mathbf{P}^c z]$.*

Remark: let us stress that **R-Tran** imposes also transitivity in the similarity between options, that is, it prevents the following situation: x is sufficiently close to y and $x\mathbf{R}^c y$, y is sufficiently close to z and $y\mathbf{R}^c z$, but x and z are too different to consider the possibility of x reacting to the absence of z.

Consider an option y that never reacts to the absence of any other option, but whose absence triggers reactance from the DM by choosing x—i.e. $x\mathbf{R}^c y$. This means that as long as the DM has access to y, she never reacts to some limitation of her freedom of choice by choosing y. At the same time, when y is no more available, she is ready to restore her threatened freedom by choosing x. Our interpretation of such an option is that it satisfies the DM's freedom requirement, or alternatively, the choice of y is never motivated by freedom concerns. Consider a third option z that is chosen over y—i.e. $z = c\{y, z\}$—and such that also $x\mathbf{R}^c z$, then z should satisfy the DM's freedom requirement at least as well as y does. Our third axiom imposes two conditions in that direction. First, any option that reacts to the absence of z must also react to the absence of y. Conversely, any option that reacts to the absence of y might not be good enough to react to the absence of z, but at least z must be chosen over this option.

Reactance Consistency (R-Con). *For any $x, y, z \in X$, if $x\mathbf{R}^c y$, $x\mathbf{R}^c z$, $z = c\{y, z\}$, and there exists no t such that $y\mathbf{R}^c t$, then for any $u \in X$:*

(i) $u\mathbf{R}^c z \implies u\mathbf{R}^c y$;

(ii) $u\mathbf{R}^c y \implies z = c\{u, z\}$.

Remark: point *(ii)* can alternatively be seen as requiring that $u\mathbf{R}^c y$ cannot be revealed through the choice with z, hence $z = c\{u, z\}$, which is consistent with the interpretation that y and z satisfy the same freedom requirements.

To motivate our last axiom, we extend the phosphate example. Suppose that both "buying phosphate on the black market" (x) and "buying phosphate in a neighbouring county" (z) react to the absence of "buying phosphate in Miami supermakets" (t). Add the third option "buying phosphate in a further county" (y): quite naturally, z is chosen over y, and assume further that y is chosen over x. Suppose that the DM considers going on the black market as a means to restore her freedom threatened by the prohibition in a further county, that is, $x\mathbf{R}^c y$. Said differently, the DM's concern for freedom is greater when x is the only phosphate detergent available than when y is. Because both x and z reacts to the absence of a common option t, then one would expect that similarly the DM's concerns for freedom be reinforced when only y is available as compared to the same situation with z available. Hence our third axiom requires that y potentially reacts to the absence of z. The second point says that if in addition

$x\mathbf{P}^c z$, that is, whenever the DM considers going in a neighbouring county as a way to restore a threatened freedom, she would also consider going on the black market if necessary, the same conclusion, that is, $y\mathbf{P}^c z$, should follow even if we only observe $x\mathbf{P}^c y$ and not necessarily $x\mathbf{R}^c y$.

REACTANCE MONOTONICITY (R-Mon). *For any $x, y, z \in X$, such that $z = c\{y, z\}$, $y = c\{x, y\}$:*

(i) if $x\mathbf{R}^c t$ and $z\mathbf{R}^c t$ for some $t \in X$, then [$x\mathbf{R}^c y \implies y\mathbf{P}^c z$];

(ii) if $x\mathbf{P}^c z$, then [$x\mathbf{P}^c y \implies y\mathbf{P}^c z$].

2 REPRESENTATION

2.1 Reactance Choice Rule

We now state and discuss our main result: a representation for choice functions that satisfy the four axioms presented above. To state it formally, let us denote $T_0^c = \{x \in X \mid \nexists\, t, x\mathbf{R}^c t \vee t\mathbf{R}^c x\}$ the set of options for which reactance never arises. Omitted proofs can be found in the appendices.

Theorem 1. *A choice function c satisfies **Exp**, **R-Tran**, **R-Con** and **R-Mon** if and only if there exist a* freedom requirement set *$F \subseteq X$, a partition $\mathcal{T}$ of the options into* types *such that $T_0^c \in \mathcal{T}$, a* utility function *$u : X \longrightarrow \mathbb{R}$, and a* reactance function *$v : X \longrightarrow \mathbb{R}$ such that:*

(i) for any $T \in \mathcal{T}$ and any $x, y \in T$, if $x \in F$ and $u(x) < u(y)$, then $y \in F$;

(ii)
- *$v(x) > u(x)$ for all $x \notin F$;*
- *$v(x) = u(x)$ for all $x \in F$;*

(iii) for any $T \in \mathcal{T}$, $v \circ u^{-1}$ is single-peaked on $u(T \setminus F)$;

(iv) for any menu A,[10]

$$c(A) = \arg\max_{x \in d(A)} v(x), \qquad (1)$$

[10] With a slight abuse of notation, if x is the unique maximizer of a function f on the set E, we simply write $x = \arg\max_{y \in E} f(y)$, instead of $\{x\} = \arg\max_{y \in E} f(y)$.

where:

$$d(A) = \bigcup_{T \in \mathcal{T}} \arg\max_{x \in T \cap A} u(x).$$

We name a choice function that satisfy our four axioms a **reactance choice rule (RCR)**. When the conditions *(i)–(iv)* of Theorem 1 are satisfied, we say that the ordered tuple $< \mathcal{T}, F, u, v >$ is a **reactance structure** that represents c.

Our interpretation of Theorem 1 is the following. The DM evaluates options either through an instrumental (or 'welfarist') preference, represented by the *utility function* u, or through the lens of a *reactance function* v, that combines welfare and freedom motives and thus exceeds (weakly) the former. Options are partitioned into classes of similar *types*. Each type comprises options that provide comparable consumption experience but with different levels of welfare, or satisfaction—i.e. they are ranked by u. Hence, it reveals a specific freedom. One clear example of this is when these are actually the same good but obtained or consumed through different channels. For instance, buying phosphate laundry in a supermarket is less costly than getting it from the black market, but both goods are of similar type. Within each type, welfare maximization is the only motive for choice. Therefore, the binary comparisons are transitive—and thus represented by u—and freedom concerns cannot yield reactance cycles.

The *freedom requirement set* consists of options that meet the DM's freedom requirements; that is, for any T, $F \cap T$ captures her demand regarding this specific freedom. Point *(i)* and *(ii)* express how these requirements depend on welfare. Point *(i)* says that if an option x meets a freedom requirement of the DM (i.e. $x \in T \cap F$), so do options of the same type that increase the DM's welfare (i.e. any $y \in T$ such that $u(y) > u(x)$). Hence, when all options in $F \cap T$ are unavailable, the DM deems that she does not have access to a sufficiently good option regarding the freedom associated to T. Since, in this case, the DM feels threatened in her freedom, point *(ii)* says that she must be even more willing to choose options of this type; that is, $v(x) > u(x)$ for $x \notin F$. Conversely, when an option in $F \cap T$ is available, the DM does not feel threatened regarding the freedom embodied by T. Hence, freedom is not a reason for choosing these options, which is captured by the fact that $u(.) = v(.)$. In this sense, $x^{T,F} \equiv \arg\min_{x \in T \cap F} u(x)$ provides the minimal welfare requirement of the type of freedom associated to T.

Choices represented by (1) happen sequentially: within a menu, only the best options (according to u) from each type are retained—this is the set $d(A)$—and then compared according to v.[11] The reasoning process can actually be decomposed into two maximization stages separated by an interim stage in which the DM evaluates for each type the threat to her freedom. More precisely, at the interim stage, if the best available option from a type T is worse than $x^{T,F}$, then freedom becomes an additional reason to choose this option. Hence, while welfare alone is relevant to describe the first maximization stage, the second one is also driven by the freedom concerns introduced at the interim stage.

Finally, point *(iii)* imposes a specific shape of the reactance function v with respect to the utility function u. Specifically, if x, y, z are in type T but not in F and $u(x) > u(y) > u(z)$, it prevents that $v(y) = \min_{t \in \{x,y,z\}} v(t)$. It reflects the DM's increasing willingness to react—up to a certain point—as the limitations on their freedom is tightened. In our representation, this indicates that the less welfare the DM can obtain from a type of options, the more she is willing to restore her freedom and thus the more reactance is at work. Of course, this is true up to a certain point where welfare motives might weigh more in the trade-off between welfare and freedom—i.e. you reach a point where you are no more willing to sacrifice welfare in the name of your liberty.

Figure 1 illustrates our model for a given type, say T. The six options are in the same type, but only e and f are in F, which is shown by the equality between the functions u and v, and $e = x^{T,F}$. Conversely, a, b, c, d are not in F. The fact that $v(b) > v(c) > v(d)$ whereas $u(b) < u(c) < u(d)$ illustrates that freedom reasons are playing an increasingly important role the more welfare the DM is denied the access to. The welfare motives however outweigh the freedom ones for option a.

[11]Note that an RCR is a specific case of a *rational shorlist method* (RSM) à la Manzini and Mariotti (2007). Indeed, one can define the two orders $\succ_1$ and $\succ_2$ in the following way: $x \succ_1 y \iff \exists T,\ x, y \in T \wedge x = c\{x,y\}$; $x \succ_2 y \iff x = c\{x,y\}$. In that case, if c is an RCR, then for any menu A, $c(A) = \max(\max(A, \succ_1), \succ_2)$. That is, the DM chooses as if she first keeps only options that are the best in each available type, and second, she chooses the best remaining one according to the binary comparisons. As a consequence, although we do not use it in our characterization, an RCR also satisfies the second property of RSMs, namely Weak WARP.

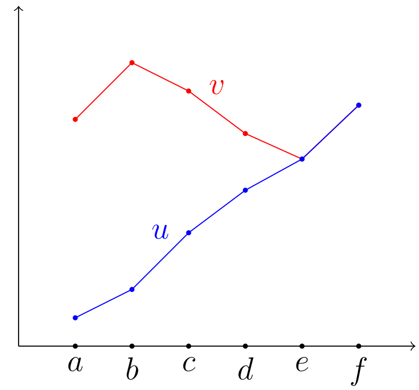

Figure 1: Reactance structure in one type.

In the light of the representation, it is worth noting that the absence of preference reversal (or binary cycles) is to be interpreted as a lack of traceable reactance. This does not necessarily mean that the DM has no concern for freedom related to her opportunity set. Rather, this means that her freedom concerns (if any) are either too weak, or too aligned with her welfare to be identified as a force counterbalancing welfare.

2.2 Uniqueness of Reactance Choice Rules

This section elaborates on the extent to which the ingredients of a reactance structure may be uniquely identified. We first provide a uniqueness result regarding the types and the freedom requirement set. It states that the collection of types is unique and that the union of two freedom requirement sets can also rationalize the DM's choices.

Proposition 1. *If c is an RCR represented both by $< \mathcal{T}, F, u, v >$ and $< \tilde{\mathcal{T}}, \tilde{F}, \tilde{u}, \tilde{v} >$, then $\mathcal{T} = \tilde{\mathcal{T}}$ and there exist $\hat{u}, \hat{v}$ such that $< \mathcal{T}, F \cup \tilde{F}, \hat{u}, \hat{v} >$ also represents c.*[12]

We say that a reactance structure $< \mathcal{T}, F, u, v >$ is **maximal** if the freedom requirement set F is *maximal* according to the inclusion relation $\subseteq$, that is, if for any other reactance structure $< \mathcal{T}, \tilde{F}, \tilde{u}, \tilde{v} >$ that represents the same choice

[12]Note that the uniqueness of the collection of types relies on the requirement in our theorem 1 that T_0^c is one type. Hence, proposition 1 alternatively says that the collection of types is unique on the set of options on which reactance phenomena are observed.

function, $\tilde{F} \subseteq F$. Proposition 1 ensures that for any RCR c, there exists an unique maximal reactance structure that represents it.

The next proposition shows that the main characteristic of this maximal structure is that the relation $\mathbf{R}^c$ depends, for options in type T, on whether they provide more welfare than $x^{T,F}$. Indeed, suppose that the option just below $x^{T,F}$ in T according to u—denote it x—does not react to the absence of $x^{T,F}$. Then, x does not react to the absence of any option in T better than $x^{T,F}$, and hence any option at all.[13] This means that the presence of x in T comes from the fact that there exists y such that $y\mathbf{R}^c x$. Namely, while x is never an effective way to restore a lost freedom, the DM feels threatened when he is denied the feasibility of x. Hence it would seem natural that x be included in the freedom requirement set. This turns out to be necessarily true for the maximal freedom requirement set.

Proposition 2. *Let $< \mathcal{T}, F, u, v >$ be a reactance structure that represents an RCR c and define for any $T \neq T_0^c$, $x^{T,F} \equiv \arg\min_{x \in T \cap F} u(x)$. $< \mathcal{T}, F, u, v >$ is maximal if and only if $T_0^c \subset F$, and for any $T \neq T_0^c$, $x \in T$:*

(i) if $u(x) < u(x^{T,F})$, then $x\mathbf{R}^c x^{T,F}$;

(ii) if $u(x) > u(x^{T,F})$, then there exists y such that $y\mathbf{R}^c x$ and $y\mathbf{R}^c x^{T,F}$.

The maximal freedom requirement set is therefore the relevant one to evaluate the DM's freedom demands and the thresholds $x^{T,F}$s can then appropriately be interpreted as the minimal welfare requirements to satisfy these demands.

We now turn to a discussion about the uniqueness of the functions u and v. Let c be an RCR represented by the reactance structure $< \mathcal{T}, F, u, v >$. What are the joint conditions on functions $\tilde{u}, \tilde{v}$ that ensures that $< \mathcal{T}, F, \tilde{u}, \tilde{v} >$ also represents c? One obvious sufficient condition is if there exists an increasing function $f : \mathbb{R} \longrightarrow \mathbb{R}$ such that $\tilde{u} = f \circ u$ and $\tilde{v} = f \circ v$. Now let suppose that there exist two functions $f, g : \mathbb{R} \longrightarrow \mathbb{R}$ such that $\tilde{u} = f \circ u$ and $\tilde{v} = g \circ v$. One clear necessary condition is that $f_{|u(F)} = g_{|u(F)}$ and that both functions be increasing on $u(F)$. Because u is not directly used as a choice rule on options that are not in F, it is not necessary that f be increasing on $u(X)$. Yet, it represents choices within types, which implies that f is increasing on $u(T)$ for every

[13]See proposition 4 below (page 17) for a more precise statement of why this is true.

T. Because v is ultimately the function through which choices are made, one might be tempted to say that g must be increasing on $v(X)$. This is however not exact because within types, the function v is never used to make choices. This problem does not arise as long as we impose one additional condition regarding the reactance function on certain pairs of options of similar types. This is captured by the following definition.

Definition 3. *A reactance structure* $< \mathcal{T}, F, u, v >$ *is a* **reactance structure*** *if for any* $T \in \mathcal{T}$ *and any* $x, y \in T$*, such that* $\{x, y\} \nsubseteq F$*,* $c\{x, y\} = x$ *and for all* $z \notin T$*,* $c\{x, z\} = z \iff c\{y, z\} = y$*, we have* $v(y) > v(x)$*.*

The next proposition states first that reactance structure* exists and second that if we restrict ourselves to reactance structure*, the conditions regarding the utility and the reactance functions stated above are not only sufficient, but also necessary.

Proposition 3. *Let* c *be an RCR.*

(i) There exists a reactance structure* $< \mathcal{T}, F, u, v >$ *that represents it.*

(ii) Furthermore, let $f, g : \mathbb{R} \longrightarrow \mathbb{R}$ *be two real-mappings,* $< \mathcal{T}, F, f \circ u, g \circ v >$ *also represents* c *if and only if* f *is increasing on* $u(T)$ *for every* $T \in \mathcal{T}$*,* g *is increasing on* $v(X)$ *and* $f_{|u(F)} = g_{|u(F)}$*.*

2.3 Behavioral Properties of Reactance Choice Rules

The relation between our behavioral definition of reactance and our representation is a legitimate concern. According to the sequential reasoning of an RCR, options are first compared according to u within types, and then according to v across types. Because u and v do not rank options in X similarly, choice reversals may happen when removing options in a type that are better ranked according to u but worse according to v. Point *(i)* of the next proposition shows that these are are necessary conditions. Conversely, they might not be sufficient. Hence point *(ii)* states conditions under which these are sufficient conditions for x to potentially react to the absence of y.

Proposition 4. *Let* c *be an RCR represented by the reactance structure* $\mathcal{S} =< \mathcal{T}, F, u, v >$*. For any* $x, y \in X$*:*

(i) if $x\mathbf{R}^c y$, then there exists $T \in \mathcal{T}$ such that $x, y \in T$, $x \notin F$, $u(x) < u(y)$ and $v(x) > v(y)$;

(ii) if $\mathcal{S}$ is maximal and there exists $T \in \mathcal{T}$ such that $x, y \in T \setminus F$, $u(x) < u(y)$ and $v(x) > v(y)$, then $x\mathbf{P}^c y$.

Proof. (i) Let c be an RCR. Consider x, y, z such that $z = c\{x, y, z\}$ and $x = c\{x, z\}$, so $x\mathbf{R}^c y$. One can easily check that it is not possible that x, y, z are either all in the same type, or all in different types. Hence exactly two among them must be of the same type: denote it T. To allow such a choice behavior, this means that either the valuation of z changes from $\{x, y, z\}$ to $\{x, z\}$, or the valuation of x changes. But given that z is chosen in the bigger menu, this means that it was the best, according to u, among the options of the same type in this menu, which is necessarily true also in a smaller menu. Hence it is the valuation of x that increases from $\{x, y, z\}$ to $\{x, z\}$, meaning that it is evaluated according to u in the former and according to v in the latter. This is only possible $x, y \in T$ and $u(x) < u(y)$. Furthermore, the choices imply that $v(x) > v(z) > v(y) \geq u(y) > u(x)$, from which we conclude that $x \notin F$ and $v(x) > v(y)$.

(ii) Because $\mathcal{S}$ is maximal, $x, y \in T \setminus F$ implies that both $x\mathbf{R}^c x^{T,F}$ and $y\mathbf{R}^c x^{T,F}$. Because $u(x) < u(y)$, $y = c\{x, y\}$, and at the same time for any $z \in T$ such that $y\mathbf{R}^c z$, $u(z) > u(y) > u(x)$ and there exists t such that $v(y) > v(t) > v(z)$, hence $v(x) > v(t)$ and $x\mathbf{R}^c z$. Therefore, $x\mathbf{P}^c y$. □

3 Ranking of Opportunity Sets

The freedom of choice literature has mainly tackled the issue of valuing freedom through the ranking of opportunity sets. Starting with Jones and Sugden (1982) and Pattanaik and Xu (1990), a wide diversity of measures have been characterized (see Baujard (2007) for a survey of this literature). Two dimensions have been pointed out as relevant to the agents' valuation of their freedom: their (potential) preferences over options (see Pattanaik and Xu (1998)—henceforth PX98—and Sen (1993)) and the similarity between different options (see Pattanaik and Xu (2000)—henceforth PX00—and Nehring and Puppe (2002)).

We propose a ranking of opportunity sets based on reactance structures that arguably offers novel insights and helps reconcile these two aspects.

According to a (maximal) reactance structure, this is through the interaction between the types and the freedom requirement set that freedom concerns impact choices. Hence it suggests that these two channels should impact the DM's assessment of freedom offered by a given menu. The types represent classes of similar options,[14] suggesting that adding options of a similar type should not increase the DM's freedom of choice. At the same time, the maximal freedom requirement set represents the DM's freedom demands. Hence, it seems natural that adding options increases the DM's valuation of freedom only if it gives access to items of a type from which no option in the freedom requirement set were available.

We characterize with two axioms a rule that reflects these arguments. As before, we denote X a finite set of options and $\mathcal{X} := 2^X \setminus \emptyset$ the collection of menus of options in X. Let $< \mathcal{T}, F, u, v >$ be a maximal reactance structure defined on X and $\succsim$ is a complete and transitive binary relation defined on $\mathcal{X}$.

To state our two axioms, we need to introduce the following definition. A menu A is **richer than** a menu B if for any $T \in \mathcal{T}$, if $T \cap F \cap A = \emptyset$, then $T \cap F \cap B = \emptyset$. So A is richer than B means that any type from which no element in F is available in A must also have no feasible options in $F \cap B$. Furthermore we say that A is **strictly richer than** B if A is richer than B but the reverse is not true.

Our first axiom, R-DOMINANCE says that (strictly) richer sets are always (strictly) preferred and imposes that it is an equivalence for singletons.

R-DOMINANCE.

(i) For any $A, B \in \mathcal{X}$: A richer than $B \implies A \succsim B$, with a strict preference if A strictly richer;

(ii) For any $x, y \in X$: $\{x\} \succ \{y\} \implies \{x\}$ strictly richer than $\{y\}$.

Note that part (i) of the axiom implies monotonicity in the sense of Kreps

[14] It is actually a specific case of PX00's analysis where the equivalence classes induced by the similarity relation form a partition.

(1979): for any $A, B \in \mathcal{X}$, $A \supseteq B \implies A \succsim B$. Indeed, in this case, A is trivially richer than B. Part (ii) is an adaptation of Pattanaik and Xu (1990)'s *Indifference Between no Choice Situations*, which simply imposes an indifference between every singleton. They argue that singletons do not offer any freedom of choice, hence they cannot be strictly ranked. This is still true in our case, except if only one the two options is in F, which is exactly what is implied by (ii).

Our second axiom, R-COMPOSITION, is an adaptation of the composition axioms used in Pattanaik and Xu's series of papers.

R-COMPOSITION. *For any $A, B, C, D \in \mathcal{X}$, such that $A \cap C = B \cap D = \emptyset$, $C \subseteq T$ and $D \subseteq T'$ for some $T, T' \in \mathcal{T}$, and A is not richer than C: if $A \succsim B$ and $C \succsim D$, then $A \cup C \succsim B \cup D$.*

Combining menus that do not overlap should preserve the ranking. This is however true only if combining really provides additional freedom, which is captured by the requirement that A is not richer that C (see PX00 for a complete discussion of their axiom).

For any menu A, we define $\Phi(A) = \{\mathcal{T}(x) \cap F \cap A | x \in A\}$, the collection of subsets of A containing every option of one type from F that is available in A.

Theorem 2. *$\succsim$ satisfies* R-DOMINANCE *and* R-COMPOSITION *iff for any menu A and B:*

$$A \succsim B \iff \#\Phi(A) \geq \#\Phi(B). \tag{2}$$

The interpretation is the following: what matters for the DM is to have access to more options, but only dissimilar objects—as captured by the distinct types—are valued. On top of that, within a certain class of similar options, the DM demands a minimal level of satisfaction to meet her freedom requirements, which is captured by the freedom requirement set F.

This measure is close to PX00's one. In addition to their representation, there is a role for preferences in this evaluation that is captured through the set F. Although PX98 also incorporate preferences, let us stress the key difference. Their starting point is a collection of possible preferences (i.e. complete and transitive orderings over the options) that a reasonable person may have. The resulting measure simply counts in a menu the number of options that are a maximiser of at least one of these preferences over the given menu. This approach integrates preferences relatively to a menu, simply attributing values to options that *could be* chosen in this menu. In contrast, in our approach, preferences are integrated in a more absolute way, in the following sense: below a certain level of satisfaction, even though the DM will have to choose an option, he does not attribute any freedom value to these potentially chosen items.[15] Even more, keeping the reactance choice rule in mind, some options that might be chosen later on, simply because of reactance, will not matter in the assessment of freedom, while some unchosen ones will matter.

4 Applications

4.1 Conspiracy Theories

As Sensenig and Brehm (1968) suggest, reactance has its counterpart in the realm of beliefs, namely the boomerang effect for psychologists (Hovland, Janis and Kelley, 1953) or the backfire effect for political scientists (Nyhan and Reifler, 2010; Wood and Porter, 2019).[16] In the wake of Covid 19 pandemics, scholars argued such effects to be closely related to the formation of conspiracy theories and extreme beliefs (Adiwena, Satyajati and Hapsari, 2020).[17] We propose to accommodate this mechanism by adapting Che and Mierendorff (2019)'s single period model of attention allocation with reactance.

A DM must choose from two actions, l or r, whose payoffs depend on an unknown state $i \in \{L, R\}$. His prior belief that the state is R is denoted p and we assume that $p \in (0, 1/2]$. Before choosing his action, the DM acquires information. To that purpose, he can allocate his attention across four sources

[15] To illustrate this, our measure can be equal to 0 on some non-empty menus, which is impossible either in PX98 or in PX00.

[16] The boomerang effect is "a situation in which a persuasive message produces attitude change in the direction opposite to that intended". The backfire effect is a concept from political science that refers to a situation in which evidence contradicting the subjects' prior belief may reinforce their belief in the opposite direction.

[17] The fact that mass media did not give any credit to conspiracy theories has been pointed out as playing a role in reinforcing such theories through reactance.

of information (e.g. newspapers). Two of them are *L-biased* and the two others are *R-biased*.

The sources are represented by statistical experiments, or signals. The L-biased ones, denoted σ^{LL} and σ^{L}, can only reveal the state R. Symmetrically, the R-biased ones, denoted σ^{RR} and σ^{R}, can only reveal the state L. For $i = L, R$, σ^{ii} is an *extreme* source, whereas σ^{i} is a *moderate* one, i.e. the former is more biased than the latter. Formally, σ^{i} sends signal s^{i} with probability 1 in state i and with probability $1-\lambda$ in state $-i$, and σ^{ii} sends signal s^{i} with probability 1 in state i and with probability $1-\delta$ in state $-i$. We assume that $3/4 > \lambda > \delta = 1/2$. The experiments induced by the moderate sources σ^{L} and σ^{R} are described in table 1. The signals σ^{LL} and σ^{RR} are obtained by replacing λ with δ.

σ^{L}

State/signal	s^L	s^R
L	1	0
R	$1-\lambda$	λ

σ^{R}

State/signal	s^L	s^R
L	λ	$1-\lambda$
R	0	1

Table 1: Experiments induced by the moderate sources.

Initially the DM faces the complete menu $M = \{\sigma^{LL}, \sigma^{L}, \sigma^{R}, \sigma^{RR}\}$. In terms of our representation, the set of L-biased sources and the one of R-biased sources each represent a type of options. For $i = L, R$, σ^{i} is strictly more Blackwell informative than σ^{ii}, therefore the DM will never choose any of the extreme sources when his opportunity set is M, that is: $d(M) = \{\sigma^{L}, \sigma^{R}\}$. The DM's demands from freedom are satisfied when the moderate sources are available, that is, his freedom requirement set is $F = \{\sigma^{L}, \sigma^{R}\}$. When facing the menu M, the DM foresees that his payoff from choosing action $a \in \{l, r\}$ in state $i \in \{L, R\}$ is u_a^i where : $u_r^R = u_l^L = 1$, $u_l^R = u_r^L = -1$. Hence the DM will prefer action r if and only if his posterior belief is greater than $1/2$. One can show that the DM's optimal allocation of attention is to choose the "own-biased news source", namely the signal biased toward one's prior: in our case σ^{L} given that $p \leq 1/2$. The rationale for this is the following. The prior indicates action l as the optimal one. Hence, a breakthrough signal s^R from σ^{L} is more valuable than a breakthrough signal s^L from σ^{R}. And the biased signal s^L from σ^{L} is more aligned with the DM's prior belief than s^R from σ^{R}. Hence, he is better off allocating his attention to σ^L (see Che and Mierendorff, 2019, pp. 2999-3000, for the complete argument).

In the next period, the moderate R-biased source σ^R is no more available, either because the government actually banned this newspaper or simply because the DM perceives that this source is no more existing: only L-biased or extremely R-biased ones are present. The DM now faces the menu $N = \{\sigma^{LL}, \sigma^L, \sigma^{RR}\}$. He interprets this removal as revealing that the disutility from making a mistake in state L—i.e. choosing action r—is lower than expected: he now foresees it to be $v_r^L = 0$. σ^{RR} is no more removed from consideration by σ^R, hence $d(N) = \{\sigma^L, \sigma^{RR}\}$. His anticipated utility from choosing σ^L is unchanged while the one attached to σ^{RR} is $v(\sigma^{RR}) = p + (1-p)\delta$ (for p sufficiently close to $1/2$ such that after signal s^R from σ^{RR}, the DM chooses action r).

As a consequence, some DMs with prior beliefs sufficiently close to $1/2$, who would have chosen news source σ^L in menu M, will choose the extreme source σ^{RR} in menu N and their default option becomes r.

Proposition 5. *There exists $p^\star < 1/2$ such that if $p \in [p^\star, 1/2]$:*

(i) The DM prefers σ^{RR} to σ^L in menu N;

(ii) After a realisation of signal s^R from σ^{RR}, the DM chooses action r.

This is in strong opposition as what would be obtained without reactance. Indeed, if the DM does not modify his anticipated utility when the menu shrinks, by removing σ^R, some DMs with prior belief strictly higher than $1/2$ would now choose the source σ^L instead and action l after a signal s^L.

4.2 Integration Policy Backlash

Can forced assimilation policy foster the integration of immigrants communities? While Alesina and Reich (2015)'s theory of nation building assumes that repressing the cultural practices of minorities spurs homogeneity, Bisin and Verdier (2001) suggest that the success of such policy may be mitigated by an increasing effort of parents to influence their children's cultural trait. In this application we show that, with reactance, one can even predict this policy to yield a backlash effect: the repressed immigrants react to repression by becoming more prompt to self-isolation. This additionally provides a rational to the persistence of reactance as an evolutionary efficient behavior.

Such a backslash effect has been recently documented by several papers. Some evidence suggests that the "burkha ban" in France in 2004 has strengthened the religious identity of French-Muslims (Abdelgadir and Fouka, 2020). Fouka (2020) shows that, in states which prohibited German Schools in the aftermath of World War I, German-Americans "were less likely to volunteer in World War II and more likely to marry within their ethnic group and to choose decidedly German names for their offspring".

To show how this backlash operates, we complement Bisin and Verdier (2001)'s account of cultural transmission with a reactance mechanism: as the repression increases, parents' educational freedom decreases and, reacting to this repression, they may endeavour to influence their children even more.[18] There are two cultural traits $\{m, M\}$—for minority and Majority. The proportion of the minority q is assumed to be positive but lower than $1/2$. Each generation is composed of parents who have only one child. Intergenerational transmission results from two socialization mechanisms. First, by vertical socialization the parents may directly transmit their cultural trait i with probability d^i. If, with probability $1 - d^i$, vertical socialization fails, then horizontal transmission occurs and the child adopts the traits of a random individual in society. Hence, the probability that a child from the minority be socialized by her parent's trait is:

$$P(d^i) \equiv d^i + (1 - d^i)q. \tag{3}$$

As Bisin and Verdier (2001), we argue that parents endeavour to influence their child. They have a unit of time to allocate between their effort to fix d^i—which costs $(d^i)^\beta$ unit of time, with $\beta > 1$—and a leisure activity $t^i \in [0, 1]$, whose cost and utility are t^i. In addition, the government can implement a repressive policy $g^i \geq 1$ that may increase the parents' cost of influencing their child: a pair (t^i, d^i) costs $t^i + (d^i)^\beta g^i$ units of time for the parents. We posit that

[18]For simplicity, we adopt a continuous setting, while our own framework is discrete. The ideas would be exactly the same with a discrete setting.

parents get a utility of 0 when their child is socialized to the other trait, while they get a utility $V(g^i)$ when she is socialized to their own trait. Hence, their expected utility of their child's socialization is $P(d^i)V(g^i)$. This means that, given a repressive policy, parents choose options $(t^i, d^i) \in [0,1]^2$ from the menu

$$K_{g^i} \equiv \{(t^i, d^i) : t^i + (d^i)^\beta g^i \leq 1\},$$

to maximize

$$t^i + P(d^i)V(g^i), \tag{4}$$

In what follows, we also assume that V has the following shape:

$$V(g) = \begin{cases} \hat{V} & \text{if } \hat{g} \geq g \\ \hat{V}\frac{g^\lambda}{\hat{g}} & \hat{g} < g \end{cases}$$

For some $\hat{g} > 1$ with $\lambda > 1$ and $\hat{V} > 1$. Hence, after a threshold $\hat{g}$, the more repressive is the policy g, the greater is $V(g)$. The interpretation is that parents react to the repressive policy when they feel that their freedom to educate their child is threatened. In other words, more repression may create incentives to dedicate more resources to transmit their traits to their children. Note that λ represents some kind of reactance rate since as it increases, parents' willingness to influence their child also increases.

From the first order condition, we obtain that the unique equilibrium educational effort—the program (4) being concave—must satisfy:

$$d^{i\star}(g^i, q) = \left(\frac{1-q}{\beta}\frac{V(g^i)}{g^i}\right)^{\frac{1}{\beta-1}} \tag{5}$$

Given the shape of V, $d^\star$ strictly decreases with g on $(1, \hat{g})$ and strictly increases with g on $(\hat{g}, +\infty)$. In other words, when the repressive policy exceeds $\hat{g}$, the more repression, the more parents invest in having their child socialized to their own trait. This suggests that reactance is at work in this model. In the following lemma, we establish the precise connection between this model and our reactance framework.

Lemma 1. *The function C defined on $\{K_g\}_{g\geq 1}$, such that for all g*

$$C(K_g) = \{(t,d) \in K_g : (t,d) \text{ solves } (4)\}.$$

is a well-defined choice function and there exists an RCR C' defined on all compact subsets of $[0,1]^2$ such that $C(K_g) = C'(K_g)$ for all $g \geq 1$.[19]

Assuming the repressive policy to solely concern the minority (i.e. $g^M = 1$), what does reactance imply for the population dynamics in this model? Let time $\tau \in [0, +\infty)$ be continuous and q_τ be the share of the population with the minority cultural trait at time τ. Then, we have[20]

$$\dot{q} = q(1-q)\Big(d^{m\star}\big(g^m, q\big) - d^{M\star}\big(1, 1-q\big)\Big).$$

Given (5), d satisfies the *cultural substitution property*.[21] This implies that q converges to some $q^\star \in (0,1)$, which satisfies $d^{m\star}\big(g^m, q\big) = d^{M\star}\big(1, 1-q\big)$ (see Bisin and Verdier, 2001, Proposition 1). Hence,

$$q^\star(g^m) = \frac{V(g^m)/g^m}{V(1) + V(g^m)/g^m} \tag{6}$$

Given that $V(g)/g$ increases with g when $g \in (\hat{g}, +\infty)$ this means that repressive policy increases the size of the minority. This prediction contrasts with Alesina and Reich (2015)'s suggestions.

Noting that reactance is presumably a characteristic cultural trait (Jonas et al., 2009), this model also provides a rational for why reactance can be evolutionary efficient. Minorities which are more prompt to exhibit reactance are more likely survive to repressive attempts to hinder their cultural practices.

To make precise this comparative statics statement, consider two minorities: one with a high reactance rate λ^H and one with a low reactance rate $\lambda^L < \lambda^H$. Denoting by $q^\star_L(.)$ and $q^\star_H(.)$ the equilibrium population share for these two

[19] For convenience, we construct a reactance structure on this infinite collection of compact sets. Obviously, analogous results could be obtained by making the set of possible policies g and the menus K_g finite.

[20] See Bisin and Verdier (2001, equation (3), footnote 9) for discussions about this differential equation.

[21] In Bisin and Verdier (2001, Definition 1), this property states that d is continuous, decreasing with q, and $d = 0$ when $q = 1$.

minorities, the following proposition establishes that $q^\star$ is always higher for the high-reactance minority.

Proposition 6. *For all $g > \hat{g}$, $q^\star_H(g) > q^\star_L(g)$.*

4.3 Optimal Delegation and Reactance

We consider a typical delegation problem (see Holmstrom, 1980; Alonso and Matouschek, 2008, for a detailed review of the literature): a principal can constrain the decision set of an informed but biased agent, but cannot commit to contingent monetary transfers. In any organization (administrations, companies, etc.), many rules govern what agents can or cannot do, with the purpose of reducing agency costs incurred by principals while benefiting as much as possible from better-informed agents. One can think for instance of a head of a company who delegates stock management to plant managers, a regulator who delegates pricing decisions to a monopolist with unknown costs, or a manager who delegates pricing decision to sales persons.

Formally, a *principal* (she) has the legal right to take an action among a finite set $A = \{a^{LL}, a^L, a^R, a^{RR}\}$. The payoffs delivered by each action depends on the realization of a binary state of the world $\theta \in \{L, R\}$. While the principal only knows the probability $p \in [0, 1]$ that the state is R, an *agent* (he) is privately informed of the realization θ. The principal cannot use contingent transfers and must decide the set of actions among which the agent will choose.

Preferences. The principal's payoff for action a in state θ is the real number $\pi_\theta(a)$. Her preferred action is a^θ in state θ and her second favorite action $a^{\theta\theta}$. Her payoffs are written in table 2. The agent behaves according to a reactance structure with state-dependent utility and reactance functions. In both states, the types are $T^L = \{a^{LL}, a^L\}$ and $T^R = \{a^R, a^{RR}\}$ and the freedom requirement set is $F = \{a^{LL}, a^{RR}\}$. The utility functions u_L, u_R and the reactance functions v_L, v_R are such that the agent reacts to the absence of $a^{\theta\theta}$ by choosing a^θ. In both states, he is more prone to restore the absence of a^{RR}. The functions are specified in table 3.

Principal	R	$\pi_R(a^R) > \pi_R(a^{RR}) > \pi_R(a^L) > \pi_R(a^{LL})$
	L	$\pi_L(a^L) > \pi_L(a^{LL}) > \pi_L(a^R) > \pi_L(a^{RR})$

Table 2: Principal's Payoffs.

Agent	R	$v_R(a^R) > v_R(a^L) > u_R(a^{RR}) > u_R(a^{LL}) > u_R(a^R) > u_R(a^L)$
	L	$v_L(a^L) > v_L(a^R) > u_L(a^{LL}) > u_L(a^{RR}) > u_L(a^L) > u_L(a^R)$

Table 3: Agent's Utility and Reactance Functions.

Optimal Delegation. Denote $\mathcal{A} = \mathcal{P}(A) \setminus \emptyset$ the set of *menus* of action. For any $M \in \mathcal{A}$, $a_\theta(M)$ is the (unique) action chosen by the agent in state θ when facing menu M. For any prior belief $p \in [0, 1]$, the objective of the principal is to solve the following maximization program, whose value is denoted $V(p)$:

$$V(p) \equiv \max_{M \in \mathcal{A}} (1-p)\pi_L\big(a_L(M)\big) + p\pi_R\big(a_R(M)\big). \tag{7}$$

A **delegation strategy** is a mapping from the set of beliefs to the set of menus: $\sigma : [0, 1] \longrightarrow \mathcal{A}$. If for any p, $(1-p)\pi_L\big(a_L(\sigma(p))\big) + p\pi_R\big(a_R(\sigma(p))\big) = V(p)$, we say that the delegation strategy σ is **optimal**.

We are interested in the effect of reactance on optimal delegation strategies by the principal, and consequently on the agent's material welfare (as measured by his utility function). Without reactance, given that the agent's interest is sufficiently aligned with the principal's ($u_R(a^R) > u_R(a^L)$ and $u_L(a^L) > u_L(a^R)$), for any prior $p \in [0, 1]$, the optimal delegation is to let the agent choose among the set of actions $\{a^L, a^R\}$. This strategy cannot be optimal with reactance because the agent would always choose a^R and therefore, for p sufficiently close to 0, offering a^L as the only possible action is better for the principal. For moderate p, it might be better to let the agent choose among the whole set of actions (or equivalently among his preferred actions $\{a^{LL}, a^{RR}\}$) given that in state $\theta = L, R$, $a^{\theta\theta}$ is the second best action for the principal. It happens that it depends on the magnitude of the principal's payoffs, as summarized

Proposition 7. Define the following beliefs:

$$\bar{p} = \frac{\pi_L(a^{LL}) - \pi_L(a^R)}{\pi_L(a^{LL}) - \pi_L(a^R) + \pi_R(a^R) - \pi_r(a^{RR})},$$
$$\underline{p} = \frac{\pi_L(a^L) - \pi_L(a^{LL})}{\pi_L(a^L) - \pi_L(a^{LL}) + \pi_R(a^{RR}) - \pi_R(a^L)},$$
$$\hat{p} = \frac{\pi_L(a^L) - \pi_L(a^R)}{\pi_L(a^L) - \pi_L(a^R) + \pi_R(a^R) - \pi_R(a^L)}.$$

Proposition 7. *An optimal delegation strategy $\sigma^\star$ must induce the following actions.*

1. *If $\underline{p} < \bar{p}$:*

   (i) $a_L(\sigma^\star(p)) = a_R(\sigma^\star(p)) = a^L$ *for* $p < \underline{p}$;

   (ii) $a_L(\sigma^\star(p)) = a^{LL}$ *and* $a_R(\sigma^\star(p)) = a^{RR}$ *for* $\underline{p} < p < \bar{p}$;

   (iii) $a_L(\sigma^\star(p)) = a_R(\sigma^\star(p)) = a^R$ *for* $p > \bar{p}$;

   and it can induce either of the two possibilities respectively at boundary beliefs $\underline{p}$ *and* $\bar{p}$.

2. *If $\underline{p} \geq \bar{p}$:*

   (i) $a_L(\sigma^\star(p)) = a_R(\sigma^\star(p)) = a^L$ *for* $p < \hat{p}$;

   (ii) $a_L(\sigma^\star(p)) = a_R(\sigma^\star(p)) = a^R$ *for* $p > \hat{p}$;

   and it can induce either of the two possibilities at boundary belief $\hat{p}$.

Two possible optimal strategies are depicted in figure 2, implementing the actions described in proposition 7. In each strategy, the principal is indifferent between the two possible menus at boundary beliefs $\underline{p}, \bar{p}$ and $\hat{p}$. These strategies are the most direct ones, in the sense that each menu does not contain irrelevant actions that are never chosen by the agent.

Agent's Welfare. If we measure the agent's material welfare through the utility functions u_L and u_R, one can see from proposition 7 that the effect of reactance is ambiguous. In the case where $\underline{p} \geq \bar{p}$, the effect is only negative, as the agent only has access to a unique action that is not among her best actions. But

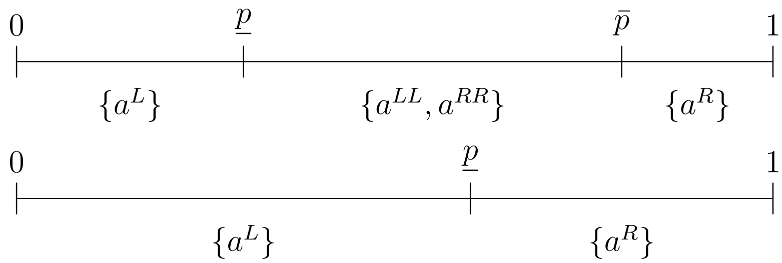

Figure 2: Optimal Delegation Strategies.

if $\underline{p} < \bar{p}$, then while there is still this negative effect when $p \leq \underline{p}$ or $p \geq \bar{p}$, on the contrary, for middle beliefs, reactance forces the principal to let the agent choose among her best options $\{a^{LL}, a^{RR}\}$. The logic behind this result is that reactance makes the agent's threat to choose bad actions (for himself) credible. Hence, the principal reacts either by constraining even more the agent's opportunity set; or on the contrary by offering him a greater satisfaction.

APPENDICES

A Proof of Theorem 1

Proof of the necessity. Let $< \mathcal{T}, F, u, v >$ be a reactance structure that represents the RCR c. We denote $T(x)$ for the type of option x. First, we need to state the following lemma, that simply says that WARP is satisfied for each collection of menus that contains options of the same type.

Lemma 2. *For any $T \in \mathcal{T}$, $A \subset B \subseteq T$, $x \in X$, if $x = c(B)$ and $x \in A$, then $x = c(A)$.*

Proof. Given that $B \subseteq T$, $x = c(B)$ implies that $c(B) = d(B)$, that is, $u(x) > u(y)$ for any $y \neq x$, $y \in B$. Because $A \subset B$, it means that $x = d(A)$, and therefore $x = c(A)$. □

To show **Exp.**, let $x \in X$ and $A, B \in \mathcal{X}$ such that $x = c(A) = c(B)$. This means that $x \in d(A) \cap d(B)$. Hence, $u(x) > u(y)$ for all $y \neq x$ such that $y \in (A \cup B) \cap T(x)$, which implies that $x \in d(A \cup B)$. Moreover, $x = c(A) = c(B)$ implies that $v(x) > v(z)$ for all $z \neq x$ such that $z \in d(A) \cup d(B)$. Besides,

$d(A \cup B) \subseteq d(A) \cup d(B)$, hence $v(x) > v(z)$ for all $z \neq x$, $z \in d(A \cup B)$. Hence, $x = c(A \cup B)$.

To show **R-Tran.** Let $x, y, z \in X$ such that $x\mathbf{R}^c y$ and $y\mathbf{R}^c z$. By definition,

$$x\mathbf{R}^c y \implies \exists t \in X, t = c\{x, y, t\} \text{ and } x = c\{x, t\},$$
$$y\mathbf{R}^c z \implies \exists t' \in X, t' = c\{y, z, t'\} \text{ and } y = c\{y, t'\}.$$

Proposition 4 implies that $T(x) = T(y) = T(z)$. Coupled with lemma 2, this shows that $T(t') \neq T(x) = T(y) = T(z) \neq T(t)$. Given that **Exp** is satisfied, we also know that $z = c\{z, y\}$ and $y = c\{y, x\}$. Hence, $u(z) > u(y) > u(z)$ and $v(x) > v(t) > v(y) > v(t') > v(z)$. Therefore, $t = c\{x, z, t\}$ and $x = c\{x, t\}$, which means that $x\mathbf{R}^c z$.

Now let $x, y, z \in X$ such that $y = c\{x, y\}$, $z = c\{y, z\} = c\{x, z\}$, $\neg[x\mathbf{R}^c y]$ and $\neg[y\mathbf{R}^c z]$. Assume by contradiction that $x\mathbf{R}^c z$. Then there exists t such that $t = c\{x, z, t\}$ and $x = c\{x, t\}$ and, by proposition 4 and lemma 2, $T(x) = T(z) \neq T(t)$. Moreover, we have that $v(x) > v(t) > v(z)$ so that $v(x) > v(z)$. Hence, if $T(y) \neq T(x)$, then $z = c\{y, z\} = c\{x, z\}$ imply that

$$v(y) \underbrace{>}_{y=c\{x,y\}} v(x) > v(z) \underbrace{>}_{z=c\{y,z\}} v(y) \tag{8}$$

A contradiction. Now if $T(y) = T(x)$, then $u(z) > u(y) > u(x)$. But then either $v(y) > v(t)$, and then $y\mathbf{R}^c z$, or $v(t) > v(y)$ and then $x\mathbf{R}^c y$. In both cases we have a contradiction.

Finally, let $x, y, z \in X$ such that $y = c\{x, y\}$, $z = c\{y, z\} = c\{x, z\}$, $\neg[x\mathbf{P}^c y]$ and $\neg[y\mathbf{P}^c z]$. Assume by contradiction that $x\mathbf{P}^c z$. Hence, there exists t such that $z\mathbf{R}^c t$ and for any such t, $x\mathbf{R}^c t$. By proposition 4, $T(z) = T(t) = T(x) \equiv T$, $u(t) > u(z) > u(x)$, $v(z) > v(t)$, and $v(x) > v(t)$, which implies that $x, z \notin F$. Suppose that $y \notin T$, then by proposition 4, $\neg[x\mathbf{R}^c y]$ and $\neg[y\mathbf{R}^c z]$. We have already proved that this implies $\neg[x\mathbf{R}^c z]$. Therefore, for any $t \notin T$, $z = c\{z, t\} \implies x = c\{x, t\}$. But then, given that $z = cz, t$ and $y = cx, y$, it must be that $y \in T$. A contradiction. Hence, $y \in T$, which means that $u(z) > u(y) > u(x)$. Because $z \notin F$, then $y \notin F$. Given that $v \circ u^{-1}$ is single peaked on $u(T \backslash F)$ we have that $v(y) > \min\{v(x), v(z)\}$. If $v(y) > v(z)$, then proposition 4 implies that $y\mathbf{P}^c z$, a contradiction. Hence $v(z) > v(y) > v(x)$. Let t be such that, $z\mathbf{R}^c t$, then $x\mathbf{R}^c t$, which means that $v(x) > v(t)$, and therefore $v(y) > v(t)$, and hence $y\mathbf{R}^c t$. This proves that $y\mathbf{P}^c z$, again a contradiction.

To show **R-Con.**, let $x, y, z \in X$ such that $x\mathbf{R}^c y$, $x\mathbf{R}^c z$, $z = c\{y, z\}$, and such that there exists no t with $y\mathbf{R}^c t$. Proposition 4 implies that $T(z) = T(x) = T(y) \equiv T$ and $u(z) > u(y)$. Furthermore, by proposition 1, it is without loss of generality to assume that F is maximal, and by proposition 2, $y\mathbf{R}^c t$ for no t implies that $y \in F$, and hence $z \in F$. Let $u \in X$ such that $u\mathbf{R}^c z$, so there exists $t \notin T$ such that $v(u) > v(t) > v(z) = u(z) > u(u)$. Hence, $u \notin F$ which, together with $y \in F$ and $u \in T$ (since $u\mathbf{R}^c z \in T$) , implies that $u(y) > u(u)$. This means that $v(u) > v(t) > v(y) = u(y) > u(u)$. Hence, $u\mathbf{R}^c y$. This completes the proof of *(i)* in **R-Con**. Now assume that $u\mathbf{R}^c y$. This means that $u \in T$ and $u(z) > u(y) > u(u)$, which proves *(ii)* in **R-Con**.

To show **R-Mon.**, let $x, y, z \in X$ such that $z = c\{y, z\}$, $y = c\{x, y\}$, $x\mathbf{R}^c t$, and $z\mathbf{R}^c t$ for some t. Assume that $x\mathbf{R}^c y$. By proposition 4, this means that $T(x) = T(y) = T(t) = T(z) \equiv T$, $u(z) > u(y) > u(x)$, $v(y) < v(x)$ and $x, y, z \notin F$. The single peakedness of $v \circ u^{-1}$ on $u(T \backslash F)$ implies that $v(y) > v(z)$, from which we can conclude that $y\mathbf{P}^c z$. This proves *(i)*. Assume now that $x\mathbf{P}^c z$ and $x\mathbf{P}^c y$. By proposition 4, $T(x) = T(y) = T(z) \equiv T$, $u(z) > u(y) > u(x)$ and $x, y, z \notin F$. The fact that $v \circ u^{-1}$ is single-peaked on $u(T \backslash F)$ implies that $v(y) > \min\{v(x), v(z)\}$. If $v(y) > v(x)$, then for any t such that $z\mathbf{R}^c t$, given that $x\mathbf{R}^c t$, also $y\mathbf{R}^c t$ and hence $y\mathbf{P}^c z$. Similarly, if $v(y) > v(z)$, $y\mathbf{P}^c z$, which ends the proof of *(ii)*. □

Proof of the sufficiency. Let define the binary relation $\succ \subset X^2$ by $x \succ y$ if and only if $x = c\{x, y\}$ or $x = y$. It is clear that $\succ$ is complete and antisymmetric. For any transitive and complete binary relation $>$ defined on a set A, we write $\max(A, >) \equiv \{x \in A \mid x > y, \forall\, y \in A\}$. When $>$ is a linear order, with a slight abuse of notation, when no confusion can be made, we indifferently write $\max(A, >)$ for the singleton or for the element of the singleton.

Lemma 3. *Let K a subset of X such that,*

$$\big((x, y) \in K^2 \iff \neg[x\mathbf{R}^c y] \text{ and } \neg[y\mathbf{R}^c x]\big), \tag{9}$$

then $\succ$ restricted to K^2 is linear order and for all $K' \subseteq K$, $c(K) \succ y$ for all $y \in K'$.

Proof. Let K satisfying (9), $x, y, z \in K$ and $x \succ y \succ z$. Suppose by contradiction that $z \succ x$. If $x = c\{x, y, z\}$, then $z\mathbf{R}^c y$, which contradicts the fact that $(y, z) \in K^2$ and K satisfies (9). The same reasoning applies if either $y = c\{x, y, z\}$ or $z = c\{x, y, z\}$. Hence, we conclude that $x \succ z$.

Moreover, let $K' \subseteq K$, the transitivity of $\succ$ on K, implies that there exists $x \in K'$ such that $x \succ y$ for any $y \in K'$. By **Exp**, $x = c(K')$. □

Define now

$$X^{\downarrow} = \bigcup_{y \in X} \{x \in X : x\mathbf{R}^c y\}$$
$$X^{\uparrow} = \bigcup_{y \in X} \bigcap_{t \in X} \{x \in X : y\mathbf{R}^c x, \neg[x\mathbf{R}^c t]\}$$

Let $\tilde{X} = X^{\uparrow} \cup X^{\downarrow}$ and for all $x \in X^{\downarrow}$, $R^{\uparrow}(x) = \{y \in X^{\uparrow} : x\mathbf{R}^c y\}$.

Lemma 4. *If $x \in X^{\downarrow}$, then $R^{\uparrow}(x) \neq \emptyset$.*

Proof. Let $x \in X^{\downarrow}$, i.e. $x\mathbf{R}^c y$ for some $y \in X$. If $y \in X^{\uparrow}$, this terminates the proof. Suppose that $y \notin X^{\uparrow}$, then there exists z_1 such that $y\mathbf{R}^c z_1$, which by **R-Tran** implies that $x\mathbf{R}^c z_1$. Either $z_1 \in X^{\uparrow}$, which ends the proofs, or there exists z_2 such that $z_1\mathbf{R}^c z_2$, which again by **R-Tran** implies that $x\mathbf{R}^c z_2$. At each step k, we replicate the same reasoning. Because X is finite, there must exist n such that for all $t \in X$, $\neg[z_n\mathbf{R}^c t]$, i.e., $z_n \in X^{\uparrow}$. Yet, **R-Tran** also implies that $x\mathbf{R}^c z_n$. Hence, $R^{\uparrow}(x) \neq \emptyset$. □

Note that lemma 3 implies that $\succ$ is transitive on $X^{\uparrow}$. Hence, lemma 4 implies the existence, for all $x \in X^{\downarrow}$, of $m(x)$, defined by:

(10) $$\{m(x)\} \equiv \min(R^{\uparrow}(x), \succ) = \{y \in R^{\uparrow}(x) \mid z \succ y, \forall\, z \in R^{\uparrow}(x)\}.$$

Lemma 5. *For all $x, y \in X^{\downarrow}$, if $R^{\uparrow}(x) \cap R^{\uparrow}(y) \neq \emptyset$, then $m(x) = m(y)$;*

Proof. Let $x, y \in X^{\downarrow}$. Assume there exists $t \in R^{\uparrow}(x) \cap R^{\uparrow}(y)$ and let $t' = m(x)$. We show that $t' \in R^{\uparrow}(y)$. If $t = t'$ there is nothing to prove. If $t \neq t'$, then by definition of t' and since $t \in R^{\uparrow}(x)$, we have $t \succ t'$. Given that $t' \in X^{\uparrow}$ we have that $\neg[t'\mathbf{R}^c z]$ for any $z \in X$. Since $x\mathbf{R}^c t$, $x\mathbf{R}^c t'$ and $t \succ t'$, by **R-Con**(i) $y\mathbf{R}^c t$, implies that $y\mathbf{R}^c t'$, i.e. $t' \in R^{\uparrow}(y)$.

We prove symmetrically that $m(y)$ belongs to $R^{\uparrow}(x)$. Hence, if $m(x) \neq m(y)$ it would be, by definition, that $m(x) \succ m(y)$ and $m(y) \succ m(x)$. A contradiction. Hence, $m(x) = m(y)$. □

Since X is finite there exists n^* such that we can index the set $\{m(x) : x \in X^{\downarrow}\}$ of every minimal option by a sequence $(m(i))_{1 \leq i \leq n^*}$ such that $i \neq j \iff m(i) \neq m(j)$. Define now for all $1 \leq i \leq n^*$:

$$T^{\downarrow}(i) = \{x \in X^{\downarrow} : x\mathbf{R}^c m(i)\},$$
$$T^{\uparrow}(i) = \{x \in X^{\uparrow} : \exists y \in X^{\downarrow}, y\mathbf{R}^c m(i), y\mathbf{R}^c x\}, \text{ and}$$
$$T(i) = T^{\uparrow}(i) \cup T^{\downarrow}(i).$$

Define finally:

$$T(0) \equiv T_0^c = X \backslash \tilde{X} = \bigcap_{y \in X} \bigcap_{t \in X} \{x \in X : \neg[x\mathbf{R}^c y], \neg[t\mathbf{R}^c x]\}$$

These will define the types. We denote $\mathcal{T} = \{T(i) : 0 \leq i \leq n^*\}$ the collection of types.

Lemma 6. *$\mathcal{T}$ forms a partition of X.*

Proof. Given the definition of $T(0)$, it is sufficient to show that the collection $\{T(i) : 1 \leq i \leq n^*\}$ partitions $\tilde{X}$.

We first show that $\tilde{X} = \bigcup_{1 \leq i \leq n^*} T(i)$. Note that for all $1 \leq i \leq n^*$, if $x \in T(i)$, then there exists y such that $x\mathbf{R}^c y$ or $y\mathbf{R}^c x$, so that $x \in \tilde{X}$. Hence, $\bigcup_{i \leq n} T(i) \subseteq \tilde{X}$. Similarly, if $x \in \tilde{X}$, then either $x \in X^{\downarrow}$ or $x \in X^{\uparrow}$. If $x \in X^{\downarrow}$, then $x\mathbf{R}^c y$ for some $y \in X$ and by (10), $x\mathbf{R}^c m(x)$, i.e. $x \in T^{\downarrow}(i)$ for some $1 \leq i \leq n^*$. If $x \in X^{\uparrow}$, then $y\mathbf{R}^c x$ for some $y \in X$ and $\neg[x\mathbf{R}^c z]$ for all $z \in X$. But then $x \in R^{\uparrow}(y)$ and (10) implies that $y\mathbf{R}^c m(y) = m(i)$ for some $1 \leq i \leq n^*$. Therefore $x \in T^{\uparrow}(i)$. Hence, in either cases, $x \in \bigcup_{1 \leq i \leq n^*} T(i)$.

We now assume that for some $1 \leq i, j \leq n^*$, $x \in T(i) \cap T(j)$, and show that this implies $i = j$.

Case 1: Assume $x \in T^{\downarrow}(i)$. Then, because $X^{\uparrow}$ and $X^{\downarrow}$ are disjoint, x necessarily belongs to $T^{\downarrow}(j)$. $x \in T^{\downarrow}(i)$ means that $x\mathbf{R}^c m(i)$. By definition of the $m(i)$'s, there exists y such that $m(y) = m(i)$. Applying lemma 5, we conclude that $m(x) = m(y) = m(i)$. Similarly, we prove that $m(x) = m(j)$. Hence $m(i) = m(j)$ and therefore $i = j$.

Case 2: Assume $x \in T^{\uparrow}(i)$. Then, because $X^{\uparrow}$ and $X^{\downarrow}$ are disjoint, $x \in T^{\uparrow}(j)$. Hence, there exists $y_i, y_j \in X^{\downarrow}$ such that $y_i\mathbf{R}^c m(i)$, $y_j\mathbf{R}^c m(j)$, $y_i\mathbf{R}^c x$, and $y_j\mathbf{R}^c x$. Hence, $x \in R^{\uparrow}(y_i) \cap R^{\uparrow}(y_j)$, which by lemma 5, implies that $m(y_i) = m(y_j)$. Using the same argument as in *case 1*, we conclude that $m(i) = m(y_i) = m(y_j) = m(j)$ which means that $i = j$. □

Note that, given lemma 6, $T(x)$ is well defined as the type of the option $x \in X$, i.e. $T(x) = T(i) \iff x \in T(i)$. We now prove that $\succ$ is transitive on every type $T(i)$.

Lemma 7. *For all $0 \leq i \leq n^*$, the relation $\succ$ is transitive on $T(i)$.*

Proof. That $\succ$ is transitive on $T(0)$ is a direct consequence of lemma 3. We now focus on $1 \leq i \leq n^*$.

First, we show that for all $x \in T^{\downarrow}(i)$ and $y \in T^{\uparrow}(i)$, $x \prec^C y$. If $y = m(i)$ this follows directly. If $y \neq m(i)$, there exists $z \in X$ such that $z\mathbf{R}^c y$ and $z\mathbf{R}^c m(i)$. Hence, $y, m(i) \in R^{\uparrow}(z)$ and $y \succ m(i)$. Moreover, $y \in X^{\uparrow}$ so that $\neg[m(i)\mathbf{R}^c y]$. Since $x\mathbf{R}^c m(i)$, **R-Con**(ii) implies that $y \succ x$.

Second, we show that $\succ$ is transitive on $T^{\downarrow}(i)$. Let $x, y, z \in T^{\downarrow}(i)$ such that $x \succ y \succ z$. Assume by contradiction that $z \succ x$. Suppose (w.l.o.g) that $x = c\{x, y, z\}$. In this case, $z\mathbf{R}^c y$. Given that $x\mathbf{R}^c m(i)$, $z\mathbf{R}^c m(i)$, and $x \succ y \succ z$, **R-Mon *(i)*** entails $y\mathbf{P}^{\mathbf{c}} x$. But since $y, z \in X^{\downarrow}$, $z\mathbf{R}^c y$ implies $z\mathbf{P}^c y$. Hence, **R-Tran (ii)** yields $z\mathbf{P}^c x$ which contradicts $z \succ x$.

Finally, we prove that $\succ$ is transitive on each type. Let i and $x, y, z \in T(i)$ such that $x \succ y \succ z$. If $x \in T^{\downarrow}(i)$ then, according to the first part of the proof, $y \in T^{\downarrow}(i)$ and therefore similarly $z \in T^{\downarrow}(i)$. Similarly, if $z \in T^{\uparrow}(i)$, the first part of the proof implies that $y \in T^{\uparrow}(i)$, which in turn also triggers that $x \in T^{\uparrow}(i)$. In both cases, we already proved that $\succ$ is transitive on $T^{\downarrow}(i)$ (second part of the proof) and on $T^{\uparrow}(i)$ (a consequence of lemma 3). The last case are if $x \in T^{\uparrow}(i)$ and $z \in T^{\downarrow}(i)$, but then from the first part of the proof we obtain $x \succ z$. □

For any menu A we define:

$$d(A) \equiv \{x \mid x = \max(T(x) \cap A, \succ)\}.$$

A direct implication of lemma 3 is that $\succ$ is transitive on $d(A)$. Hence we can state the following lemma:

Lemma 8. *For any $A \in \mathcal{X}$,*

$$c(A) = \max(d(A), \succ) \tag{11}$$

Proof. For any menu A, denote $i(A) = \#\{i \mid T(i) \cap A \neq \emptyset\}$. We prove that for any $1 \leq n \leq n^* + 1$, for any A such that $i(A) = n$, (11) holds.

If $i(A) = 1$, the conclusion follows from lemma 7. Assume now that $i(A) = 2$. Let $x, y \in A$ be such that $T(x) \cap T(y) = \emptyset$, $x = \max(T(x) \cap A, \succ)$, $y = \max(T(y) \cap A, \succ)$, and $y \succ x$. Assume by contradiction that $y \neq c(A)$. First note that by definition of y, **Exp**, and lemma 7, $y \succ z$ for any $z \in T(y) \cap A$. Hence, there must exist $z \in T(x)$ such that $z \succ y$ and $y \neq c\{x, y, z\}$; otherwise **Exp** would imply that $y = c(A)$. This implies that $x \succ z \succ y \succ x$. Since $y \neq c\{x, y, z\}$, this is only possible if either $y\mathbf{R}^c z$ or $x\mathbf{R}^c y$, which in any case contradicts that $x, z \notin T(y)$ (given that, according to lemma 6, types partition X). Hence we conclude that $y = c(A)$.

Then fix $3 \leq n \leq n^* + 1$ and let A a menu such that $i(A) = n$. We denote $y = \max(d(A), \succ)$. Given the preceding proof for any menu A' such that $i(A') = 2$, for any $z \in A$, $y = c\Big(\big(T(y) \cup T(z)\big) \cap A\Big)$. This implies by **Exp** that $y = C(A)$. $\square$

Let $\mathbf{Q}^c = \mathbf{R}^c \cup \mathbf{P}^c$.

Lemma 9. *$\mathbf{Q}^c$ is asymmetric and transitive.*

Proof. By definition for no $x, y \in X$, $x\mathbf{R}^c y$ and $y\mathbf{P}^c x$; otherwise $x \succ y$ and $y \succ x$. Hence, given that both $\mathbf{R}^c$ and $\mathbf{P}^c$ are asymmetric, so is $\mathbf{Q}^c$.

To show that $\mathbf{Q}^c$ is transitive, let $x, y, z \in X$ such that $x\mathbf{Q}^c y$ and $y\mathbf{Q}^c z$.

If $y\mathbf{R}^c z$, then, when $x\mathbf{R}^c y$, the conclusion directly follows from **R-Tran**. When $x\mathbf{P}^c y$, for any t such that $y\mathbf{R}^c t$, we have $x\mathbf{R}^c t$. Hence, $y\mathbf{R}^c z$ implies $x\mathbf{R}^c z$, i.e. $x\mathbf{Q}^c z$.

If $y\mathbf{P}^c z$, then there exists t such that $z\mathbf{R}^c t$ and for any of such t, $y\mathbf{R}^c t$. If $x\mathbf{P}^c y$, then the conclusion follows from the transitivity of $\mathbf{P}^c$. If $x\mathbf{R}^c y$, **R-Tran** implies that $x\mathbf{R}^c t$ for any t such that $y\mathbf{R}^c t$, and therefore for any t such that $z\mathbf{R}^c t$, i.e. $x\mathbf{P}^c z$ and $x\mathbf{Q}^c z$. □

For each $0 \leq i \leq n^*$ define the relation $\triangleright_i$ on $T(i)$ such that for all $x, y \in T(i)$, $x \triangleright_i y$ if and only if either $x\mathbf{Q}^c y$ or ($x \succ y$ and $\neg[y\mathbf{Q}^c x]$).

Lemma 10. *For all $0 \leq i \leq n^*$, $x \triangleright_i y$ is a linear order.*

Proof. Note that $\triangleright_0 = \succ \cap T(0)^2$; hence, when $i = 0$ the conclusion follows from lemma 3.

Let $1 \leq i \leq n^*$. Showing that $\triangleright_i$ is antisymmetric and reflexive is straightforward and thus left to the reader. Regarding the connectedness, for any $x, y \in T(i)$, $x \neq y$, if $x \succ y$, then it is not possible that $x\mathbf{Q}^c y$, so either $y\mathbf{Q}^c x$, and therefore $y \triangleright_i x$, or not, and therefore $x \triangleright_i y$. The proof is symmetric if $y \succ x$. Given that $\succ$ is connected, this terminates the proof of the connectedness of $\triangleright_i$.

We finally show the transitivity. Let $x, y, z \in T(i)$ such that $x \triangleright_i y \triangleright_i z$. We have to deal with several cases separately.

(1) If $z \succ y \succ x$, then $x\mathbf{Q}^c y$ and $z\mathbf{Q}^c x$. Thus, by lemma 9, $x\mathbf{Q}^c z$ and $x \triangleright_i z$.

(2) If $x \succ y \succ z$, then $\neg[y\mathbf{Q}^c x]$ and $\neg[z\mathbf{Q}^c y]$. Given that lemma 7 implies $x \succ z$, we need to show that $\neg[z\mathbf{Q}^c y]$. This directly follows from **R-tran**(ii).

(3) If $x \succ z \succ y$, then $x \succ y$ (lemma 7) and thus $\neg[y\mathbf{Q}^c x]$ and $y\mathbf{Q}^c z$. Assume by contradiction that $z\mathbf{Q}^c x$. Then lemma 9 implies that $y\mathbf{Q}^c x$, a contradiction. So $\neg[z\mathbf{Q}^c x]$ and $x \triangleright_i z$.

(4) If $y \succ x \succ z$, then $y \succ z$ and thus $x\mathbf{Q}^c y$ and $\neg[z\mathbf{Q}^c y]$. The similar reasoning as in (3) gives the expected conclusion.

(5) If $z \succ x \succ y$, then $z \succ y$ and thus $\neg[y\mathbf{Q}^c x]$ and $y\mathbf{Q}^c z$. Suppose that $\neg[x\mathbf{Q}^c z]$, then **R-Tran**(ii) implies that $\neg[y\mathbf{Q}^c z]$, a contradiction. Hence, $x\mathbf{Q}^c z$ and $x \triangleright_i z$.

(6) If $y \succ z \succ x$, then $y \succ x$ and thus $x\mathbf{Q}^c y$ and $\neg[z\mathbf{Q}^c y]$. The similar reasoning as in (5) gives the expected conclusion.

This completes that proof that $\triangleright_i$ is transitive. $\square$

Denote $\tilde{\triangleright} = \bigcup_i \triangleright_i$. Let $\triangleright$ be the relation on X defined by:

$$\forall x, y \in X, x \triangleright y \iff \begin{cases} x \,\tilde{\triangleright}\, y & \text{if } x \in T(y) \\ x \succ y & \text{if } x \notin T(y) \end{cases}$$

Lemma 11. *The relation $\triangleright$ is a linear order.*

Proof. Given that $\succ$ and $\tilde{\triangleright}$ are both antisymmetric and complete, so is $\triangleright$. We now prove the transitivity. Let us consider x, y, z such that $x \triangleright y \triangleright z$. If there exists i such that $x, y, z \in T(i)$, then this follows from lemma 10. If $T(x) \cap T(y) = T(x) \cap T(z) = T(y) \cap T(z) = \emptyset$, then this follows from lemma 3.

Suppose we are in the case $T(x) = T(y) \neq T(z)$. Note that this implies that $y \succ z$. Assume first that $x\mathbf{Q}^c y$ and suppose by contradiction that $z \succ x$. Given that $T(x) = T(y) \neq T(z)$, we have $\neg[z\mathbf{Q}^c y]$ and $\neg[x\mathbf{Q}^c z]$. Moreover, by assumption, $y \succ z \succ x$, so that **R-Tran**(ii) implies $\neg[x\mathbf{Q}^c y]$. A contradiction. Assume now that $x \succ y$ and $\neg[y\mathbf{Q}^c x]$. Suppose by contradiction that $z \succ x$, then, since $\neg[y\mathbf{Q}^c x]$, we have either $x\mathbf{R}^c z$ or $z\mathbf{R}^c y$. In any case a contradiction arises since $T(x) = T(y) \neq T(z)$. We deal with the case $T(y) = T(z) \neq T(x)$ similarly.

Suppose finally that we are in the case $T(x) = T(z) \neq T(y)$. Note that this implies that $x \succ y$ and $y \succ z$. Assume by contradiction that $z \triangleright x$. If $x \succ z$, then $z\mathbf{Q}^c x$. But given that $\neg[z\mathbf{Q}^c y]$, $\neg[y\mathbf{Q}^c x]$, $x \succ y$, and $y \succ z$, **R-Tran**(ii) implies that $\neg[z\mathbf{Q}^c x]$, a contradiction. Hence, $z \triangleright x$ is possible only if $z \succ x$ and $\neg[x\mathbf{Q}^c y]$. But, given that $\neg[x\mathbf{Q}^c z]$, this would imply either $y\mathbf{R}^c x$ or $z\mathbf{R}^c y$. Given that $T(x) = T(z) \neq T(y)$, both cases lead to a contradiction. $\square$

Now let $F = \bigcup_{1 \leq i \leq n^*} T^{\uparrow}(i) \cup T(0)$. Given that $\succ$ is transitive on F (lemma 3), there exists a function $w : F \longrightarrow \mathbb{R}$ that represents $\succ$ on F. Furthermore, for every $i = 0, \ldots, n^*$, $\succ$ is transitive on every $T(i)$ (lemma 7), hence there exists a function $u_i : T(i) \longrightarrow \mathbb{R}$ representing $\succ$ on $T(i)$, and such that $u_T(x) = w(x)$ for every $x \in F$. We now define the function $u : X \longrightarrow \mathbb{R}$ such that for every i,

$x \in T(i)$, $u(x) = u_i(x)$. We clearly have, for any menu A,

$$d(A) = \bigcup_{T \in \mathcal{T}} \arg\max_{x \in T \cap A} u(x)$$

Lemma 12. *There exists a function v representing $\triangleright$ such that for any $x \in F$, $v(x) = u(x)$ and for any $x \notin F$, $v(x) > u(x)$.*

Proof. Note first that, restricted to F^2, $\succ = \triangleright$. Indeed let $x, y \in F$ such that $x \succ y$. If $x \notin T(y)$, this directly follows from definition of $\triangleright$. If $x \in T(y)$, then if, by contradiction, $y \triangleright x$, then we would have $x\mathbf{Q}^c y$, in contradiction with $x \in F$.

Now given that $\triangleright$ is transitive and complete on X, there exists (up to a monotone transformation) a function v representing $\triangleright$. Given that $\succ = \triangleright$ on F, we can set this function such that $u(x) = v(x)$ for all $x \in F$.

Next, if $x \notin F$, then $x\mathbf{R}^c m(x)$, which implies $v(x) > v(m(x))$ and $m(x) \succ x$, which means that $u(m(x)) > u(x)$. Given that $m(x) \in F$, this means that $v(x) > v(m(x)) = u(m(x)) > u(x)$ □

Lemma 13. *For any menu A,*

$$c(A) \underbrace{=}_{lemma\ 8} \max\big(d(A), \succ\big) = \arg\max_{x \in d(A)} v(x).$$

Proof. Let $x, y \in d(A)$, $T(x) \neq T(y)$ and suppose (w.l.o.g) that $x \succ y$. Given that v represents $\triangleright$ and $T(x) \neq T(y)$, this means that $x \succ y \implies x \triangleright y \implies v(x) > v(y)$. Hence, we just established that $\max\big(d(A), \succ\big) \subseteq \arg\max_{x \in d(A)} v(x)$. Given that $\max\big(d(A), \succ\big) \neq \emptyset$ and $\arg\max_{x \in d(A)} v(x)$ is a singleton (since $\triangleright$ is antisymmetric), this proves that $\max\big(d(A), \succ\big) = \arg\max_{x \in d(A)} v(x)$. □

To complete the proof of theorem 1, we show that we can construct a function $\hat{v}$ such that $\hat{v} \circ u^{-1}$ is single-peaked on $u(T(i) \setminus F)$ for any $1 \leq i \leq n^\star$. For any $1 \leq i \leq n^\star$, let define $M(i) \equiv \max(T(i), \triangleright_i)$. Note that for any $z \in T^\uparrow(i)$, there exists t such that $t\mathbf{R}^c z$, i.e. $t \triangleright_i z$, hence necessarily $M(i) \in T^\downarrow(i)$.

The following lemma shows that v as constructed above satisfies single-peakedness on the "right" of $M(i)$, that is:

Lemma 14. *For all $1 \leq i \leq n^\star$, $x, y \in T^\downarrow(i)$, if $u(y) > u(x) > u\big(M(i)\big)$, then $v(x) > v(y)$.*

Proof. $u(y) > u(x) > u\big(M(i)\big)$ means that $y \succ x \succ M(i)$ and $y \succ M(i)$. We know that $y\mathbf{R}^c m(i)$, $M(i)\mathbf{R}^c m(i)$. Given that $\mathbf{R}^c \cap T^{\downarrow}(i)^2 \subseteq \mathbf{P}^c \cap T^{\downarrow}(i)^2$ and the definition of $M(i)$, it must be that $M(i)\mathbf{P}^c y$, $M(i)\mathbf{P}^c x$. Therefore, **R-Mon** (ii) implies that $x\mathbf{P}^c y$, which implies $v(x) > v(y)$. □

For any $1 \leq i \leq n^\star$, we define the set $\underline{T}^{\downarrow}(i) \equiv \{x \in T^{\downarrow}(i) \mid u(x) < u(M(i))\}$. We order this set $\underline{T}^{\downarrow}(i) = \{x_1^i, \ldots, x_{K^i}^i\}$ such that for any $1 \leq k \leq K^i - 1$, $u(x_k^i) > u(x_{k+1}^i)$.

Lemma 15. *For all $1 \leq i \leq n^\star$, if $x, y \in \underline{T}^{\downarrow}(i)$, $u(x) > u(y)$, and $v(x) < v(y)$, then for any $z \notin T(i)$, either $v(z) < v(x)$ or $v(y) < v(z)$.*

Proof. Let $1 \leq i \leq n^\star$ such that there exists $x, y \in \underline{T}^{\downarrow}(i)$ with $u(x) > u(y)$, and $v(x) < v(y)$. Assume by contradiction that there exists $z \notin T$ such that $v(x) < v(z) < v(y)$. Hence, $y\mathbf{R}^c x$. But then, by **R-Mon (i)**, because $y\mathbf{R}^c m(i)$ and $M(i)\mathbf{R}^c m(i)$, $x\mathbf{P}^c M(i)$, which contradicts the definition of $M(i)$. □

Now let us define for all $i \leq n^\star$ and all $x \in \underline{T}^{\downarrow}(i)$,

$$a_+^i(x) = \min_{y:\notin T(x):v(y)>v(x)} v(y)$$
$$a_-^i(x) = \max_{y:\notin T(x):v(y)<v(x)} v(y)$$

Lemma 16. *for all $i \leq n^\star$ $x, y, z \in \underline{T}^{\downarrow}(i)$,*

1. *$a_+^i(x) = a_+^i(y)$ if and only if $a_-^i(x) = a_-(y)$;*
2. *if $u(x) > u(y) > u(z)$ and $v(z) \in]a_-^i(x), a_+^i(x)[$, then $v(y) \in]a_-^i(x), a_+^i(x)[=]a_-^i(y), a_+^i(y)[$.*

Proof. The first claim stems from the fact that $a_+(x) > a_+(y)$ implies that $v(x) > a_+(y)$ so that $a_-(x) \geq a_+(y) > a_-(y)$. Hence, $a_-(x) = a_-(y)$ implies $a_+(x) = a_+(y)$. The reverse implication is similar. The second claim is a direct consequence of lemma 15. □

We now define a function $\hat{v}$ which satisfies the conclusion of lemma 13 and is single- peaked as required by our representation. First, $\hat{v}(x) = v(x)$ for any

$x \in \bigcup_{1\le i\le n^\star}[T(i)\setminus \underline{T}^{\downarrow}(i)]\cup T(0)$. Second, for each $1 \le i \le n^\star$, set $\hat{v}^i$ on $\underline{T}^{\downarrow}(i)$ such that:

$$\hat{v}^i(x) \in]a^i_-(x), a^i_+(x)[\tag{12}$$

$$\hat{v}^i(x) \in [\min_{y\in \underline{T}^{\downarrow}(i):v(y)\in]a^i_-(x),a^i_+(x)[} v(y), \max_{y\in \underline{T}^{\downarrow}(i):v(y)\in]a^i_-(x),a^i_+(x)[} v(y)] \tag{13}$$

$$\hat{v}^i(x) > \hat{v}^i(y) \iff u(x) > u(y) \tag{14}$$

Finally, let $\hat{v}_{|\underline{T}^{\downarrow}(i)} = \hat{v}^i$ for each i.

By construction $\hat{v}$ is single-peaked. It remains to check that for all A, $c(A) = \arg\max_{x\in d(A)} \hat{v}(x)$. To see why this is the case assume that $\arg\max_{x\in d(A)} \hat{v}(x) \neq \arg\max_{x\in d(A)} v(x)$ for some A. This means that there exist $x, y \in A$ with $x \notin T(y)$ such that $v(x) > v(y)$ and $\hat{v}(x) < \hat{v}(y)$. Given lemma 13, this is possible only if $x \in \underline{T}^{\downarrow}(i)$ or $y \in \underline{T}^{\downarrow}(i)$ for some i. Assume that $x \in \underline{T}^{\downarrow}(i)$. Then, this would imply that $v(x) > a^i_-(x) \ge v(y)$. If $y \notin \underline{T}^{\downarrow}(i)$ for all $i \le n^\star$, then $v(x) > a^i_-(x) \ge v(y) = \hat{v}(y)$, while $\hat{v}(x) < \hat{v}(y) \le a^i_-(x)$, in contradiction with (12). If $y \in \underline{T}^{\downarrow}(j)$ for some $j \le n^\star$, then $v(x) \ge a^j_+(y) > v(y)$ and $v(y) \le a_-(x) < v(x)$. Note that $a^i_-(x) \ge a^j_+(y)$ would be in contradiction with (12), hence $a^i_-(x) < a^j_+(y)$. This is possible only if $a^j_+(y) = v(x')$ and $a^i_-(x) = v(y')$ for some $x' \in \underline{T}^{\downarrow}(i)$ and some $y' \in \underline{T}^{\downarrow}(j)$. Note also that $v(x') \in]a^i_-(x), a^i_+(x)[$ and $v(y') \in]a^j_-(y), a^j_+(y)[$. Moreover, it must be that

$$v(x') = \min_{z\in \underline{T}^{\downarrow}(i):v(z)\in]a^i_-(x),a^i_+(x)[} v(z),$$

as otherwise this would contradict the definition of $a_+(y)$. Similarly, it must be that

$$v(y') = \max_{y\in \underline{T}^{\downarrow}(i):v(z)\in]a^i_-(y),a^i_+(y)[} v(z).$$

Hence, by (13) we have $\hat{v}(x) \ge v(x') > v(y') \ge \hat{v}(y)$. A contradiction.

□

B Proof of Proposition 1

We first prove the following corollary of Proposition 4.

Corollary 1. *Let c be an RCR represented by a reactance structure $< \mathcal{T}, F, u, v >$. For any $T \in \mathcal{T}$: $T \subset F \implies T = T_0^c$.*

Proof. $T \subset F$ implies that $\forall x \in T$, $u(x) = v(x)$. Given proposition 4 *(i)*, this implies that for any $y \in X$ $\neg[x\mathbf{R}^c y]$. If, by contradiction, there exists z such that $z\mathbf{R}^c x$, then proposition 4 *(i)* implies again that there exists T' such that $x, z \in T'$, $z \notin F$. Given that $\mathcal{T}$ forms a partition, this means that $T' = T$, and thus $T \backslash F = T' \backslash F \neq \emptyset$. A contradiction. Hence, $T = T_0$. □

Proof of Proposition 1. Assume that c is represented both by $< \mathcal{T}, F, u, v >$ and $< \tilde{\mathcal{T}}, \tilde{F}, \tilde{u}, \tilde{v} >$. We first prove that $\mathcal{T} = \tilde{\mathcal{T}}$. For that purpose, we define the binary relation $\mathbf{E}^c \subset X^2$ as the reflexive extension of $\mathbf{R}^c$, that is: for any $x, y \in X$, $x\mathbf{E}^c y \iff [x\mathbf{R}^c y \vee y\mathbf{R}^c x]$. We simply show that for any $x, y \in X$, $y \in T(x) \neq T_0^c$ if and only if there exists a sequence $(x_k)_{k=0}^{n+1}$ such that $x_0 = x$, $x_{n+1} = y$, and for any $i \in \{0, \ldots, n\}$, $x_i\mathbf{E}^c x_{i+1}$. Or equivalently, we show that the collection of types on $X \setminus T_0^c$ is defined by the collection of the components of the graph generated by the binary relation $\mathbb{E}^c$ on the set $X \setminus T_0^c$.

Only if. Consider $x, y \in T \in \mathcal{T}$ with $T \neq T_0^c$. We show there exists a sequence $(x_k)_{k=0}^{n+1}$ such that $x_0 = x$, $x_{n+1} = y$, and for any $i \in \{0, \ldots, n\}$, $x_i\mathbf{E}^c x_{i+1}$.

By corollary 1, $T \setminus F \neq \emptyset$. First, define $x^\star \equiv \arg\max v(T \setminus F)$. Given that $v \circ u^{-1}$ is single-peaked on $u(T \setminus F)$, for any $s, t \in T \setminus F$, if $u(s) < u(t) \leq u(x^\star)$, then $v(s) < v(t) \leq v(x^\star)$. Hence there exists no z such that $z\mathbf{R}^c s$ (or equivalently $z\mathbf{R}^c t$). Hence, $T \neq T_0^c$ implies that $s\mathbf{R}^{\mathbf{c}} z$ for some $z \in T$ with $u(z) > u(x^\star)$.

Second, define $x_\star = \arg\min v\big(\{x \in T \mid u(x) \geq u(x^\star)\}\big)$. Given the definition of the freedom requirement set F and the fact that $v \circ u^{-1}$ is single-peaked on $u(T \setminus F)$, for any $s, t \in T$, if $u(s) > u(t) \geq u(x_\star)$, then $v(s) > v(t) \geq v(x_\star)$. Hence there exists no z such that $s\mathbf{R}^c z$ (or equivalently $t\mathbf{R}^c z$). Hence, $T \neq T_0^c$ implies that $z\mathbf{R}^{\mathbf{c}} s$ for some $z \in T$ with $u(z) < u(x_\star)$.

Hence, for any $s \in T$, if $s\mathbf{R}^c z$ for some $z \in T$, then $u(s) < u(x_\star)$. Therefore, given the definition of $x_\star$, and the single-peakedness of $v \circ u^{-1}$ on $u(T \setminus F)$, for any such s, this is also the case that $s\mathbf{R}^c x_\star$. Indeed, this means that $u(s) < u(z)$, $u(z) > u(x^\star)$ (from our first point made above) and there exists $t \notin T$ such that $v(s) > v(t) > v(z)$. But note then that $u(x_\star) > u(s)$ and $v(s) > v(t) > v(z) > v(x_\star)$, hence $s\mathbf{R}^c x_\star$.

Finally, there are three cases to consider for x, y. (1) If both $u(x) \geq u(x_\star)$ and $u(y) \geq u(x_\star)$, then there exist s, t such that $s\mathbf{R}^c x$ and $t\mathbf{R}^c y$. Both $s\mathbf{R}^c x_\star$ and $t\mathbf{R}^c x_\star$. Hence $x\mathbf{E}^c s\mathbf{E}^c x_\star \mathbf{E}^c t\mathbf{E}^c y$. (2) If $u(x) \geq u(x_\star) > u(y)$, then there exists s such that $s\mathbf{R}^c x$ and $s\mathbf{R}^c x_\star$. Furthermore, either $y\mathbf{R}^c z$, in which case $y\mathbf{R}^c x_\star$, or $z\mathbf{R}^c y$, in which case $z\mathbf{R}^c x_\star$, for some z. In both cases, we can conclude similarly as in (1). (3) If both $u(x) < u(x_\star)$ and $u(y) < u(x_\star)$, then either $\alpha\mathbf{R}^c z$, in which case $\alpha\mathbf{R}^c x_\star$, or $z\mathbf{R}^c \alpha$, in which case $z\mathbf{R}^c x_\star$, for $\alpha = x, y$, for some z. In any of the four possible combinations, we can similarly conclude.

<u>If</u>. Consider $x, y \in X$ such that there exists a sequence $(x_k)_{k=0}^{n+1}$ such that $x_0 = x$, $x_{n+1} = y$, and for any $i \in \{0, \dots, n\}$, $x_i \mathbf{E}^c x_{i+1}$. By proposition 4, for any $i \in \{0, \dots, n\}$, $x_i \mathbf{E}^c x_{i+1}$ implies that there exists T such that $x_i, x_{i+1} \in T$, from which we conclude that there exists T such that $x, y \in T$.

We now prove that there exists $\hat{u}, \hat{v}$ such that $< \mathcal{T}, F \cup \tilde{F}, \hat{u}, \hat{v} >$ also represents c. First note that given the definition of a freedom requirement set, for any type $T \in \mathcal{T}$, either $F \cap T \subseteq \tilde{F} \cap T$ or $\tilde{F} \cap T \subseteq F \cap T$. Let us suppose w.l.o.g that $\tilde{F} \cap T_0^c \subseteq F \cap T_0^c$. Then, for any T such that $\tilde{F} \cap T \subseteq F \cap T$, we simply define $\hat{u}_{|T} = u_{|T}$ and $\hat{v}_{|T} = v_{|T}$.

Now, let T be such that $F \cap T \subsetneq \tilde{F} \cap T$ and define when they exist $x^\star(T) = \arg\min u(F \cap T)$ and $x_\star(T) = \arg\max u(T \setminus \tilde{F})$. Suppose first that $x_\star(T)$ does not exist, this means that $T \subset \tilde{F}$, in which case, by corollary 1, $T = T_0^c$. But we assumed that $\tilde{F} \cap T_0^c \subseteq F \cap T_0^c$. Hence, $x_\star(T)$ necessarily exists.

Suppose now that $x^\star(T)$ exists. There exists a sequence of options $(x_k)_{k=1}^{n(T)}$ such that $T \cap (\tilde{F} \setminus F) = \{x_1, \dots, x_{n(T)}\}$ and $u(x^\star(T)) > u(x_1) > \dots > u(x_{n(T)}) > u(x_\star(T))$. Note that this must be that $v(x_\star(T)) > v(x_{n(T)}) > \dots > v(x_1) > v(x^\star(T))$. Indeed, suppose by contradiction that $v(x_\star(T)) < v(x_{n(T)})$, then point *(iii)* of theorem 1 implies that for any $x \in T$ such that $u(x) < u(x_\star(T))$, $v(x) < v(x_{n(T)})$. If there exists y such that $x\mathbf{R}^c y$, then this would imply that $x_{n(T)}\mathbf{R}^c y$, which is not possible given proposition 4 and the fact that $x_{n(T)} \in \tilde{F}$. This implies that there exists no y such that $x\mathbf{R}^c y$, which in turn triggers that for any $y \in T$ there exists no z such that $y\mathbf{R}^c z$, from which we obtain that $T = T_0^c$, a contradiction. Hence we conclude that $v(x_\star(T)) > v(x_{n(T)}) > \dots > v(x_1) > v(x^\star(T))$. At the same time, $\tilde{v}(x_\star(T)) < \tilde{v}(x_{n(T)} < \dots < \tilde{v}(x_1) < \tilde{v}(x^\star(T))$. Given that $< \mathcal{T}, F, u, v >$ and $< \tilde{\mathcal{T}}, \tilde{F}, \tilde{u}, \tilde{v} >$ both represent c, this implies that for any $k \in \{1, \dots, n(T)\}$ and any $z \notin T$, $c\{x^\star(T), z\} = x^\star(T) \iff c\{x_k, z\} = x_k$. Let $\nu^T > 0$ be sufficiently small and define $\hat{u}(x_k) = \hat{v}(x_k) = u(x^\star(T)) - k\nu^T$ for every $k \in \{1, \dots, n\}$.

Let us now consider the case where $x^\star(T)$ does not exist. This means that $T \cap F = \emptyset$. There exists similarly a sequence of options $(x_k)_{k=1}^{n(T)}$ such that $T \cap (\tilde{F} \setminus F) = \{x_1, \dots, x_{n(T)}\}$ and $u(x_1) > \cdots > u(x_{n(T)}) > u(x_\star(T))$. Following a similar reasoning, one can show that $v(x_\star(T)) > v(x_{n(T)}) > \cdots > v(x_1)$ while $\tilde{v}(x_\star(T)) < \tilde{v}(x_{n(T)} < \cdots < \tilde{v}(x_1)$. Hence for any $k \in \{2, \dots, n(T)\}$ and any $z \notin T$, $c\{x_1, z\} = x_1 \iff c\{x_k, z\} = x_k$. Let $\nu^T > 0$ be sufficiently small and define $\hat{u}(x_1) = \hat{v}(x_1) = v(x_1)$ and $\hat{u}(x_k) = \hat{v}(x_k) = v(x_1) - (k-1)\nu^T$ for every $k \in \{2, \dots, n\}$.

In both cases, finally define $\hat{u}_{|T \setminus (\tilde{F} \setminus F)} = u_{|T \setminus (\tilde{F} \setminus F)}$ and $\hat{v}_{|T \setminus (\tilde{F} \setminus F)} = v_{|T \setminus (\tilde{F} \setminus F)}$. Repeat these operations for any T such that $F \cap T \subsetneq \tilde{F} \cap T$. One can easily check that by appropriately choosing the ν^Ts, $\hat{u}, \hat{v}$ so defined are such that the reactance structure $< \mathcal{T}, F \cup \tilde{F}, \hat{u}, \hat{v} >$ represents c. □

C Proof of Proposition 2

Proof of the necessity. Let $< \mathcal{T}, F, u, v >$ be a a maximal reactance structure that represents c. The fact that $T_0^c \subset F$ directly follows from our proof of theorem 1. Let $T \neq T_0^c$, $x \in T$ and define $x^{T,F}$ as in the proposition. Note that $T \not\subset F$, as otherwise, by proposition 4, it would be the case that $T = T_0^c$. Hence there exist a sequence of options $(x_k)_{k=1}^n$ such that $T \setminus F = \{x_1, \dots, x_n\}$ and $u(x^{T,F}) > u(x_1) > \cdots > u(x_n)$.

First, this must be that $v(x_1) > u(x^{T,F})$, as otherwise, one could redefine $\hat{u}$ by $\hat{u}(x_1) = v(x_1)$, $\hat{u} = u$ elsewhere, and $\hat{F} = F \cup \{x_1\}$ such that $< \mathcal{T}, \hat{F}, \hat{u}, v >$ also represents c, which contradicts that $< \mathcal{T}, F, u, v >$ is maximal.

Second, there exists $k^\star \in \{1, \dots, n\}$ such that $v(x_{k^\star}) = \max v(T \setminus F)$. Given that $v \circ u^{-1}$ is single-peaked on $u(T \setminus F)$, for any $k' < k < k^\star$, $v(x_{k'}) < v(x_k) < v(x_{k^\star})$. Hence, it is sufficient to show that $x_1 \mathbf{R}^c x^{T,F}$. Suppose by contradiction that it is not the case. This means that for any $z \notin T$, $c\{x^{T,F}, z\} = x^{T,F} \implies c\{x_1, z\} = x_1$. But $v(x_1) > u(x^{T,F})$ implies that $c\{x_1, z\} = x_1 \implies c\{x^{T,F}, z\} =$

$x^{T,F}$. A contradiction.

Third, the single-peakness of $v \circ u^{-1}$ on $u(T \setminus F)$ implies that for any $k' > k > k^\star$, $v(x_{k'}) < v(x_k) < v(x_{k^\star})$. Hence, let $k \geq k^\star$, by proposition 4, there exists no x such that $x\mathbf{R}^c x_k$. This implies that there exists $x \in T$, with $u(x) > u(x_{k^\star})$, such that $x_k\mathbf{R}^c x$, as otherwise, $x \in T_0^c$ a contradiction. In any case we have $v(x_k) > v(x) > v(x^{T,F})$. Indeed, if $x = x_{\hat{k}}$ for some $\hat{k} < k^\star$, then $v(x_{\hat{k}}) > v(x_1) > v(x^{T,F})$ by the previous step. If $x \in F$, then $v(x) = u(x) > u(x^{T,F}) = v(x^{T,F})$, by definition of $x^{T,F}$. But applying the same reasoning as for x_1, it can be shown that $x_k\mathbf{R}^c x^{T,F}$. This concludes the proof of *(i)*.

Now let us consider $x \in T$ such that $u(x) > u(x^{T,F})$. Hence, $x \in F$, therefore, there exists no y such that $x\mathbf{R}^c y$ (by proposition 4). Given that $T \neq T_0^c$, there must exist $y \in T$ such that $y\mathbf{R^c}x$. Again by proposition 4, this can be possible only if $u(y) < u(x)$ and $v(y) > v(x)$, which is possible only if $u(y) < u(x^{T,F})$. Point *(i)* implies that $y\mathbf{R}^c x^{T,F}$, which ends the proof of *(ii)*. □

Proof of the sufficiency. Let $< \mathcal{T}, F, u, v >$ be a reactance structure that represents the choice function c and that satisfies the conditions stated in the proposition. Suppose by contradiction that it is not maximal. Therefore, there exists $T \neq T_0^c$ such that $T \setminus F \neq \emptyset$, and there exists $< \tilde{\mathcal{T}}, \tilde{F}, \tilde{u}, \tilde{v} >$ that represents c with $\tilde{F} \supset F$ and $F \cap T \subsetneq \tilde{F} \cap T$. But this contradicts the fact that for any x with $u(x) < u(x^{T,F})$, i.e. $x \in T \setminus F$, $x\mathbf{R}^c x^{T,F}$. □

D Proof of Proposition 3

Proof. The proof of *(i)* directly follows from the proof of theorem 1. The proof of the *if* part of *(ii)* is not complicated and thus left to the readers. We only prove the *only if* part.

The fact that f must be increasing on $u(T)$ for every $T \in \mathcal{T}$ simply follows from the fact that the function u represents the binary choices within each type. $f_{|u(F)} = g_{|u(F)}$ is a direct consequence of the requirement that utility and the reactance functions be equal on the freedom requirement set.

We now prove that g must be increasing on $v(X)$. Suppose by contradiction that there exists $x, y \in X$ such that $v(x) > v(y)$ but $g \circ v(x) \leq g \circ v(y)$. Note that there must exist a type, denote it T, such that $x, y \in T$, as otherwise $v(x) >$

$v(y) \implies x = c\{x,y\}$, which cannot be accommodated by $< \mathcal{T}, F, f \circ u, g \circ v >$ if $g \circ v(x) \leq g \circ v(y)$. Define $x^\star = \arg\max v(T \setminus F)$ and $x^{T,F}$ as in proposition 2.

(1) Consider the case where $u(x) > u(y)$. If $u(y) \geq u(x^{T,F})$, this would mean that $x, y \in F$, in which case, given that $u_{|F} = v_{|F}$, $f_{|u(F)} = g_{|u(F)}$ and f is increasing on $u(T)$, it is impossible that $g \circ v(x) \leq g \circ v(y)$. If $u(y) \leq u(x^{T,F})$, it means that $y \notin F$. The fact that $v(x) > v(y)$ implies that there exists $z \notin T$ such that $x = c\{x,z\}$ while $z = c\{y,z\}$, that is $v(x) > v(z) > v(y)$. Hence, $g \circ v(x) \leq g \circ v(y)$ cannot accommodate these choices.

(2) Consider the case where $u(x) < u(y)$. Then necessarily $u(x) < u(x^{T,F})$, that is $x \notin F$. Given that $v(x) > v(y)$, there exists no $z \notin T$ such that $z = c\{x,z\}$ while $y = c\{y,z\}$. Conversely, if there exists $z \notin T$ such that $x = c\{x,z\}$ while $z = c\{y,z\}$, that is $v(y) > v(z) > v(x)$, then again $g \circ v(x) \leq g \circ v(y)$ cannot accommodate these choices. If it is note the case, that is for every $z \notin T$ such that $x = c\{x,z\} \iff y = c\{y,z\}$, then $g \circ v(x) \leq g \circ v(y)$ does not satisfy the requirement of reactance structure*. □

E Proof of Theorem 2

Proof. The necessity part of the theorem is left to the readers. We only prove the sufficiency.

(a) We first show that for any A, B such that $A \subseteq T$ and $B \subseteq T'$ for some $T, T' \in \mathcal{T}$, $A \succ B \iff A \cap F \neq \emptyset = B \cap F$. If $T = T'$, this is simply a consequence of part *(i)* of R-Dominance (RD).

Suppose now that $T \neq T'$. Let denote $A' = A \setminus F = \{a_1, \ldots, a_n\}$ and $B' = B \setminus F = \{b_1, \ldots, b_l\}$ and suppose that both are non-empty. By RD, $\{a_1\} \sim A'$, because both are richer than each other. Similarly $\{b_1\} \sim B'$. Furthermore, RD *(ii)* implies that $\{a_1\} \sim \{b_1\}$; hence, by transitivity, $A' \sim B'$.

Let denote $A'' = A \setminus A'$ and $B'' = B \setminus B'$. If $A'' = B'' = \emptyset$, we conclude from the previous argument that $A \sim B$. Suppose that $A'' \neq \emptyset = B''$, so $B = B'$. By a simple application of RD *(i)*, A is strictly richer than A', so $A \succ A'$, and by transitivity, $A \succ B$.

Assume now that $B'' \neq \emptyset$. By a similar reasoning as for A' and B', one can easily show that $A'' \sim B''$. If $B' = \emptyset$, then $B = B''$, hence $A \sim B$. If $B' \neq \emptyset$, note that $A' \cap A'' = B' \cap B'' = \emptyset$ and neither A' is richer than A'' nor B' is richer than B''. Hence applying twice R-Composition (RC), we obtain that $A \sim B$.

(b) We next show that for any A, B, if $\#\Phi(A) = \#\Phi(B)$, then $A \sim B$. Denote $\Phi(A) = \{A_1, \ldots, A_n\}$ and $\Phi(B) = \{B_1, \ldots, B_n\}$. By (a), we know that for any i, $A_i \sim B_i$. Noting that $A_1 \cap A_2 = B_1 \cap B_2 = \emptyset$, and neither A_1 is richer than A_2 nor B_1 is richer than B_2, by applying twice RC, we get that $A_1 \cup A_2 \sim B_1 \cup B_2$. By reiterating the same argument, we obtain that $\bigcup_i A_i \sim \bigcup_i B_i$. Finally, note that A is richer that $\bigcup_i A_i$ and conversely $\bigcup_i A_i$ is richer than A, hence, by RD, $A \sim \bigcup_i A_i$; similarly $B \sim \bigcup_i B_i$. Therefore, by transitivity, we obtain that $A \sim B$.

(c) We finally prove that for any A, B, if $\#\Phi(A) > \#\Phi(B)$, then $A \succ B$. Denote $\Phi(A) = \{A_1, \ldots, A_n\}$ and $\Phi(B) = \{B_1, \ldots, B_k\}$, with $k < n$. By (b) $\bigcup_{i=1}^k A_i \sim B$. Furthermore, by RD, $\bigcup_{i=1}^n A_i \succ \bigcup_{i=1}^k A_i$. A similar argument as before shows that $A \sim \bigcup_{i=1}^n A_i$ and $B \sim \bigcup_{i=1}^k B_i$. Hence by transitivity $A \succ B$.

□

F Proofs of Section 4

Proof of Proposition 5. (i) Denote $u(\sigma^L)$ and $v(\sigma^{RR})$ the DM's anticipated utility from choosing respectively σ^L and σ^{RR} in the menu N:

$$u(\sigma^L) \leq v(\sigma^{RR}) \iff (1-p) + p\lambda - p(1-\lambda) \leq p + (1-p)\delta$$
$$\iff p \geq \frac{1-\delta}{3-2\lambda-\delta} = \frac{1/2}{5/2-2\lambda}$$

We define $p^\star := \frac{1/2}{5/2-2\lambda}$ and verify that $p^\star < 1/2$:

$$p^\star < 1/2 \iff \lambda < \frac{3}{4}$$

which is true by assumption.

(ii). We first compute the value $q^\star$ of the posterior such that for any $q \geq q^\star$, action r is preferred. $q^\star$ solves $(1-q) - q = q$, hence $q^\star = 1/3$.

Then we simply compare the posterior obtained after the realisation of a signal s^r from the news source σ^{RR} with $1/3$. The posterior is, $\frac{p}{p+(1-p)1/2}$, which is greater or equal than p. We are in the case where $p \geq p^\star$, hence it is sufficient to show that $p^\star \geq 1/3$: $p^\star \geq \frac{1}{3} \iff \lambda \geq \delta$ which is true by assumption. $\square$

Proof of Lemma 1. The maximand of the program (4) is strictly concave and the set K_g is compact. Hence, C is well-defined (Weierstrass theorem) and is a choice function.

Now let us build the reactance structure $< \mathcal{T}, F, u, v >$ that represents C. Given (5), $d^\star$ strictly increases with g if and only if $g > \hat{g}$. Let $g(t,d)$ be the g such that $t + gd^\beta = 1$.

Let us introduce the three following sets:

$$D^\uparrow = \bigcup_{\substack{t \in [0,1] \\ g > \hat{g}}} \{d \in [0,1] : (t,d) = C(K_g)\},$$
$$\forall d \in D^\uparrow, T(d) = \bigcup_{t \in [0,1]} \{(t,d)\},$$
$$T_0 = \bigcup_{\substack{d \notin D^\uparrow \\ t \in [0,1]}} \{(t,d)\}.$$

From these sets we can define the set of types and the freedom set

$$\mathcal{T} = \{T_0\} \cup \bigcup_{d \in D^\uparrow} \{T(d)\} \text{ and } F = T_0 \cup \bigcup_{d \in D^\uparrow} \{(t,d) \in T(d) : g(t,d) \leq \hat{g}\}$$

Now let us define u and v. For each (t,d) we posit $u(t,d) = t + P(d)\hat{V}$ and $v(t,d) = t + P(d)V\big(g(t,d)\big)$.

Given the uniqueness of $g(t,d)$ for each (t,d), v is well-defined. It can easily be shown that $< \mathcal{T}, F, u, v >$ is a reactance structure. Consider the choice function C' which is the RCR defined on the compact subsets of $[0,1]^2$ and represented by $< \mathcal{T}, F, u, v >$. We claim that for all g, $C(K_g) = C'(K_g)$. To check this claim let (t,d) and g such that $(t,d) = C(K_g)$ and (t',d') such that $(t',d') = C'(K_g)$. Note that $(t,d) = C(K_g)$ implies $g = g(t,d)$. Similarly, $(t',d') = C'(K_g)$ implies that $u(t',d') \geq u(t'',d')$ for all t'' such that $(t'',d') \in K_g$, that is, for all $t'' \leq t'$. Hence, $g = g(t',d')$.

Assume first that $g \leq \hat{g}$. Then, note that $(t',d') \in F$. Suppose that there exists $(t'',d'') \in K_g \setminus F$, this means that $g(t'',d'') > \hat{g}$, and hence there exists

$t''' > t''$, such that $g(t''', d'') = g$, which implies $(t''', d'') \in K_g$ and $u(t''', d'') > u(t'', d'')$. Therefore, only elements in F can be considered for choices in K_g according to the choice procedure (1). Hence, both (t, d) and (t', d') are elements of $\arg\max u(K_g)$. Because the latter is a singleton, $(t, d) = (t', d')$.

Assume next that $g(t, d) > \hat{g}$. Suppose that $(t', d') \in F$, then this implies that $d' \notin D^{\uparrow}$, that is $d' \neq d$. Because, $g(t, d) = g(t', d')$, this in turn implies that $t' \neq t$. Furthermore, by definition, $(t, d) \notin F$, and from $(t', d') = C'(K_g)$, we deduce that $u(t', d') > v(t, d)$. We also know that $v(t', d') \geq u(t', d')$, so $v(t', d') > v(t, d)$, which contradicts that $(t, d) = C(K_g)$. Therefore, $(t', d') \notin F$. Because $(t, d) \notin F$, $(t', d') = C'(K_g)$ and $(t, d) = C(K_g)$ imply that $v(t', d') \geq v(t, d) = \max v(K_g)$. Therefore, $(t', d') \in \arg\max v(K_g)$, and given that this set is a singleton, this implies that $(t', d') = (t, d)$. □

Proof of Proposition 6. This is a straightforward consequence of (6). □

Proof of Proposition 7. Action a^L can only be implemented in the absence of both a^R and a^{LL}. In any case, if a^L is chosen in a menu M by the agent, it is chosen in both states L and R, which gives the principal the expected payoff:

$$(1 - p)\pi_L(a^L) + p\pi_R(a^L). \tag{15}$$

Similarly, action a^R can only be implemented in the absence of a^{RR}, in which case it is chosen in both states L and R, giving the principal the expected payoff:

$$(1 - p)\pi_L(a^R) + p\pi_R(a^R). \tag{16}$$

From this we can deduce the existence of $p_\star \in (0, 1)$ and $p^\star \in (0, 1)$ such that: for any $p < p_\star$, the principal strictly prefers a menu M (e.g. $\{a^L\}$) such that $a_\theta(M) = a^L$ for $\theta = L, R$; for any $p > p^\star$, the principal strictly prefers a menu M (e.g. $\{a^R\}$) such that $a_\theta(M) = a^R$ for $\theta = L, R$. Furthermore, there exists $\hat{p}$, the unique belief such that (15) = (16).

Only actions a^{LL} and a^{RR} can be simultaneously implemented respectively in state L and R. Given that $\pi_L(a^{LL}) > \pi_L(a^{RR})$ and $\pi_R(a^{RR}) > \pi_R(a^{LL})$, the principal will always prefer a menu implementing both actions (e.g. $\{a^{LL}, a^{RR}\}$) than a menu implementing only one of them. In this, the principal's expected payoff is:

$$(1-p)\pi_L(a^{LL}) + p\pi_R(a^{RR}). \tag{17}$$

Hence there exist $\underline{p}$ and $\bar{p}$ such that: (15) $\geq$ (17) if and only if $p \leq \underline{p}$; and (16) $\geq$ (17) if and only if $p \geq \bar{p}$.

The conclusions of the proposition follows easily from these observations.

□

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