Reactance: a Freedom-Based Theory of Choice

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- Agents' propensity to reverse their choice as a reaction to a threat to their freedom.

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  - $\mathcal{X} = 2^X \setminus \varnothing$ the collection of non-empty subsets of X, that is, the **menus**.
  - A **choice function** $c : \mathcal{X} \longrightarrow X$ associates to each menu the option chosen by the DM in this menu. Namely, for any menu A, $c(A) \in A$.

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- Derive preferences over menus (not today).
- Three applications: formation of conspiracy theories (today), backlash of integration policies targeted towards minority, principal-agent's delegation problem (not today).

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The **removal** of y entails **reactance** and the DM chooses x as a way to **restore** this threatened freedom.

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3. In a menu A, the DM first retains the best options from each type according to her **welfare criterion** u ($u(y) > u(x)$), forming the set $d(A)$:

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4. Finally, the DM chooses in $d(A)$ the best option according to her welfare u + a boost v, the **reactance function**, with $v(F) = \{0\}$ and $u(y) < u(z) < u(x) + v(x)$, so that,

$$c\{x, y, z\} = z \quad ; \quad c\{x, z\} = x.$$

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$$c(A) = \arg\max_{x \in d(A)} v(x) + u(x), \text{ where } d(A) = \bigcup_{T \in \mathcal{T}} \arg\max_{x \in T \cap A} u(x); \quad (1)$$

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(v) for any $T \in \mathcal{T}$, $(u + v) \circ u^{-1}$ is single-peaked on $u(T \setminus F)$. Curves

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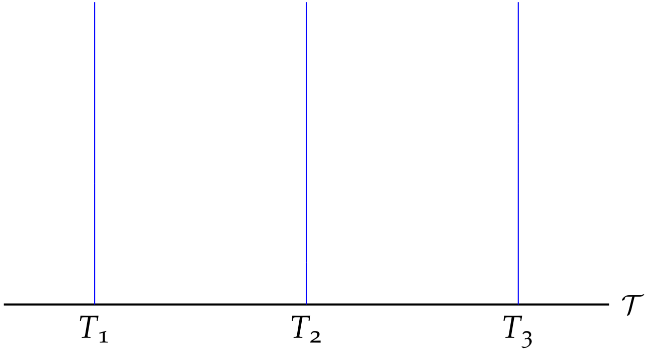

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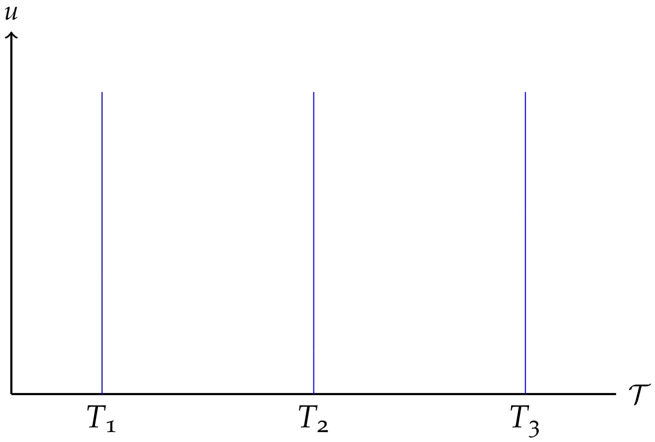

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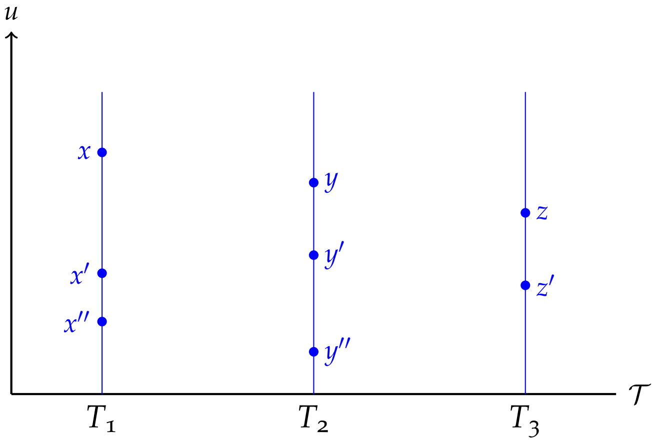

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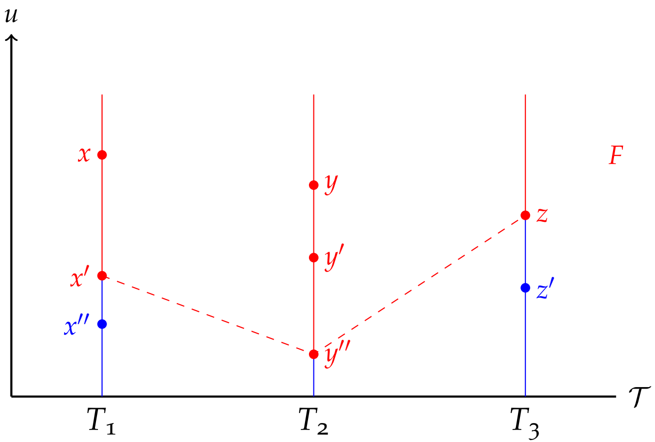

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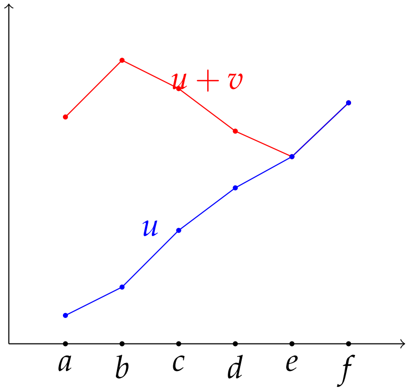

Reactance and Utility Functions in One Type.

Prop 3

Expansion & Revealed Reactance

Expansion. For any $x \in X$, $A, B \in \mathcal{X}$, if $x = c(A) = c(B)$, then $x = c(A \cup B)$.

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  $\Longrightarrow$ we write $x\mathbf{P}^c y$ for x ***potentially reacts to the absence of*** y.

Characterization

In addition to **Expansion**, we use three axioms, all structuring $\mathbf{R}^c$ and $\mathbf{P}^c$.

- Transitivity of $\mathbf{R}^c$, non-$\mathbf{R}^c$, and non-$\mathbf{P}^c$. (**Reactance Transitivity**)
- A form a consistency over the options for which no *reactance-driven choice* is observed (which are revealed to be in the freedom requirement set). (**Reactance Consistency**)
- A form of consistency over the options which reacts to common options. (**Reactance Monotonicity**)

Reactance Transitivity.

For any $x, y, z \in X$, ***(i)*** if $x\mathbf{R}^c y$ and $y\mathbf{R}^c z$, then $x\mathbf{R}^c z$, ***(ii)*** let $y = c\{x, y\}$, $z = c\{y, z\} = c\{x, z\}$: if $\neg[x\mathbf{R}^c y]$ and $\neg[y\mathbf{R}^c z]$, then $\neg[x\mathbf{R}^c z]$; if $\neg[x\mathbf{P}^c y]$ and $\neg[y\mathbf{P}^c z]$, then $\neg[x\mathbf{P}^c z]$.

Reactance Consistency.

For any $x, y, z \in X$, if $z = c\{y, z\}$, there exists no t such that $y\mathbf{R}^c t$ or $z\mathbf{R}^c t$, and $x\mathbf{R}^c y$, $x\mathbf{R}^c z$, then for any $u \in X$: ***(i)*** $u\mathbf{R}^c z \implies u\mathbf{R}^c y$; ***(ii)*** $u\mathbf{R}^c y \implies z = c\{u, z\}$.

Reactance Monotonicity.

For any $x, y, z \in X$, such that $z = c\{y, z\}$, $y = c\{x, y\}$: ***(i)*** if $x\mathbf{R}^c t$ and $z\mathbf{R}^c t$ for some $t \in X$, then $[x\mathbf{R}^c y \implies y\mathbf{P}^c z]$; ***(ii)*** if $x\mathbf{P}^c z$, then $[x\mathbf{P}^c y \implies y\mathbf{P}^c z]$.

Main Results

Theorem 1

c satisfies Expansion, Reactance-Transitivity, Reactance-Consistency and Reactance-Monotonicity if and only if there exist a reactance $< \mathcal{T}, F, u, v >$ *that represents c.*

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Proposition 2

If c is represented by $< \mathcal{T}, F, u, v >$ *and* $< \mathcal{T}', F', u', v' >$*, then:*

1. $\mathcal{T} = \mathcal{T}'$;
2. *there exist* u'', v'' *such that* $< \mathcal{T}, F \cup F', u'', v'' >$ *represents c.*

Reactance in the Representation

Proposition 3

Let c be an RCR represented by the reactance structure $\mathcal{S} =< \mathcal{T}, F, u, v >$. *For any* $x, y \in X$:

(i) *if* $x\mathbf{R^c}y$, *then there exists* $T \in \mathcal{T}$ *such that* $x, y \in T$, $x \notin F$, $u(x) < u(y)$ *and* $u(x) + v(x) > u(y) + v(y)$;

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(ii) *if $\mathcal{S}$ is maximal and there exists $T \in \mathcal{T}$ such that $x, y \in T \setminus F$, $u(x) < u(y)$ and $u(x) + v(x) > u(y) + v(y)$, then $x\mathbf{P}^{c}y$.*

Curves

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The model

- 2 states of the world: L or R
- 2 actions: l or r
- Payoffs : $u_r^R = u_l^L = 1$, $u_l^R = u_r^L = -1$.
- $p \in (0, 1/2]$: prior belief that the state is R.

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- $p \in (0, 1/2]$: prior belief that the state is R.
- Before choosing his action, the DM acquires information by allocating his attention across 4 biased sources of information (e.g newspapers).

- The sources are represented by statistical experiments.
  - The L-biased ones, denoted σ^{LL} and σ^{L}.
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- $\{\sigma^{LL}, \sigma^{L}\}$ and $\{\sigma^{RR}, \sigma^{R}\}$ each represents a **type**.
- The DM's demands of freedom are satisfied when the moderate sources are available, that is, his **freedom requirement set** is $F = \{\sigma^{L}, \sigma^{R}\}$.

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Observation 1 (See Che and Mierendorff, 2019)

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Proposition 4

There exists $p^\star < 1/2$ such that if $p \in [p^\star, 1/2]$:

(i) The DM prefers σ^{RR} to σ^L in menu N;

(ii) After a realisation of signal s^R from σ^{RR}, the DM chooses action r.

Thank You!