Should we increase or decrease public debt? Optimal fiscal policy in heterogeneous agents

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EEA, Milano, August 2022

Introduction

- **new public spending shocks** should be expected (climate policy, health, military spending)
- **How should we finance them?** : Capital tax, progressive tax, public debt?

Old public finance question : heterogeneous-agent models seem the perfect tool.

- Realistic amount of inequality to consider redistributive issue.
- A relevant fiscal system to quantify distortions.
- Non-Ricardian environment : Public debt "well-defined".

Yet...

Many new normative results can be expected in this environment.

However, many questions...

1. Is capital tax positive ? (Chien, Chien, Wen, Yang, 2021)
2. Does the steady-state exist? (Straub-Werning 2020; Auclert and Rognlie, 2022).
3. Is public debt well defined ? (critics of Aiyagari and Mc Grattan 1998; Bhandari et al, 2017).

Actually, less work on optimal fiscal policy than on optimal monetary policy, whereas more important (Martin-Baillon, Le Grand Ragot, 2022).

What we do

Study optimal (Ramsey) fiscal policy in heterogeneous-agent model with capital, and with aggregate shocks and capital tax, non-linear labor tax (HSV), public debt. We prove that the equilibrium is well defined.

- capital tax and public debt are generally positive (Woodford, 1990)
- steady-state is stable

Main new result: After a positive public spending shock (given NPV)

- Public debt should **increase** if the persistence is low ("Keynesian")
- Public debt should **decrease**, front-load with taxes, if the persistence is high ("Classical")

Intuition : high persistence, you have to pay both interest payment on public debt and public spending.

Consistent with US data.

Other literature on Optimal policy in HA model

1. Linear-quadratic approach Woodford, 2003; Bilbiie 2008, Bilbiie and Ragot, 2021; Mckay and Wolf, 2022
2. Transitions Dyrda and Pedroni (2021)
3. Continuous-time techniques (Achdou et al 2022; Nuno and Thomas, 2022 among others)
4. Primal Approach + time-varying perturbations (Bhandari, et al. 2022)
5. Lagrangian approach + truncation (Legrand Ragot, 2022a, 2022b, (see also Acikgoz et al 2021).

Outline of the presentation

1. The Simple Model
2. General model
3. US data

1 - The Simple Model

As Woodford (1990) and

1. GHH utility function

$$U(c,l) = \log\left(c - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}\right),$$

2. Two types of agents, alterning (deterministically) between productivve and unproductive states.
   - Agents A Earn wage w_t in even periods **employed**, (nothing odd periods, **unemployed**).
   - Agents B Earn wage w_t in odd periods (nothing even periods).
3. All agents face credit constraints $a_t \geq -\bar{a}$
4. save in a risless asset (capital or public debt)
5. Standard production sector

$$Y_t = F(K_{t-1}, L_t) = K_{t-1}^{\alpha} L_t^{1-\alpha} - \delta K_{t-1},$$

Tax system

Three instruments: linear labor tax: τ_t^L; Linear capital tax: τ_t^K; Public debt B_t

$$G_t + (1+\tilde{r}_t)B_{t-1} = \tau_t^L \tilde{w} L_t + \tau_t^K \tilde{r}_t (B_{t-1} + K_{t-1}) + B_t.$$

Market equilibrium

$$L_t = l_{e,t} \text{ and } B_t + K_t = a_{e,t} + a_{u,t}. \tag{1}$$

The utilitarian planner objective:

$$W_0 = \sum_{t=0}^{\infty} \beta^t \left[\log\left(c_t^u\right) + \log\left(c_t^e - \chi^{-1} \frac{l_{e,t}^{1+1/\varphi} d}{1+1/\varphi} \right) \right], \tag{2}$$

Program

We show that unemployed agents are credit constraint, solve in post-tax price (Chamley, 1986). Define $R_t = 1 + (1 - \tau_t^K)\tilde{r}$ and $w_t = (1 - \tau_t^L)\tilde{w}$

$$\max_{(B_t, w_t, R_t)_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left((1+\beta) \log \left(\frac{1}{1+\beta} \frac{w_t(\chi w_t)^{\varphi}}{\varphi + 1} \right) + \log\left(\beta R_t\right) \right),$$

$$\text{s.t. } G_t + B_{t-1} + (R_t - 1)\frac{\beta}{1+\beta}\frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi} + w_t(\chi w_t)^{\varphi} = F\left(\frac{\beta}{1+\beta}\frac{w_{t-1}(\chi w_{t-1})^{\varphi}}{1+\varphi} - B_{t-1}, (\chi w_t)^{\varphi}\right) + B_t,$$

Some Issues:

- Kuhn-Tucker, non linear-constraints : Qualificaiton of the constraints.
- Second-order conditions : Gobal concavity.
- Local stability of the equilibrium

Result; Steady State

Three thresholds $\overline{G}_1, \overline{G}_{SW}$ and $\overline{G}_{La}$,

Proposition

When $\overline{G}_1 \leq G$, $G \leq \overline{G}_{SW}$, $G \leq \overline{G}_{pos}$,and $G < \overline{G}_{La}$, there exists a steady-state equilibrium, where B, τ^L and τ^K are positive.

Key equations :

- Modified Golden Rule (Aigarti, 1995), with $B = S^{private} - K$ with

$$F(K, L) = \frac{1}{\beta} - 1$$

- Tradeoff between taxes

$$\tau^K = \varphi \frac{1+\beta}{1-\beta} \frac{\tau^L}{1-\tau^L}$$

Optimal Dynamics

Analytical results after a MIT public spending shock

$$\widehat{G}_t = \begin{cases} \sigma_G & \text{if } t = 0, \\ \rho_G \widehat{G}_{t-1} & \text{if } t > 0, \end{cases}$$

Then $\widehat{K}_t = \rho_K \widehat{K}_{t-1} + \sigma_K \widehat{G}_t$

Proposition

Denoting by $\widehat{B}_0$ the public debt variation on impact, we have:

$$\frac{\partial \widehat{B}_0}{\partial \rho_G} < 0.$$

Optimal Dynamics

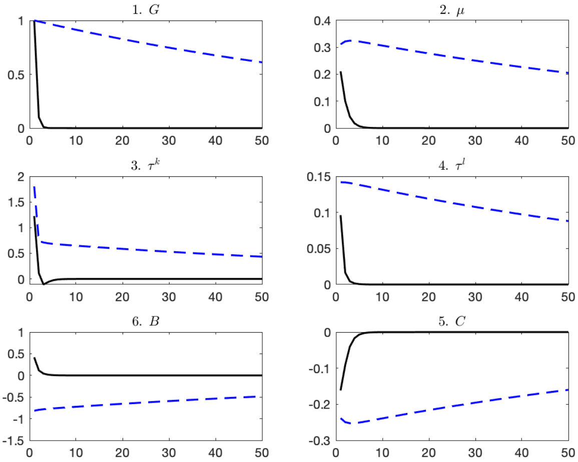

Figure: Fiscal variables for two persistence $\rho_G = 0.1$ (black line) and $\rho_G = 0.99$ (blue dashed line). Parameters are $\alpha = 0.4, \beta = 0.97, \varphi = .5, G = 0.05$.

2 - General model

1. Utility function $u(c) - v(l)$, (Chetty et al., 2011)
2. General income process, first-order Markov chain, (Mitman, Krueger, perri, 2018)
3. Fiscal system : labor tax HSV (Heathcote et al. 2017):
   $T_t(\tilde{w}yl) := \tilde{w}yl - \kappa_t(\tilde{w}yl)^{1-\tau_t}$

The Program

$$\max_{\left(r_t, w_t, B_t, K_t, L_t, (a_t^i, c_t^i, l_t^i, \nu_t^i)_i\right)_{t\geq 0}} \sum_{t=0}^{\infty} \beta^t \int_i \omega\left(y_t^i\right)\left(u(c_t^i) - v(l_t^i)\right)\ell(di),$$

$$G_t + R_t B_{t-1} + (R_t - 1)K_{t-1} + w_t \int_i (y_t^i l_t^i)^{1-\tau_t} \ell(di) = F(K_{t-1}, L_t) + B_t$$

for all $i \in \mathcal{I}$: $a_t^i + c_t^i = R_t a_{t-1}^i + w_t (y_t^i l_t^i)^{1-\tau_t}$,

$$a_t^i \geq -\bar{a},\ \nu_t^i(a_t^i + \bar{a}) = 0,\ \nu_t^i \geq 0,$$

$$U_c(c_t^i, l_t^i) = \beta \mathbb{E}_t\left[R_t U_c(c_{t+1}^i, l_{t+1}^i)\right] + \nu_t^i,$$

$$-U_l(c_t^i, l_t^i) = (1-\tau_t) w_t y_t^i (y_t^i l_t^i)^{-\tau_t} U_c(c_t^i, l_t^i),$$

$$K_t + B_t = \int_i a_t^i \ell(di),\ L_t = \int_i y_t^i l_t^i \ell\left(di\right),$$

Parameter values

Parameter	Description	Value
	Preference and technology	
β	Discount factor	0.99
α	Capital share	0.36
δ	Depreciation rate	0.025
$\bar{a}$	Credit limit	0
χ	Scaling param. labor supply	0.05
φ	Frisch elasticity labor supply	0.5
	Shock process	
ρ_y	Autocorrelation idio. income	0.993
σ_y	Standard dev. idio. income	0.082
	Tax system	
τ^K	Capital tax	36%
κ	Scaling of Labor tax	0.75
τ	Progressivity of tax	0.181

The model is solved using the truncation method, estimating Pareto weight, using the specification of Heathcote and Tsujiyama, 2021.

Dynamics (1/2)

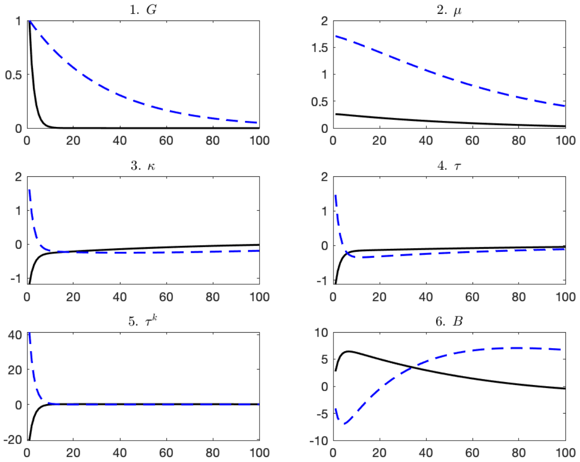

Figure: The black solid line is for the persistence $\rho_G = 0.6$. The blue dashed line is for persistence $\rho_G = 0.97$.

Dynamics (2/2)

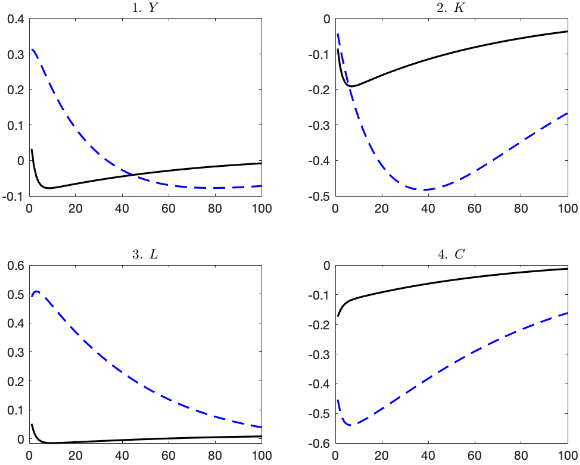

Figure: The blue dashed line is for persistence $\rho_G = 0.97$. All variables are in proportional deviations.

3 - US data

Using data of Ramey and Zubairy, 2018 : shocks and path of public spending.

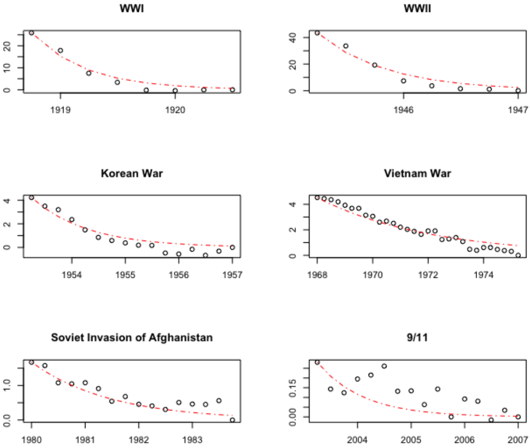

Figure: The blue dashed line is for persistence $\rho_G = 0.97$. All variables are in

Change in debt

Event	Quart. Pers.(%)	Dates		ΔDebt/G$_{NPV}$(%)
		Beg.	End	
WWI	59	1914:Q3	1920:Q3	7.0
WWII	66	1939:Q3	1947:Q1	6.7
9/11	74	2001:Q3	2007:Q1	1.1
Korean War	78	1950:Q3	1957:Q1	-3.7
Soviet Inv. of Afg.	84	1980:q1	1983:Q4	2.2
Vietnam War	94	1965:Q1	1975:Q2	-1.5

Table: Estimated persistence of public spending in percent for the six events, in increasing order and change in public debt divided by the net present value of public spending.

Conclusion

- Can be either procyclical or countercyclical depending on persistence.
- Tax and progressitivity can be procyclical or countercyclical depending on persistence.
- Public debt is slow moving
- Consider mny other frictions.