Regret-Free Truth-Telling in School Choice with Consent

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Motivation

Two desirable properties in school choice:

→ Stability

→ Pareto Efficiency

Impossibility Result (Balinski and Sönmez, 1999):
No assignment procedure always chooses a matching that is both stable and efficient.

Problem: Proposed solutions which address the incompatibility are not strategy-proof.

This Paper

- We study the incentive properties of mechanisms that address the efficiency and stability trade-off.
- Motivated by two key features of school assignment, we follow a regret-based approach:
  - *Incomplete information*: Students information on other students' scores, preferences and reports is often incomplete (privacy protection).
  - *Feedback*: Students observe
    1. Features of the Final Matching
    2. Cutoffs
- The incentive criterion is regret-free truth-telling (Fernandez, 2020)—which is weaker than strategy-proofness.

Candidate Mechanisms

- The Efficiency Adjusted Deferred Acceptance Matching Rule (EDA):
  1. Asks for students' consent for being exposed to justified envy.
  2. Improves efficiency on a stable matching only with students' consent.
  3. Provides no incentive to refuse a consent.
- Stable Dominating Matching Rules without Consent Option:
  1. Do <u>not</u> ask for students' consent for being exposed to justified envy.
  2. Students complaining about justified envy can be offered a remedy (a stable matching) that makes herself and all others weakly worse off.

Main Results

1. EDA is regret-free truth-telling.
2. No stable dominating rule which is efficient is regret-free truth-telling.
3. There exist stable dominating rules without consent option which are regret-free truth-telling and not equivalent to DA.

Primitives and Preferences

- Finite set of students I. [fixed]
- Finite set of schools S, where each school $s \in S$ has a capacity of seats q_s. [fixed]
- We collect all capacities in $q = (q_s)_{s \in S}$.
- Outside option $\emptyset$ with infinite capacity of seats.

- For each student $i \in I$, let P_i be a strict preference ranking over $S \cup \{\emptyset\}$ (with domain $\mathcal{P}$ and corresponding weak relation R_i)
- $P = (P_i)_{i \in I}$ is a preference profile for all students (domain $\mathcal{P}_I$).
- P_{-i} is a preference profile without i (domain $\mathcal{P}_{-i}$).

School $s \in S \cup \{\emptyset\}$ is acceptable to student i if $s \; R_i \; \emptyset$.

Scores

For each $i \in I$ and $s \in S$, let $g_i^s \in (0,1)$ be *i's score at s* and

- $g_i = \{g_i^s\}_{s \in S}$ are all scores of i.
- for any $j \in I$ with $j \neq i$ it holds $g_i^s \neq g_j^s$.
- $g = (g_i)_{i \in I}$ is a score structure (domain $\mathcal{G}_I$). [fixed]

Matchings

A matching is a function $\mu : I \mapsto S \cup \{\emptyset\}$ where for each school $s \in S$, $|\mu^{-1}(s)| \leq q_s$ and

- μ_i is the school assigned to student $i \in I$.
- μ_s is the set of students assigned to school s.
- $\mathcal{M}$ is the domain of all matchings.

Stability

Consider a matching $\mu \in \mathcal{M}$ and a preference profile $P \in \mathcal{P}_I$:

- Student $i \in I$ has justified envy towards $j \in I$ at school μ_j if $\mu_j P_i \mu_i$ and $g_i^{\mu_j} > g_j^{\mu_j}$.
- μ is individually rational if for each $i \in I$, μ_i is acceptable.
- μ is non-wasteful if there is no $i \in I$ and $s \in S$ s.t. $sP_i\mu_i$ and $|\mu_s| < q_s$.

μ is *stable* if there is no justified envy, it is individually rational and non-wasteful.

Efficiency

- Matching μ weakly Pareto dominates another matching μ' if for all $i \in I$, $\mu_i R_i \mu'_i$.
- Matching μ Pareto dominates μ' if μ weakly Pareto dominates μ' and for some $j \in I$, $\mu_j P_j \mu'_j$.

μ is Pareto efficient if there does not exist another matching μ' which Pareto dominates μ.

Consent

A weaker notion of justified envy (Kesten, 2010):

- For each student $i \in I$, student i's consent is parameterized through $c_i \in \{0, 1\}$ where $c_i = 1$ means that i consents and otherwise not.
- A matching μ violates the priority of student i given c_i if and only if
  1. $c_i = 0$ and;
  2. there is some $j \in I$ such that i has justified envy towards j at μ.
- $c = (c_i)_{i \in I}$ is a consent profile (domain $\mathcal{C}_I$).

School Choice with Consent

- A school choice problem with consent is a tuple (I, S, q, g, P, c). [fixed]
- A matching rule $f : \mathcal{G}_I \times \mathcal{P}_I \times \mathcal{C}_I \mapsto \mathcal{M}$ maps any triple of a score structure, preference profile and consent profile into a matching.
- A matching rule f is stable (Pareto efficient) if each outcome of the matching rule is stable (Pareto efficient)

Consent-Invariance and Strategy-Proofness

Consent-Invariance

A matching rule f is consent-invariant if, given any problem and for all i, it holds that $f_i(g, P, (c_i, c_{-i})) = f_i(g, P, (c'_i, c_{-i}))$ for all c_i, c'_i.

We restrict attention to rules that are consent-invariant.

Strategy-Proofness

A matching rule f is strategy-proof if, given any problem and for all i, we have $f_i(g, (P_i, P_{-i}), c)\ R_i\ f_i(g, (P'_i, P_{-i}), c)$ for any $P'_i \in \mathcal{P}$.

Stable Dominating Rules

Stable Dominating Rules

A stable dominating matching rule always implements a matching which weakly Pareto dominates a stable matching (Alva and Manjunath, 2019).

Main Objection:
Stable dominating rules are not strategy-proof, except it is DA (Alva and Manjunath, 2019).

DA Algorithm (Gale and Shapley, 1962) — Take any report profile $(\hat{P}, \hat{c})$:

Step t: Each student who was rejected at the previous step applies to her most preferred school not yet applied to. Each school considers all the new applicants together with those who are tentatively assigned to it at the previous step and now tentatively accepts the highest ranked applicants up to its capacity, rejecting all others.

The algorithm terminates with the tentative assignments of the first step in which no student is rejected.

Interrupting Pairs

A pair $(i, s) \in I \times S$ is an interrupting pair at step t'' if

1. student i is tentatively accepted by s at some step t' and rejected by s at some later step $t'' > t'$ and;

2. another student is rejected by s at some step t^* with $t' \leq t^* < t''$.

EDA

EDA Algorithm (Kesten, 2010) — Take any report profile $(\hat{P}, \hat{c})$:

Round 0: Run DA.

Round, $m \geq 1$:

1. If in the DA application process of the previous round $m-1$, there are no consenting interrupters, then the algorithm terminates and outputs the DA matching of Round $m-1$.
2. Otherwise, find the last step of the DA process with consenting interrupters and collect all associated interrupting pairs.
   - For each collected interrupting pair (i, s), update student i's report such that school s is ranked last, leaving everything else unchanged.
   - Then, run DA with the updated report profile and proceed to the next round.

Running Example - EDA

Setting:

- $S = \{a, b\}$ with $q_a = q_b = 1$
- $I = \{i, j, k\}$

Scores at a: $g_i^a > g_k^a > g_j^a$

Scores at b: $g_j^b > g_i^b > g_k^b$

Students' preferences and consents are:

P_i : $b \; P_i \; \emptyset \; P_i \; a$, and $c_i = 1$.

P_j : $a \; P_j \; b \; P_j \; \emptyset$, and $c_j = 1$.

P_k : $b \; P_k \; a \; P_k \; \emptyset$. and $c_k = 1$.

Round 0 - Identifying Interrupters

Students submit $(\hat{P}, \hat{c}) = (P, c)$ and we reach matching

$$\{(i, \emptyset), (j, b), (k, a)\}$$

at Round 0 of EDA.

Student i is the lastly rejected interrupter at school b in Step 3.

Step	a	b	$\emptyset$
1	$\boxed{j}$	$\boxed{i}$, k	
2	j, $\boxed{k}$		
3		i, $\boxed{j}$	
4			$\boxed{i}$

Table: DA Applications Round 0

Round 1 - Improvements through Consent

Since $\hat{c}_i = 1$, we run DA with updated reported preferences $\hat{P}_i^{new}$ where

$$\emptyset \ \hat{P}_i^{new} \ a \ \hat{P}_i^{new} \ b$$

and the EDA algorithm stops with

$$\{(i, \emptyset), (j, a), (k, b)\}.$$

Step	a	b	$\emptyset$
1	$\boxed{j}$	$\boxed{k}$	$\boxed{i}$

Table: DA Applications Round 1

Note:

Compared to Round 0 of EDA

- j improved her match from b to a.
- k improved her match from a to b.

School Choice under Incomplete Information

1. Fix a student $i \in I$ and a matching rule f.
2. Students select to submit report profile $(\hat{P}, \hat{c})$.

Before student $i \in I$ submits $(\hat{P}_i, \hat{c}_i)$, she knows

- the market features (I, S, q, g_i).
- the matching rule f and how it works.

Student i's Observation

After submission of the report profile $(\hat{P}, \hat{c})$, student i makes observation $(\mu, \pi(\mu, g))$ with

1. final matching $\mu = f(g, \hat{P}, \hat{c})$ and;
2. cutoffs $\pi(\mu, g)$, where for each $s \in S \cup \{\emptyset\}$
   - $\pi_s(\mu, g) = \min_{i \in \mu_s} g_i^s$, when $|\mu_s| = q_s$ and;
   - $\pi_s(\mu, g) = 0$, otherwise.

Student i's Inferences

A *scenario* $(g'_{-i}, P'_{-i}, c'_{-i}) \in \mathcal{P}_{-i} \times \mathcal{C}_{-i} \times \mathcal{G}_{-i}$ is plausible if

1. $\pi(\mu, g) = \pi(\mu, (g_i, g'_{-i}))$
2. and $f((g_i, g'_{-i}), (P_i, P'_{-i}), (c_i, c'_{-i})) = \mu$.

Student i's inference set is the set of plausible scenarios.

Regret

Let $\mathcal{M}|_{(\hat{P}_i, \hat{c}_i)}$ be the set of matchings student i can possibly observe under f given her own report $(\hat{P}_i, \hat{c}_i)$.

Definition (Fernandez, 2020)

Fix consent decision $\hat{c}_i$. Student i regrets submitting $\hat{P}_i$ at $\mu \in \mathcal{M}|_{(\hat{P}_i, \hat{c}_i)}$ through P_i' under f if

1. for all plausible $(g'_{-i}, P'_{-i}, c'_{-i})$, we have $f_i((g_i, g'_{-i}), (P_i', P'_{-i}), (\hat{c}_i, c'_{-i}))\ R_i\ \mu_i$.
2. there exists a plausible $(\tilde{g}_{-i}, \tilde{P}_{-i}, \tilde{c}_{-i})$ where $f_i((g_i, \tilde{g}_{-i}), (P_i', \tilde{P}_{-i}), (\hat{c}_i, \tilde{c}_{-i}))\ P_i\ \mu_i$.

Regret-Freeness

Definition (Fernandez, 2020)

Fix consent decision $\hat{c}_i$. Submitting $\hat{P}_i$ is regret-free under f if there does not exist a pair $(\mu, P_i') \in \mathcal{M}|_{(\hat{P}_i, \hat{c}_i)} \times \mathcal{P}$ such that i regrets $\hat{P}_i$ at μ through P_i'.

Regret-Free Truth-Telling

We refer to any $\tilde{P}_i \in \mathcal{P}$ as truth-telling (w.r.t. true preferences P_i), if *relative* to P_i the ranking $\tilde{P}_i$ keeps

1. the order of schools that are acceptable as before and;
2. the sets of acceptable and unacceptable schools the same.

Definition (Fernandez, 2020)

A matching rule f is regret-free truth-telling if for each problem and for each student, truth-telling is regret-free under f.

Results

Theorem 1

EDA is regret-free truth-telling.

Proposition

For any non truth-telling report, there exists an observation at which the student regrets it through truth-telling.

Our Approach to Theorem 1

- Let student $i \in I$ report her true preferences P_i and observe μ.
- We divide misreports (relative to P_i) into four categories of elementary variations:
  (1) Altering the order among schools ranked above μ_i on P_i.
  (2) Ranking some schools that are preferred to μ_i on P_i as less preferred.
  (3) Ranking some schools that are less preferred to μ_i on P_i as more preferred.
  (4) Altering the order among schools that rank below μ_i on P_i.

Types (3) and (4)

(3) Ranking some schools that are less preferred to μ_i on P_i as more preferred.

(4) Altering the order among schools that rank below μ_i on P_i.

$\rightarrow$ Both types of variations can be profitable misreports under EDA.

Key Task:
If the misreport is profitable in a plausible scenario, then we show that there is a plausible scenario with a detrimental consequence.

Student i's Observation and Inferences

$S = \{a, b\}$, $q_a = q_b = 1$ and $I = \{i, j, k\}$.

Scores of a: $g_i^a > g_k^a > g_j^a$

Scores of b: $g_j^b > g_i^b > g_k^b$

P_i : $b \; P_i \; \emptyset \; P_i \; a$, and $c_i = 1$,

P_j : $a \; P_j \; b \; P_j \; \emptyset$, and $c_j = 1$,

P_k : $b \; P_k \; a \; P_k \; \emptyset$, and $c_k = 1$.

Observation: If students report truthfully, student i observes:

1. $\mu = \{\boldsymbol{(i, \emptyset)}, (j, a), (k, b)\}$,
2. $g_i^a > \pi_a(\mu, g)$ and $g_i^b > \pi_b(\mu, g)$.

Inferences: The true report profile of others and the true scores are always plausible.

Round 0 - A Misreport

With scores g, student i's misreport P'_i and truthfully submitted preferences of j and k:

P'_i: $b \; P'_i \; a \; P'_i \; \emptyset$,
$\hat{P}_j$: $a \; \hat{P}_j \; b \; \hat{P}_j \; \emptyset$,
$\hat{P}_k$: $b \; \hat{P}_k \; a \; \hat{P}_k \; \emptyset$,

we reach the matching

$$\{(\boldsymbol{i}, \boldsymbol{a}), (j, b), (k, \emptyset)\}$$

at Round 0 of EDA.

Student k is the lastly rejected interrupter at a in Step 4.

Step	a	b	$\emptyset$
1	$\boxed{j}$	$\boxed{i}$, k	
2	j, $\boxed{k}$		
3		i, $\boxed{j}$	
4	$\boxed{i}$, k		
5			$\boxed{k}$

Table: DA Applications in Round 0

Round 1 - Two Plausible Scenarios

Case 1: If $\hat{c}_k = 1$, then run DA with update reported preferences $\hat{P}_k^{new}$ where

$$b \ \hat{P}_k^{new} \ \emptyset \ \hat{P}_k^{new} \ a$$

and the EDA algorithm stops with

$$\{(\boldsymbol{i}, \boldsymbol{b}), (j, a), (k, \emptyset)\}.$$

$\Rightarrow$ i is strictly better off with her top choice school b.

Step	a	b	$\emptyset$
1	$\boxed{j}$	$\boxed{i}$, k	
2			$\boxed{k}$

Table: DA Applications Case 1

Round 1 - Two Plausible Scenarios

Case 2: If $\hat{c}_k = 0$, then the EDA algorithm must stop with

$$\{(\boldsymbol{i}, \boldsymbol{a}), (j, b), (k, \emptyset)\}.$$

$\Rightarrow$ i is strictly worse off with her least preferred school a.

Take Away:
Uncertainty about consent decision of k implies that student i does not regret submitting the truth P_i through misreport P_i' at the observation.

Stable Dominating Rules without Consent Option

Theorem 2

There is no (Pareto) efficient stable dominating rule which is regret-free truth-telling.

Proposition

There exists a stable dominating rule without consent option that is not equivalent to DA and which is regret-free truth-telling.

Related Literature

Regret in Matching Markets
Fernandez (2020), Immorlica et al. (2020)

EDA
Troyan and Morrill (2020), Cerrone et al. (2022), Kesten (2010), Dur et al. (2019), Reny (2021), Kwon and Shorrer (2019), Alva and Manjunath (2019), Tang and Yu (2014), Bando (2014), Troyan et al. (2020)

Cutoffs
Azevedo and Leshno (2016), Immorlica et al. (2020), Abdulkadiroğlu et al. (2015), Leshno and Lo (2017), Hakimov and Raghavan (2020), Bo and Hakimov (2019)

Conclusion

- Telling the truth is the unique guaranteed regret-free option under EDA.
- Efficient stable dominating rules are not regret-free truth-telling.
- There exist stable dominating rules without consent option which are regret-free truth-telling.

Open Questions:

- Do other matching rules which are not strategy-proof satisfy regret-free truth-telling in related models?
- Is regret-free truth-telling of EDA preserved if we allow for more flexible choice functions for schools as in Ehlers and Morrill (2019)?