Block-Recursive Non-Gaussian SVAR: Identification, Efficiency, and Moment Selection

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SVAR: $u_t = B_0 \varepsilon_t \quad \rightarrow$ Example: $\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$

Identification is about imposing an a priori structure on the interaction and the shocks.

- Traditional approaches assume that the structural shocks are uncorrelated and impose restrictions on the interaction: Sims (1980), Blanchard and Quah (1989), or Uhlig (2005).
- More recent approaches require no assumptions on the interaction but impose more structure on the stochastic properties of the shocks, e.g. independent and non-Gaussian shocks: Gouriéroux et al. (2017), Lanne et al. (2017).

Figure 1: Examples of Different Block-Recursive SVAR Models.

$$\tilde{u}_{p_1}\left\{\begin{bmatrix}u_1\\ u_2\\ u_3\\ u_4\end{bmatrix}\right. = \begin{bmatrix}b_{11} & b_{12} & b_{13} & b_{14}\\ b_{21} & b_{22} & b_{23} & b_{24}\\ b_{31} & b_{32} & b_{33} & b_{34}\\ b_{41} & b_{42} & b_{43} & b_{44}\end{bmatrix}\left.\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \varepsilon_4\end{bmatrix}\right\}\tilde{\epsilon}_{p_1}$$

(a) One Block

$$\left[\begin{array}{r}\tilde{u}_{p_1}\left\{\begin{array}{c}u_1\\ u_2\end{array}\right.\\ \tilde{u}_{p_2}\left\{\begin{array}{c}u_3\\ u_4\end{array}\right.\end{array}\right] = \begin{bmatrix}b_{11} & b_{12} & 0 & 0\\ b_{21} & b_{22} & 0 & 0\\ b_{31} & b_{32} & b_{33} & b_{34}\\ b_{41} & b_{42} & b_{43} & b_{44}\end{bmatrix}\left[\begin{array}{l}\left.\begin{array}{c}\varepsilon_1\\ \varepsilon_2\end{array}\right\}\tilde{\epsilon}_{p_1}\\ \left.\begin{array}{c}\varepsilon_3\\ \varepsilon_4\end{array}\right\}\tilde{\epsilon}_{p_2}\end{array}\right]$$

(b) Two Blocks

$$\left[\begin{array}{r}\tilde{u}_{p_1}\left\{u_1\right.\\ \tilde{u}_{p_2}\left\{u_2\right.\\ \tilde{u}_{p_3}\left\{\begin{array}{c}u_3\\ u_4\end{array}\right.\end{array}\right] = \begin{bmatrix}b_{11} & 0 & 0 & 0\\ b_{21} & b_{22} & 0 & 0\\ b_{31} & b_{32} & b_{33} & b_{34}\\ b_{41} & b_{42} & b_{43} & b_{44}\end{bmatrix}\left[\begin{array}{l}\left.\varepsilon_1\right\}\tilde{\epsilon}_{p_1}\\ \left.\varepsilon_2\right\}\tilde{\epsilon}_{p_2}\\ \left.\begin{array}{c}\varepsilon_3\\ \varepsilon_4\end{array}\right\}\tilde{\epsilon}_{p_3}\end{array}\right]$$

(c) Three Blocks

$$\left[\begin{array}{r}\tilde{u}_{p_1}\left\{u_1\right.\\ \tilde{u}_{p_2}\left\{u_2\right.\\ \tilde{u}_{p_3}\left\{u_3\right.\\ \tilde{u}_{p_4}\left\{u_4\right.\end{array}\right] = \begin{bmatrix}b_{11} & 0 & 0 & 0\\ b_{21} & b_{22} & 0 & 0\\ b_{31} & b_{32} & b_{33} & 0\\ b_{41} & b_{42} & b_{43} & b_{44}\end{bmatrix}\left[\begin{array}{l}\left.\varepsilon_1\right\}\tilde{\epsilon}_{p_1}\\ \left.\varepsilon_2\right\}\tilde{\epsilon}_{p_2}\\ \left.\varepsilon_3\right\}\tilde{\epsilon}_{p_3}\\ \left.\varepsilon_4\right\}\tilde{\epsilon}_{p_4}\end{array}\right]$$

(d) Four Blocks

Illustration: Independent shocks and higher-order moment conditions

Bivariate SVAR: $\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} = \begin{bmatrix} 1 & A_{12} \\ A_{21} & 1 \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$

Moment conditions uncorrelated shocks: $E[\varepsilon_{1t}\varepsilon_{2t}] = 0$

Moment conditions mean independent shocks:

$$E[\varepsilon_{1t}^2\varepsilon_{2t}] = 0 \qquad E[\varepsilon_{1t}\varepsilon_{2t}^2] = 0 \qquad E[\varepsilon_{1t}^3\varepsilon_{2t}] = 0 \qquad E[\varepsilon_{1t}\varepsilon_{2t}^3] = 0$$

Identification: The SVAR is identified (up to sign and permutation) if all shocks are mean independent at most one shock is Gaussian, see Lanne and Luoto (2021), or Keweloh (2021).

Block-Recursive Non-Gaussian SVAR: Identification

One block (unrestricted):

$$\begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix} = \begin{bmatrix} 1 & b_{12} & b_{13} & b_{14} \\ b_{21} & 1 & b_{23} & b_{24} \\ b_{31} & b_{32} & 1 & b_{34} \\ b_{41} & b_{42} & b_{43} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{bmatrix}$$

Assumptions to ensure identification:

1. Uncorrelated shocks ε_t.
2. At most one Gaussian shock per block.
3. Mean independent shocks within blocks, i.e. $E[\varepsilon_{it}|\varepsilon_{-it}] = 0$.

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Identifying coskewness conditions:

$E[\varepsilon_1^2\varepsilon_2] = 0$	$E[\varepsilon_1^2\varepsilon_4] = 0$
$E[\varepsilon_1\varepsilon_2^2] = 0$	$E[\varepsilon_1\varepsilon_4^2] = 0$
$E[\varepsilon_1^2\varepsilon_3] = 0$	$E[\varepsilon_2^2\varepsilon_4] = 0$
$E[\varepsilon_1\varepsilon_3^2] = 0$	$E[\varepsilon_2\varepsilon_4^2] = 0$
$E[\varepsilon_2^2\varepsilon_3] = 0$	$E[\varepsilon_3^2\varepsilon_4] = 0$
$E[\varepsilon_2\varepsilon_3^2] = 0$	$E[\varepsilon_3\varepsilon_4^2] = 0$
$E[\varepsilon_1\varepsilon_2\varepsilon_3] = 0$	$E[\varepsilon_1\varepsilon_2\varepsilon_4] = 0$
$E[\varepsilon_2\varepsilon_3\varepsilon_4] = 0$	$E[\varepsilon_1\varepsilon_3\varepsilon_4] = 0$

... plus 25 cokurtosis conditions

Block-Recursive Non-Gaussian SVAR: Identification

Two blocks (e.g. two countries):

$$\begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix} = \begin{bmatrix} 1 & b_{12} & 0 & 0 \\ b_{21} & 1 & 0 & 0 \\ b_{31} & b_{32} & 1 & b_{34} \\ b_{41} & b_{42} & b_{43} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{bmatrix}$$

Assumptions to ensure identification:

1. Uncorrelated shocks ε_t.
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Identifying coskewness conditions:

$$E[\varepsilon_1^2\varepsilon_2] = 0 \qquad E[\varepsilon_3^2\varepsilon_4] = 0$$
$$E[\varepsilon_1\varepsilon_2^2] = 0 \qquad E[\varepsilon_3\varepsilon_4^2] = 0$$

... plus 4 cokurtosis conditions

Assumptions to ensure identification:

1. Uncorrelated shocks ε_t.
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3. Mean independent shocks within blocks, i.e. $E[\varepsilon_{it}|\varepsilon_{-it}] = 0$.

Block-Recursive Non-Gaussian SVAR: Identification

Four blocks (recursive):

$$\begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ b_{21} & 1 & 0 & 0 \\ b_{31} & b_{32} & 1 & 0 \\ b_{41} & b_{42} & b_{43} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{bmatrix}$$

Assumptions to ensure identification:

1. Uncorrelated shocks ε_t.
2. At most one Gaussian shock per block.
3. Mean independent shocks within blocks, i.e. $E[\varepsilon_{it}|\varepsilon_{-it}] = 0$.

Recursive Non-Gaussian SVAR: Efficiency

In a recursive SVAR with independent shocks identified by covariance conditions it holds that:

1. $E[\varepsilon_i \varepsilon_j \varepsilon_k]$ is redundant.
2. $E[\varepsilon_i^2 \varepsilon_j]$ is redundant for the q-th column of B if $q \neq j$.
3. $E[\varepsilon_i^2 \varepsilon_j]$ is redundant for the j-th column of B if and only if

$$\frac{2E[\epsilon_j^3]}{E[\epsilon_j^4]-1} a_{jj} + E[\epsilon_i^3] a_{ij} = 0,$$

$$E[\epsilon_i^3] a_{iz} = 0, \qquad z = j+1, \dots, i,$$

where a_{ij} denote the elements of $A = B^{-1}$ and $i \neq j \neq k$.

Block-Recursive Non-Gaussian SVAR: Data-driven moment selection

Lasso moment selection based on Cheng and Liao (2015):

$$\{\widehat{B}, \widehat{\beta}\} := \underset{\{B,\beta\}\in\{\mathbb{B},\mathbb{R}^k\}}{\arg\min} \begin{bmatrix} g_{\mathbf{N}}(B) \\ g_{\mathbf{D}}(B) - \beta \end{bmatrix}' W \begin{bmatrix} g_{\mathbf{N}}(B) \\ g_{\mathbf{D}}(B) - \beta \end{bmatrix} + \lambda \sum_{j=1}^{k} \omega_j \, |\beta_j|, \tag{1}$$

where

- $\mathbb{B}$ is the set of B matrices satisfying a given block-recursive order,
- $g_{\mathbf{N}}(B)$ is the sample average of the identifying conditions,
- $g_{\mathbf{D}}(B)$ is the sample average of the overidentifying conditions,
- k is the number of overidentifying moment conditions,
- λ is a tuning parameter,
- ω_j for $j = 1, ..., k$ are moment specific weights.

Block-Recursive Non-Gaussian SVAR: Data-driven moment selection

$$\{\widehat{B}, \widehat{\beta}\} := \underset{\{B,\beta\}\in\{\mathbb{B},\mathbb{R}^k\}}{\arg\min} \begin{bmatrix} g_{\mathbf{N}}(B) \\ g_{\mathbf{D}}(B) - \beta \end{bmatrix}' W \begin{bmatrix} g_{\mathbf{N}}(B) \\ g_{\mathbf{D}}(B) - \beta \end{bmatrix} + \lambda \sum_{j=1}^{k} \omega_j \, |\beta_j| \tag{2}$$

For an overidentifying moment D_j use the adaptive weights

$$\omega_j = \frac{\mu_j^{r_1}}{|\gamma_j{}^{r_2}|}, \quad \text{with } r_1 \geq r_2 \geq 0, \tag{3}$$

with a measure of relevance μ_j and a measure of validity γ_j.

$\rightarrow$ Allows to select relevant and valid overidentifying moment conditions.

Summary block-recursive SVAR:

- A block-recursive structure appears in many applications, e.g., SVARs with two countries, SVARs with macroeconomic and financial variables, and proxy VARs.
- Block-recursive restrictions allow to relax the independence and non-Gaussianity assumptions. The bias and variance of the estimator decreases with the number of restrictions.
- Overidentifying coskewness and cokurtosis moment conditions can increase the asymptotic efficiency.
- Relevant and valid overidentifying moment conditions can be selected with Lasso.

The impact of stock market news shocks on the oil price

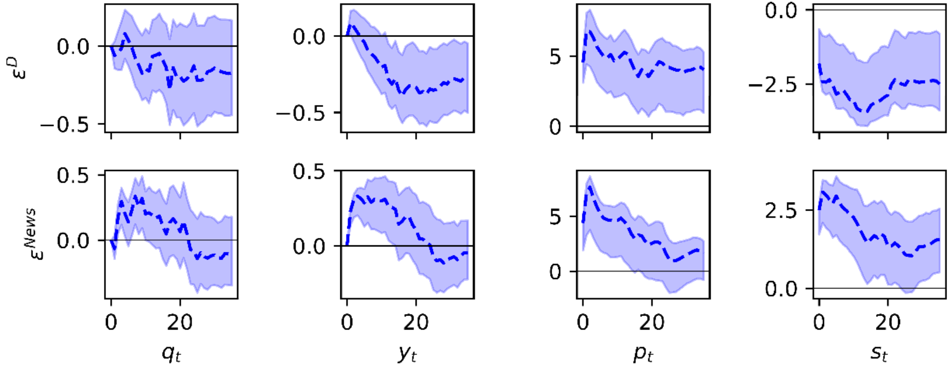
Block-recursive SVAR:

$$\begin{bmatrix} q_t \\ y_t \\ p_t \\ s_t \end{bmatrix} = \alpha + \sum_{i=1}^{24} A_i \begin{bmatrix} q_{t-i} \\ y_{t-i} \\ p_{t-i} \\ s_{t-i} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{41} \end{bmatrix} \begin{bmatrix} \varepsilon_{S,t} \\ \varepsilon_{Y,t} \\ \varepsilon_{D,t} \\ \varepsilon_{News,t} \end{bmatrix}.$$

Monthly data from January 1975 to March 2022:

- q_t: Δ log global oil production
- y_t: Δ log industrial production:
- p_t: log of real oil price
- s_t: real stock returns

Figure 2: Blue: Block-recursive SVAR. 90% confidence bands.



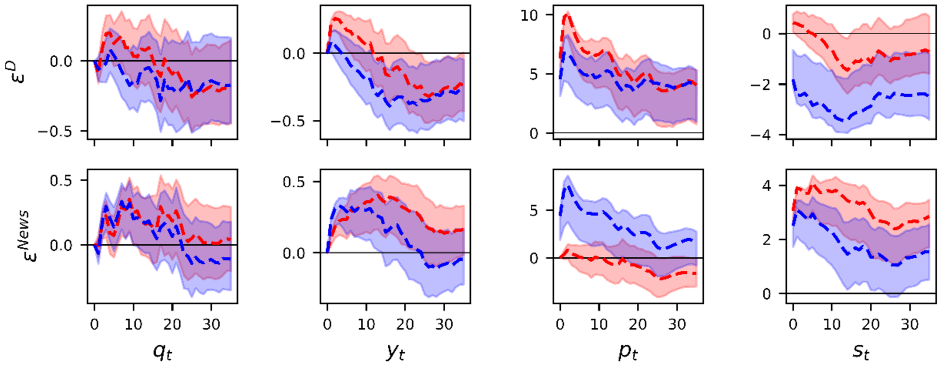
The impact of stock market news shocks on the oil price

Recursive SVAR:

$$\begin{bmatrix} q_t \\ y_t \\ p_t \\ s_t \end{bmatrix} = \alpha + \sum_{i=1}^{24} A_i \begin{bmatrix} q_{t-i} \\ y_{t-i} \\ p_{t-i} \\ s_{t-i} \end{bmatrix} + \underbrace{\begin{bmatrix} b_{11} & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & 0 \\ b_{41} & b_{42} & b_{43} & b_{41} \end{bmatrix}}_{\text{Kilian and Park (2009)}} \begin{bmatrix} \varepsilon_{S,t} \\ \varepsilon_{Y,t} \\ \varepsilon_{D,t} \\ \varepsilon_{News,t} \end{bmatrix}.$$

The impact of stock market news shocks on the oil price

Figure 3: Red: Recursive SVAR. Blue: Block-recursive SVAR. 90% confidence bands.



The impact of stock market news shocks on the oil price

Table 1: One year ahead forecast error variance decomposition

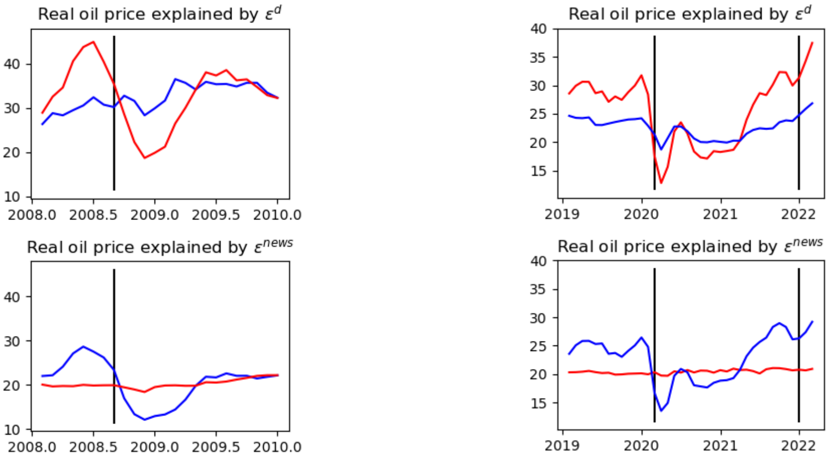
Recursive SVAR

	ε_S	ε_Y	ε_D	ε_{News}
p	0.11	0.1	0.79	0.00
s	0.01	0.03	0.04	0.92

Block-recursive SVAR

	ε_S	ε_Y	ε_D	ε_{News}
p	0.11	0.1	0.39	0.40
s	0.01	0.03	0.35	0.61

Figure 2: Red: Recursive SVAR. Blue: block-recursive SVAR.



Summary application

- The oil price reacts simultaneously to news shocks. News shocks explain 40% of the oil price variation compared to 0% in the recursive model.
- Stock returns react simultaneously to oil-specific demand shocks. Oil-specific demand shocks explain 35% of the variation in stock returns compared to 4% in the recursive model.
- News shocks retrieved from stock returns were important drivers of the oil price following the collapse of Lehman Brothers and during the COVID-19 pandemic.

References

Blanchard, O. J. and D. Quah (1989). The Dynamic Effects of Aggregate Demand and Supply Disturbances. The American Economic Review 79(4), 655–673.

Cheng, X. and Z. Liao (2015). Select the Valid and Relevant Moments: An Information-Based LASSO for GMM with Many Moments. Journal of Econometrics 186(2), 443–464.

Gouriéroux, C., A. Monfort, and J.-P. Renne (2017). Statistical Inference for Independent Component Analysis: Application to Structural VAR Models. Journal of Econometrics 196(1), 111–126.

Keweloh, S. A. (2021). A Generalized Method of Moments Estimator for Structural Vector Autoregressions Based on Higher Moments. Journal of Business & Economic Statistics 39(3), 772–782.

Kilian, L. and C. Park (2009). The Impact of Oil Price Shocks on the US Stock Market. International Economic Review 50(4), 1267–1287.

Lanne, M. and J. Luoto (2021). GMM Estimation of Non-Gaussian Structural Vector Autoregression. Journal of Business & Economic Statistics 39(1), 69–81.

Lanne, M., M. Meitz, and P. Saikkonen (2017). Identification and Estimation of Non-Gaussian Structural Vector Autoregressions. Journal of Econometrics 196(2), 288–304.

Sims, C. A. (1980). Macroeconomics and Reality. Econometrica: Journal of the Econometric Society, 1–48.

Uhlig, H. (2005). What are the Effects of Monetary Policy on Output? Results from an Agnostic Identification Procedure. Journal of Monetary Economics 52(2), 381–419.