Optimal Monetary Policy in HANK

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Motivation

- **Heterogeneity** in the model adds relevance but also complexity
  - How aggregate variables react?
  - How distribution reacts?
  - More relevance for the Optimal Policy
- **Redistributive motives** in NK model
  - Affects differently constrained and unconstrained households
  - Affects share of constrained

Related literature

- New Keynesian model, with **reduced form heterogeneity**

  Bilbiie (2008); Debortoli and Galí (2017); Bilbiie (2019); Challe (2020)

- **HANK** models

  Kaplan, Moll, and Violante (2018); Le Grand and Ragot (2022); Werning (2015); McKay, Nakamura, and Steinsson (2016)

- **Optimal policy** in HANK
  - *Nuño and Thomas (2022)*

    Small open economy
  - *González, Nuño, Thaler, and Albrizio (WP)*

    Firms heterogeneity
  - *Bhandari, Evans, Golosov, and Sargent (2021)*

    Both monetary and fiscal, but no binding borrowing constraint

- **Contribution**
  - Transition to and from boundary constraint opens **new channel** for the policy
  - Optimal policy is **qualitatively different** from the RANK and TANK models

Empirical Evidence*

Probability to be constrained.
Conditional mean and s.d. over time

Credit Score	*Prob. Constrained* mean	s.d.
< 620	73.7	4.1
$620 - 679$	54.7	4.4
$680 - 719$	37.8	5.7
$720 - 760$	23.4	3.7
> 760	11.8	2.0

Correlation of real interest rate and the share of constrained households

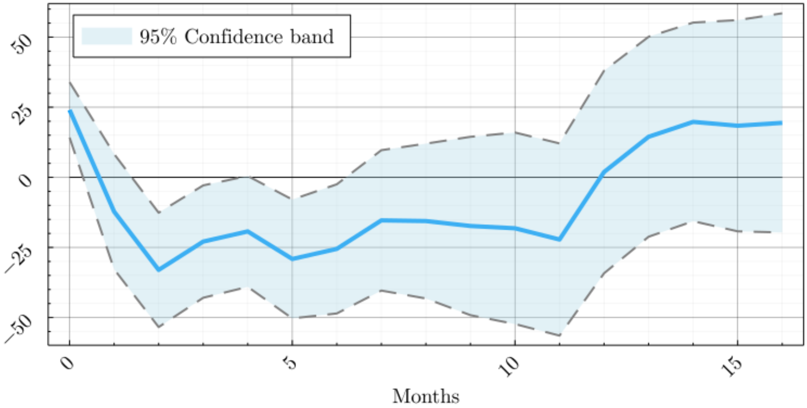
	Prob. Constrained
real rate	-0.53
	(0.81)
real rate x *credit score* < 620	-2.42^{**}
	(1.08)
real rate, 1 year lag	-0.85
	(0.91)
real rate, 1 year lag x *credit score* < 620	2.34^{**}
	(1.16)
R-squared	0.2471
N	18,431

Note: $^{*} p < 0.1$; $^{**} p < 0.05$; $^{***} p < 0.01$
GDP, CPI, time trend^2*, individual controls*

*Survey of Consumer Expectations (SCE) Credit Access Survey

Empirical Evidence (Supply*)

Change of willingness to provide consumer installment loans after a contractionary monetary policy shock



*Senior Loan Officer Opinion Survey on Bank Lending Practices

Households overview

- Continuum of households $i \in [0,1]$, each solving the problem:

$$\max_{\{c_{i,t}, l_{i,t}, \dot{b}_{i,t}\}_t} \int_0^{+\infty} e^{-\rho t} \left(\frac{c_{i,t}^{1-\nu}}{1-\nu} - \varphi \frac{l_{i,t}^{1+\gamma}}{1+\gamma} \right) dt$$

$$\text{s.t.} \quad c_{i,t} + \dot{b}_{i,t} = \lambda W_t l_{i,t} \varepsilon_{i,t} + d_t + T_t + r_t^b b_{i,t}$$

  - **Idiosyncratic productivity** $\varepsilon_{i,t}$ follows the process:

$$\varepsilon_{i,t} = exp\{e_{i,t}\}; \; de_{i,t} = \rho_e(\bar{e} - e_{i,t})dt + \sigma_e dW_{e,i,t}$$

  - Where $b_{i,t} \geq \underline{b}$ are individual holdings of **nominal bonds** expressed in real terms
    With real return: $r_t^b = i_t - \pi_t$

- RANK: $\varepsilon_{i,t} = 1, \forall i \Rightarrow b_{i,t} = 0, \forall i$

Supply side

Optimality conditions

- Result of the household problem is given by equations:

$$c_{i,t} = \left(\frac{\partial \mathcal{V}_{i,t}}{\partial b}\right)^{-\frac{1}{\nu}} \qquad \text{(Consumption)}$$

$$\rho \mathcal{V}_{i,t} = \frac{c_{i,t}^{1-\nu}}{1-\nu} - \varphi \frac{l_{i,t}^{1+\gamma}}{1+\gamma} + \mathcal{A}_{i,t}\mathcal{V}_{i,t} + \frac{\partial \mathcal{V}_{i,t}}{\partial t} \qquad \text{(HJB)}$$

$$l_{i,t}^{\gamma} c_{i,t}^{\nu} = \frac{\lambda W_t \varepsilon_{i,t}}{\varphi} \qquad \text{(Labor supply)}$$

- **Evolution of the distribution** is given by Fokker–Planck / Kolmogorov forward equation more

$$\frac{\partial f_{i,t}}{\partial t} = \mathcal{A}_{i,t}^{*} f_{i,t}$$

Competitive Equilibrium

Optimal Policy

- The **Ramsey problem** is solved by maximizing the Lagrangian:

$$\mathcal{L}[f, \mathcal{V}, c, l, W, Y, \pi, r^b, T] = \int_0^\infty e^{-\rho t} \left[\left\langle \frac{c_{i,t}^{1-\nu}}{1-\nu} - \varphi \frac{l_{i,t}^{1+\gamma}}{1+\gamma}, f_{i,t} \right\rangle \right.$$

$$\left. + \textit{(costate variables)} \times \textit{(competitive equilibrium equations)} \right] dt$$

more

- Why **continuous time**?

  Distribution law of motion has simple functional form

$$\frac{\partial f_{i,t}}{\partial t} = \mathcal{A}_{i,t}^* f_{i,t}$$

  derivative of $\mathcal{L}$ can be calculated using Calculus of Variations more

Solution algorithm

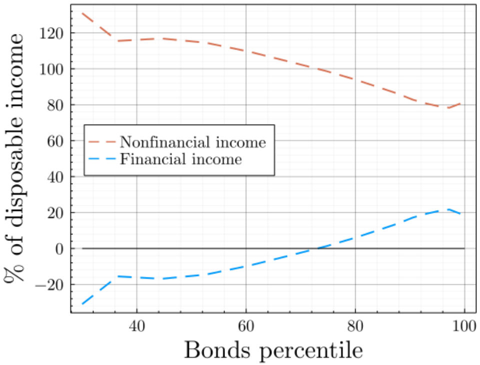
Solving for the equilibrium response to the **deterministic path** of the shock under the optimal policy

- Solving dynamics given a **candidate path of π**
  - Guess bonds prices, wages and dividends
  - Solve the household problem
  - Calculate implied distribution and market clearing prices
- **Costate dynamics**
  - Solve a system of linear differential equations
- Check the first order condition wrt π_t, otherwise iterate

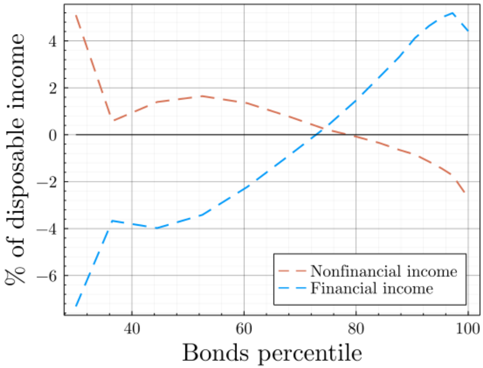
Calibration

Solving for optimal **stabilization policy**

Monetary policy shock ($r^b \uparrow$)



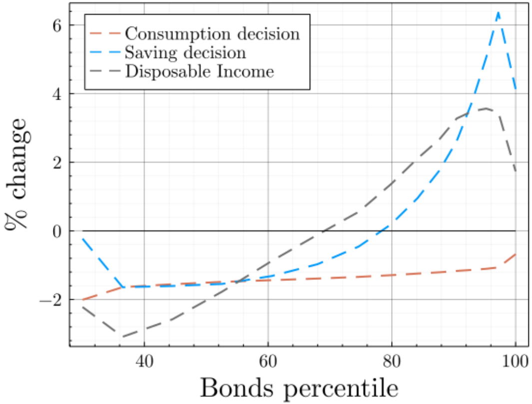
(a) Cash flow shares in Steady State



(b) Cash flow shares change after MP shock

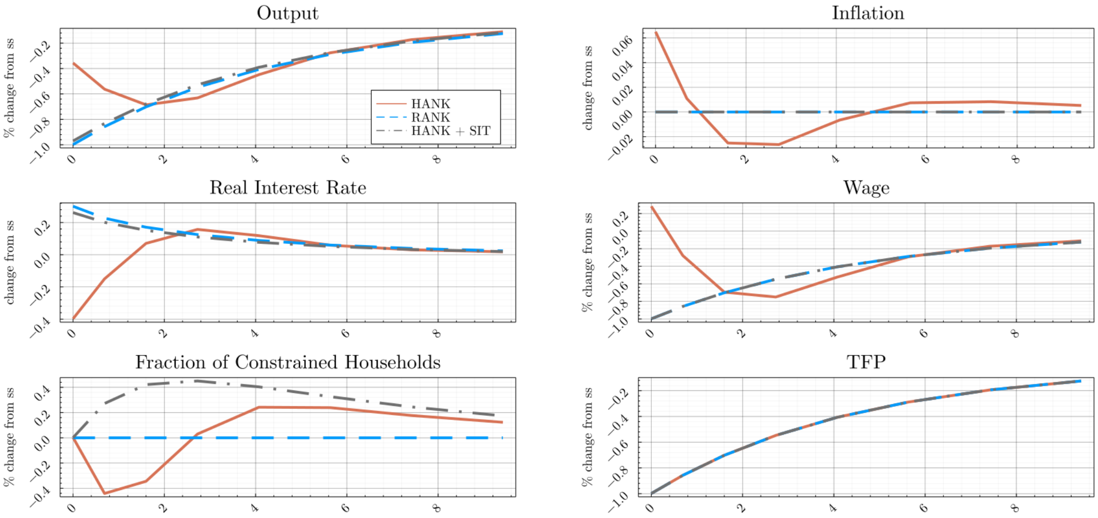
- Distribution of bonds has the **point mass 0.3** at the constraint
- Borrowers suffer from higher interest rates
- *Countercyclical inequality* through interest rates exposure

Monetary policy shock ($r^b \uparrow$)



- Borrowers have decline in income and can't smooth consumption
- Having high interest rates is clearly harming borrowers

Optimal policy in response to TFP shock more

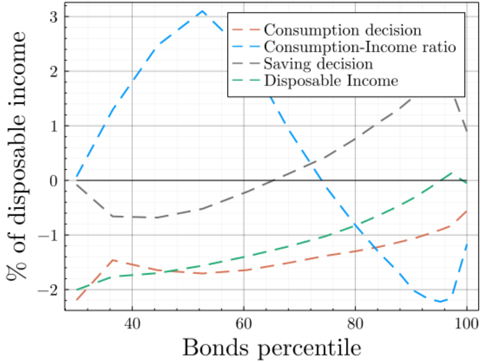


Optimal policy: lower the real interest rate to create redistribution from wealthy to poor

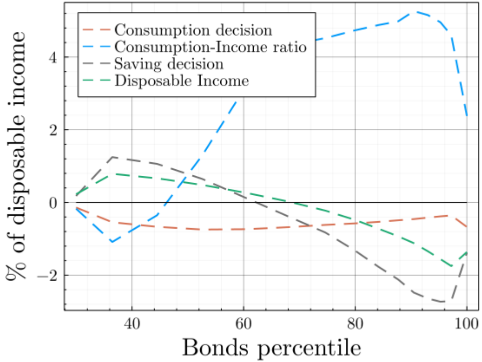
SIT vs Optimal policy

- Policy affects **households' income** differently
- Looking at **differential impact** of two policies
- **SIT vs Ramsey** in the first quarter after TFP shock

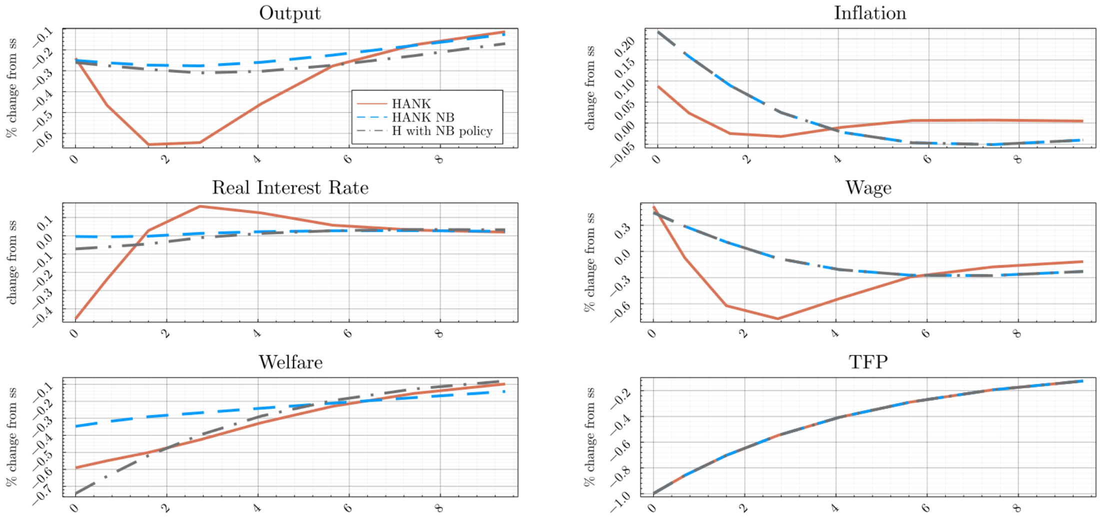
(a) SIT



(b) Ramsey OP

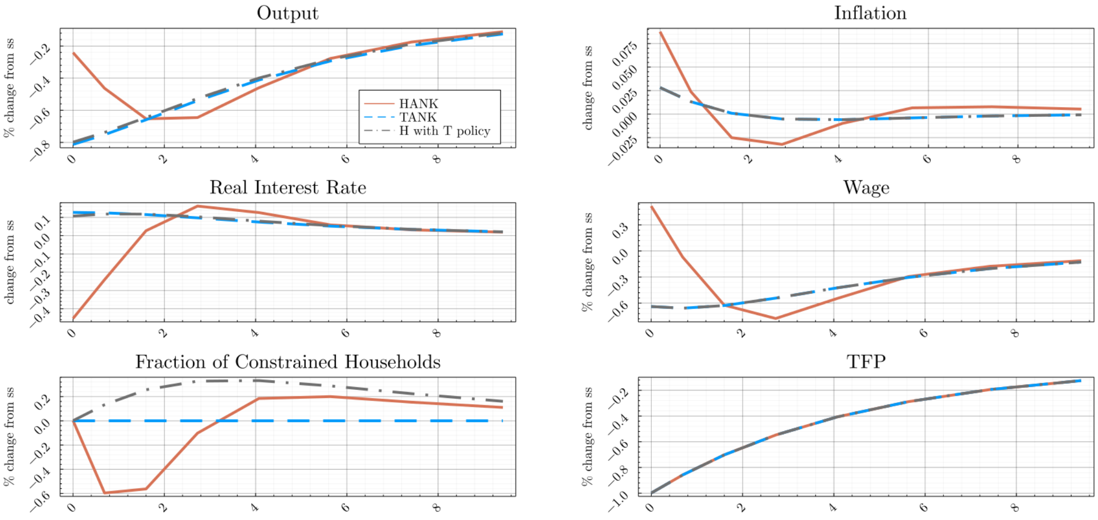


Optimal policy Natural Borrowing Constraint



With natural borrowing constraint in the model, only partial redistributive motive applies, and there is almost no response of real interest rate

Optimal policy TANK



TANK model does not have the redistributive motive

Conclusion

- **Heterogeneity in is a needed** extension but brings a lot of complexity
- Optimal **policy is significantly different** in HANK model
- Changing the **fraction of constrained agents** has the first order effect on Optimal policy

Thank You

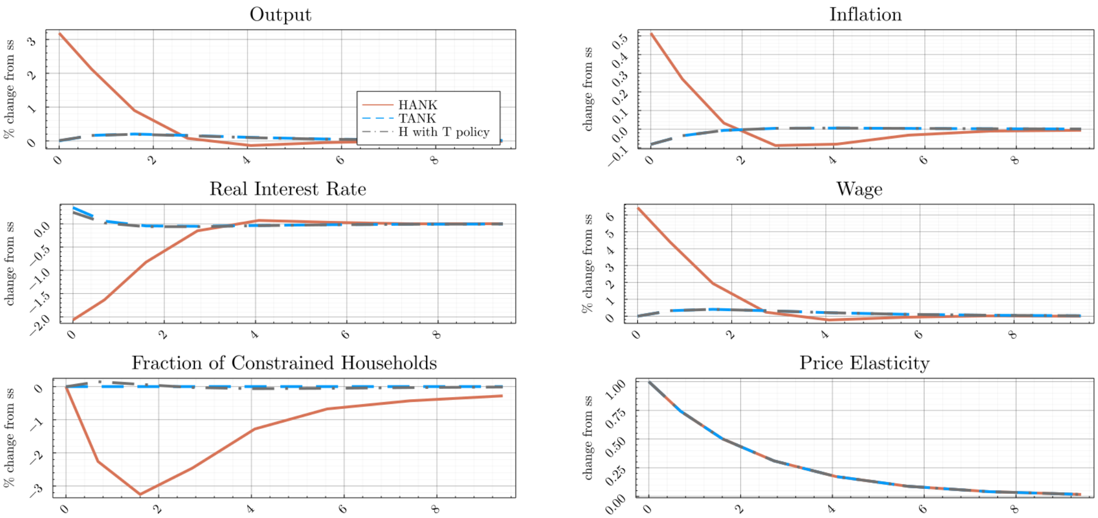
Model

- Standard sticky price model in continuous time
- Supply side gives **Phillips Curve**
- Government provides labor subsidy to balance the inefficiency caused by monopolistic competition with no additional redistribution
- Household side:
  - Idiosyncratic productivity shocks drive **heterogeneity in income and wealth**
  - Bonds constrained by the **borrowing limit** $\underline{b} < 0$
- Planner chooses interest rate path to **maximize aggregate welfare**
- Study response to TFP and Markup shocks

back

Optimal policy in response to Markup shock

more



Supply side back

- Final good producers:
  - Produced by competitive firms with **CES production function**

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\phi} Y_t$$

- Intermediate firms are **monopolistic producers** and have linear production function and quadratic price adjustment costs more
- Solution gives the **Phillips Curve**

$$\left(r_t^b - \frac{\dot{Y}_t}{Y_t}\right)\pi_t = \frac{\phi - 1}{\psi}\left(\frac{\phi}{\phi - 1}m_t - 1\right) + \dot{\pi}_t$$

Competitive equilibrium back

- Household problem (for $i \in [0,1]$)
  - **HJB** $\rho \mathcal{V}_{i,t} = \frac{c_{i,t}^{1-\nu}}{1-\nu} - \varphi \frac{l_{i,t}^{1+\gamma}}{1+\gamma} + \mathcal{A}_{i,t}\mathcal{V}_{i,t} + \frac{\partial \mathcal{V}_{i,t}}{\partial t}$
  - **Consumption** $c_{i,t}^{-\nu} = \frac{\partial \mathcal{V}_{i,t}}{\partial b}$
  - **Household budget constraint** $c_{i,t} + \dot{b}_{i,t} = \lambda W_t l_{i,t} \varepsilon_{i,t} + d_t + T_t + r_t^b b_{i,t}$
  - **Labor supply** $l_{i,t}^{\gamma} c_{i,t}^{-\nu} = \frac{\lambda W_t \varepsilon_{i,t}}{\varphi}$
- **Distribution law of motion** $\mathcal{A}_{i,t}^{*} f_{i,t} = \frac{\partial f_{i,t}}{\partial t}$
- Supply side
  - **Aggregate output** $Y_t = \theta_t \left\langle l_{i,t}\varepsilon_{i,t}, f_{i,t} \right\rangle$
  - **Phillips Curve** $\frac{\phi-1}{\psi}\left(\frac{\phi}{\phi-1}\frac{W_t}{\theta_t} - 1\right) + \dot{\pi}_t = \left(r_t^b - \frac{\dot{Y}_t}{Y_t}\right)\pi_t$
  - **Dividends** $d_t = Y_t - W_t \left\langle l_{i,t}\varepsilon_{i,t}, f_{i,t} \right\rangle - \frac{\psi}{2}\pi_t^2$
- **Bond market clearing** $\left\langle b_{i,t}, f_{i,t} \right\rangle = 0$
- **Feasibility constraint** $C_t = Y_t - \frac{\psi}{2}\pi_t^2$
- **Monetary policy**

Intermediate firms back

$$\mathcal{J}_j = \max_{\{\dot{p}_{j,t}\}_t} \mathbb{E} \int_0^\infty e^{-\int_t^\infty r_t^b dt} \left[\left(\frac{p_{jt}}{P_t} - m_t \right) \left(\frac{p_{j,t}}{P_t} \right)^{-\phi} Y_t - \frac{\psi}{2} \left(\frac{\dot{p}_{j,t}}{p_{j,t}} \right)^2 Y_t \right] dt$$

The Bellman equation for the firms problem has the following form

$$r_t^b \mathcal{J}_{j,t} = \max_{\dot{p}_{j,t}} \left(\frac{p_{j,t}}{P_t} - m_t \right) \left(\frac{p_{j,t}}{P_t} \right)^{-\phi} Y_t - \frac{\psi}{2} \left(\frac{\dot{p}_{j,t}}{p_{j,t}} \right)^2 Y_t + \dot{p}_{j,t} \frac{\partial \mathcal{J}_t}{\partial p} + \frac{\partial \mathcal{J}_t}{\partial t}$$

$$\Rightarrow \begin{cases} \pi_t = \frac{P_t}{\psi Y_t} \frac{\partial \mathcal{J}_t}{\partial p} \\ r_t^b \frac{\partial \mathcal{J}_t}{\partial p} = -\phi(1 - m_t) \frac{Y_t}{P_t} + \frac{Y_t}{P_t} + \pi_t \frac{\partial \mathcal{J}_t}{\partial p} + P_t \pi_t \frac{\partial^2 \mathcal{J}_t}{\partial p^2} + \frac{\partial^2 \mathcal{J}_t}{\partial p \partial t} \end{cases}$$

This implies the Phillips Curve

$$\left(r_t^b - \frac{\dot{Y}_t}{Y_t} \right) \pi_t = \frac{\phi - 1}{\psi} \left(\frac{\phi}{\phi - 1} m_t - 1 \right) + \dot{\pi}_t$$

Infinitesimal generator back

$$\begin{cases} \mathcal{A}_{i,t}\mathcal{V}_{i,t} = \dot{b}_{i,t}\frac{\partial \mathcal{V}_{i,t}}{\partial b} + \rho_\varepsilon \varepsilon_{i,t}(\bar{e} - e_{i,t})\frac{\partial \mathcal{V}_{i,t}}{\partial \varepsilon} + \varepsilon_{i,t}\frac{\sigma_\varepsilon^2}{2}\frac{\partial^2 \mathcal{V}_{i,t}}{\partial \varepsilon^2} \\ \dot{b}_{i,t} = \lambda W_t l_{i,t}\varepsilon_{i,t} + r_t^b b_{i,t} + T_t + d_t - c_{i,t} \end{cases}$$

$$\mathcal{A}_{i,t}\mathcal{V}_{i,t} = \left(\lambda W_t l_{i,t}\varepsilon_{i,t} + r_t^b b_{i,t} + T_t + d_t - c_{i,t}\right)\frac{\partial \mathcal{V}}{\partial b} + \rho_\varepsilon \varepsilon_{i,t}(\bar{e} - e_{i,t})\frac{\partial \mathcal{V}}{\partial \varepsilon} + \varepsilon_{i,t}\frac{\sigma_\varepsilon^2}{2}\frac{\partial^2 \mathcal{V}}{\partial \varepsilon^2}$$

Infinitesimal generator $\mathcal{A}_{i,t}$ of HJB equation is adjacent to the $\mathcal{A}_{i,t}^*$ of the Fokker–Planck equation more

Fokker–Planck / Kolmogorov forward equation back

$$\langle g, \mathcal{A}^* h\rangle = \int_\varepsilon \int_b g\mathcal{A}^* h db d\varepsilon =$$

$$= \int_\varepsilon \int_b g \left(-\frac{\partial}{\partial b}\left\{ \left(\lambda W l \varepsilon + d + T + r^b b - c \right) h \right\} - \frac{\partial}{\partial \varepsilon}\rho_\varepsilon \varepsilon(\bar{e} - e)h + \frac{\partial^2}{\partial \varepsilon^2}\varepsilon \frac{\sigma_\varepsilon^2}{2} h \right) db d\varepsilon =$$

$$= -\int_\varepsilon \int_b g \frac{\partial}{\partial b}\dot{b} h db d\varepsilon - \int_b \int_\varepsilon g \frac{\partial}{\partial \varepsilon}\dot{\varepsilon} h d\varepsilon db + \int_b \int_\varepsilon g \frac{\partial^2}{\partial \varepsilon^2}\frac{\sigma_\varepsilon^2}{2} h d\varepsilon db =$$

$$= \langle \mathcal{A} g, h\rangle - \int_\varepsilon \left[g\dot{b}h\Big|_{\underline{b}}^{\infty} \right] d\varepsilon - \int_b \left[g\dot{\varepsilon}h\Big|_0^{\infty} \right] db + \int_b \left[g\frac{\partial}{\partial \varepsilon}\frac{\sigma_\varepsilon^2}{2} h \Big|_0^{\infty} \right] db - \int_b \left[\frac{\sigma_\varepsilon^2}{2} h \frac{\partial}{\partial \varepsilon} g \Big|_0^{\infty} \right] db$$

$$\mathcal{A}_{i,t}\mathcal{V}_{i,t} = \dot{b}_{i,t}\frac{\partial \mathcal{V}_{i,t}}{\partial b} + \rho_\varepsilon \varepsilon_{i,t}(\bar{e} - e_{i,t})\frac{\partial \mathcal{V}_{i,t}}{\partial \varepsilon} + \varepsilon_{i,t}\frac{\sigma_\varepsilon^2}{2}\frac{\partial^2 \mathcal{V}_{i,t}}{\partial \varepsilon^2}$$

$$\mathcal{A}_{i,t}^* f_{i,t} = -\frac{\partial}{\partial b}\dot{b}_{i,t} f_{i,t} - \frac{\partial}{\partial \varepsilon}\rho_\varepsilon \varepsilon_{i,t}(\bar{e} - e_{i,t}) f_{i,t} + \frac{\partial^2}{\partial \varepsilon^2}\varepsilon_{i,t}\frac{\sigma_\varepsilon^2}{2} f_{i,t}$$

Optimal Policy back

$$\mathcal{L}[f, \mathcal{V}, c, l, W, Y, \pi, r^b, T] =$$

$$= \int_0^\infty e^{-\rho t} \left[\left\langle \frac{c_{i,t}^{1-\nu}}{1-\nu} - \varphi \frac{l_{i,t}^{1+\gamma}}{1+\gamma}, f_{i,t} \right\rangle + \left\langle \zeta_{i,t}, \mathcal{A}_{i,t}^* f_{i,t} - \frac{\partial f_{i,t}}{\partial t} \right\rangle \right. \qquad \textbf{(Objective); (Distribution LOM)}$$

$$+ \left\langle \varrho_{i,t}, \frac{c_{i,t}^{1-\nu}}{1-\nu} - \varphi \frac{l_{i,t}^{1+\gamma}}{1+\gamma} + \mathcal{A}_{i,t} \mathcal{V}_{i,t} + \frac{\partial \mathcal{V}_{i,t}}{\partial t} - \rho \mathcal{V}_{i,t} \right\rangle \qquad \textbf{(Household HJB)}$$

$$+ \left\langle \mu_{i,t}, c_{i,t}^{-\nu} - \frac{\partial \mathcal{V}_{i,t}}{\partial b} \right\rangle + \left\langle \kappa_{i,t}, l_{i,t}^{\gamma} c_{i,t}^{-\nu} - \frac{\lambda W_t \varepsilon_{i,t}}{\varphi} \right\rangle \qquad \textbf{(Consumption); (Labor supply)}$$

$$+ \eta_{b,t} \langle b_{i,t}, f_{i,t} \rangle + \eta_{Y,t} \left(Y_t - \theta_t \langle l_{i,t} \varepsilon_{i,t}, f_{i,t} \rangle \right) \qquad \textbf{(Bond market); (Output)}$$

$$+ \eta_{T,t} \left(T_t - \left(1 - \frac{\psi}{2} \pi_t^2 - \lambda \frac{W_t}{\theta_t} \right) Y_t \right) \qquad \textbf{(Government budget constraint)}$$

$$\left. + \eta_{\pi,t} \left(\frac{\phi - 1}{\psi} \left(\frac{\phi}{\phi - 1} \frac{W_t}{\theta_t} - 1 \right) + \dot{\pi}_t - \left(r_t^b - \frac{\dot{Y}_t}{Y_t} \right) \pi_t \right) \right] dt \qquad \textbf{(Phillips Curve)}$$

Calculus of Variations back

- Maximization with respect to functions
- Control over inflation:
  - **Weak form** (looking at total variation of v_t)

$$\frac{\delta \mathcal{L}}{\delta \pi} = \int_0^\infty e^{-\rho t} \left[\eta_{T,t} \psi \pi_t v_t Y_t + \eta_{\pi,t} \left(\dot{v}_t - \left(r_t^b - \frac{\dot{Y}_t}{Y_t} \right) v_t \right) \right] dt = 0$$

Using integration by parts to substitute $\dot{v}_t$

$$\int_0^\infty e^{-\rho t} \left[\eta_{T,t} \psi \pi_t v_t Y_t - \eta_{\pi,t} \left(r_t^b - \frac{\dot{Y}_t}{Y_t} \right) v_t - \dot{\eta}_{\pi,t} v_t + \rho \eta_{\pi,t} v_t \right] dt + e^{-\rho t} \eta_{\pi,t} v_t \Big|_0^\infty = 0$$

  - **Strong form** (Since v_t can be chosen freely, every part of the function has to be zero)

$$\eta_{T,t} \psi \pi_t Y_t - \eta_{\pi,t} \left(r_t^b - \frac{\dot{Y}_t}{Y_t} - \rho \right) - \dot{\eta}_{\pi,t} = 0$$

$$\eta_{\pi,0} = 0$$

Duality

- Symmetry between the original problem and the OP costate variables problem
- Phillips Curve is a **forward looking** differential equation in π_t (has to be solved backward)
  Solution uniqueness is given by the boundary constraint at $t \to \infty$

$$\dot{\pi}_t = \left(r_t^b - \frac{\dot{Y}_t}{Y_t} \right) \pi_t - \frac{\phi - 1}{\psi} \left(\frac{\phi}{\phi - 1} \frac{W_t}{\theta_t} - 1 \right)$$
$$\lim_{t \to \infty} \pi_t = \pi$$

- Associated costate equation is a **backward looking** differential equation in $\eta_{\pi,t}$ (hast to be solved forward)

$$\dot{\eta}_{\pi,t} = \eta_{\pi,t} \left(\rho + \frac{\dot{Y}_t}{Y_t} - r_t^b \right) + \eta_{T,t} \psi \pi_t Y_t$$
$$\eta_{\pi,0} = 0$$

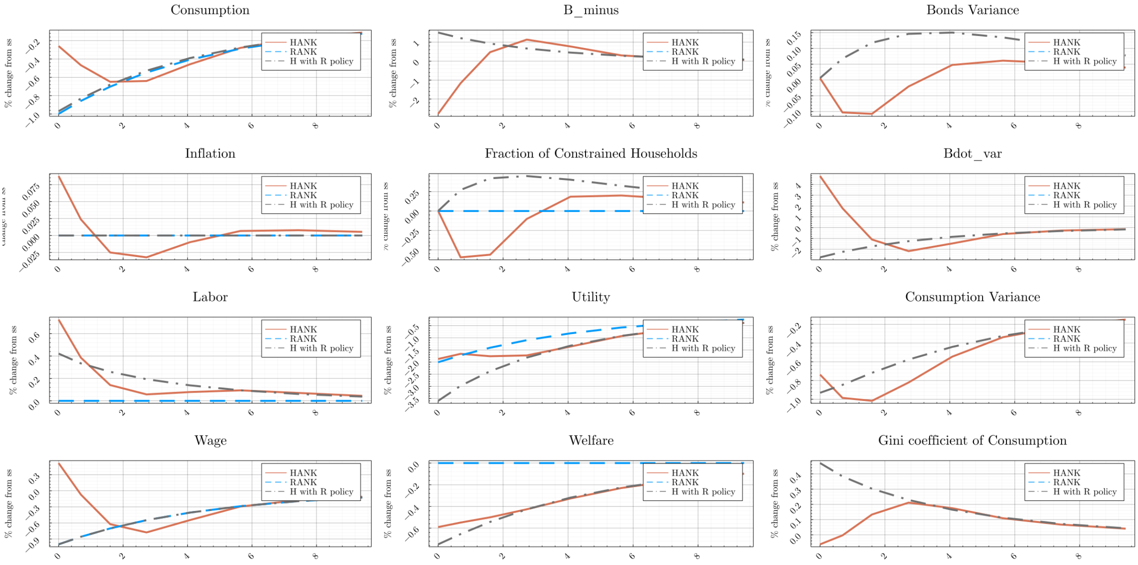
- Same duality holds for the rest of the differential equations constraints
- Importantly, for the **HJB on the borrowing limit**

Calibration

back

Fixed	Description	Value		
ν	Risk aversion	1		
$1/\gamma$	Frisch elasticity of labor supply	1		
ϕ	Price elasticity of demand	10	(slope of the Phillips Curve	
ψ	Price adjustment cost	100	$\phi/\psi = 0.1$)	
Fitted	**Description**	**Value**	**Moment**	**Value**
ρ	Discount rate	0.067	real return	3%
$\underline{b}$	Borrowing limit	-3.54	% constrained	30%
ρ_e	Mean reversion	0.1	var $log(average\ LI)$	0.7
σ_e	Volatility	0.32	var $\Delta(average\ LI)$	0.23

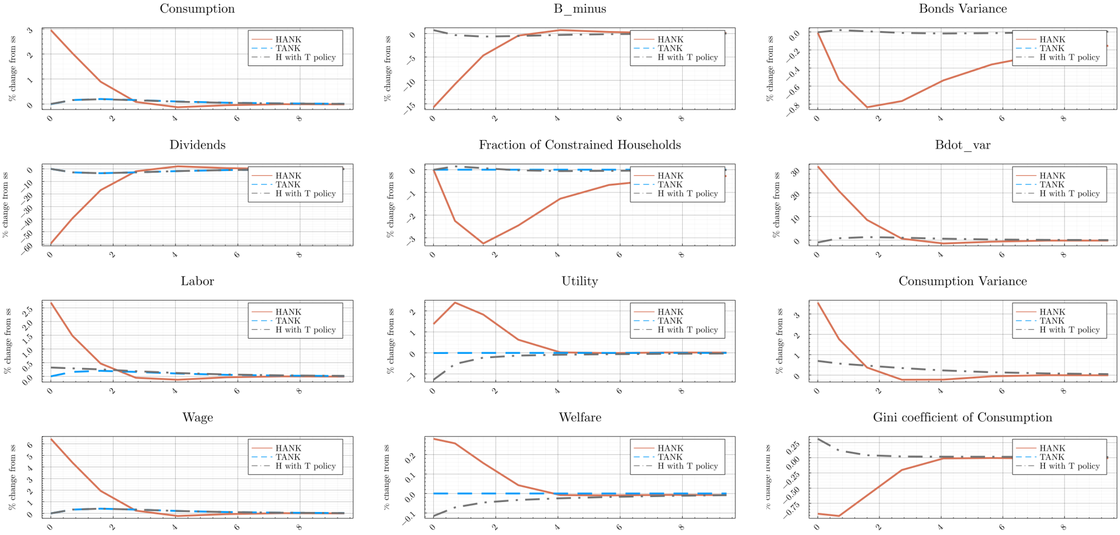
TFP back



Consumption
B_minus
Bonds Variance
Inflation
Fraction of Constrained Households
Bdot_var
Labor
Utility
Consumption Variance
Wage
Welfare
Gini coefficient of Consumption
HANK
RANK
H with R policy
% change from ss

Markup

back



Consumption
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HANK
TANK
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