

U.S. Monetary Policy and Indeterminacy

Giovanni Nicolò
Federal Reserve Board

Econometric Society, European Meeting

August 23, 2022

Disclaimer: The views expressed in this paper are those of the author and do not necessarily reflect those of the Federal Reserve Board or the Federal Reserve System.

Motivation

Two common approaches to study role of U.S. monetary policy (MP) in post-war period:

- 1 Models with *stylized* structure (i.e. small-scale)
 - Use method of Lubik and Schorfheide (2004) (LS) to allow for 'passive' MP;
- Before 1979: 'passive' MP → failure to stabilize inflation and output → indeterminacy

Motivation

Two common approaches to study role of U.S. monetary policy (MP) in post-war period:

① Models with *stylized* structure (i.e. small-scale)

- Use method of Lubik & Schorfheide (2004) (LS) to allow for 'passive' MP;
- Before 1979: 'passive' MP → failure to stabilize inflation and output → indeterminacy

② Models with *rich* structure (i.e. medium-scale)

- Few studies overcome technical challenges to implement LS and allow for 'passive' MP;
- **Contrasting results on role of MP (Justiniano & Primiceri, 2008; Hirose et al., 2021).**

Motivation

However, two choices are relevant when studying the conduct of U.S. monetary policy:

- 1 The choice of small-scale models (Beyer and Farmer, 2007);
 - Under **passive** MP, expectations affect macroeconomy by introducing:
 - **More persistence** in the propagation of *all* shocks (expectations are fundamental drivers);
 - **More volatility** (due to newly-introduced sunspot shocks);

→ **Stylized models may favor evidence of passive MP.**
- 2 The choice of existing solution methods;
 - Technical and computational difficulties could challenge the search for a global maximum;

→ **Alternative methods may deliver different results, especially in rich models.**

This paper

- 1 I show that two features can have relevant empirical implications for findings:
 - Rich structural model + novel method of Bianchi and Nicolò (2021) (BN).
- 2 I study the systematic conduct of U.S. monetary policy in the post-war period;
 - Estimate the medium-scale, New Keynesian model of Smets and Wouters (SW, 2007);
 - Implement BN to allow for passive MP;
 - Use Hybrid Metropolis-Hastings algorithm to efficiently explore entire parameter space.
- 3 I verify robustness of results to alternative model specifications and data used.

Findings

- 1 The adoption of medium-scale model is key:
 - Simulate determinate version of SW model;
 - Estimation of small-scale model → incorrect evidence of indeterminacy
- 2 The adoption of BN is also key:
 - Simulate indeterminate version of SW model;
 - Estimation of SW model with LS → incorrect evidence of determinacy
- 3 Monetary policy was passive before 1979;
 - Robust to alternative specifications of SW model and data used.
- 4 The evidence of active monetary policy after 1979 is overturned if:
 - Volcker-disinflation period is excluded;
 - SW model allows for time-varying inflation target (also with data on infl. expectations).

Model, data, solution and estimation methods

- Smets and Wouters (2007) model (SW)
 - Medium-scale DSGE model with nominal and real frictions + seven exogenous shocks [Details](#)
- Three sub-samples
 - Pre-1979: 1955:Q4 - 1979:Q2
 - Post-1979: 1979:Q3 - 2007:Q3 (Boivin & Giannoni, 2006; Leduc et al., 2007)
 - Post-1982: 1982:Q4 - 2007:Q3 (as in LS)
- Solution method: Bianchi and Nicolò (2021)
 - Augmented representation to solve a LRE model over entire parameter space [Details](#)
- Estimation method: Hybrid Metropolis-Hastings algorithm in Bianchi and Nicolò (2021)
 - Explicitly accounts for local peaks in various regions of the parameter space [Details](#)

Feature #1: Relevance of adopting a medium-scale model

Under a passive monetary policy, the propagation of *all* shock is more persistent, and additional sunspot shocks are introduced.

These features may erroneously favor evidence of passive monetary policy, especially in small-scale models (Beyer and Farmer, 2007):

- 1 Richer *dynamic* structure could explain the *persistence* in macro data without recurring to the additional persistence due to role of expectations as fundamental business-cycle drivers;
- 2 Richer *stochastic* structure could explain the *volatility* in macro data without recurring to the additional sunspot shocks.

Feature #1: Relevance of adopting a medium-scale model

Consider small-scale Del Negro and Schorfheide (2004) model → Three main differences:

- Richness of the structural model;
- Taylor rules used in the two models;
- Additional time series used for the estimation of SW model.

Feature #1: Relevance of adopting a medium-scale model

Consider small-scale Del Negro and Schorfheide (2004) model → Three main differences:

- Richness of the structural model;
- Taylor rules used in the two models;
- Additional time series used for the estimation of SW model.

To disentangle these channels,

- Generate a long simulation using a determinate version of SW model;
- Use the last 500 observations to estimate the following models:
 - 1 Del Negro and Schorfheide (2004);
 - 2 Del Negro and Schorfheide (2004) with Taylor rule as in SW;
 - 3 SW model estimated using only the same three time series used for the small-scale model.

DS model details

Feature #1: Relevance of adopting a medium-scale model

| Model | Posterior mode | | Prob.Determ. |
|--|----------------|---------------|--------------|
| | Determinacy | Indeterminacy | |
| Del Negro and Schorfheide | -623.5 | -581.7 | 0 |
| Del Negro and Schorfheide + Taylor rule as in SW | -621.7 | -581.6 | 0 |
| SW model + time series as in Del Negro and Schorfheide | -523.1 | -534.5 | 1 |

- The two versions of the small-scale model erroneously favor evidence of a passive monetary policy.
- When a richer model structure is considered, the results correctly indicate that the simulated data is consistent with an active monetary policy.

Feature #2: Relevance of adopting BN

- Most studies that allow for passive monetary policy adopt the solution method of LS;
- Because of the technical and computational challenges to implement this method, small-scale models are generally adopted;
- Few exceptions:
 - Contrasting evidence (Justiniano and Primiceri, 2008; Hirose et al., 2021);
 - Adopt method of LS.
- However, the choice of solution method/baseline solution is relevant for the study of the conduct of monetary policy, especially in rich models.

Details

Feature #2: Relevance of adopting BN

To show the empirical relevance of the choice of solution method,

- Generate a long simulation using an indeterminate version of SW model;
- Use the last 500 observations to estimate the SW model using both alternative methods.

Feature #2: Relevance of adopting BN

To show the empirical relevance of the choice of solution method,

- Generate a long simulation using SW model + posterior means from pre-Volcker period;
- Use the last 500 observations to estimate the SW model using both alternative methods.

| Model | Posterior mode | | Prob.Determ. |
|----------------------------------|----------------|---------------|--------------|
| | Determinacy | Indeterminacy | |
| SW model + Bianchi and Nicolò | -2663.3 | -2590.9 | 0 |
| SW model + Lubik and Schorfheide | -2661.7 | -2769.6 | 1 |

- The model estimation using LS erroneously favors evidence of an active monetary policy.
- When the model is estimated using BN, the results correctly indicate that the simulated data is consistent with a passive monetary policy.

U.S. Monetary policy in the post-war period

Three sub-samples:

- Pre-1979: 1955:Q4 - 1979:Q2
- Post-1979: 1979:Q3 - 2007:Q3 (Boivin & Giannoni, 2006; Leduc et al., 2007)
- Post-1982: 1982:Q4 - 2007:Q3 (as in LS)

| | Posterior mode | | Prob. Determ. |
|-----------|----------------|---------------|---------------|
| | Determinacy | Indeterminacy | |
| Pre-1979 | -546.3 | -525.1 | 0 |
| Post-1979 | -567.1 | -584.8 | 1 |
| Post-1982 | -377.1 | -375.3 | 0 |

→ Pre-1979: indeterminacy result carries over to medium-scale SW model.

→ Post-1979: determinacy result overturned if Volcker-disinflation period is excluded.

U.S. Monetary policy in the post-war period

| Coefficient | Description | Pre-1979 | | Post-1979 | | Post-1982 | |
|----------------|---|----------|-------------|-----------|-------------|-----------|-------------|
| | | Mean | [5 , 95] | Mean | [5 , 95] | Mean | [5 , 95] |
| r_π | Taylor rule inflation | 0.86 | [0.68,0.97] | 2.10 | [1.76,2.47] | 0.75 | [0.34,0.96] |
| r_y | Taylor rule output gap | 0.14 | [0.08,0.21] | 0.07 | [0.03,0.11] | 0.10 | [0.03,0.18] |
| $r_{\Delta y}$ | Taylor rule $\Delta(\text{output gap})$ | 0.16 | [0.12,0.21] | 0.20 | [0.15,0.25] | 0.14 | [0.09,0.20] |
| ρ | Taylor rule smoothing | 0.85 | [0.78,0.92] | 0.78 | [0.73,0.82] | 0.88 | [0.81,0.93] |

- Relative to pre-1979, MP response in post-1979: $\uparrow r_\pi$, $\downarrow r_y$ and $\downarrow \rho$
- Relative to post-1979, MP response in post-1982: $\downarrow r_\pi$ and $\uparrow \rho$
- Similar estimates of Taylor-rule coefficients for pre-1979 and post-1982.

U.S. Monetary policy in the post-war period

| Coefficient | Description | Pre-1979 | | Post-1979 | | Post-1982 | |
|--------------|---------------------------|----------|-------------|-----------|-------------|-----------|-------------|
| | | Mean | [5 , 95] | Mean | [5 , 95] | Mean | [5 , 95] |
| σ_a | Technology shock | 0.56 | [0.49,0.64] | 0.39 | [0.34,0.44] | 0.38 | [0.33,0.42] |
| σ_b | Risk premium shock | 0.17 | [0.11,0.23] | 0.22 | [0.19,0.26] | 0.17 | [0.06,0.22] |
| σ_g | Government sp. shock | 0.52 | [0.46,0.59] | 0.47 | [0.42,0.52] | 0.40 | [0.35,0.45] |
| σ_I | Investment-specific shock | 0.52 | [0.40,0.65] | 0.39 | [0.32,0.46] | 0.38 | [0.28,0.51] |
| σ_r | Monetary policy shock | 0.18 | [0.15,0.20] | 0.23 | [0.20,0.26] | 0.12 | [0.10,0.14] |
| σ_p | Price mark-up shock | 0.30 | [0.25,0.35] | 0.10 | [0.07,0.12] | 0.14 | [0.10,0.18] |
| σ_w | Wage mark-up shock | 0.28 | [0.23,0.32] | 0.31 | [0.25,0.37] | 0.30 | [0.24,0.37] |
| σ_ν | Sunspot shock | 0.13 | [0.03,0.22] | 0.50 | [0.05,0.95] | 0.14 | [0.08,0.21] |

- Relative to pre-1979:
 - Some exog. shocks in post-1979 have smaller std. dev. (vs higher: risk premium and MP).
 - *All* exog. shocks in post-1982 have similar or smaller std. dev.
 - Std. dev. of sunspot shock is only identified in pre-1979 and post-1982.

Posterior distributions

Conclusions

- 1 Two key features for the study of the systematic conduct of U.S. monetary policy:
 - *Both* a rich structural model and the method of BN.
- 2 Evidence of passive monetary policy before 1979 is strong and robust.
- 3 Evidence of active monetary policy after 1979 is weak and sensitive to:
 - Exclusion of Volcker-disinflation period;
 - Adoption of time-varying inflation target (even when using data on inflation expectations).

The Smets-Wouters Model in a Nutshell

- Medium-scale DSGE model with both nominal and real frictions:
 - External habit formation in consumption;
 - Variable capital utilization;
 - Investment adjustment cost;
 - Fixed cost in production
 - Sticky nominal prices and wages;
 - Indexation to past inflation.
- Seven exogenous shocks to match the seven observables used in the estimation:
 - “Demand” shocks: Exogenous gov. spending, risk-premium and investment-specific;
 - “Supply” shocks: Productivity, price mark-up and wage mark-up;
 - Monetary policy shock:

$$R_t = \rho R_{t-1} + (1 - \rho)\{r_\pi \pi_t + r_Y(y_t - y_t^p)\} + r_{\Delta y}[(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \varepsilon_t^r$$

Data and Measurement Equations

- The measurement equation used to estimate the data is:

$$Y_t^{obs} = \begin{bmatrix} dIGDP_t \\ dICONS_t \\ dIINV_t \\ dIWAG_t \\ lHours_t \\ dIP_t \\ FEDFUNDS_t \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{\gamma} \\ \bar{l} \\ \bar{\pi} \\ \bar{r} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ l_t \\ \pi_t \\ r_t \end{bmatrix}$$

- Three sub-samples:
 - Pre-1979: 1955:Q4 - 1979:Q2
 - Post-1979: 1979:Q3 - 2007:Q3 (Boivin & Giannoni, 2006; Leduc et al., 2007)
 - Post-1982: 1982:Q4 - 2007:Q3 (as in LS)

Solution method: BN

I solve the SW model by implementing the novel method of BN.

The approach proposes an augmented representation of a LRE model and:

- Solves the model over the *entire* parameter space;
- Is applicable when region of determinacy and degrees of indeterminacy are *unknown*;
- Can be easily combined with standard as well as sophisticated estimation algorithms.

Solution Method: Building the Intuition

Consider a classical monetary model described by the Fisher Equation

$$R_t = r_t + E_t(\pi_{t+1}), \quad r_t \sim \mathcal{N}(0, \sigma_r^2) \quad (1)$$

the Taylor rule

$$R_t = \phi_\pi \pi_t \quad (2)$$

and $\eta_t \equiv \pi_t - E_{t-1}(\pi_t)$.

Combining (1) and (2), the model becomes a univariate LRE model

$$\begin{cases} E_t(\pi_{t+1}) = \phi_\pi \pi_t - r_t \\ \eta_t = \pi_t - E_{t-1}(\pi_t) \end{cases}$$

Augmented Representation

The approach proposes to solve the augmented system:

$$\begin{cases} E_t(\pi_{t+1}) = \phi_\pi \pi_t - r_t \\ \omega_t = \left(\frac{1}{\alpha}\right) \omega_{t-1} - \nu_t + \eta_t \end{cases}$$

Solution in Augmented Model

The approach proposes to solve the augmented system:

$$\begin{cases} E_t(\pi_{t+1}) = \phi_\pi \pi_t - r_t \\ \omega_t = \left(\frac{1}{\alpha}\right) \omega_{t-1} - \nu_t + \eta_t \end{cases}$$

Suppose $|\phi_\pi| > 1$.

- If $|\frac{1}{\alpha}| \leq 1 \rightarrow$ the solution for the augmented representation is

$$\begin{cases} \pi_t = \frac{1}{\phi_\pi} r_t \\ \omega_t = \left(\frac{1}{\alpha}\right) \omega_{t-1} - \nu_t + \eta_t \end{cases}$$

- If $|\frac{1}{\alpha}| > 1 \rightarrow$ *No solution for the augmented representation (boundedness is violated).*

Solution in Augmented Model

The approach proposes to solve the augmented system:

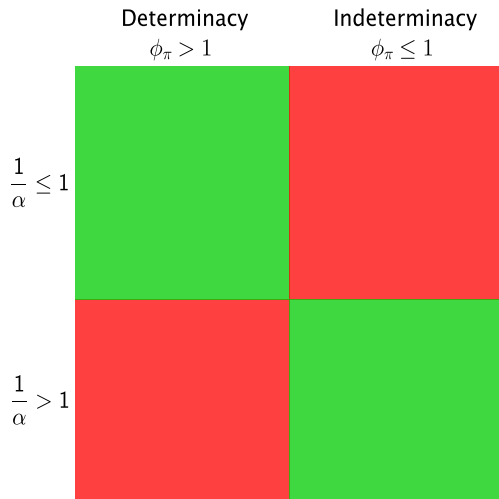
$$\begin{cases} E_t(\pi_{t+1}) = \phi_\pi \pi_t - r_t \\ \omega_t = \left(\frac{1}{\alpha}\right) \omega_{t-1} - \nu_t + \eta_t \end{cases}$$

Suppose $|\phi_\pi| \leq 1$.

- If $|\frac{1}{\alpha}| \leq 1 \rightarrow$ *No solution for the augmented representation (BK condition).*
- If $|\frac{1}{\alpha}| > 1 \rightarrow$ the solution for the augmented representation is

$$\begin{cases} \pi_t = \phi_\pi \pi_{t-1} - r_{t-1} + \eta_t \\ \omega_t = 0 \end{cases}$$

Solution in Augmented Model



Solution in Augmented Model

| | Determinacy $\phi_\pi > 1$ | Indeterminacy $\phi_\pi \leq 1$ |
|---------------------------|--|---|
| $\frac{1}{\alpha} \leq 1$ | $\begin{cases} \pi_t = \frac{1}{\phi_\pi} r_t \\ \omega_t = \frac{1}{\alpha} \omega_{t-1} - \nu_t + r_t \end{cases}$ | |
| $\frac{1}{\alpha} > 1$ | | $\begin{cases} \pi_t = \phi_\pi \pi_{t-1} - r_{t-1} + \eta_t \\ \omega_t = 0 \end{cases}$ |

How to choose α

Consider the following two cases:

① The researcher knows the region of determinacy ($\phi_\pi > 1$)

→ Set $\alpha \equiv \phi_\pi$.

② The region of determinacy is completely unknown

→ Set uniform prior for α over the interval $[1 - a, 1 + a]$, where $a \in (0, 1)$.

◀ Back

Estimation method: Hybrid Metropolis-Hastings algorithm

- At the boundaries between the regions of (in)determinacy, the posterior often presents possibly severe discontinuities.
- Also, these models often have local peaks in the posterior.
- Methods, such as the sequential Monte Carlo algorithm (Herbst and Schorfheide, 2015), tend to overcome these challenges.

→ In this paper, I adopt the Hybrid Metropolis-Hastings algorithm developed in BN.

Hybrid Metropolis-Hastings algorithm: Steps

- 1 In each region of (in)determinacy $j = 1, \dots, J$, apply a numerical optimization procedure to search for modes $\tilde{\theta}_{(j)}$ of the posterior density and compute the inverse of the Hessian, $\tilde{\Sigma}_{(j)}$.

Hybrid Metropolis-Hastings algorithm: Steps

- 1 In each region of (in)determinacy $j = 1, \dots, J$, apply a numerical optimization procedure to search for modes $\tilde{\theta}_{(j)}$ of the posterior density and compute the inverse of the Hessian, $\tilde{\Sigma}_{(j)}$.
- 2 Let $q_j(\theta)$ be the density of a multivariate distribution obtained mixing two normals:

$$q_j(\theta) = z^l N\left(\tilde{\theta}_{(j)}, c_j^l \tilde{\Sigma}_{(j)}\right) + (1 - z^l) N\left(\tilde{\theta}_{(j)}, c_j^s \tilde{\Sigma}_{(j)}\right), \quad c_j^s < c_j^l \text{ and } z^l \in [0, 1].$$

Hybrid Metropolis-Hastings algorithm: Steps

- 1 In each region of (in)determinacy $j = 1, \dots, J$, apply a numerical optimization procedure to search for modes $\tilde{\theta}_{(j)}$ of the posterior density and compute the inverse of the Hessian, $\tilde{\Sigma}_{(j)}$.
- 2 Let $q_j(\theta)$ be the density of a multivariate distribution obtained mixing two normals:

$$q_j(\theta) = z^l N\left(\tilde{\theta}_{(j)}, c_j^l \tilde{\Sigma}_{(j)}\right) + (1 - z^l) N\left(\tilde{\theta}_{(j)}, c_j^s \tilde{\Sigma}_{(j)}\right), \quad c_j^s < c_j^l \text{ and } z^l \in [0, 1].$$

- 3 After choosing a starting value $\theta^{(0)}$, follow these steps for $s = 1, \dots, nsim$:

- 1 Defining $q(\theta) \equiv \sum_{j=1}^J \pi_j q_j(\theta)$, make a draw ϑ from the following proposal distribution:

$$\tilde{q}(\vartheta | \theta^{(s-1)}) = w^{RW} N\left(\theta^{(s-1)}, c^{RW} \tilde{\Sigma}_{(j)}\right) + (1 - w^{RW}) q(\theta), \quad w^{RW} \in [0, 1].$$

- 2 Accept the jump from $\theta^{(s-1)}$ to ϑ ($\theta^{(s)} = \vartheta$) with probability $\min\{1, r_j(\theta^{(s-1)}, \vartheta | Y)\}$, otherwise reject the proposed draw and set $\theta^{(s)} = \theta^{(s-1)}$, where

$$r_j\left(\theta^{(s-1)}, \vartheta | Y\right) = \frac{\mathcal{L}(\vartheta | Y) p(\vartheta) / \tilde{q}(\vartheta | \theta^{(s-1)})}{\mathcal{L}(\theta^{(s-1)} | Y) p(\theta^{(s-1)}) / \tilde{q}(\theta^{(s-1)} | \vartheta)}$$

Identification Problem

Suppose a researcher uses two alternative models to study the dynamics of the inflation rate.

Model 1:

$$\pi_t = a E_t(\pi_{t+1}),$$

where the forecast error is

$$\eta_t \equiv \pi_t - E_{t-1}(\pi_t).$$

Identification Problem

Suppose a researcher uses two alternative models to study the dynamics of the inflation rate.

Model 1:

$$\pi_t = a E_t(\pi_{t+1}),$$

where the forecast error is

$$\eta_t \equiv \pi_t - E_{t-1}(\pi_t).$$

BK condition: Unique, determinate sol if \neq unstable roots equals \neq expectational variables.

The roots of the system are 0 and $\lambda \equiv a^{-1}$.

When $|\lambda| > 1 \Rightarrow$ **Unique, determinate** solution: $\pi_t = 0$.

When $|\lambda| \leq 1 \Rightarrow$ **Multiple, indeterminate** solution defined by **any** process of the form

$$\pi_t = \lambda\pi_{t-1} + \eta_t.$$

Identification Problem

Suppose a researcher uses two alternative models to study the dynamics of the inflation rate.

Model 2:

$$\pi_t = a E_t(\pi_{t+1}) + b \pi_{t-1} + \varepsilon_t, \quad \varepsilon_t \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_\varepsilon^2)$$

where the forecast error is

$$\eta_t \equiv \pi_t - E_{t-1}(\pi_t).$$

Identification Problem

Suppose a researcher uses two alternative models to study the dynamics of the inflation rate.

Model 2:

$$\pi_t = a E_t(\pi_{t+1}) + b \pi_{t-1} + \varepsilon_t, \quad \varepsilon_t \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_\varepsilon^2)$$

where the forecast error is

$$\eta_t \equiv \pi_t - E_{t-1}(\pi_t).$$

Denote the two roots of the system by $\{\lambda, \theta\}$ as a function of the structural parameters $\{a, b\}$.

Consider the case, $|\lambda| < 1$ and $|\theta| > 1$.

\Rightarrow **Unique, determinate** solution defined by

$$\pi_t = \lambda \pi_{t-1} + \frac{\lambda + \theta}{\theta} \varepsilon_t.$$

Identification Problem

The two models are observationally equivalent.

Model 1:

$$\pi_t = a E_t(\pi_{t+1}).$$

→ Indeterminate solution

$$\pi_t = \lambda \pi_{t-1} + \eta_t.$$

Model 2:

$$\pi_t = a E_t(\pi_{t+1}) + b \pi_{t-1} + \varepsilon_t.$$

→ Determinate solution

$$\pi_t = \lambda \pi_{t-1} + \frac{\lambda + \theta}{\theta} \varepsilon_t.$$

⇒ The adoption of the richer Model 2 overturns the results.

Identification Problem

The univariate model provides an analytical example of the identification problem due to observational equivalence.

A similar example *cannot* be analytically derived for more complicated models.

⇒ In this paper, I show empirically that the choice of the model structure impacts the conclusions on the conduct of monetary policy.

Del Negro and Schorfheide (2004) model

The Del Negro and Schorfheide (2004) model allows for a BGP and is described by:

- Dynamic IS curve

$$y_t = E_t(y_{t+1}) - \tau^{-1}(R_t - E_t(\pi_{t+1})) + (1 - \rho_g)g_t + \rho_z\tau^{-1}z_t$$

- NKPC

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa(y_t - g_t), \quad \beta = \frac{\gamma}{r^*}$$

- Monetary policy

$$R_t = \rho_R R_{t-1} + (1 - \rho_R)(\phi_\pi \pi_t + \phi_y y_t) + \varepsilon_{R,t}$$

- Demand and supply shocks: $g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$ and $z_t = \rho_z z_{t-1} + \varepsilon_{z,t}$.

◀ Back

Feature #2: Relevance of adopting BN

Consider a classical monetary model described by the Fisher Equation

$$R_t = r_t + E_t(\pi_{t+1}), \quad r_t \sim \mathcal{N}(0, \sigma_r^2) \quad (3)$$

the Taylor rule

$$R_t = \phi_\pi \pi_t \quad (4)$$

and $\eta_t \equiv \pi_t - E_{t-1}(\pi_t)$.

Combining (1) and (2), the model becomes a univariate LRE model

$$\begin{cases} E_t(\pi_{t+1}) = \phi_\pi \pi_t - r_t \\ \eta_t = \pi_t - E_{t-1}(\pi_t) \end{cases}$$

Feature #2: Relevance of adopting BN

$$\begin{cases} E_t(\pi_{t+1}) = \phi_\pi \pi_t - r_t, & r_t \sim \mathcal{N}(0, \sigma_r^2) \\ \eta_t = \pi_t - E_{t-1}(\pi_t) \end{cases}$$

- If $|\phi_\pi| > 1$ → Active monetary policy: $\left\{ \pi_t = \frac{1}{\phi_\pi} r_t, E_{t-1}(\pi_t) = 0, \eta_t = \frac{1}{\phi_\pi} r_t \right\}$.
- If $|\phi_\pi| \leq 1$ → Passive monetary policy:

Feature #2: Relevance of adopting BN

$$\begin{cases} E_t(\pi_{t+1}) = \phi_\pi \pi_t - r_t, & r_t \sim \mathcal{N}(0, \sigma_r^2) \\ \eta_t = \pi_t - E_{t-1}(\pi_t) \end{cases}$$

- If $|\phi_\pi| > 1 \rightarrow$ Active monetary policy: $\left\{ \pi_t = \frac{1}{\phi_\pi} r_t, E_{t-1}(\pi_t) = 0, \eta_t = \frac{1}{\phi_\pi} r_t \right\}$.
- If $|\phi_\pi| \leq 1 \rightarrow$ Passive monetary policy:

① Using BN:

$$\begin{cases} \pi_t = \phi_\pi \pi_{t-1} - r_{t-1} + \nu_t \\ \eta_t \equiv \nu_t \end{cases} \quad \begin{pmatrix} r_t \\ \nu_t \end{pmatrix} \sim \mathcal{MVN} \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_r^2 & \rho_{r,\nu} \\ \rho_{r,\nu} & \sigma_\nu^2 \end{pmatrix} \right).$$

② Using LS:

$$\begin{cases} \pi_t = \phi_\pi \pi_{t-1} - r_{t-1} + \eta_t \\ \eta_t \equiv M_r r_t + \zeta_t \end{cases} \quad \begin{pmatrix} r_t \\ \zeta_t \end{pmatrix} \sim \mathcal{MVN} \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\zeta^2 \end{pmatrix} \right).$$

Feature #2: Relevance of adopting BN

1 Using BN:

$$\begin{cases} \pi_t = \phi_\pi \pi_{t-1} - r_{t-1} + \nu_t \\ \eta_t \equiv \nu_t \end{cases} \quad \begin{pmatrix} r_t \\ \nu_t \end{pmatrix} \sim \mathcal{MVN} \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_r^2 & \rho_{r,\nu} \\ \rho_{r,\nu} & \sigma_\nu^2 \end{pmatrix} \right).$$

2 Using LS:

$$\begin{cases} \pi_t = \phi_\pi \pi_{t-1} - r_{t-1} + \eta_t \\ \eta_t \equiv M_r r_t + \zeta_t \end{cases} \quad \begin{pmatrix} r_t \\ \zeta_t \end{pmatrix} \sim \mathcal{MVN} \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\zeta^2 \end{pmatrix} \right).$$

The methods have two important differences:

- 1 In BN, the correlation $\rho_{r,\nu}$ has a well-defined domain.
- 2 In LS, the centering of the baseline solution around the determinate solution ($M_r = 1/\phi_\pi$) poses technical and computational difficulties, especially in rich models.

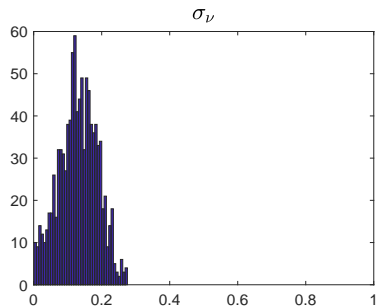
Prior distributions

| Coefficient | Description | Distr. | Mean | Std. Dev |
|-----------------------|-----------------------------------|--------|-------------|-------------|
| ϕ | Adjustment cost | Normal | 4.00 | 1.50 |
| σ_c | IES | Normal | 1.50 | 0.37 |
| h | Habit Persistence | Beta | 0.70 | 0.10 |
| σ_l | Labor supply elasticity | Normal | 2.00 | 0.75 |
| ξ_w | Wage stickiness | Beta | 0.50 | 0.10 |
| ξ_p | Price Stickiness | Beta | 0.50 | 0.10 |
| ι_w | Wage Indexation | Beta | 0.50 | 0.15 |
| ι_p | Price Indexation | Beta | 0.50 | 0.15 |
| ψ | Capacity utilization elasticity | Beta | 0.50 | 0.15 |
| Φ | Share of fixed costs | Normal | 1.25 | 0.12 |
| α | Share of capital | Normal | 0.30 | 0.05 |
| $\bar{\pi}$ | S.S. inflation rate (quart.) | Gamma | 0.62 | 0.10 |
| $100(\beta^{-1} - 1)$ | Discount factor | Gamma | 0.25 | 0.10 |
| \bar{l} | S.S. hours worked | Normal | 0.00 | 2.00 |
| $\bar{\gamma}$ | Trend growth rate | Normal | 0.40 | 0.10 |
| r_π | Taylor rule inflation | Normal | 1.00 | 0.35 |
| r_y | Taylor rule output gap | Normal | 0.12 | 0.05 |
| $r_{\Delta y}$ | Taylor rule Δ (output gap) | Normal | 0.12 | 0.05 |
| ρ | Taylor rule smoothing | Beta | 0.75 | 0.10 |

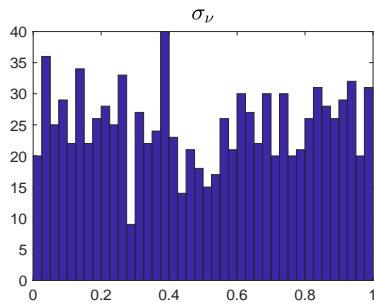
Prior distributions

| Coefficient | Description | Distr. | Mean | Std. Dev |
|----------------|---------------------------------|---------------|------|----------|
| σ_a | Technology shock | Invgamma | 0.10 | 2.00 |
| σ_b | Risk premium shock | Invgamma | 0.10 | 2.00 |
| σ_g | Government sp. shock | Invgamma | 0.10 | 2.00 |
| σ_I | Investment-specific shock | Invgamma | 0.10 | 2.00 |
| σ_r | Monetary policy shock | Invgamma | 0.10 | 2.00 |
| σ_p | Price mark-up shock | Invgamma | 0.10 | 2.00 |
| σ_w | Wage mark-up shock | Invgamma | 0.10 | 2.00 |
| σ_ν | Sunspot shock | Uniform[0,1] | 0.50 | 0.29 |
| ρ_a | Persistence technology | Beta | 0.50 | 0.20 |
| ρ_b | Persistence risk premium | Beta | 0.50 | 0.20 |
| ρ_g | Persistence government sp. | Beta | 0.50 | 0.20 |
| ρ_I | Persistence investment-specific | Beta | 0.50 | 0.20 |
| ρ_r | Persistence monetary policy | Beta | 0.50 | 0.20 |
| ρ_p | Persistence price mark-up | Beta | 0.50 | 0.20 |
| ρ_w | Persistence wage mark-up | Beta | 0.50 | 0.20 |
| μ_p | Mov. Avg. term, price mark-up | Beta | 0.50 | 0.20 |
| μ_w | Mov. Avg. term, wage mark-up | Beta | 0.50 | 0.20 |
| ρ_{ga} | $Cov(\sigma_a, \sigma_g)$ | Normal | 0.50 | 0.25 |
| $\rho_{\nu p}$ | $Corr(\sigma_\nu, \sigma_p)$ | Uniform[-1,1] | 0 | 0.57 |

U.S. Monetary policy in the post-war period



(a) Pre-1979



(b) Post-1979

→ Std. dev. of sunspot shock is only identified in pre-1979 and post-1982 (not shown).

Robustness Analysis

- Time-varying inflation target.
- Time-varying inflation target with inflation expectations.
- Real-time data.

◀ Back

Time-varying Inflation Target

- A time-varying inflation target better captures low-frequency movements in inflation.
- Haque (2021): a time-varying target rules out indeterminacy with a small scale model.
 - Response to inflation gap is more aggressive when target is time-varying rather than fixed.

→ Add a **time-varying inflation target** to the SW model (Del Negro and Schorfheide, 2013):

$$R_t = \rho R_{t-1} + (1 - \rho) \{ r_\pi (\pi_t - \pi_t^*) + r_y (y_t - y_t^p) \} + r_{\Delta y} [(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + u_{R,t}$$

where

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \varepsilon_{\pi^*,t} \quad 0 < \rho_{\pi^*} < 1$$

◀ Back

Time-varying Inflation Target

| Model | Pre-1979 | | | Post-1979 | | |
|------------|------------------------|---------------|-----------|------------------------|---------------|-----------|
| | Posterior mode Det. | Indet. | Prob.Det. | Posterior mode Det. | Indet. | Prob.Det. |
| SW model | -546.3 | -525.1 | 0 | -567.1 | -584.8 | 1 |
| SW π^* | -534.4 | -520.5 | 0 | -564.4 | -557.1 | 0 |

→ Model with time-varying inflation target fits the data better.

Time-varying Inflation Target

| Model | Pre-1979 | | | Post-1979 | | |
|------------|------------------------|---------------------|-----------|------------------------|---------------------|-----------|
| | Posterior mode Det. | Prob.Det. Indet. | Prob.Det. | Posterior mode Det. | Prob.Det. Indet. | Prob.Det. |
| SW model | -546.3 | -525.1 | 0 | -567.1 | -584.8 | 1 |
| SW π^* | -534.4 | -520.5 | 0 | -564.4 | -557.1 | 0 |

- Model with time-varying inflation target better fits the data.
- Inclusion of time-varying target can affect findings on stance of U.S. monetary policy:
 - Pre-1979: indeterminacy result **persists**.
 - Post-1979: determinacy result is **overturned**.
- Differences from Haque (2021) can be attributed to model scale and solution method.

Time-varying Inflation Target and Inflation Expectations

- Data on inflation expectations can improve measurement of π_t^* .

→ Augment measurement equation of SW π^* model as in Del Negro and Schorfheide (2013):

$$\pi_t^{e,J} = \bar{\pi} + \mathbb{E}_t \left[\frac{1}{J} \sum_{k=1}^J \pi_{t+k} \right]$$

Time-varying Inflation Target and Inflation Expectations

- Data on inflation expectations can improve measurement of π_t^* .

→ Augment measurement equation of SW π^* model as in Del Negro and Schorfheide (2013):

$$\pi_t^{e,J} = \bar{\pi} + \mathbb{E}_t \left[\frac{1}{J} \sum_{k=1}^J \pi_{t+k} \right]$$

- Short-term inflation expectations ($\pi_t^{e,4}$):
 - Survey Professional Forecasters (SPF): 1-year-ahead avg. GDP price infl. (since 1970:Q2)
- Longer-term inflation expectations ($\pi_t^{e,40}$):
 - Blue Chip Economic Indicators survey: 10-year-ahead avg. CPI infl. (since 1979:Q4)
 - SPF: 10-year-ahead avg. GDP price infl. (since 1991:Q4)

Time-varying Inflation Target and Inflation Expectations

| Model | Pre-1979 | | | Post-1979 | | |
|------------------------|----------------|--------|-----------|----------------|--------|-----------|
| | Posterior mode | | Prob.Det. | Posterior mode | | Prob.Det. |
| | Det. | Indet. | | Det. | Indet. | |
| SW model | -546.3 | -525.1 | 0 | -567.1 | -584.8 | 1 |
| $SW\pi^*$ | -534.4 | -520.5 | 0 | -564.4 | -557.1 | 0 |
| $SW\pi^* + \pi^{e,4}$ | -584.8 | -571.1 | 0 | -485.2 | -451.0 | 0 |
| $SW\pi^* + \pi^{e,40}$ | - | - | - | -472.2 | -466.6 | 0 |

- Both periods: Estimation of $SW\pi^*$ model with $\pi^{e,4}$ points to indeterminacy.
 - Post-1979: same conclusion can be reached with $\pi^{e,40}$.
- Relative to $SW\pi^*$ model, inclusion of short- or longer-term inflation expectations does not affect the indeterminacy result.

Real-Time Data

- Orphanides (2001): use of real-time data can affect conclusions about conduct of MP.
 - Policymakers overestimated potential output in the 1970s (Orphanides 2002, 2003).

Real-Time Data

- Orphanides (2001): use of real-time data can affect conclusions about conduct of MP.
 - Policymakers overestimated potential output in the 1970s (Orphanides 2002, 2003).

→ Estimate the SW model using real-time data.

- Real-time data sourced from Philadelphia Fed's Real-Time Data Research Center.
- Pre- and post-1979 vintages of real-time data (as available at the end of the sample):
 - Real output, personal consumption expenditure, non-residential private domestic investment, and output price index.
- Remaining time series use most recent data available.

◀ Back

Real-Time Data

| Model | Pre-1979 | | | Post-1979 | | |
|---------------------------|------------------------|--------|-----------|------------------------|--------|-----------|
| | Posterior mode Det. | Indet. | Prob.Det. | Posterior mode Det. | Indet. | Prob.Det. |
| SW model | -546.3 | -525.1 | 0 | -567.1 | -584.8 | 1 |
| SW model + Real-time data | -543.9 | -522.8 | 0 | -598.2 | -605.7 | 1 |

→ Results with real-time data support findings of baseline SW model with most recent data.

◀ Back