Implications of Endogenous Cognitive Discounting

James Moberly[1]

EEA-ESEM Congress

25th August 2022

[1]University of Oxford

Motivation

Cognitive discounting (Gabaix (2020)) is very powerful in resolving new Keynesian puzzles

Gabaix provides a method to calculate choice of attention, but takes as given when analysing the model as a simplification. This is the prevailing approach in the literature

Question: Do the results change materially when the discount factor is endogenous?

This paper: Yes. Policy analysis and estimation results change very considerably when discount factor is endogenous.

Methodology

- Take the Gabaix derivation for the optimal choice of attention given macroeconomic dynamics
- Generalize to a broader class of models
- Define "attention equilibrium"
- Existence of equilibrium in simple version of the model
- Develop algorithm to estimate the model

Theoretical Applications

Today:

- Determinacy condition: Absent further assumptions, there always exists an indeterminate equilibrium when RE Taylor principle is violated, very different to exogenous discounting.

See the paper:

- Changes in the Taylor Rule
- Average inflation targeting

Identification & Empirical Applications

Today:

- Identification: Exogenous discounting model suffers from weak identification, and endogenous discounting resolves this
- Great Inflation: Indeterminacy ruled out as a possible cause, unlike previous studies with exogenous discounting

See the paper:

- External validation

Overview: cognitive discounting

The subjective time t expectation of output in $t+h$ is the rational expectation discounted by $m \in [0,1]$:

$$\tilde{E}_t y_{t+h} = m^h E_t y_{t+h}$$

Where y_t is the deviation of output from its steady state.

Allow separate discount factors for consumers m_c and firms m_f

Gabaix shows that this generates the aggregate Phillips and IS curves:

$$\pi_t = \beta M_f E_t \pi_{t+1} + \kappa y_t$$
$$y_t = M_c E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1})$$

Simple Model: Exogenous Attention (1)

$$\pi_t = \kappa y_t$$
$$y_t = M_c E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1})$$
$$i_t = \phi_\pi \pi_t$$

Solving the model:

$$\Rightarrow y_t = \delta(M_c) E_t y_{t+1}$$
$$\delta(M_c) = \frac{M_c + \kappa\sigma}{1 + \kappa\sigma\phi_\pi}$$

Determinacy condition:

$$\delta(M_c) < 1 \Rightarrow \phi_\pi > 1 - \frac{1 - M_c}{\kappa\sigma}$$

Simple Model: Exogenous Attention (2)

Determinacy:

$$y_t = 0$$

Indeterminacy:

$$y_t = \underbrace{\delta(M_c)^{-1}}_{\substack{\text{Persistence} \\ \text{of sunspot}}} y_{t-1} + \underbrace{\zeta_t}_{\substack{\text{Sunspot} \\ \text{shock}}}$$

Standard result for variance of an AR(1):

$$Var(y_t) = \frac{Var(\zeta_t)}{1 - \delta(M_c)^{-2}}$$

As $\delta(M_c)$ approaches 1, $Var(y_t)$ becomes unboundedly large

Simple Model: Endogenous Attention

Now let's suppose attention is endogenously chosen. Following Gabaix (2020), attention is chosen as:

$$\min_{m_c \in [m_{c,d},1]} \underbrace{L_c(m_c, M_c, \chi)}_{\text{Losses from inattention}} + \underbrace{C(m_c, \boldsymbol{\xi_c})}_{\text{Cost of attention}}$$

Refer to the optimal choice of attention using the mapping g_c:

$$g_c(M_c, \chi, \boldsymbol{\xi_c}) = \arg \min_{m_c \in [m_{c,d},1]} L_c(m_c, M_c, \chi) + C(m_c, \boldsymbol{\xi_c})$$

Simple Model: Endogenous Attention (2)

Using Gabaix's suggested linear cost of attention:

$$g_c(M_c, \boldsymbol{\chi}, \boldsymbol{\xi_c}) = \max\left(1 - \frac{k^2}{E\left[\left(\frac{\partial c}{\partial m}\right)^2\right]}, m_{c,d}\right)$$

Simple Model: Attention Equilibrium

Definition: An *equilibrium choice of attention* is a choice of attention $M_c(\chi, \boldsymbol{\xi_c})$ such that:

$$M_c(\chi, \boldsymbol{\xi_c}) = g_c(M_c(\chi, \boldsymbol{\xi_c}), \chi, \boldsymbol{\xi_c}) \tag{1}$$

Intuition: A choice of attention which implies a set of macroeconomic dynamics that in turn justify that choice of attention as optimal.

Simple Model: $\phi_\pi < 1$

Let's suppose $\phi_\pi < 1$. What equilibria exist?

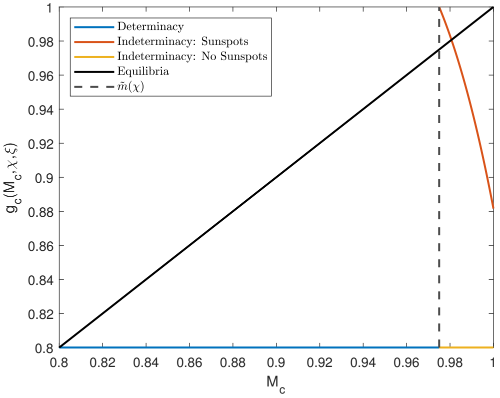


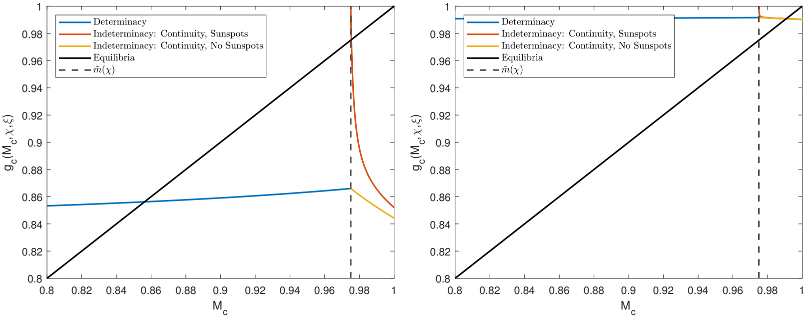
Figure: Attention equilibria: $\phi_\pi < 1$

Simple Model: Adding Fundamental Shocks

Now add an AR(1) fundamental shock, $\tilde{v}_t$, with persistence $\rho_v > 0$

$$y_t = \delta E_t y_{t+1} - \tilde{v}_t$$

Does a determinate equilibrium exists? It depends on $Var(\tilde{v}_t)$:



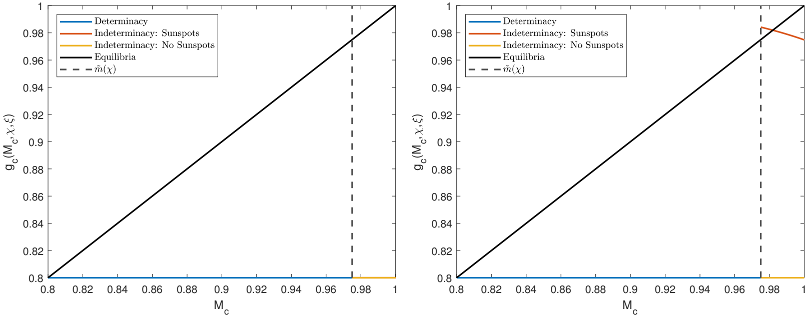
(a) Small fundamental shocks (b) Large fundamental shocks

Figure: Attention equilibria with fundamental shocks

Simple Model: Equilibrium Refinement

Suppose that the economy was in determinate equilibrium in $t-1$, so $y_{t-1}=0$. If the economy "jumps" to an indeterminacy in t:

$$y_t = \delta^{-1} y_{t-1} + \zeta_t$$
$$Var(y_t) = Var(\zeta_t)$$



(a) Smaller sunspot shocks

(b) Very large sunspot shocks

Figure: Attention equilibria, assuming determinacy in the previous period

Richer Model: Overview

Now let's allow for expectations terms in the Phillips curve:

$$\pi_t = \beta M_f(\boldsymbol{\chi}, \boldsymbol{\xi}) E_t \pi_{t+1} + \kappa x_t + \eta_t$$
$$x_t = M_c(\boldsymbol{\chi}, \boldsymbol{\xi}) E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + \tilde{z}_t$$
$$i_t = \rho_i i_{t-1} + (1 - \rho_i)(\phi_\pi \pi_t + \phi_x x_t) + \varepsilon_t^v$$

Now, the g mapping maps a vector $[m_c \; m_f]'$ into a set of macroeconomic dynamics into the optimal choices of m_c and m_f.

$$\boldsymbol{M}(\chi, \xi) = \boldsymbol{g}(\boldsymbol{M}(\chi, \xi), \chi, \xi)$$

Requires extension of Gabaix framework to allow for interest rate smoothing and indeterminacy to calculate.

Identification: Setup

I take a calibrated example (specific calibration does not matter much).

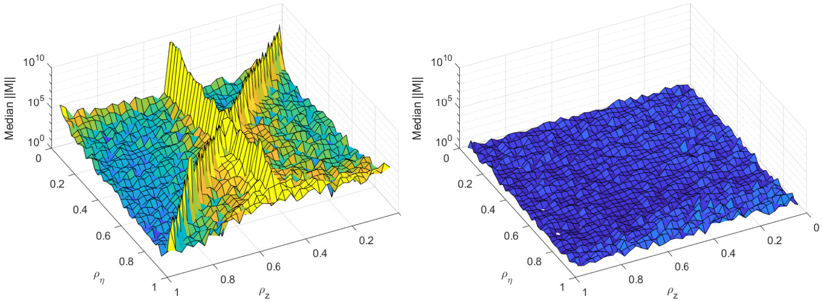
I look for points of weak identification:

- Search over different values of ρ_z and ρ_η
- For each calibration, simulate the model
- Calculate the measure of weak identification proposed by Andrews and Mikusheva (2014)

Two different assumptions: (i) M_c and M_f are exogenous, (ii) both are endogenous.

Identification: Results

Weak identification throughout much of the parameter space with exogenous attention. Much stronger under endogenous attention.



(a) Exogenous Attention (b) Endogenous Attention

Figure: Strength of Identification under Exogenous and Endogenous Attention

Estimation: Great Inflation

Method: Bayesian estimation of endogenous attention model

Priors: Taken from Lubik and Schorfheide (2004) where possible

Attention problem: Set $k_f = 1.5$, $k_c = 4.5$, $m_{c,d} = m_{f,d} = 0.85$

Data: From FRED. De-meaned inflation (GDP deflator) and nominal interest rate (Fed Funds Rate). Negative of HP-filtered unemployment rate as output gap proxy. Three sub-samples: 1960:I to 1979:II, 1984:I to 2007:IV and 1990:I to 2007:IV

Other assumptions: Impose continuity solution and "attainability" refinement

Estimation Results (Great Inflation)

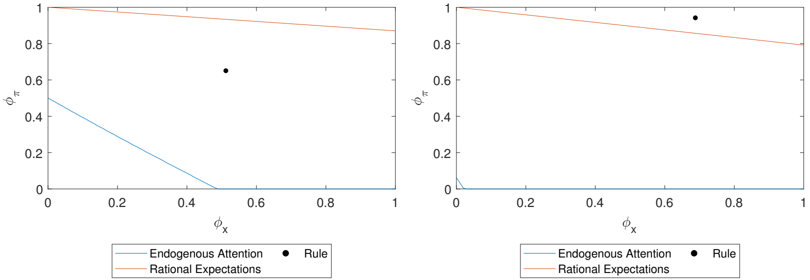
	Great Inflation (1960:I to 1979:II)			
	Determinacy		Indeterminacy	
Parameter	Mean	90-pct. interval	Mean	90-pct. interval
ϕ_π	0.65	[0.55,0.76]	0.53	[0.52,0.62]
ϕ_x	0.51	[0.39,0.65]	0.48	[0.37,0.60]
θ	0.87	[0.83,0.92]	0.86	[0.82,0.89]
γ	2.90	[2.13,3.78]	3.42	[2.57,4.39]
ρ_η	0.74	[0.67,0.81]	0.86	[0.81,0.90]
ρ_z	0.80	[0.73,0.86]	0.76	[0.69,0.83]
ρ_i	0.56	[0.45,0.66]	0.56	[0.46,0.65]
σ_η^ε	0.12	[0.08,0.17]	0.08	[0.05,0.11]
σ_z^ε	0.48	[0.36,0.63]	0.62	[0.47,0.81]
σ_v^ε	0.16	[0.14,0.19]	0.16	[0.14,0.19]
σ_ζ			0.12	[0.06,0.20]
m_f	0.93	[0.85,0.98]	0.98	[0.96,0.99]
m_c	0.85	[0.85,0.85]	0.85	[0.85,0.85]
Log ML	-53.0		-67.8	

Estimation Results (Great Moderation)

	1984:I to 2007:IV Determinacy		1990:I to 2007:IV Determinacy	
Parameter	Mean	90-pct. interval	Mean	90-pct. interval
ϕ_π	1.55	[1.00,2.17]	0.94	[0.49,1.47]
ϕ_x	0.73	[0.52,0.95]	0.69	[0.51,0.88]
θ	0.90	[0.87,0.93]	0.91	[0.88,0.93]
γ	4.02	[3.10,5.04]	3.49	[2.63,4.46]
ρ_η	0.66	[0.57,0.76]	0.63	[0.53,0.73]
ρ_z	0.91	[0.88,0.94]	0.90	[0.85,0.94]
ρ_i	0.84	[0.80,0.87]	0.83	[0.78,0.87]
σ_η^ε	0.08	[0.06,0.09]	0.08	[0.06,0.10]
σ_z^ε	0.28	[0.22,0.35]	0.28	[0.22,0.36]
σ_v^ε	0.12	[0.10,0.14]	0.10	[0.09,0.12]
σ_ζ				
m_f	0.85	[0.85,0.89]	0.85	[0.85,0.88]
m_c	0.85	[0.85,0.85]	0.85	[0.85,0.85]
Log ML	84.4		64.0	

Note: Log ML under indeterminacy in the two sub-samples is 23.6 and 20.2 respectively.

Determinacy Regions



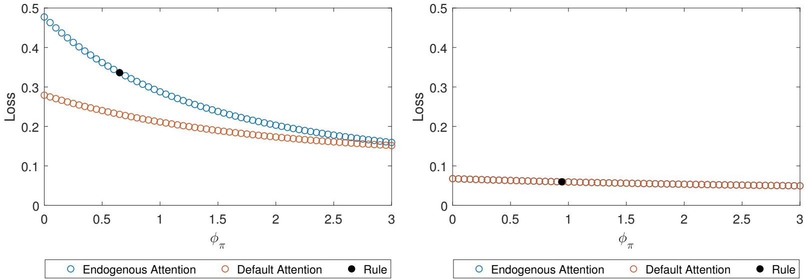
(a) Great Inflation

(b) Great Moderation (Post-1990)

Figure: Counterfactual Determinacy Regions: Baseline Specification

Great Inflation: Counterfactual Central Bank Losses

I assume $L = V(\pi) + \xi V(x)$. I use $\xi = 0.0625$ (based on commentary in Debortoli et al. (2019)).



(a) Great Inflation (b) Great Moderation

Figure: Counterfactual Central Bank Losses

Conclusions

Endogenizing cognitive discount factors materially changes policy analysis and estimation results

- Neglecting endogeneity of discount factors may be misleading, because we often use the model to entertain large changes in policy or macro environment
- The source of discounting matters: these are not just different ways of getting to the same model
- Adding exogenous discount factors leads to weak identification
- More empirical work to validate sources of discounting and attention costs is highly important