Optimal and Fair Prizing in Sequential Round-Robin Tournaments: Experimental Evidence

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Round-Robin Tournaments

Definition: A **round-robin tournament (RRT)** is a form of competition in which each participant is

- matched with each other participant in a sequence of pairwise contests,
- ranked according to the number of matches won,
- awarded a prize according to this ranking.

Applications in sports:

- *Multi-player:* sports leagues like the major European football leagues in England, Spain, Germany, and Italy.
- *4-player:* first round (group stage) of the FIFA World Cup (since 1950) and UEFA European Championship (since 1980)
- *3-player:* second round of the FIFA World Cup (1982), first round of the FIFA World Cup (from 2026 on)

Tournament Design

Objectives: Organizers of contests (in music, arts, or sports) usually aim at a **fair/balanced/close** and/or an **intense** competition.

Reasons: Fairness and intensity attract attention and exert positive externalities on the market for

- tickets (viewers),
- TV-contracts (broadcasters),
- merchandizing (sponsors).

Instruments: Effort is incentivized via (possibly multiple) prizes; e.g., in the group stage of the FIFA World Cup,

- two out of four (2026ff.: three) teams advance to the next round,
- each winner of a group is matched with a runner-up from a different group.

Motivation

Common wisdom: "In theory, a round-robin tournament is the fairest way to determine the champion from among a known and fixed number of contestants. Each contestant, whether player or team, has equal chances against all other opponents [...]. The element of luck is seen to be reduced as compared to a knockout system [...]." (*Wikipedia, accessed on 21/08/2022*)

Problem: Many RRT have a sequential structure

- in a canonical way (e.g. 3-player tournaments),
- due to technical constraints (e.g. capacity of the venue),
- due to economic considerations (e.g. more broadcasting time).

Questions:

- Are **sequential** RRT fair?
- Which prize structure induces the most intense RRT?

Theoretical Predictions

Theoretical Predictions

Laica et al (GEB, 2021) investigate the question theoretically:

- arbitrary number of (possibly heterogeneous) players,
- matches organized as general Tullock contests (including the APA-case),
- multiple arbitrary rank-dependent prizes.

Fairness: A RRT is

- **ex-ante fair**, if the players' ex-ante winning probabilities and expected payoffs depend only on their types.
- **completely fair**, if the winning probabilities and expected payoffs of the two players *in each match* depend only on their types..
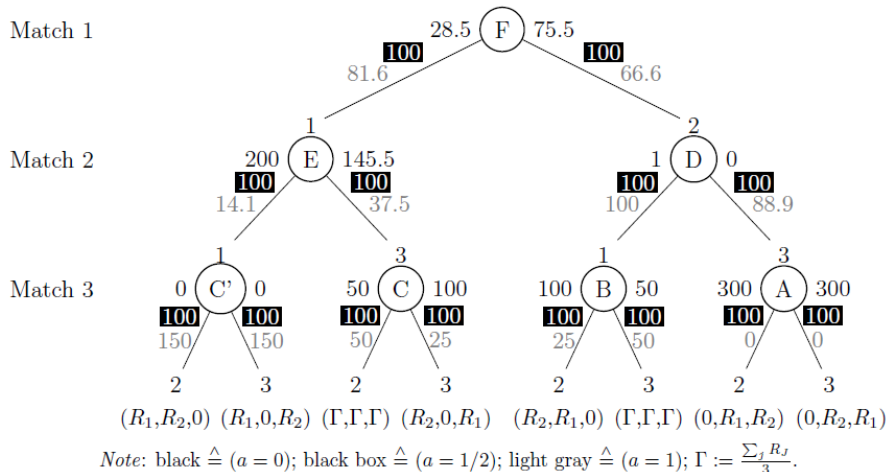
Theoretical Predictions (ctd.)

Main findings of Laica et al (2021):

- RRT with 3 players are fair if and only if the second prize equals half of the first prize.

  Intuition:

    ► only first prize: *discouragement effect* for trailing players
    ► positive second prize: *lean-back effect* for leading players
    ► effects cancel out, if 2nd prize equals half of 1st prize (1st place requires twice as many wins as 2nd place)

- With more than three players, there is no prize-structure for which a sequential RRT is perfectly fair.

Theoretical Predictions by Match

Match 1   28.5 (F) 75.5

100   100

81.6   66.6

1   2

Match 2   200 (E) 145.5   1 (D) 0

100   100   100   100

14.1   37.5   100   88.9

1   3   1   3

Match 3   0 (C') 0   50 (C) 100   100 (B) 50   300 (A) 300

100   100   100   100   100   100   100   100

150   150   50   25   25   50   0   0

2   3   2   3   2   3   2   3

$(R_1,R_2,0)$ $(R_1,0,R_2)$ (Γ,Γ,Γ) $(R_2,0,R_1)$ $(R_2,R_1,0)$ (Γ,Γ,Γ) $(0,R_1,R_2)$ $(0,R_2,R_1)$

Note: black $\triangleq (a=0)$; black box $\triangleq (a=1/2)$; light gray $\triangleq (a=1)$; $\Gamma := \frac{\sum_j R_j}{3}$.

Experiment

Experimental Design

This paper: We test Laica et al's (2021) predictions in an experiment with the following features:

- RRT with three symmetric players,
- individual matches organized as all-pay-auctions,
- three treatments in which second prize equals 0%, 50%, or 100% of first prize;
- total prize money in the RRT equals 6 Euro = 600 points in each treatment,
- subjects play 20 repetitions with random rematching, but fixed player roles,
- elicitation of risk preferences and cognitive reflection levels,
- 4 sessions in the 0%-treatment and 3 sessions each in the 50%- and 100%-treatment.

Hypotheses

Let $a \in \{0, 0.5, 1\}$ denote the ratio of the second tho the first prize:

Hypothesis 1: The tournament is most (least) intense, if $a = 0.5$ ($a = 0$).

Hypothesis 2: The tournament is most (least) fair, if $a = 0.5$ ($a = 0$).

Hypothesis 3: A prize structure with $a = 0.5$ induces (i) a fair ranking, (ii) fair payoffs, and (iii) fair matches.

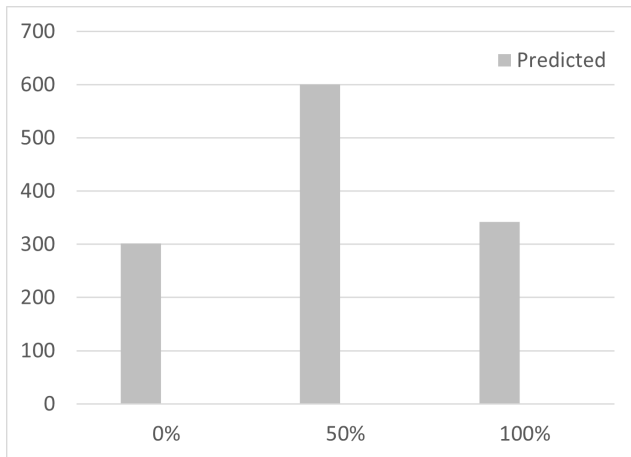**Hypothesis 4:** The late moving player 3 will be (i) advantaged, if $a = 1$, and (ii) disadvantaged, if $a = 0$.

Hypothesis 5: After winning the first match,

(i) each player will decrease effort in her second match, if $a = 1$,

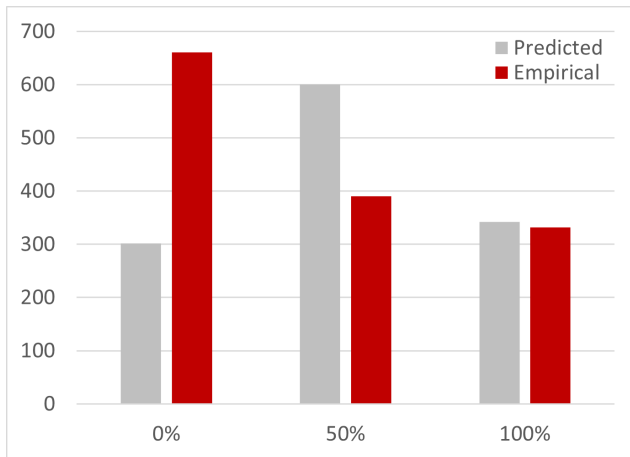(ii) players 1 and 2 will increase effort in their second match, if $a = 0$.

Experimental Results
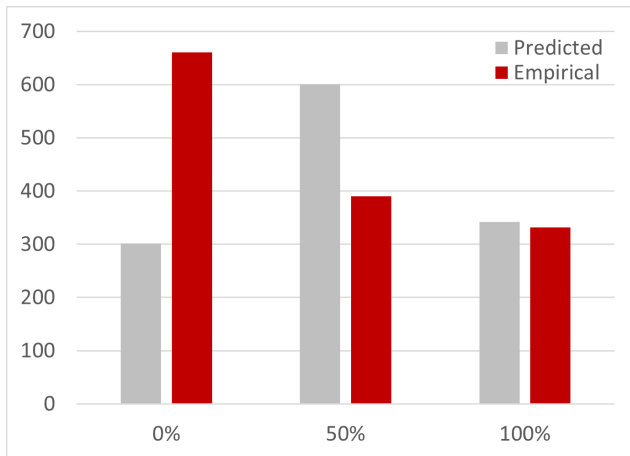($\rightarrow$ focus on final 13 rounds)
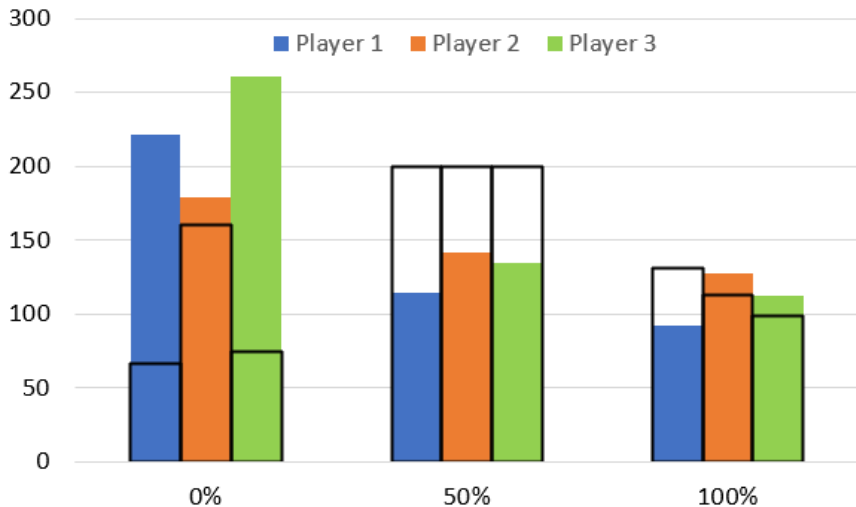
Total Effort
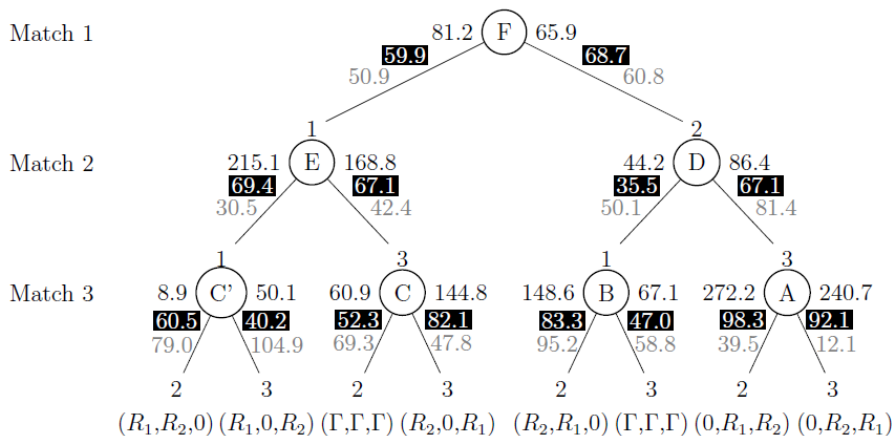
Total Effort

Total Effort

Result 1: The tournament is most (least) intense, if the second prize equals zero (the first prize).

Total Effort by Player

Efforts by Match

Match 1

$$81.2 \quad \text{(F)} \quad 65.9$$
$$\boxed{59.9} \qquad \boxed{68.7}$$
$$50.9 \qquad 60.8$$

Match 2

1

$$215.1 \quad \text{(E)} \quad 168.8$$
$$\boxed{69.4} \qquad \boxed{67.1}$$
$$30.5 \qquad 42.4$$

2

$$44.2 \quad \text{(D)} \quad 86.4$$
$$\boxed{35.5} \qquad \boxed{67.1}$$
$$50.1 \qquad 81.4$$

Match 3

1

$$8.9 \quad \text{(C')} \quad 50.1$$
$$\boxed{60.5} \qquad \boxed{40.2}$$
$$79.0 \qquad 104.9$$

3

$$60.9 \quad \text{(C)} \quad 144.8$$
$$\boxed{52.3} \qquad \boxed{82.1}$$
$$69.3 \qquad 47.8$$

1

$$148.6 \quad \text{(B)} \quad 67.1$$
$$\boxed{83.3} \qquad \boxed{47.0}$$
$$95.2 \qquad 58.8$$

3

$$272.2 \quad \text{(A)} \quad 240.7$$
$$\boxed{98.3} \qquad \boxed{92.1}$$
$$39.5 \qquad 12.1$$

2   3   2   3   2   3   2   3

$(R_1,R_2,0) \ (R_1,0,R_2) \ (\Gamma,\Gamma,\Gamma) \ (R_2,0,R_1) \quad (R_2,R_1,0) \ (\Gamma,\Gamma,\Gamma) \ (0,R_1,R_2) \ (0,R_2,R_1)$
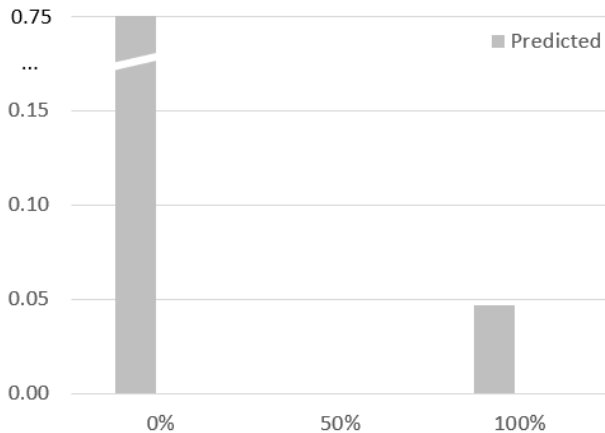
Note: black $\hat{=}$ 0%-treatment; black box $\hat{=}$ 50%-treatment; light gray $\hat{=}$ 100%-treatment; $\Gamma := \frac{\sum_J R_J}{3}$.

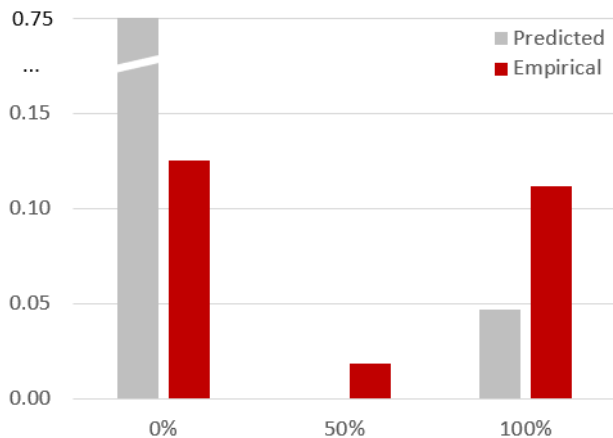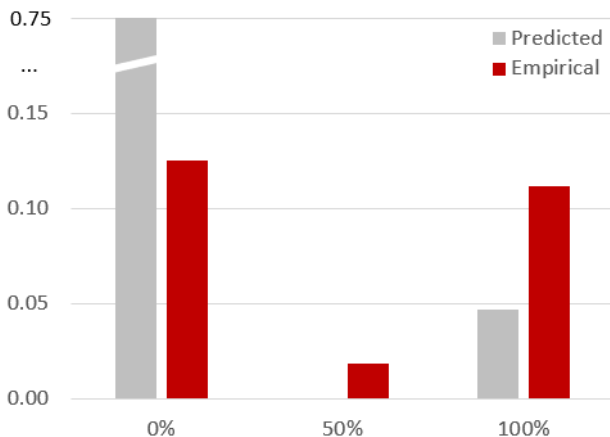## Average Winnings
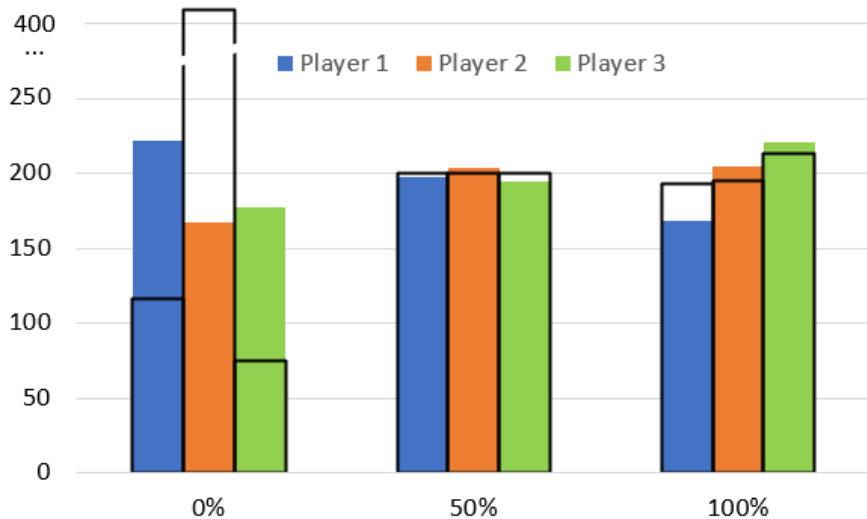
Average Winnings: Relative Standard Deviation

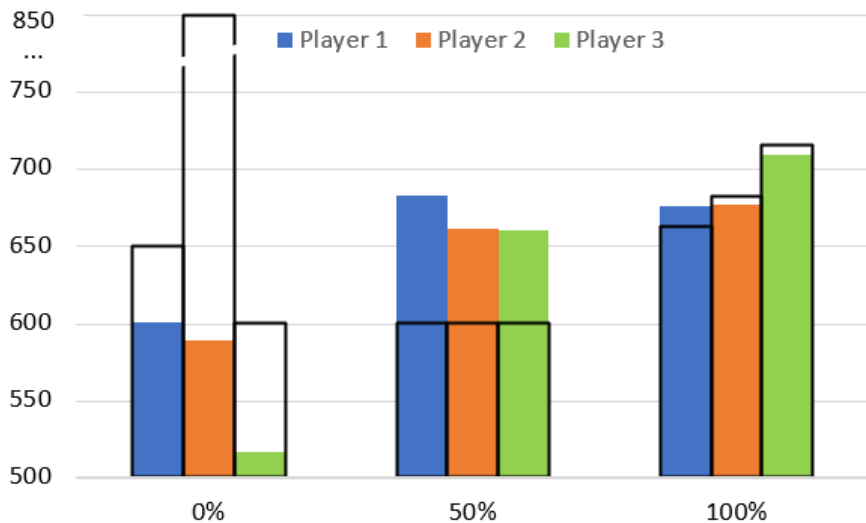# Average Winnings: Relative Standard Deviation
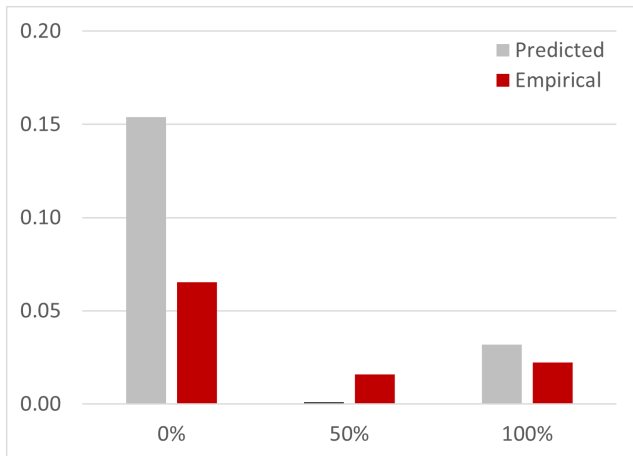
Average Winnings: Relative Standard Deviation

Result 2a: The distribution of average winnings is most (least) fair, if the second prize equals 50% of the first prize (zero).

Average Payoffs

Average Payoffs: Relative Standard Deviation

Result 2b: The distribution of average payoffs is most (least) fair, if the second prize equals 50% of the first prize (zero).

Fairness in the Theoretically Fair Treatment

Result 3: The 50%-tournament is not perfectly fair:

(a) Player 3 wins significantly less than players 1 and 2.

(b) Player 1 earns (significantly) more than player 2 (player 3).

(c) Only three out of seven possible matches are fair:
- ▸ the first match (player 1 vs. player 2),
- ▸ the second match of player 1 after she won her first match,
- ▸ the match of player 2 and player 3, after both won their first match.

Further Results (ctd.)

Result 4: The late moving player 3 is

- advantaged in terms of winnings and payoffs in the 100%-treatment,
- disadvantaged in terms of payoffs (but not winnings) in the 0%-treatment.

Result 5: After winning the first match, subjects exhibit

- a **lean-back effect** in the 100%-treatment,
- a **strategic momentum** in the 0%-treatment when acting as player 2.
- We obtain mixed results for the 50%-treatment. → **Psychological momentum**?

Discussion

- First RRT experiment.

- Results reveal a trade-off between intensity & fairness:
  50%-treatment is fairest, 0%-treatment is most intense.

- Variation of over- & underbidding with treatment and player role
  needs to be better understood.

- Behavioral dynamics within a given RRT are likely driven by strategic
  and psychological effects, that must be disentangled.

- Extensions:
  - ▶ more than three players
  - ▶ endogenous sequences of matches