Public Persuasion in Elections: Single-Crossing Property and the Optimality of Censorship*

Junze Sun[†] Arthur Schram[‡] Randolph Sloof[§]

This version: August 18, 2022. Click here for the latest version.

Abstract

We study public persuasion in elections with binary outcomes, such as referendums. In our model, one or multiple information designers attempt to influence the election outcome by manipulating public information about a payoff-relevant state. We allow for a wide class of designer preferences, ranging from pursuing pure self-interest to maximizing any social welfare function that can be expressed as a rank-dependent weighted sum of voter payoffs (e.g., utilitarian). Our main result identifies a *single-crossing property* and shows that it ensures the optimality of *censorship policies* – which reveal intermediate states while censoring extreme states – in large elections under both monopolistic and competitive persuasion. The single-crossing property holds for an information designer if either (i) the designer is self-interested, or (ii) the distribution of voters' preferences satisfies a mild regularity condition. We characterize the asymptotically optimal censorship policy and a designer's payoff as electorate size goes to infinity. We also analyze how the structure of the optimal censorship policy varies with a designer's preference and voting rules. Our results shed new lights on whether media competition maximizes voter welfare.

JEL Codes: D72, D82, D83

Keywords: voting, single-crossing property, censorship, Bayesian persuasion, competition in persuasion, information design

*We thank Zeinab Aboutalebi, Ricardo Alonso, Giacomo Calzolari, Piotr Dworczak, Navin Kartik, Nenad Kos, David Levine, Elliot Lipnowski, Laurent Mathevet, Niko Matouschek, Savatore Modica, Daniel Monte, Mallesh Pai, Marzena Rostek, Alex Smolin, Erik Snowberg, Jakub Steiner, Shoshana Vasserman, Asher Wolinsky, Takuro Yamashita, Andriy Zapechelnyuk, Jidong Zhou, and Zhen Zhou for helpful comments and discussions. We also thank audiences at the 5th International Conference on "The Political Economy of Democracy and Dictatorship" (PEDD) in Germany (virtual), the Young Economists' Meeting 2022 in Brno, the 12th Conference on Economic Design in Padova, the Max Weber program multidisciplinary research workshop on "Voters' Preferences and Parties' Performance in Politics" at the EUI, Econometric Society Summer Meetings (Asia, Australia), the 33rd Stony Brook International Conference on Game Theory, and seminar participants at the Nanyang Technological University and the European University Institute for their comments and suggestions. All errors are our own.

[†]Corresponding author. Department of Economics, European University Institute. Email: Junze.Sun@eui.eu.

[‡]Amsterdam School of Economics, University of Amsterdam. Email: A.J.H.C.Schram@uva.nl.

[§]Amsterdam School of Economics, University of Amsterdam. Email: R.Sloof@uva.nl.

1 Introduction

In modern democracies, important choices are often made through collective decisions. For instance, presidents are selected via general elections and many important policies are determined in referendums. In general, many different individuals and organizations have diverse interests over the outcomes of such collective decisions; think of (possibly foreign) governments, politicians, mass media outlets, interest groups, representatives of industry or community leaders. Anyone with a stake in the outcome may try to influence the election outcome through manipulating public information, e.g., via public announcements or debate.

This paper studies the strategic provision of public information in elections with binary outcomes, such as referendums. We model the environment of interest as a public Bayesian persuasion problem (Kamenica and Gentzkow, 2011), in which information designers strategically choose public information policies to maximize their own expected payoffs.[1] Relative to existing works in the literature, our paper has two important and distinguishing features. First and foremost, we allow for a wide class of utility functions for information designers that embed both the pursuit of self-interest and the maximization of utilitarian social welfare as special cases. Second, we characterize information provision in equilibrium under both monopolistic persuasion with a single information designer and competitive persuasion with multiple designers. We do all of this in a single, unified framework. Our central research question is: given the (possibly different) objectives of information designers, what public information will be provided in equilibrium?

Answering this question is important from both the positive and normative perspectives. From the positive view, this helps us to understand the equilibrium behavior of actor(s) interested in manipulating public information to influence the election outcome. From the normative view, our result sheds light on the structure of the ideal public information policy for a social planner whose objective is to maximize (some weighted average of) voters' payoffs.

To illustrate our model, consider a referendum where voters collectively decide between passing a reform and maintaining the status quo. An ex-ante unknown state k, which is drawn from a commonly known prior supported on a bounded interval (say $[-1,1]$), determines the quality of the reform relative to the status quo. Each voter is characterized by a private 'threshold of acceptance', such that her utility is linear in k and she prefers the reform if and only if k exceeds this threshold.[2] We refer to this threshold as the voter's *type*. Voters' types are independently drawn from a commonly known prior distribution. Voters with higher type realizations receive lower payoffs if reform is passed.

This setup fits into many real-world scenarios in which the adoption of the reform can bring a

[1] Kamenica (2019) and Bergemann and Morris (2019) provide comprehensive overviews of this literature.
[2] Throughout this paper, we will refer to voters as feminine and information designers as masculine.

public good of uncertain value, while at the same time induce idiosyncratic payoff shocks to voters. For example, consider a referendum on climate change in which the reform is a tax policy aiming at reducing missions of greenhouse gasses, such as a car fuel levy and a tax on air tickets.[3] The state k then represents the effectiveness of this tax policy in reducing emissions of greenhouse gases, which benefits the whole society. Voters' private types can, for instance, reflect the income shocks brought by the tax policy to them. Magnitudes of these shocks depend on many idiosyncractic individual characteristics, such as a voter's occupation, employment status, wealth level, etc. Those with higher types experience greater negative shocks in income due to such policy reform.[4]

There is a finite set of information designers, who can provide voters public information about (the ex-ante unknown) state k. We interpret an information designer as anyone with the interest and ability to manipulate voters' public information (e.g., governments, mass media outlets, interest groups, etc). We also view a designer as an abstract social planner who maximizes voter welfare when we are interested in normative implications. Like voters, each designer's utility is also linear in state k and he prefers the reform if and only if k is above some threshold, say ϕ (which can differ across designers). For a *self-interested* designer who does not take voters' welfare into account, his threshold of acceptance for reform is independent of voters' realized types. Alternatively, a designer can also be *prosocial*; in this case he cares about voters' welfare and hence his threshold ϕ will depend on voters' private types. For example, a utilitarian planner's threshold ϕ equals voters' average type; he prefers the reform if and only if voters are on average better off under the reform than under the status quo. More generally, we allow each designer's utility function to be any convex combination of on the one hand his self-interest and on the other hand any weighted average of all voters' payoffs. This generates a broad spectrum of designer preferences.

Without knowing the realizations of either the state or voters' types, each designer simultaneously chooses an *information policy*, which maps any state realization k to a (distribution of) public signal. After observing their private types and the public signals jointly sent by (all) designer(s), voters simultaneously decide to vote for either the reform or the status quo. The reform will be adopted if and only if the fraction of votes it receives exceeds a cutoff that is determined by the voting rule. For

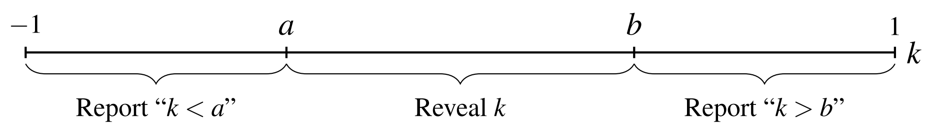
[3] Such referendum indeed occurred in practice. For instance, in June 2021 the Swiss People's Party launched a referendum on the Federal Act on the Reduction of Greenhouse Gas Emissions (CO2 Act). The goal of this act is to reduce emissions of carbon dioxide and other greenhouse gases in Switzerland by 50% (compared to 1990 levels) by 2030, using mainly tax policies. See https://en.wikipedia.org/w/index.php?title=2021_Swiss_referendums&oldid=1100537027.

[4] Our model also fits into many other contexts beyond referendums. For example, many papers in the political economics literature adopt similar models to ours in studying the electoral competition between two politicians (Groseclose, 2001; Ashworth and De Mesquita, 2009; Chakraborty and Ghosh, 2016; Chakraborty, Ghosh and Roy, 2020; Alonso and Câmara, 2016*b*; Sun, Schram and Sloof, 2021). Here, the state can be interpreted as candidates' valences or competences, which are attributes commonly appreciated by all voters. Each voter's private type is interpreted as her idiosyncratic ideology. Another example is committee voting such as the share holders' meeting in a corporate. The board members decide whether to invest in a project, whose profit is the ex-ante unknown state. Private types measure each board member's reservation value.

example, under simple majority rule this cutoff is 50%. Since information transmission is public and voters' payoffs are linear in state, they must share the same posterior expectation about state realization (which is sufficient to determine their voting behavior and expected payoffs).

One class of information policies that will prove to be particularly important is the so-called *censorship policy*, which has a simple interval-revelation structure as illustrated in Figure 1. With this policy, an information designer will precisely reveal the realized state k if it lies in interval $[a,b]$, but only report that "$k < a$" if the realization is below a and only report that "$k > b$" if the realization is above b. It is in this sense that state realizations outside of the revelation interval $[a,b]$ are 'censored'. Under such a policy, voters' posterior expected state equals k whenever it lies within the revelation interval $[a,b]$, and it equals $\mathbb{E}[k|k > b]$ for $k > b$ and $\mathbb{E}[k|k < a]$ for $k < a$. The latter two values depend on the (commonly known) prior distribution of the state k and the thresholds a and b chosen by the information designer.

Figure 1: Censorship Policy



The main result of our paper is the following. We identify a sufficient condition that ensures that in sufficiently large elections it is without loss of optimality for a designer to focus on censorship policies of the kind described in Figure 1. This is true under both monopolistic persuasion with a single information designer and competitive persuasion with multiple designers.

Our sufficient condition can be interpreted as a *single-crossing property* over the designer's and the pivotal voter's *indifference curves*, which are derived as follows. Under any cutoff voting rule the election outcome is essentially determined by choice of the *pivotal voter*, whose realized type (i.e., threshold of acceptance for the reform) is denoted by x.[5] The pivotal voter thus prefers the reform to the status quo if and only if $k \geq x$. His indifference is therefore the 45-degree line on a two-dimensional plane with horizontal axis the pivotal voter's realized type x, and the vertical axis the realized state k. Now we draw an information designer's indifference curve on the same plane. This task is straightforward for a self-interested designer; his indifference curve is simply a flat line in this plane, because his threshold of acceptance for the reform is independent of all voters' types (which of course include the pivotal voter's type x). Deriving the indifference curve for a prosocial designer is more subtle. The key tension here is that a prosocial designer's preference over the election outcome depends on voters' private types, which are however unobservable to him. In this case, the designer must *infer* his preference by exploiting the *statistical correlations* between

[5] For example, under simple majority rule the pivotal voter is the median voter, whose type x equals the sample median of voters' realized types.

the pivotal voter's type and the types of other voters. For example, suppose that the designer is a utilitarian social planner and the election outcome is determined by simple majority rule (so that the median voter is pivotal). Then, given any realized type profile of voters, the planner's threshold of acceptance for the reform is given by the average type (denoted by $\tilde{v}$) while the pivotal voter's threshold of acceptance is the median type (denoted by v^m). Therefore, conditional on the pivotal voter's type being x, the designer rationally infer his expected threshold of acceptance to be $\mathbb{E}[\tilde{v}|v^m = x]$, whose value depends on x, the distribution of voter's types, and the electorate size. This gives his indifference curve for all possible type realizations x of the pivotal voter. This inference procedure similarly applies to more general social preferences and voting rules. The wedge between the indifference curves of the pivotal voter and a designer determines their conflict of interests, which is critical in shaping a designer's optimal information policy.

We are now ready to introduce our single-crossing property. Informally speaking (in Section 4 we present the formal definition), the single-crossing property holds for an information designer if, in sufficiently large elections, his indifference curve crosses the pivotal voter's indifference curve at most once, and if so only from above.[6] This implies that if the pivotal voter weakly prefers the reform in some state k, the designer must strictly prefer the reform in all higher states, in which the reform has a higher quality. Conversely, if the pivotal voter weakly prefers the status quo in state k, then the designer must strictly prefer the status quo in all lower states, namely when the quality of the reform is lower. We show that the single-crossing property holds under very broad conditions. It is (i) always satisfied if the designer is self-interested, or (ii) satisfied for all designer preferences and voting rules under a mild assumption for the distribution of voter preferences.

In Section 5 we analyze monopolistic persuasion by a single information designer for whom the single-crossing property holds. In this case we show that some censorship policy with revelation interval $[a,b]$ (as in Figure 1) must be uniquely optimal for this designer in sufficiently large elections (cf. Theorem 1).[7] The optimal choices of boundaries a and b are driven by the tradeoff between the capability of manipulating voters' beliefs in more states on the one hand (providing incentives to censor more states), and the effectiveness of belief manipulation on the other hand (reducing censoring incentives). The resolution of this tradeoff depends on all model primitives: the designer's preference, the prior distributions of the state and voters' types, the electorate size, and the voting rule. In Section 6 we further characterize and discuss properties of the designer's optimal censorship policy and payoff as the electorate size goes to infinity (cf. Theorem 2). We also derive comparative statics regarding how the structure of the optimal censorship policy varies with the

[6] The fact that our single-crossing property can be interpreted in terms of indifference curves is reminiscent of the Spence-Mirrlees condition in the signaling and mechanism design literature. However, the interpretation and application of the Spence-Mirrlees condition are very different than ours.

[7] In Theorem 1 we also give conditions under which the censorship policies are uniquely optimal independent of the electorate size. Hence, when these conditions hold, our result applies to small-size elections such as committee voting.

designer's preference and the voting rule (cf. Propositions 1 to 3).

In Section 7 we study competitive persuasion where multiple information designers simultaneously choose their public information policies as in Gentzkow and Kamenica (2017*b*). In this case we show that if the single-crossing property holds for a designer and the electorate size is sufficiently large, then it is *without loss of optimality* for this designer to restrict attention to a subset of censorship policies in the following sense: for any feasible pure strategy profile chosen by other designers (which need not be censorship policies), this designer can always find a censorship policy from this subset as his best response (cf. Theorem 3). Suppose that the single-crossing property holds for all designers. Then, under a weak regularity condition, in the minimally informative equilibrium the public information jointly provided by all designers can be equivalently reproduced by a censorship policy, whose revelation interval is simply the convex hull of the revelation intervals that would be optimal for each of the designers under monopolistic persuasion (cf. Theorem 4). In fact, this outcome is the unique equilibrium outcome in pure and weakly undominated strategies if all designers commit to using censorship policies from their best-response sets only. We also characterize a sufficient condition under which competition in persuasion must induce full information revelation in all equilibria.

We finally apply our results to study the welfare implications of media competition. In our model, the competition between two partisan and opposite-minded media outlets can induce full information revelation in any equilibrium. Nevertheless, perhaps surprisingly, such full disclosure is in general suboptimal from the welfare perspective. We compare voters' utilitarian welfare under the second-best benchmark (i.e., under the information policy that maximizes utilitarian welfare) and under full information disclosure as the electorate size goes to infinity. We show that the former is always larger than the latter, and the gap can be substantial if the ex-ante conflict of interests between the average voter and the pivotal voter is large. These results imply that it is important to account for the distribution of voters' preferences and voting rules – which jointly determine the ex-ante conflict of interests between the average and pivotal voters – when evaluating the welfare effects of media competition.

The remainder of this paper is organized as follows. The next section reviews the related literature. Section 3 lays out our model. Section 4 introduces the single-crossing property, explains its economic implications, and provides sufficient conditions for it to hold. Section 5 presents and proves our main result for monopolistic persuasion, which relates the single-crossing property to the optimality of censorship policies. Section 6 characterizes a monopoly designer's optimal censorship policy and payoff as the electorate goes to infinity. It also analyzes how the structure of the designer's optimal censorship policy responds to variations in his preference or in the voting rule. Section 7 extends our main results to competition in persuasion with multiple designers and discusses an application on media competition and voter welfare. Section 8 concludes.

2 Related literature

This paper speaks to several strands of literature. First of all, our paper belongs to a strand of literature that studies information transmission in elections using the Bayesian persuasion or information design approach.[8] Aside from a few exceptions discussed below, most papers in this literature study monopolistic persuasion problems by a single designer whose goal is to sway the election outcome in favor of his preferred alternative (Wang, 2013; Alonso and Câmara, 2016*a*,*b*; Bardhi and Guo, 2018; Chan et al., 2019; Ginzburg, 2019; Kerman, Herings and Karos, 2020; Heese and Lauermann, 2021).[9] Our paper complements these works by allowing for a wider class of designer preferences – ranging from pursuing self-interest to maximizing any social welfare function that can be expressed as a weighted average of voters' payoffs – while at the same time analyzing both monopolistic and competitive persuasion in a unified framework.

The two studies closest to ours are Alonso and Câmara (2016*b*) and Kolotilin, Mylovanov and Zapechelnyuk (2022). The models in both papers can be interpreted as a monopoly designer persuading a privately informed representative voter. In Alonso and Câmara (2016*b*), the designer is an incumbent party leader who aims at maximizing the re-election probability. They show that, under some regularity conditions, the optimal information policy is upper censorship if the distribution of the representative voter's private type has a log-concave density. Kolotilin, Mylovanov and Zapechelnyuk (2022) characterize sufficient and necessary conditions for the optimality of upper censorship for general linear persuasion problems. They show that the same log-concavity density assumption ensures this optimality for a wider class of designer preferences, ranging from maximizing the winning probability to maximizing the payoff of the representative voter.

Our paper enriches and generalizes the results of both papers to an environment that allows for multiple designers and voters. Looking at a setup with multiple voters instead of a single representative voter enables us to (i) model a wider class of social preferences for designers, and (ii) study the influence of voting rules on the optimal information policy. We show that in large elections the optimality of censorship can be ensured under much weaker assumptions regarding the underlying distribution of voter types than those made in previous studies. More, we establish that the same conditions that ensure the optimality of censorship for a designer under monopolistic persuasion continue to do so under competitive persuasion with multiple designers.

[8] Of course, strategic information transmission in elections has been extensively studied under various other communication protocols, such as cheap talk (Schnakenberg, 2015, 2017; Kartik and Van Weelden, 2019; Sun, Schram and Sloof, 2021) and verifiable disclosure (Liu, 2019). One important feature that separates our paper from these is that we can also address the normative question regarding the optimal information policy for a social planner.

[9] Some of these papers (e.g., Heese and Lauermann (2021)) allow the designer's preferred alternative to be state-dependent. They do not, however, allow for the designer's utility to depend on voters' payoffs. Moreover, all these papers except Alonso and Câmara (2016*a*,*b*) and Ginzburg (2019) study targeted persuasion in which the designer can privately communicate to voters (Bergemann and Morris, 2016; Taneva, 2019; Mathevet, Perego and Taneva, 2020). Our paper instead focuses on public persuasion whereby a designer must send the same message to all voters.

Alonso and Câmara (2016*a*) study public persuasion in elections by a monopoly designer in a model similar to ours. A crucial difference between our paper and theirs is that we allow voters to have private types, whereas in their model the designer perfectly knows voters' preferences. This difference is important in two ways. First, the structures of the optimal information policies are very different depending on whether the designer knows voters' preferences. Second, we show that when a designer cares about social welfare and is imperfectly informed about voters' preferences, varying the voting rule can affect his optimal information policy through a novel designer-preference effect. This effect is absent if the designer has perfect information about voters. Van der Straeten and Yamashita (2020) and Ferguson (2020) study monopolistic persuasion problems in which the designer maximizes voters' utilitarian welfare. They do so in models different from ours. Both papers show that full information disclosure is suboptimal from the utilitarian perspective. Our paper extends this insight to general social welfare functions. Finally, Innocenti (2021) and Mylovanov and Zapechelnyuk (2021) study competition in Bayesian persuasion by two opposite-minded designers with pure persuasion motives. The former does so in a model where each voter can only hear from one designer. The latter, like ours, consider public persuasion a la Gentzkow and Kamenica (2017*b*). Our paper allows for a much richer set of designer preferences compared to theirs.

Second, methodologically, our paper relates to a recent strand of literature that develops the duality approach to solve linear persuasion problems in which designers' utility functions depend only on the posterior expected state (Kolotilin, 2018; Dworczak and Martini, 2019; Dworczak and Kolotilin, 2019; Dizdar and Kováč, 2020; Kolotilin, Mylovanov and Zapechelnyuk, 2022; Sun, 2022*a*,*b*).[10] In particular, Dworczak and Martini (2019) show that the problem of finding an equilibrium outcome under competitive persuasion can be converted to solving the monopolistic persuasion problems of each designer with modified utility functions. This allows us to treat monopolistic and competitive persuasion in a unified framework. Kolotilin, Mylovanov and Zapechelnyuk (2022) exploit the duality method to show that upper (resp. lower) censorship policies are uniquely optimal if the designer's utility function is strictly S-shaped (resp. inverse S-shaped) in posterior expectation. Sun (2022*a*) extends this observation to competition in persuasion a la Gentzkow and Kamenica (2017*b*); he shows that if a designer's utility function is strictly S-shaped (resp. inverse S-shaped), then given any pure strategy profile of others, there exists an upper (resp. lower) censorship policy as the designer's best response. Sun (2022*b*) use the duality method to derive a sufficient condition for full information under competition in persuasion in linear persuasion games. We build on these findings to establish our main results.

[10] Several papers study linear persuasion problems using other methods. For instance, Gentzkow and Kamenica (2016) and Kolotilin et al. (2017) characterize the set of implementable outcomes under public and private signals, respectively, using an implication of Blackwell's theorem. More recently, Arieli et al. (2020), Ivanov (2020) and Kleiner, Moldovanu and Strack (2021) develop methods based on extreme points and majorization to characterize structures of solutions to linear persuasion problems.

Finally, our results also relate to papers studying competition in Bayesian persuasion with multiple senders (Gentzkow and Kamenica, 2017*b*,*a*; Cui and Ravindran, 2020; Au and Kawai, 2020, 2021; Li and Norman, 2021; Mylovanov and Zapechelnyuk, 2021; Sun, 2022*b*). An important theme of this literature is to identify conditions under which full information disclosure is the unique equilibrium outcome. We contribute to this research agenda by providing such a sufficient condition in the context of publicly persuading voters. In contrast to many earlier works but consistent with Sun (2022*b*), we show that strong conflicts of interests between competing senders are not necessary to sustain full information disclosure as the unique equilibrium outcome.

3 Framework

We consider an election in which $n+1$ voters collectively decide between two options, which, for ease of reference, we label *Reform* and *Status quo*. The outcome is determined by a cutoff rule with threshold $q \in (0,1)$; the reform is adopted if and only if it obtains strictly more than nq votes. For instance, $q = 0.5$ corresponds to simple majority rule. For ease of exposure, we assume that nq is an integer (unless explicitly mentioned otherwise).

An ex-ante unknown but payoff relevant state k is drawn from a common prior F that admits a positive and continuous density f on $[-1,1]$. Without loss of generality, we normalize all players' payoffs to zero if the status quo is maintained. If the reform is adopted, each voter i's payoff equals $k - v_i$, where v_i is her private type. In this way, voter i's payoffs attributed to the reform (relative to the status quo) consist of a common value, k (think of the 'quality' of the reform), and her private threshold of acceptance for the reform, v_i. We assume that each v_i is independently drawn from a commonly known distribution G, which admits a positive and twice continuously differentiable density g on $[\underline{v}, \overline{v}]$ with $\underline{v} < -1$ and $\overline{v} > 1$. For any profile of type realizations $v = (v_1, \cdots, v_{n+1})$, we let $v^{(1)} \leq v^{(2)} \leq \cdots \leq v^{(n+1)}$ be its ascending permutation. Since $k - v_i$ decreases in v_i, voters with lower type realizations receive higher ex-post payoffs if the reform is adopted. Because voter i prefers the reform if and only if $k \geq v_i$, her private type represents her threshold of acceptance of the reform.

Consider first a monopoly designer; in Section 7 we extend our model to allow for multiple designers competing in persuading voters. The designer's payoff under the reform is given by

$$u(k,v) = \rho \sum_{j=1}^{n+1} w_j \cdot \left(k - v^{(j)}\right) + (1-\rho)(k-\chi) \tag{1}$$

where $\rho \in [0,1]$, $\chi \in \mathbb{R}$ and $(w_1, \cdots, w_{n+1})$ is a non-negative vector of weights that sum up to 1. Parameter ρ captures the extent to which the designer cares about 'voter welfare' relative to his 'self-interest'. If $\rho = 0$ then $u(k,v) = k - \chi$ so that the designer prefers reform to be adopted if and only if $k \geq \chi$. In this case, the designer is *self-interested* in the sense that his preference over alternatives is independent of voters' interests.[11]

Conversely, if $\rho = 1$ then $u(k,v) = \sum_{j=1}^{n+1} w_j \cdot \left(k - v^{(j)}\right)$ is a weighted average of voters' realized payoffs when the reform is adopted. For each $j = 1, \cdots, n+1$, w_j is the *rank-dependent welfare weight* the designer assigns to the voter whose payoff under reform is ranked the j-th highest under the realized type profile v.[12] The vector $(w_1, \cdots, w_{n+1})$ is generated by a *weighting function* $w(\cdot)$ that is non-decreasing, absolutely continuous on $[0,1]$ and satisfies $w(0) = 0$ and $w(1) = 1$. Hence, $w(\cdot)$ is the cumulative distribution function (cdf) of a random variable on $[0,1]$.[13] For any integer $n \geq 0$ and $j \in \{1, \cdots, n+1\}$, element w_j is uniquely generated by

$$w_j = w\left(\frac{j}{n+1}\right) - w\left(\frac{j-1}{n+1}\right) \tag{2}$$

This setup captures a wide class of social welfare functions in a unified way. For instance, the utilitarian welfare function can be obtained by letting $\rho = 1$ and $w(x) = x$ for all $x \in [0,1]$. With this $w(\cdot)$, it follows from (2) that $w_j = \frac{1}{n+1}$ for each j so that the welfare weights are equal across voters. If $w(\cdot)$ is not the cdf of a uniform distribution on $[0,1]$, then it represents the preference of some non-utilitarian social planner who may discriminate voters according to the ranking of their ex-post payoffs. We will discuss some examples in Section 5.

The designer can affect voters' information about k by designing an *information policy*. Following the convention of the Bayesian persuasion literature, we define an information policy π by a pair (S,σ), where S is a sufficiently rich signal space and $\sigma : [-1,1] \mapsto \Delta(S)$ maps each state realization k to a probability distribution on S. Let Π denote the set of all feasible information policies.

The timing of the game is as follows. First, prior to observing state k, the designer chooses an information policy $\pi \in \Pi$. Second, state k is realized and a public signal is drawn according to π. Observing the realized public signal, voters simultaneously decide to vote for either the reform or the status quo. The reform is adopted if and only if its vote tally strictly exceeds nq. All players' payoffs then realize. Throughout, we focus on equilibria in weakly undominated strategies.[14]

[11] This captures transparent persuasion motives (which are most extensively explored in the literature) as limiting cases. For instance, the preference of a designer whose aim is to maximize the winning probability of reform (resp. status quo) independent of state realizations can be captured by letting $\chi \to -\infty$ (resp. $\chi \to \infty$).

[12] Under our assumption that each v_i is independently drawn from a common distribution G, it is without loss of generality to let the welfare weight depend only on a voter's ex-post payoff ranking rather than her identity. This is because, due to ex-ante homogeneity, maximizing any particular voter's payoff is equivalent to maximizing voters' ex-ante average payoff, which is the case where $w_1 = \cdots = w_{n+1} = 1/(n+1)$.

[13] $w(\cdot)$ is reminiscent of the probability weighting function in rank-dependent utility theory (Quiggin, 1982). As we will see in Section 6.2, there is a natural link between first order stochastic dominance ordering of $w(\cdot)$ and the designer's social preference.

[14] It is well known that the voting game at the second stage has a plethora of uninteresting equilibria in weakly

3.1 Voting behavior and election outcome

Because voters have a common prior F and information transmission is public, they must share a common posterior about the state realization after hearing from the information designer. Since voters' payoffs under reform are linear in state k, their expected payoffs depend only on their posterior expectation θ and are given by $\theta - v_i$ for all i. It is then a weakly dominant strategy for i to vote for reform if and only if $\theta \geq v_i$. Therefore, under the cutoff voting rule with threshold q, the election outcome is determined by the choice of the *pivotal* voter, whose type realization is $v^{(nq+1)}$. Note that $v^{(nq+1)}$ is a random variable and let $\hat{G}_n(\cdot;q)$ denote its cumulative distribution function. Since reform is adopted only if $v^{(nq+1)} \leq \theta$, $\hat{G}_n(\theta;q)$ gives the winning probability of reform. Appendix A offers a formal expression and useful properties of $\hat{G}_n(\theta,q)$.

Lemma 1. *$\hat{G}_n(\cdot;q)$ is strictly increasing. $v^{(nq+1)}$ converges in probability to $v_q^* := G^{-1}(q)$.*

Lemma 1 says that the winning probability of reform strictly increases in θ. Moreover, as $n \to \infty$ the reform will be adopted almost surely if $\theta > v_q^*$, while the status quo will be maintained almost surely if $\theta < v_q^*$.

4 Indifference curves and the single-crossing property

In this section we introduce the single-crossing property and discuss its implications for a designer's temptation to manipulate voters' beliefs. We also characterize sufficient conditions for our single-crossing property. All derivations and proofs are relegated to Appendix B.

4.1 Indifference curves and the inference from pivotal voter's choice

Given any realization of voter type profile v, it follows from (1) that the designer weakly prefers the reform if and only if

$$k \geq \varphi_n(v) := \rho \sum_{j=1}^{n+1} w_j \cdot v^{(j)} + (1-\rho)\chi \ .$$

$\varphi_n(v)$ is the designer's threshold of acceptance for the reform. Note that this depends on voters' realized type profile v whenever $\rho > 0$. Importantly, however, at the time of choosing his information policy, any designer with $\rho > 0$ cannot precisely observe $\varphi_n(v)$ because realized types are voters' private information. Nevertheless, the election outcome, which is essentially the choice of the pivotal voter, is informative about the realization of $\varphi_n(v)$.

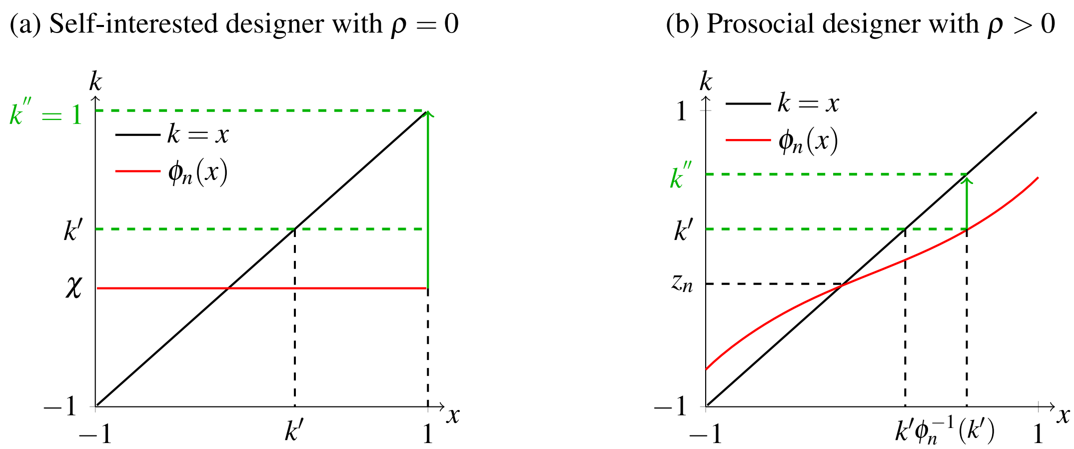
dominated strategies. For example, whenever $n > 0$ it is an equilibrium for all voters to vote for reform regardless of their private types or the public information they obtain, because no single vote can unilaterally change the outcome. In this case, any information policy π can be sustained in equilibrium as well because they have no influence. The restriction to weakly undominated strategies rules out such uninteresting equilibria.

To make this point clear, it is instructive to draw the indifference curves of the pivotal voter and the designer in the same plane, as in Figure 2. In each panel, the horizontal axis x represents the pivotal voter's type realization $v^{(nq+1)}$ and the vertical axis denotes the realized state k. The pivotal voter's indifference curve is simply the 45-degree line; she is indifferent between alternatives if and only if $k = x$. Let

$$\phi_n(x) := \mathbb{E}\left[\varphi_n(v)\middle|v^{(nq+1)} = x\right] = \rho \sum_{j=1}^{n+1} w_j \cdot \mathbb{E}\left[v^{(j)}\middle|v^{(nq+1)} = x\right] + (1-\rho)\chi \tag{3}$$

denote the expectation of $\varphi_n(v)$ conditional on event $v^{(nq+1)} = x$. Then, if the designer only knows that $v^{(nq+1)} = x$, he would be indifferent between alternatives if and only if $k = \phi_n(x)$. For this reason, we refer to $\phi_n(x)$ as the designer's *indifference curve*.

Figure 2: Indifference Curves and the Single-Crossing Property



Note: In both panels the horizontal axis x denotes the pivotal voter's type realization $v^{(nq+1)}$, the black line denotes the pivotal voter's indifference curve, and the red line denotes the designer's indifference curve.

Panel (a) of Figure 2 depicts the indifference curve of a self-interested designer with $\rho = 0$. In this case it is obvious from (3) that $\phi_n(x) = \chi$ for all x. The preference of a self-interested designer is thus independent of the pivotal voter's type realization.

Panel (b) of Figure 2 depicts the indifference curve of a prosocial designer with $\rho > 0$. In this case, we show in Appendix B (Proposition B.2 therein) that $\phi_n(x)$ is strictly increasing in x for all $n \geq 0$ and weighting function $w(\cdot)$. This is because the pivotal voter's type realization $v^{(nq+1)}$ is positively associated with all other order statistics $v^{(j)}$ for $j = 1, \cdots, n+1$. Therefore, no matter how the designer assigns his welfare weights, the pivotal voter's type realization is either *directly relevant* or *indirectly informative* about the designer's threshold of acceptance for the reform. It is in this way that the inference from the pivotal voter's choice is important for any prosocial designer.

Two remarks are in place. First, the inference problem here is conceptually different from the inference about the state conditional on the event of being pivotal, which is central to the literature on information aggregation in voting (e.g., Feddersen and Pesendorfer (1996, 1997)). In our model voters have no private information about state k, so the information aggregation issue is absent. Second, for our inference problem to be relevant, it is necessary that voters' types are their private information. Therefore, our inference problem disappears in models where the designer has complete information, such as Alonso and Câmara (2016*a*).

4.2 The single-crossing property and its economic implications

To formally define our *single-crossing property* we need the following lemma, which characterizes the limit of $\phi_n(\cdot)$ as $n \to \infty$.

Lemma 2. *For $x \in [\underline{v}, \overline{v}]$, define*

$$\phi(x) := \rho \left[\int_0^q G^{-1}\left(\frac{y}{q}G(x)\right) dw(y) + \int_q^1 G^{-1}\left(\frac{y-q}{1-q} + \frac{1-y}{1-q}G(x)\right) dw(y) \right] + (1-\rho)\chi \tag{4}$$

As $n \to \infty$, $\phi_n(x)$ and its partial derivative $\phi_n'(x)$ converge uniformly to $\phi(x)$ and $\phi'(x)$, respectively, on $[\underline{v}, \overline{v}]$. Moreover, $\varphi_n(v)$ converges almost surely to

$$\phi^* := \phi(v_q^*) = \rho \int_0^1 G^{-1}(y) dw(y) + (1-\rho)\chi \tag{5}$$

For any continuously differentiable function $h(\cdot)$, we say that $h(\cdot)$ is *single-crossing* on interval $[l, r]$ if (i) $h(x)$ crosses zero at most once and if so from below on $[l, r]$, and (ii) $h'(x) > 0$ whenever $h(x) = 0$ and $x \in [l, r]$.

Definition 1. *We say that the* single-crossing property *holds for a designer if $x - \phi(x)$ is single-crossing on $[-1, 1]$.*

By Lemma 2, $\phi_n(x)$ and $\phi_n'(x)$ converge uniformly to $\phi(x)$ and $\phi'(x)$, respectively, on $[\underline{v}, \overline{v}]$. Therefore, when the single-crossing property holds for the designer, there exists a threshold $\tilde{n}$ such that for all $n \geq \tilde{n}$ function $x - \phi_n(x)$ is single-crossing on $[-1, 1]$; that is, $\phi_n(x)$ crosses the pivotal voter's indifference curve $k = x$ at most once and if so only from above. For such $\phi_n(x)$ we can pin down a unique *switching state* z_n defined as follows

$$z_n := \begin{cases} -1 & \text{if } x > \phi_n(x) \text{ for all } x \in [-1, 1] \\ x & \text{if } x = \phi_n(x) \text{ for some } x \in [-1, 1] \\ 1 & \text{if } x < \phi_n(x) \text{ for all } x \in [-1, 1] \end{cases}. \tag{6}$$

The definition of z_n implies $k > \phi_n(k)$ for $k > z_n$ and $k < \phi_n(k)$ for $k < z_n$. Therefore, in any state $k > z_n$ the designer is more biased towards the reform than the pivotal voter in the following sense: whenever the pivotal voter is indifferent (i.e., in event $k = x$) the designer must strictly prefer the reform to be adopted because $k > \phi_n(k)$. Similarly, in any state $k < z_n$ the designer is more biased towards the status quo than the pivotal voter in that he must strictly prefer the status quo to be retained whenever the pivotal voter is indifferent.

An important economic implication of the single-crossing property is that the designer is tempted to manipulate voters' beliefs upwards (downwards) for state realizations above (below) the switching state z_n. Figure 2 illustrates this. Consider any state realization k' in $(z_n, 1)$. Under the single-crossing property $k' > \phi_n(k')$ must hold. Let $k'' = \phi_n^{-1}(k')$ if $k' \leq \phi_n(1)$ (right panel) or set $k'' = 1$ otherwise (left panel). As is evident in Figure 2, $k'' > k'$ must hold so that the designer and pivotal voter prefer different alternatives whenever $v^{(nq+1)} = x \in (k', k'')$, with the designer strictly preferring the reform. Since x is the pivotal voter's private information, the designer is tempted to lie and let the pivotal voter believe that the realized state is k'', which is higher than the true state k'. It is in this sense that the designer is tempted to manipulate voters' beliefs about state realization upwards. Following the same logic, if the state realization k' is below z_n then the designer is tempted to manipulate voters' beliefs about the state realization downwards.

4.3 Sufficient conditions for the single-crossing property

This subsection provides two easy-to-check sufficient conditions for the single-crossing property. By Definition 1, the single-crossing property is ensured if $\phi'(\cdot) < 1$, that is, the designer's indifference curve is 'uniformly flatter' than the pivotal voter's indifference curve. Lemma 3 provides two sufficient conditions for this to hold.

Lemma 3. *Suppose either (i) ρ is sufficiently close to 0, or (ii) both G and $1 - G$ are strictly log-concave.*[15] *Then $\phi'(\cdot) < 1$, and $\phi_n'(\cdot) \leq 1$ for all $n \geq 0$. These imply that the single-crossing property holds for the designer.*

Conditions (i) simply says that the designer is sufficiently self-interested. Condition (ii) requires very mild conditions on the distribution of voter preferences. These are satisfied if the density function g is strictly log-concave, which already includes a wide class of distributions (see Bagnoli and Bergstrom (2005) for examples) that are frequently assumed in applied theories. Once this mild assumption for G is satisfied, the single-crossing property holds generically for all designer preferences and voting rules.

[15] Strict log-concavity of $1 - G$ is equivalent to a strictly increasing hazard rate $g(x)/(1 - G(x))$. Strict log-concavity of G is equivalent to a strictly decreasing reversed hazard rate $g(x)/G(x)$. In fact, this condition is tight; suppose that either G or $1 - G$ is strictly log-convex on some sub-interval within $[-1, 1]$, then it is possible to construct a designer preference and voting rule under which the single-crossing property fails to hold.

5 Main result: The single-crossing property and the optimality of censorship policy

This section presents our main result, which relates the single-crossing property to the optimality of censorship policies for a monopoly designer in sufficiently large elections (Section 5.1). Formal presentations of the persuasion problem and proofs are in Sections 5.2 and 5.3, respectively.

5.1 Optimal information policy under monopolistic persuasion

Consider a monopoly designer and let $\phi_n(x)$ be his indifference curve. We assume that the single-crossing property holds, so there exists $\tilde{n} \geq 0$ such that for all $n \geq \tilde{n}$ function $x - \phi_n(x)$ is single-crossing on $[-1,1]$ and the unique switching state z_n is identified by (6).

As explained in the Introduction, a censorship policy is characterized by a revelation interval $[a,b]$ with $-1 \leq a \leq b \leq 1$ such that (i) all intermediate state realizations $k \in [a,b]$ are precisely revealed, and (ii) extreme state realizations $k > b$ and $k < a$ are censored under different pooling messages as in Figure 1. Under a censorship policy voters' (common) posterior expectation equals k for all state realizations $k \in [a,b]$ due to full revelation, and equals $\mathbb{E}_F[k|k > b]$ (resp. $\mathbb{E}_F[k|k < b]$) for all state realizations $k > b$ (resp. $k < a$). Observe that both *full disclosure* (with $a = -1$ and $b = 1$) and *no disclosure* (with $a = b \in \{-1, 1\}$) are special cases of censorship policies.

Our main result, Theorem 1, relates the single-crossing property to the optimality of censorship policies in large elections under monopolistic persuasion.

Theorem 1. *Consider a monopoly designer for whom the single-crossing property holds. Then there exists an $N \geq 0$ such that for all $n \geq N$ any optimal information policy is outcome equivalent to a censorship policy with revelation interval $[a_n, b_n]$ that satisfies $-1 \leq a_n \leq z_n \leq b_n \leq 1$.*[16] *Moreover, the following holds:*

1. *If $-1 < z_n < 1$ then $a_n < z_n < b_n$ so that the revelation interval contains the switching state z_n in its interior.*

2. *If $z_n = -1$, then $a_n = -1$ so that only sufficiently high states can be censored.*

3. *If $z_n = 1$, then $b_n = 1$ so that only sufficiently low states can be censored.*

If $g(\cdot)$ is strictly log-concave and ρ is sufficiently close to 0, then $N = 0$ so that these three properties hold for all $n \geq 0$.

[16] Two information policies are *outcome equivalent* if their induced mappings from state realization k to voters' posterior expected state are equal almost everywhere.

Theorem 1 establishes a one-to-one mapping between the three possible locations of the switching state z_n and the structure of the optimal censorship policy. If $z_n \in (-1,1)$ so that $\phi_n(x) - x$ crosses zero at some interior state, then the optimal policy has the feature of *two-sided censorship* in the sense that both very high and very low states can be censored. If instead $z_n = -1$, then $\phi_n(x) < x$ for all $x \in (-1,1)$ and the designer is uniformly more biased towards reform than the pivotal voter. In this case the optimal policy takes the form of *upper censorship* in the sense that only sufficiently high states can be censored. Finally, if $z_n = 1$, then $\phi_n(x) > x$ for all $x \in (-1,1)$ and the designer is uniformly more biased towards the status quo than the pivotal voter. In this case the optimal policy takes the form of *lower censorship* in that only sufficiently low states can be censored. Observe that Theorem 1 is robust in the sense that it applies for all continuous prior F, and for all G as long as the single-crossing property holds. By Lemma 3, this implies that Theorem 1 holds for generic G if the designer is sufficiently self-interested[17], and it holds for all designer preferences characterized by (1) if both G and $1-G$ are strictly log-concave.

Before explaining the intuition of Theorem 1, we apply this theorem to characterize the structure of the monopolistically optimal censorship policies for four examples of designer preferences (illustrated by the four panels of Figure 3).

Example 1. Self-interested designer. *Panel (a) depicts the indifference curve and structure of the optimal censorship policy for a self-interested designer with* $\rho = 0$. *By* (3), $\phi_n(x) = \chi$ *for all* $x \in [\underline{v}, \overline{v}]$. *His switching state* z_n *thus depends solely on* χ. *If* $\chi \in (-1,1)$, *then* $z_n = \chi$ *and by Theorem 1 some two-sided censorship policy with* $a_n < \chi < b_n$ *is optimal for this designer in large elections. This is the case depicted in panel (a) of Figure 3. If instead* $\chi \leq -1$ *(resp.* $\chi \geq 1$*), then he is uniformly more biased towards the reform (resp. status quo) than the pivotal voter in all states. For these cases, Theorem 1 implies the designer's optimal information policy must be either upper (if* $\chi \leq -1$*) or lower (if* $\chi \geq 1$*) censorship in large elections.*

Example 2. Utilitarian social planner. *Panel (b) considers the case of a Utilitarian planner who aims at maximizing voters' ex-post average payoffs. His indifference curve* $\phi_n(x)$ *is given by*[18]

$$\phi_n(x) = \frac{n}{n+1}\left(q\mathbb{E}_G\left[v_i|v_i \leq x\right] + (1-q)\,\mathbb{E}_G\left[v_i|v_i \geq x\right]\right) + \frac{1}{n+1}x \tag{7}$$

[17] The result would be sharply different in a setup with only one representative voter (i.e., $n = 0$). This case is studied by Kolotilin, Mylovanov and Zapechelnyuk (2022). They show that G must be uni-model (i.e., g is single-peaked) to ensure the optimality of a censorship policy for a self-interested designer with $\rho = 0$. In our setup this is not required because, as Proposition A.1 in Appendix A shows, the density function of the pivotal voter's type distribution $\hat{g}_n(\cdot;q)$ is single-peaked for sufficiently large n for all g that are positive and twice-continuously differentiable.

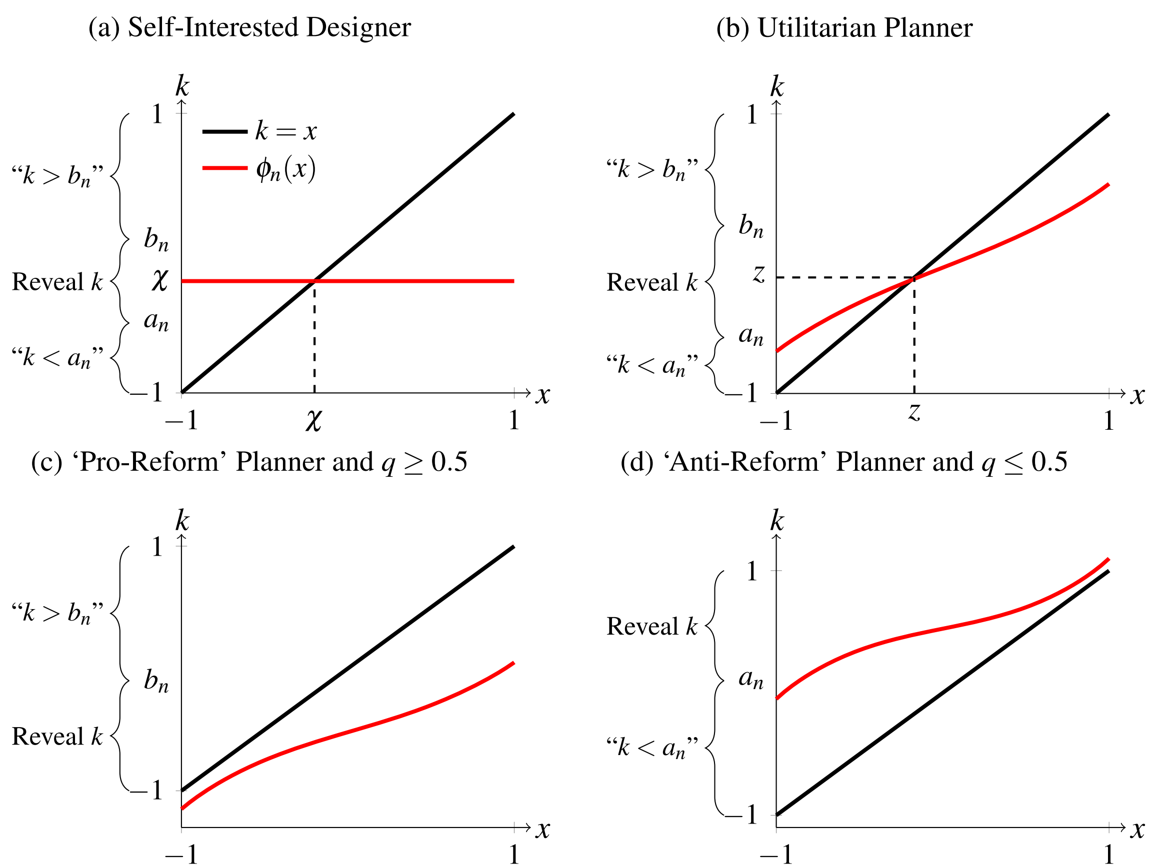
[18] To see why (7) is true, notice that for event $v^{(nq+1)} = x$ to hold, there must be one voter with type $v_i = x$, nq other voters with $v_i \leq x$, and the remaining $n(1-q)$ voters with $v_i \geq x$. Since each voter's type is independently drawn from G, the conditional expectation of any voter with $v_i \leq x$ (resp. $v_i \geq x$) equals $\mathbb{E}_G\left[v_i|v_i \leq x\right]$ (resp. $\mathbb{E}_G\left[v_i|v_i \geq x\right]$). Taking the average over the whole electorate size $n+1$ yields (7).

for $x \in [\underline{v}, \overline{v}]$*. Therefore,* $\phi_n(x) = x$ *if and only if*

$$q\mathbb{E}_G[v_i | v_i \le x] + (1-q)\mathbb{E}_G[v_i | v_i \ge x] = x \tag{8}$$

When both G *and* $1-G$ *are strictly log-concave the single crossing property holds by Lemma 3 and* (8) *admits a unique solution* z *on* $(\underline{v}, \overline{v})$*. A Utilitarian planner's switching point* z_n *thus depends only on* z*. If* $z \in (-1,1)$ *as depicted in panel (b), then* $z_n = z$ *and by Theorem 1 some two-sided censorship policy with* $a_n < z < b_n$ *is Utilitarian optimal in large elections. Interestingly, if* $z \le -1$ *(resp.* $z \ge 1$*) then even a Utilitarian planner can be uniformly more biased towards reform (resp. status quo) than the pivotal voter. For these cases the Utilitarian optimal information policy is either upper (if* $z \le -1$*) or lower (if* $z \ge 1$*) censorship in large elections.*

Figure 3: Four Examples of the Monopolistically Optimal Censorship Policies



Note: In these panels the horizontal axis x denotes the pivotal voter's type realization $v^{(nq+1)}$, the black line denotes the pivotal voter's indifference curve, and the red line denotes the designer's indifference curve.

Example 3. 'Pro-Reform' social planner. *In panel (c) we consider a non-Utilitarian social planner who aims at maximizing the average payoff of the subset of voters whose ex-post payoffs* *under reform are above the 50%-percentile.*[19] *Suppose $q \geq 0.5$ so that a strict majority is required in order to pass the reform. In this case, $\phi_n(x) < x$ must hold for all $x \in (-1,1)$; that is, the designer must be uniformly more biased towards the reform than the pivotal voter in all states. This is because he assigns positive weights only to voters who always like the reform better than the pivotal voter does. The designer thus must prefer the reform if the pivotal voter is indifferent. Therefore, by Theorem 1, in large elections some upper censorship policy must be optimal.*

Example 4. 'Anti-Reform' social planner. *In panel (d) we consider a non-Utilitarian social planner who aims at maximizing the average payoff of the subset of voters whose ex-post payoffs under reform are below the 50% percentile.*[20] *Following the same logic as in Example 3, we can show that for all $q \leq 0.5$ (i.e., a strict majority is required to maintain the status quo) $\phi_n(x) > x$ must hold for all $x \in (-1,1)$; that is, such a designer must be uniformly more biased towards the status quo than the pivotal voter in all states. Theorem 1 then implies that some lower censorship policy must be optimal in large elections.*

The intuition underlying Theorem 1 is as follows. Observe that when $\phi_n(x) - x$ crosses zero from above at an interior switching state $z_n \in (-1,1)$, the revelation interval $[a_n, b_n]$ of the optimal censorship policy must contain z_n in its interior so that voters can always perfectly distinguish between state realizations above and below z_n. Indeed, the single-crossing property implies that the designer has no incentive to hide state realization $k = z_n$. This is because at the switching state $k = z_n$ the interests of the designer and the pivotal voter are aligned; whenever the pivotal voter strictly prefers either alternative, the designer weakly prefers it. Moreover, it is always optimal for the designer to fully separate any pair of state realizations on different sides of z_n. To see why, consider any k_1 and k_2 such that $k_n < z_n < k_2$ and suppose they are not fully separated. As explained above, the designer is tempted to manipulate voters' beliefs about state realizations upwards in state k_2 while downwards in k_1. By fully separating these two states, the induced posterior expectation about state realization will indeed be lower in k_1 and higher in k_2 than in any case where they are not fully separated. The designer thus strictly benefits from such separation.

What, then, drives the optimal choices of thresholds a_n and b_n? Consider b_n first. To perfectly separate state realizations above and below z_n, $b_n \geq z_n$ must hold. Now, recall that the designer is tempted to manipulate voters' beliefs upwards for all states $k > z_n$. Suppose the designer increases threshold b_n to some $b_n + \Delta$ with $\Delta > 0$ small. Then the designer losses the opportunity to manipulate voter's beliefs for state realizations $k \in [b_n, b_n + \Delta]$ because these states are now fully revealed. Nevertheless, this expansion of b_n increases the induced posterior expectation from $\mathbb{E}_F[k|k > b_n]$ to $\mathbb{E}_F[k|k > b_n + \Delta]$ – so that the pivotal voter is more likely to be convinced to pass the reform – in

[19] The weighting function for such a 'pro-Reform' planner is given by $w(x) = \min\{2x, 1\}$ for $x \in [0,1]$.
[20] The weighting function for such an 'anti-Reform' planner is given by $w(x) = \max\{2x-1, 0\}$ for $x \in [0,1]$.

all states $k \in [b_n + \Delta, 1]$. The optimal choice of b_n therefore balances the marginal costs of losing the capability to manipulate voters' beliefs in some states with the marginal gains of a increased effectiveness of persuasion. The tradeoff governing the optimal choice of a_n is similar.

We conclude this subsection with two remarks. First, if the designer can perfectly observe voters' preferences $(v_1, \cdots, v_{n+1})$, our model would become a special case of Alonso and Câmara (2016*a*) and the optimal information policy would be a binary cutoff strategy that only reveals whether the realized k is above or below some threshold. Therefore, the fact that our optimal information policy can have a more nuanced structure (i.e., with a non-trivial revelation interval) is due to the assumption that voters have private preferences. Second, as Examples 2 to 4 illustrate, full information disclosure can be suboptimal even when the designer's goal is to maximize voters' welfare. To understand why, observe that conditional on the pivotal voter's type realization being x, in all states k between x and $\phi_n(x)$ the preferences of the designer and pivotal voters disagree. Our single-crossing property implies that $x = \phi_n(x)$ can hold for at most one $x \in [-1, 1]$ for sufficiently large n. The interim conflict of interests between the designer and the pivotal voter is thus ubiquitous. Consequently, full disclosure is in general suboptimal.

5.2 A formal presentation of the persuasion problem

We start by formally presenting the persuasion problem faced by the monopoly designer. Let θ denote the common posterior expectation about the state realization, shared by all voters and the designer. Given $\phi_n(\cdot)$, the designer's expected utility under θ is given by

$$W_n(\theta) = \int_{\underline{v}}^{\theta} (\theta - \phi_n(x))\, \hat{g}_n(x; q) dx \tag{9}$$

where $\hat{g}_n(\cdot; q) = \hat{G}_n'(\cdot; q)$ is the density function of the pivotal voter's type realization.[21] Because the designer's expected payoff depends on voters' posterior expectation θ only, it is convenient to present any information policy π by the distribution H_π of posterior means it induces. We say that a distribution of posterior means H is *feasible* if it can be induced by some information policy $\pi \in \Pi$. It is well known that given prior F, a distribution of posterior means H is feasible if and only if F is a *mean-preserving spread* of H (Gentzkow and Kamenica, 2016; Kolotilin et al., 2017; Dworczak and Martini, 2019).[22] In the sequel we write $F \succeq_{MPS} H$ if F is a mean-preserving spread of H.

The persuasion problem. For a monopoly designer, an information policy π is optimal if and

[21] To see why (9) holds, recall that x denotes the type realization of the pivotal voter. By the discussion in Section 3.1, the reform is adopted if $\theta \geq x$ and in this case the designer gets an expected payoff $\theta - \phi_n(x)$; otherwise the status quo is maintained and the designer's payoff is zero.

[22] F is a mean-preserving spread of H if $\int_{-1}^{x} H(\theta)d\theta \leq \int_{-1}^{x} F(\theta)d\theta$ for all $x \in [-1, 1]$, where equality holds for $x = \pm 1$. An alternative, equivalent definition is that $\mathbb{E}_F[\omega(\cdot)] \geq \mathbb{E}_H[\omega(\cdot)]$ for any convex function $\omega(\cdot)$.

only if H_π solves

$$\max_{H\in\Delta([-1,1])}\int_{-1}^{1} W_n(\theta)dH(\theta), \quad \text{s.t. } F \succeq_{MPS} H \tag{MP}$$

As we show in Appendix C.2, $W_n(\cdot)$ is twice-continuously differentiable and thus upper semi-continuous. Therefore, (MP) admits at least one solution (Dworczak and Martini, 2019).

Finally, observe that the distribution of the posterior expectation induced by a censorship policy with revelation interval $[a,b]$ is given by

$$H_{\mathscr{P}(a,b)}(\theta) := \begin{cases} F(a)\cdot \mathbb{1}\{\theta \geq \mathbb{E}_F[k|k<a]\}, & \text{if } \theta\in[-1,a) \\ F(\theta), & \text{if } \theta\in[a,b) \\ F(b)+[1-F(b)]\cdot\mathbb{1}\{\theta\geq \mathbb{E}_F[k|k>b]\}, & \text{if } \theta\in[b,1] \end{cases} \tag{10}$$

where $\mathbb{1}\{\cdot\}$ is the indicator function. We say that an information policy $\pi\in\Pi$ is a *censorship policy* if H_π coincides with (10) for some $-1\leq a\leq b\leq 1$ almost everywhere. In the sequel we slightly abuse notation and let $\mathscr{P}(a,b)$ denote both any specific censorship policy or the set of all censorship policies with revelation interval $[a,b]$, whenever this does not lead to confusion. For the special case $a=b$ we simply write $\mathscr{P}(a,b)$ as $\mathscr{P}(a)$ and refer to it as a *cutoff policy* because it only reveals whether the realize state is above, equal, or below cutoff a.

5.3 Proof of Theorem 1

Our proof for Theorem 1 relies critically on the following two lemmas, which are proven in Appendices C.1 and C.2, respectively. Both Lemmas establish important curvature properties of $W_n(\cdot)$, which are illustrated in the three panels of Figure 4, that help to pin down the structures of solutions to the designer's problem (MP).

In Lemma 4, we identify a novel 'increasing slope property' and show that this condition ensures that any solution H to (MP) cannot be less informative than a given cutoff policy that only reveals whether the realized state is above, equal or below a certain threshold.

Lemma 4. *Suppose that $x-\phi_n(x)$ crosses zero only once and from below at an interior point $z_n\in(-1,1)$. Then $W_n(\cdot)$ satisfies the 'increasing-slope property' at point z_n, that is,*

$$\frac{W_n(x)-W_n(z_n)}{x-z_n}\leq\frac{W_n(y)-W_n(z_n)}{y-z_n}, \forall y>x$$

and strict inequality holds if $x<z_n<y$ (cf. panel (a) of Figure 4).[23] *Moreover, any solution H to*

[23] Geometrically, a function $U(\cdot)$ satisfies the increasing-slope property at point z only if for all $x\neq z$ the line segment connecting $(x,U(x))$ and $(z,U(z))$ lies above $U(\cdot)$, as demonstrated in panel (a) of Figure 4. Note that if $U(\cdot)$ satisfies the increasing-slope at point z then it must be locally convex at z. The converse, however, is not true in general.

problem ([MP]) *must satisfy* $H \succeq_{MPS} H_{\mathscr{P}(z_n)}$. *In other words, any optimal information policy must reveal whether the state realization is above, equal to or below* z_n.

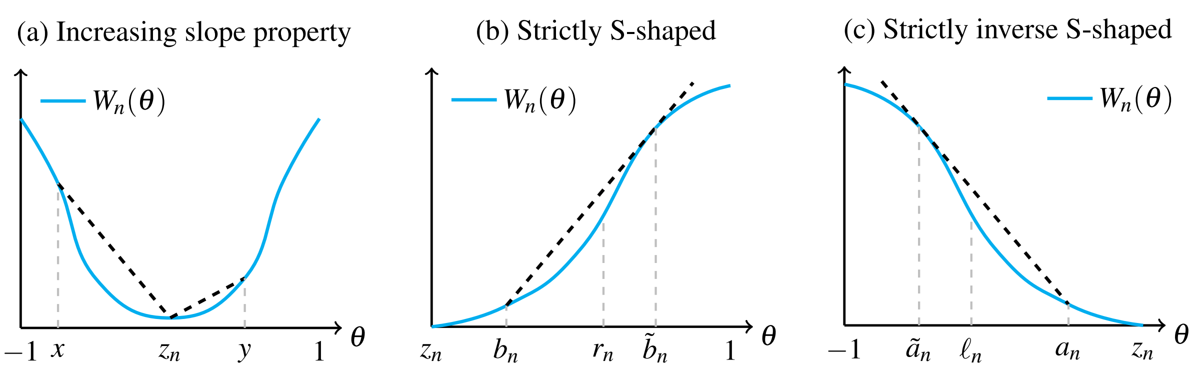
In line with standard terminology (e.g., Kolotilin, Mylovanov and Zapechelnyuk (2022)), we say that $W_n(\cdot)$ is *strictly S-shaped* on some interval if it is strictly convex below some inflection point and strictly concave above it. Likewise, $W_n(\cdot)$ is *strictly inverse-S-shaped* if it is strictly concave below some inflection point and strictly concave above it. Notice that both definitions include strictly convex and concave functions as special cases.

Lemma 5. *Suppose that the single-crossing property holds for a monopoly designer. Then there exists an* $N \geq 0$ *such that for all* $n \geq N$ *there are* ℓ_n *and* r_n *with* $-1 \leq \ell_n \leq z_n \leq r_n \leq 1$ *such that the following two properties hold:*[24]

1. $W_n(\cdot)$ *is strictly S-shaped on* $[z_n, 1]$ *with inflection point* r_n *(cf. panel (b) of Figure 4).*
2. $W_n(\cdot)$ *is strictly inverse-S-shaped on* $[-1, z_n]$ *with inflection point* ℓ_n *(cf. panel (c) of Figure 4).*

In addition, if $g(\cdot)$ *is strictly log-concave and* ρ *is sufficiently close to* 0*, then* $N = 0$ *so the above curvature properties hold for all* $n \geq 0$.

Figure 4: Graphical illustrations of Lemmas 4, 5 and the proof of Theorem 1



With these ingredients we are ready to prove Theorem 1, directly using the quantities N, ℓ_n and r_n identified in Lemma 5.

Proof of Theorem 1. Depending on the value of z_n, we distinguish between three cases.

Case 1: $\phi_n(x) - x$ crosses zero from above at a unique interior point $z_n \in (-1, 1)$. By Lemma 4, $W_n(\cdot)$ satisfies the increasing-slope property at point z_n and any solution H to problem (MP)

[24] In fact, single-crossing property is almost necessary; suppose instead that $\phi(x) - x$ crosses zero from below at some point, then this lemma no longer holds and for sufficiently large n there exists some interval $[x, y] \subset (-1, 1)$ and $\varepsilon > 0$ such that $W_n(\cdot)$ is strictly concave on $[x, y]$ but is strictly convex on $[x - \varepsilon, x]$ and $[y, y + \varepsilon]$, respectively. In this case, it follows from duality arguments in Dworczak and Martini (2019) and Kolotilin, Mylovanov and Zapechelnyuk (2022) that there exists some continuous and full-support prior F under which the optimal information policy is not censorship.

must satisfy $H \succeq_{MPS} H_{\mathscr{P}(z_n)}$. As a consequence, the monopolistic persuasion problem can be decomposed into two auxiliary problems on intervals $[-1,z_n]$ and $[z_n,1]$, respectively:

$$\max_{H\in\Delta([z_n,1])}\int_{z_n}^{1} W_n(\theta)dH(\theta), \quad \text{s.t. } F_{\mathrm{I}} \succeq_{MPS} H \tag{MP-I}$$

$$\max_{H\in\Delta([-1,z_n])}\int_{-1}^{z_n} W_n(\theta)dH(\theta), \quad \text{s.t. } F_{\mathrm{II}} \succeq_{MPS} H \tag{MP-II}$$

In these problems, F_{I} is the truncated cdf of F on $[z_n,1]$, and it equals F if $z_n=-1$. F_{II} is the truncated cdf of F on $[-1,z_n]$, and it equals F if $z_n=1$.

Recall from Lemma 5 that, for all $n\geq N$, $W_n(\cdot)$ is strictly S-shaped on $[z_n,1]$ with inflection point r_n and strictly inverse S-shaped on $[-1,z_n]$ with inflection point ℓ_n. Then, by Kolotilin, Mylovanov and Zapechelnyuk (2022), the solution to problem (MP-I) is uniquely given by a censorship policy $\mathscr{P}(z_n,b_n)$. The threshold b_n satisfies the following complementary slackness condition

$$(\tilde{b}_n-b_n)W_n'(\tilde{b}_n)\leq W_n(\tilde{b}_n)-W_n(b_n) \tag{FOC: b_n}$$

where $\tilde{b}_n=\mathbb{E}_F[k|k\geq b_n]$ and (FOC: b_n) is binding whenever $b_n\in(z_n,1)$ (cf. panel (b) of Figure 4).[25] Moreover, b_n and $\tilde{b}_n$ satisfy $b_n<r_n<\tilde{b}_n$ for $r_n\in(z_n,1)$, and $b_n=1$ if $r_n=1$. Similarly, the solution to problem (MP-II) is uniquely given by a censorship policy $\mathscr{P}(a_n,z_n)$. The threshold a_n satisfies the following complementary slackness condition

$$(a_n-\tilde{a}_n)W_n'(\tilde{a}_n)\leq W_n(a_n)-W_n(\tilde{a}_n) \tag{FOC: a_n}$$

where $\tilde{a}_n=\mathbb{E}_F[k|k\leq a_n]$ and (FOC: a_n) is binding whenever $a_n\in(-1,z_n)$ (cf. panel (c) of Figure 4). Moreover, a_n and $\tilde{a}_n$ satisfy $a_n>\ell_n>\tilde{a}_n$ for $\ell_n\in(-1,z_n)$, and $a_n=-1$ if $\ell_n=1$. Taken together, these imply that the optimal solution is uniquely given by a censorship policy $\mathscr{P}(a_n,b_n)$.

Next we show that $a_n<z_n<b_n$ must hold whenever $z_n\in(-1,1)$. By (9) we have

$$\begin{aligned} W_n''(z_n) &= \hat{g}_n(z_n;q)\left(2-\phi_n{}'(z_n)\right)+(z_n-\phi_n(z_n))\,\hat{g}_n'(z_n;q) \\ &= \hat{g}_n(z_n;q)\left(2-\phi_n{}'(z_n)\right)>\hat{g}_n(z_n;q)>0 \end{aligned}$$

The second step holds because $z_n-\phi_n(z_n)=0$ by definition of z_n, and the third step holds because the single-crossing property requires $\phi_n'(z_n)<1$ whenever $z_n-\phi_n(z_n)=0$. Therefore, $W_n(\theta)$ is strictly convex in a neighborhood around z_n and thus $r_n>z_n$. This implies that

$$W_n'(z_n)<\frac{W_n(\theta)-W_n(z_n)}{\theta-z_n} \tag{11}$$

[25] If $(\tilde{b}_n-b_n)W_n'(\tilde{b}_n)>W_n(\tilde{b}_n)-W_n(b_n)$ for all $b_n\in[z_n,1]$ then $b_n=1$. The similar result holds for a_n.

holds for all $\theta \in [z_n, r_n]$. Since $W_n(\cdot)$ satisfies the increasing-slope property at point z_n, the right-hand side of (11) is increasing in θ. Therefore, (11) must hold for all $\theta > z_n$. It follows directly that condition (FOC: b_n) cannot be binding at $b_n = z_n$ for any $z_n \in (-1,1)$. This implies that $b_n > z_n$ must hold. $a_n < z_n$ can be established analogously. This proves statement (1) of Theorem 1.

Case 2: $z_n = -1$ so that $\phi_n(x) < x$ for all $x \in (-1,1)$. By Lemma 5, $W_n(\cdot)$ is S-shaped on $[-1,1]$ with inflection point $r_n \in [-1,1]$ for all $n > N$. The monopolistically optimal information policy is therefore uniquely given by an upper-censorship policy $\mathscr{P}(-1,b_n)$, with b_n determined by condition (FOC: b_n). This proves statement (2) of Theorem 1.

Case 3: $z_n = 1$ so that $\phi_n(x) > x$ for all $x \in (-1,1)$. By Lemma 5, $W_n(\cdot)$ is strictly inverse S-shaped on $[-1,1]$ with inflection point $\ell_n \in [-1,1]$ for all $n > N$. The monopolistically optimal information policy is therefore uniquely given by a lower-censorship policy $\mathscr{P}(a_n,1)$, with a_n determined by condition (FOC: a_n). This proves statement (3) of Theorem 1. □

6 Properties of the optimal censorship policy and the designer's payoff

Our main result Theorem 1 shows that if the single-crossing property holds for a designer, then there exists threshold $N \geq 0$ such that for all $n \geq N$ a censorship policy with revelation interval $[a_n, b_n]$ is uniquely optimal under monopolistic persuasion. In this section we further explore the properties of these optimal thresholds. In Section 6.1, we characterize and discuss properties of the limits of a_n and b_n and the designer's asymptotic payoff as the electorate size $n \to \infty$. In Section 6.2 we study how the optimal thresholds a_n and b_n vary with designer preferences and voting rules.

6.1 Asymptotically optimal censorship policy and designer's payoff

In this subsection we characterize and discuss properties of the limits of a_n and b_n and the designer's payoff as $n \to \infty$. Omitted proofs for this subsection are in Appendix D.

Our main result for this subsection is Theorem 2, which shows that under single-crossing property the asymptotically optimal censorship policy and designer's payoff can be characterized with only three variables: $v_q^* = G^{-1}(q)$, $\phi^* = \phi(v^*) = \rho \int_0^1 G^{-1}(y)dw(y) + (1-\rho)\chi$, and

$$z^* := \lim_{n\to\infty} z_n = \begin{cases} -1 & \text{if } x > \phi(x) \text{ for all } x \in [-1,1] \\ x & \text{if } x = \phi(x) \text{ for some } x \in [-1,1] \\ 1 & \text{if } x < \phi(x) \text{ for all } x \in [-1,1] \end{cases} \tag{12}$$

Before formally stating Theorem 2, let us recall the economic implications of the three variables.

By Lemmas 1 and 2 we have $v^{(nq+1)} \xrightarrow{p} v_q^*$ and $\varphi_n(v) \xrightarrow{a.s.} \phi^*$, respectively. Therefore, as $n \to \infty$, the pivotal voter prefers reform (status quo) almost surely if $k > (<) v_q^*$, while the designer prefers reform (status quo) almost surely if $k > (<) \phi^*$. Finally, z^* is the limiting switching state and, following the discussion in Section 4.2, it has the following feature: for all $k > z^*$ (resp. $k < z^*$), the designer is tempted to manipulate voters' posterior expectation about k upwards (resp. downwards) even as $n \to \infty$. This reflects the effect of the inference problem from the pivotal voter's choice explained in Section 4.1. Because $\phi(\cdot)$ is non-decreasing and $\phi^* = \phi(v_q^*)$, it follows from (12) that z^* and v_q^* must locate on different sides of ϕ^* (i.e., either $z^* \leq \phi^* \leq v_q^*$ or $z^* \geq \phi^* > v_q^*$).

In what follows we assume $v_q^*, \phi^* \in [-1,1]$ to simplify the exposition.[26] Let $\{a_n\}_{n \geq N}$ and $\{b_n\}_{n \geq N}$ denote the sequences of the cutoff points of the optimal censorship policies. Define $a^* := \lim_{n\to\infty} a_n$ and $b^* := \lim_{n\to\infty} b_n$. Let W_n denote the expected payoff of a monopoly designer under his optimal information policy given electorate size n.[27] Define $W^* := \lim_{n\to\infty} W_n$. Theorem 2 shows that all these limits exist and explicitly characterize them.

Theorem 2. *Suppose that the single-crossing property holds for the monopoly designer and $v_q^*, \phi^* \in [-1,1]$. Define*

$$\overline{\phi}(v_q^*) := \sup\left\{y \in [-1,1] : \mathbb{E}_F\left[k | k \leq y\right] \leq v_q^*\right\} \tag{13}$$

$$\underline{\phi}(v_q^*) := \inf\left\{y \in [-1,1] : \mathbb{E}_F[k | k \geq y] \geq v_q^*\right\} \tag{14}$$

Then a^*, b^* and W^* are characterized as follows:[28]

1. *If $\underline{\phi}(v_q^*) \leq \phi^* \leq \overline{\phi}(v_q^*)$, then $a^* = \min\{\phi^*, z^*\}$, $b^* = \max\{\phi^*, z^*\}$, and $W^* = \int_{\phi^*}^{1} (k - \phi^*)\, dF(k)$.*
2. *If $\phi^* < \underline{\phi}(v_q^*)$, then $a^* = z^* \leq \phi^*$, $b^* = \underline{\phi}(v_q^*) > \phi^*$, and $W^* = \int_{\underline{\phi}(v_q^*)}^{1} (k - \phi^*)\, dF(k)$.*
3. *If $\phi^* > \overline{\phi}(v_q^*)$, then $a^* = \overline{\phi}(v_q^*) < \phi^*$, $b^* = z^* \geq \phi^*$, and $W^* = \int_{\overline{\phi}(v_q^*)}^{1} (k - \phi^*)\, dF(k)$.*

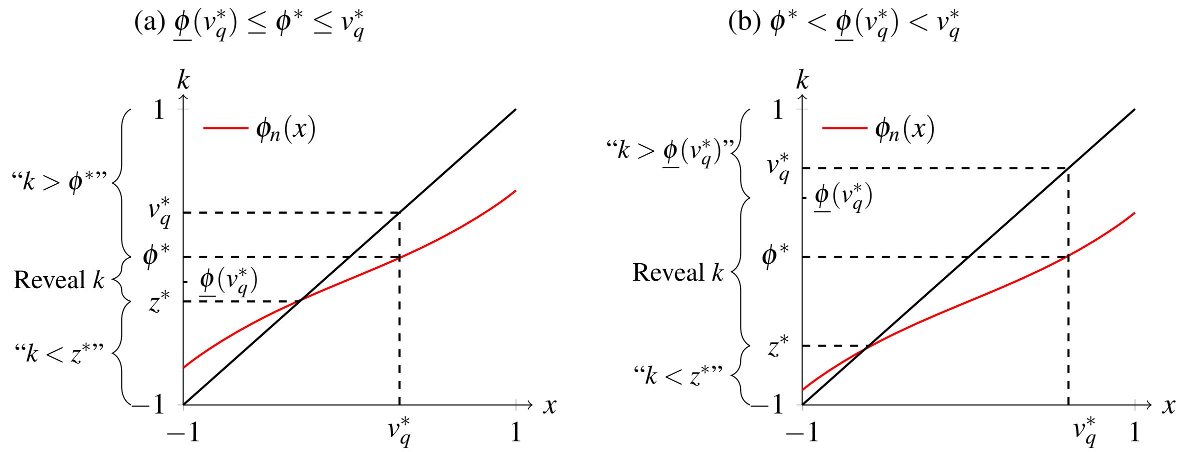
Figure 5 illustrates Theorem 2 for the case $\phi^* \leq v_q^*$ and $\rho > 0$. By Theorem 2, when $\phi^* \leq v_q^*$ we have $a^* = z^* \leq \phi^*$ and $b^* = \max\left\{\phi^*, \underline{\phi}(v_q^*)\right\} \in \left[\phi^*, v_q^*\right]$. If instead $\phi^* > v_q^*$ we have $b^* = z^* > \phi^*$ and $a^* = \min\left\{\phi^*, \overline{\phi}(v_q^*)\right\} \in \left[v_q^*, \phi^*\right]$. These together imply

$$\min\left\{v_q^*, z^*\right\} \leq a^* \leq \min\left\{\phi^*, z^*, \overline{\phi}(v_q^*)\right\} \leq \max\left\{\phi^*, z^*, \underline{\phi}(v_q^*)\right\} \leq b^* \leq \max\left\{v_q^*, z^*\right\} . \tag{15}$$

(15) suggests that both z^* and ϕ^* must lie in $[a^*, b^*]$, the limiting revelation interval as $n \to \infty$. $z^* \in [a^*, b^*]$ follows from the fact that $a_n \leq z_n \leq b_n$ for all $n \geq N$ (cf. Theorem 1) and $z^* = \lim_{n\to\infty} z_n$. The

[26] Any $\phi^* < -1$ (resp. $\phi^* > 1$) is equivalent to the case $\phi^* = -1$ (resp. $\phi^* = 1$). The same applies for v_q^*.
[27] That is, W_n is the value of persuasion problem (MP) with electorate size n.
[28] In fact, our characterizations for W^* do not rely on the single-crossing property; see Appendix D.

Figure 5: Asymptotically optimal censorship policy for the case $\phi^* \leq v_q^*$ and $\rho > 0$



reason for $\phi^* \in [a^*, b^*]$ is the following. If $k > \phi^*$ (resp. $k < \phi^*$) then as $n \to \infty$ the designer almost surely prefers Reform (resp. Status Quo) and hence would like to induce a higher (resp. lower) posterior expectation of state. This cannot be efficiently achieved if any $k \neq k'$ with $k \leq \phi^* \leq k'$ are not fully separated ex-post.

Three implications are immediate from (15). First, regardless of the designer's preference, full disclosure (i.e., $a_n = -1$ and $b_n = 1$) is generically suboptimal for sufficiently large n whenever $v_q^* \in (-1, 1)$, that is, the pivotal voter's preference is state-dependent as $n \to \infty$. Second, whenever $\phi^* \in (-1, 1)$ so that the designer's preference is state-dependent as $n \to \infty$, no disclosure (i.e., $a_n = b_n \in \{-1, 1\}$) is never optimal in sufficiently large elections. Third, if both $z^*, v_q^* \in (-1, 1)$ hold, then $-1 < a_n < b_n < 1$ can be ensured – so that the optimal policy will indeed censor both sufficiently high and low state realizations – for n large enough. Condition $z^* \in (-1, 1)$ requires that the indifference curves of the designer and pivotal voter intersect at some interior point in $(-1, 1)$ as $n \to \infty$. This is the case, for instance, if the designer is self interested with $\rho = 0$ and $\chi \in (-1, 1)$ (cf. Example 1), or if the designer is a Utilitarian planner (cf. Example 2) and (8) admits an interior solution on $(-1, 1)$. We summarize these observations in Corollary 1.

Corollary 1. *Suppose that the single-crossing property holds. If $v_q^* \in (-1, 1)$, then full disclosure is not optimal for sufficiently large n. If $\phi^* \in (-1, 1)$, then no disclosure is not optimal for sufficiently large n. If both $z^*, v_q^* \in (-1, 1)$, then $-1 < a_n < b_n < 1$ must hold for sufficiently large n.*

As we explained in the previous section, a necessary condition for the optimal censorship policy to have a non-trivial revelation interval (i.e., $a_n < b_n$) with finite n is that the designer is uncertain about the pivotal voter's type. As $n \to \infty$ such uncertainty vanishes and one might expect $a^* = b^*$ in some circumstances. Corollary 2 gives sufficient and necessary conditions for $a^* = b^*$ to hold.

Corollary 2. *Suppose that the single-crossing property holds and $v_q^* \in (-1,1)$. Let a^* and b^* be characterized in Theorem 2. The following properties hold:*

1. *If $\phi^* = v_q^*$, then $a^* = b^* = \phi^*$.*
2. *If $\phi^* \neq v_q^*$ and $\rho = 0$, then $a^* = b^*$ if and only if $\chi \in \left[\underline{\phi}(v_q^*), \overline{\phi}(v_q^*)\right]$.*
3. *If $\phi^* \neq v_q^*$ and $\rho > 0$, then $a^* < b^*$.*

To understand this corollary, note that a^* and b^* must satisfy two conditions: (i) $z^*, \phi^* \in [a^*, b^*]$, and (ii) $[a^*, b^*] \cap \left[\underline{\phi}(v_q^*), \overline{\phi}(v_q^*)\right] \neq \emptyset$ where $\overline{\phi}(v_q^*)$ and $\underline{\phi}(v_q^*)$ are defined by (13) and (14) respectively. Both conditions follow directly from (15). Condition (i) reflects the impacts of the designer's ex-ante preference and the inference based on the pivotal voter's choice on a^* and b^*, as explained above. Condition (ii) reflects the fact that there are limits in the extent to which the pivotal voter with type v_q^* can be persuaded. More specifically, notice that under censorship policy the induced election outcome in the limit can be characterized by an implementation threshold $t^* \in [-1,1]$ such that reform (status quo) is implemented almost surely if $k > (<) t^*$. To implement any interior $t^* \in (-1,1)$ it is necessary that $a^* \leq t^* \leq b^*$. Given the pivotal voter's type v_q^*, the set of feasible implementation thresholds in the limit is exactly $\left[\underline{\phi}(v_q^*), \overline{\phi}(v_q^*)\right]$.

Consider first the case $v_q^* = \phi^* \in (-1,1)$, i.e., there is no ex-ante conflict of interests between the designer and the pivotal voter. In this case we have $v_q^* = \phi^* = \phi(v_q^*) \in (-1,1)$ and thus $z^* = \phi^*$ by (12). It then follows from (15) that $a^* = b^* = \phi^*$. Therefore, perhaps surprisingly, even in the absence of ex-ante conflict of interests the asymptotically optimal information policy is not full disclosure, but instead a binary cutoff policy that only reveals whether the state realization is above, equal or below ϕ^*, the ex-ante threshold of acceptance of the designer. Such binary cutoff policy is indeed asymptotically optimal because it does induce the pivotal voter to implement the reform (status quo) almost surely as $n \to \infty$ whenever $k > (<) \phi^*$, which perfectly coincides with the designer's favored outcome ex-ante.[29]

Now suppose $\phi^* \neq v_q^*$ so that the ex-ante preferences of the designer and the pivotal voter are not fully aligned. Consider the case $\rho = 0$ first so that the designer is purely self interested. For this case, $\phi(\cdot)$ is constant and equals χ so we have $z^* = \phi^* = \chi$. The designer would prefer the reform (status quo) to be implemented in all states $k > (<) \chi$. Corollary 2 implies that $a^* = b^* = \chi$ if and only if χ lies in the feasible set of implementation thresholds $\left[\underline{\phi}(v_q^*), \overline{\phi}(v_q^*)\right]$. This is because by simply revealing whether k is above or below χ the designer can already sway the pivotal voter's

[29] Kamenica and Gentzkow (2011) made a similar observation that aligning the interests between the sender and the receiver does not necessarily imply more information disclosure by the sender. Specifically, they wrote that "*The impact of alignment on the amount of information communicated in equilibrium is also ambiguous. On the one hand, the more Receiver responds to information in a way consistent with what Sender would do, the more Sender benefits from providing information. On the other hand, when preferences are more aligned Sender can provide less information and still sway Receiver's action in a desirable direction. Hence, making preferences more aligned can make the optimal signal either more or less informative.*" (cf. pages 2604-2605 therein).

decision in the designer's preferred direction in all states k as $n \to \infty$. Any further expansion of the revelation interval is weakly harmful in the limit as it can only make the pivotal voter choose the designer's less preferred outcome. If instead χ lies outside $\left[\underline{\phi}(v_q^*), \overline{\phi}(v_q^*)\right]$, then $a^* < b^*$ so that the optimal censorship policy will contain a non-trivial revelation interval even in the limit. For instance, if $\chi < \underline{\phi}(v_q^*)$ we will have $a^* = \chi$ and $b^* = \underline{\phi}(v_q^*) > \chi$. The expansion of b^* from χ to $\underline{\phi}(v_q^*)$ is driven by the demand to effectively persuade the pivotal voter. The resulting structure is very similar to the judge example in Kamenica and Gentzkow (2011) in that the pooling message "$k > \underline{\phi}(v_q^*)$" just makes the pivotal voter indifferent ex-ante. It is the difficulty to persuade voters that produces the non-trivial revelation interval in the asymptotically optimal censorship policy.

If the designer is prosocial with $\rho > 0$, then Corollary 2 implies $a^* < b^*$, so that the optimal censorship policy will have a non-trivial revelation interval even as $n \to \infty$, whenever $v_q^* \neq \phi^*$. This is because $\phi(\cdot)$ is a strictly increasing function for $\rho > 0$ and $\phi^* = \phi(v_q^*)$ by definition. Therefore, $v_q^* > (<) \phi^*$ implies $z^* < (>) \phi^*$ (see Figure 5 for an illustration). Then, by (15) we have $a^* \leq \min\{\phi^*, z^*\} < \max\{\phi^*, z^*\} \leq b^*$. Unlike the previous case with $\rho = 0$, here the emergence of a non-trivial revelation interval in the limit is not driven by the difficulty to persuade the pivotal voter. It instead stems from the fact that the designer is uncertain about his preference (as it depends on voters' private types) and must infer this through the pivotal voter's choice. As is explained in Section 4.2, such inference produces a unique (limiting) switching state z^*, which is in general different from the designer's ex-ante threshold of acceptance ϕ^*. Importantly, the discrepancy between ϕ^* and z^* appears only when the designer is uncertain about his preferences. In our model this can happen only if both (i) voters have private information regarding their preferences, and (ii) $\rho > 0$ so that designer cares about voters' payoffs.

Finally, we compare W^* – the designer's asymptotic payoff at the optimum – with two important benchmarks $\overline{W}$ and W^{Full}. Here, $\overline{W}$ is the designer's payoff under his *omniscient* control – i.e., he directly observes state k and voters' type profile v and dictates the election outcome – as $n \to \infty$.[30] W^{Full} equals the designer's payoff under full information disclosure as $n \to \infty$. Corollary 3 gives the relative rankings of $\overline{W}$, W^* and W^{Full}.

Corollary 3. *Suppose $v_q^* \in (-1,1)$. Then $\overline{W}$, W^* and W^{Full} are ranked as follows:*

1. *If $v_q^* = \phi^*$, then $\overline{W} = W^* = W^{Full}$;*
2. *If $v_q^* \neq \phi^*$ and $\underline{\phi}(v_q^*) \leq \phi^* \leq \overline{\phi}(v_q^*)$, then $\overline{W} = W^* > W^{Full}$;*
3. *If $\phi^* < \underline{\phi}(v_q^*)$ or $\phi^* > \overline{\phi}(v_q^*)$, then $\overline{W} > W^* > W^{Full}$.*

To understand Corollary 3, consider first the asymptotic payoff under full information revelation, W^{Full}. Because a designer can always opt for full disclosure, $W^* \geq W^{\text{Full}}$ necessarily holds. Under

[30] If the designer is a social planner (i.e., $\rho = 1$) who maximizes some voter welfare function, then $\overline{W}$ corresponds to the asymptotic voter welfare under the first best scenario.

full disclosure the pivotal voter of type v_q^* implements the reform if $k > v_q^*$ and does not do so otherwise. Hence, whenever an ex-ante conflict of interests exists (i.e. $v_q^* \neq \phi^*$), the designer earns strictly less under full disclosure than under his optimal information policy (i.e., $W^* > W^{\text{Full}}$). Intuitively, full disclosure implies that the conflicting states between ϕ^* and v_q^* are fully revealed, while the optimal information policy should avoid doing so as much as possible.

Next, we relate W^* to the 'omniscient control' benchmark $\overline{W}$. Recall that $\left[\underline{\phi}\left(v_q^*\right), \overline{\phi}\left(v_q^*\right)\right]$ is the set of feasible implementation threshold t^* in the limit; that is, the designer can ensure reform (status quo) being elected with probability one for $k > (<) t^*$. Whenever ϕ^* falls within this set, he can secure his preferred outcome with probability one as $n \to \infty$ by setting $t^* = \phi^*$. In that case he receives payoff $\overline{W}$, which is what he could secure in the limit under omniscient control. Note that $\phi^* \in \left[\underline{\phi}\left(v_q^*\right), \overline{\phi}\left(v_q^*\right)\right]$ if either (i) ϕ^* is close to v_q^* such that the ex-ante conflict of interests between the designer and the pivotal voter is low, or (ii) v_q^* is close to $\mathbb{E}_F[k]$ so that, a priori, the pivotal voter is almost indifferent between the reform and the status quo. In the latter case very weak evidence is already sufficient to persuade the pivotal voter (and thus the feasible set is large). In case ϕ^* lies below the feasible set ($\phi^* < \underline{\phi}\left(v_q^*\right)$), the designer's preferred alternative will (as $n \to \infty$) almost surely not be elected when $k \in \left(\phi^*, \underline{\phi}\left(v_q^*\right)\right)$. The designer thus cannot ensure his preferred outcome being elected with certainty as $n \to \infty$ through public persuasion and therefore gets strictly less than $\overline{W}$. A similar intuition applies when $\phi^* > \overline{\phi}\left(v_q^*\right)$.

6.2 Comparative statics

In this section we derive comparative statics for how the optimal thresholds a_n and b_n vary with the designer's preference and the voting rule. Omitted proofs for this subsection are in Appendix E.

We first study the effects of shifting the designer's preference towards the reform, holding ρ fixed. Such a shift can occur, for instance, if $\rho < 1$ and χ decreases. In that case the designer's personal payoffs from reform increase and his threshold for accepting reform decreases. Proposition 1 shows how such a preference shift towards reform affects the designer's optimal censorship policy.

Proposition 1. *Suppose $\rho < 1$ and either condition (i) or (ii) in Lemma 3 holds.*[31] *Consider any $\chi_I > \chi_{II}$. Then, for sufficiently large n, as χ decreases from χ_I to χ_{II} the following holds:*

1. *a_n weakly decreases, strictly so if $a_n \in (-1,1)$ under $\chi = \chi_I$.*
2. *b_n weakly decreases, strictly so if $b_n \in (-1,1)$ under $\chi = \chi_I$.*

In words, as the designer's personal payoff from reform increases, his optimal censorship policy will censor fewer states downwards but more states upwards.

The intuition of Proposition 1 is as follows. On the one hand, as the designer's personal payoff from reform increases, he becomes more tempted to persuade voters to pass the reform. Therefore,

[31] Recall that these conditions are (i) ρ is sufficiently close to 0, or (ii) both G and $1-G$ are strictly log-concave.

b_n decreases because the designer is now tempted to manipulate voters' beliefs upwards in some states that he would previously have been willing to reveal truthfully. On the other hand, such a preference shift makes the designer less tempted to persuade voters to maintain the status quo. Consequently, a_n also decreases because the designer is now willing to truthfully reveal some states he would previously censor to manipulate voters' beliefs downwards.

For a prosocial designer with $\rho > 0$, a similar preference shift towards the reform could also occur if his welfare weighting function $w(\cdot)$ decreases in the sense of first order stochastic dominance.[32] In this way the designer systematically puts more weights on voters whose ex-post type realizations are lower and hence receive higher payoffs under reform. Such a change also makes the designer favor reform more and thus be more tempted to persuade voters to pass the reform. Following the intuition discussed above, one may expect that the result in Proposition 1 continues to hold in this case. Proposition E.1 in Appendix E shows that, under some mild conditions, this is indeed true: for sufficiently large n both a_n and b_n decrease as the designer's weighting function $w(\cdot)$ shifts from $w^{\mathrm{I}}(\cdot)$ to $w^{\mathrm{II}}(\cdot)$, where $w^{\mathrm{I}}(\cdot)$ and $w^{\mathrm{II}}(\cdot)$ are absolutely continuous cdfs on $[0,1]$ and $w^{\mathrm{I}}(\cdot)$ first order stochastically dominates $w^{\mathrm{II}}(\cdot)$.

Next we turn to the effects on a_n and b_n of changing voting rule q, the required vote share to pass the reform. The results depend critically on whether the designer is purely self-interested ($\rho = 0$) or prosocial ($\rho > 0$). We consider the self-interested case first.

Proposition 2. *Suppose $\rho = 0$ and consider any $q_I, q_{II} \in (0,1)$ with $q_{II} > q_I$. Then, for sufficiently large n, as q rises from q_I to q_{II} the following holds:*

1. *a_n weakly increases, strictly so if $a_n \in (-1,1)$ under $q = q_I$.*
2. *b_n weakly increases, strictly so if $b_n \in (-1,1)$ under $q = q_I$.*

In words, if the designer is purely self-interested, then increasing the required vote share to pass the reform makes him censor more states downwards but fewer states upwards.

Proposition 2 is driven by a *stringency effect*: as q increases it becomes harder to persuade the pivotal voter to pass the reform while easier to persuade her to maintain the status quo (Alonso and Câmara, 2016*a*). This is because the pivotal voter's threshold of acceptance $v^{(nq+1)}$ increases in q. For a self-interested designer who does not care about voter welfare, his best response would be to shift up both thresholds a_n and b_n. By raising b_n the designer makes the upward pooling message "$k > b_n$" more effective in persuading the pivotal voter – who is now harder to convince – to pass the reform. At the same time, the demand for the effectiveness of the downward pooling message "$k < a_n$" is lower because it is now easier to convince the pivotal voter to maintain the status quo. Therefore, by increasing a_n the designer can expand the set of states in which he can successfully persuade the pivotal voter to maintain the status quo at minor costs of reduced effectiveness.

[32] More precisely, Proposition B.2 in Appendix B shows that whenever $\rho > 0$ the designer's indifference curve $\phi_n(\cdot)$ systematically shifts downwards as $w(\cdot)$ decreases in the sense of first order stochastic dominance.

An important implication of Proposition 2 is that, under the stringency effect alone, both a_n and b_n increase monotonically in q for sufficiently large n. Such unambiguous effects are no longer obtained once the designer cares about voter welfare. In fact, as Proposition 3 shows, comparative statics can then go either way.

Proposition 3. *Suppose $\rho > 0$, both G and $1-G$ are strictly log-concave, n is sufficiently large, and $-1 < a_n < b_n < 1$ holds under $q = q_I$. Then there exist $q_I, q_{II} \in (0,1)$ with $q_{II} > q_I$ such that, as q increases from q_I to q_{II}, any one of the following may happen:*

1. *a_n strictly decreases and b_n strictly increases;*
2. *a_n strictly increases and b_n strictly decreases;*
3. *both a_n and b_n strictly decrease;*
4. *both a_n and b_n strictly increase.*

In words, if the designer is prosocial, then raising the required vote share to pass the reform may make him reveal more states both upwards and downwards, censor more states in both directions, or reveal more states in one direction and censor more states in the other direction.

Proposition 3 shows that for a prosocial designer an increase in the vote share required to pass the reform may shift a_n and b_n both downwards or in opposite directions. As noted above, neither case is possible under the stringency effect alone. This result is thus attributed to an additional effect. For a prosocial designer, an increase in q also affects thresholds a_n and b_n through a novel *designer-preference effect*; increasing the vote share required to pass the reform induces a shift of a prosocial designer's preference towards the reform.[33] Therefore, following the intuition discussed above (for the effects of shifts in the designer's preference), this induced designer-preference effect *per se* drives both a_n and b_n downwards as q increases. As a consequence, the net effect of an increase in q depends on the relative strengths of the stringency and designer preference effects. For instance, if the designer-preference effect dominates in driving a_n while the stringency effect dominates in driving b_n, then the net effect would be a strict expansion of the revelation interval $[a_n, b_n]$ on both sides. Conversely, if the designer-preference effect dominates in driving b_n while the stringency effect dominates in driving a_n, then the net effect would be a strict reduction of the revelation interval $[a_n, b_n]$ on both sides.

The designer-preference effect – that an increase in q shifts the designer's preference towards reform – stems from the inference problem based on the pivotal voter's choice introduced in Section 4 and can be seen as follows. Let q increase from q' to q''. The pivotal voter's type thus shifts from $v^{(nq'+1)}$ to $v^{(nq''+1)}$. Under cutoff q'' the pivotal event $v^{(nq''+1)} = x$ necessarily implies $v^{(nq'+1)} \leq x$ (that is, the pivotal voter's type must be lower than x for cutoff q'). Therefore, for any

[33] Formally, an increase in q systematically shifts the designer's indifference curve $\phi_n(\cdot)$ downwards whenever $\rho > 0$. See Proposition B.2 in Appendix B.

fixed x, the event that the pivotal voter's type equals x implies that the entire realized type profile is systematically lower (and thus voters' ex-post payoffs from reform becomes systematically higher) as q increases. This makes any prosocial designer leaning more towards reform.

It is important to note that the designer-preference effect exists if and only if the designer is both prosocial and imperfectly informed about voters' preferences. This effect is therefore absent in Alonso and Câmara (2016*a*), who study effects of voting rules on optimal persuasion strategies in a model where the designer is perfectly informed of voters' preferences.

7 Competition in persuasion with multiple designers

In this section we extend our model to allow for competition in persuasion with multiple information designers. We show that our main result for monopolistic persuasion – that the single-crossing property ensures the optimality of censorship policies in sufficiently large elections – continues to hold under competition in persuasion. As an application, we use our results to study the welfare impact of media competition.

Specifically, we consider a setup with multiple designers competing in persuading voters a la Gentzkow and Kamenica (2017*b*). Let there be a set M of designers with $|M| \geq 2$. For each designer $m \in M$, his preference is characterized by utility function (1) with parameters $\rho_m \in [0,1]$, $\chi_m \in \mathbb{R}$, and weighting function $w_m(\cdot)$. In this way, for each designer m we can obtain his indifference curve $\phi_n^m(\cdot)$ via (3). We assume that all designers' preferences are commonly known among themselves. Each designer m simultaneously chooses an information policy $\pi_m = (S_m, \sigma_m)$ from the feasible set Π, prior to observing the realization of k. Given profile $\{\pi_m\}_{m \in M}$, we denote by $\pi := \left\langle \{\pi_m\}_{m \in M} \right\rangle$ the *joint information policy* induced by observing the signal realizations from all π_m's. Notice that $\pi \in \Pi$ necessarily because it is clearly feasible. In this way, our information environment is *Blackwell-connected*; given any strategy profile $\pi_{-m} := \times_{j \in M \setminus \{m\}} \pi_j$ of other designers, each designer m can unilaterally deviate to any feasible joint information policy that is Blackwell more informative.[34] We focus on equilibria in pure and weakly undominated strategies.[35] The equilibrium derivation and proofs for all results in this section are in Appendix F.

Suppose the single-crossing property holds for a designer $m \in M$. Then, by Theorem 1, there

[34] Formally, for two feasible joint information policies $\pi, \pi' \in \Pi$, π is Blackwell more informative than π' if $H_\pi \succeq_{MPS} H_{\pi'}$; that is, the distribution of posterior expectations about states induced by π is a mean-preserving spread of that induced by π'.

[35] As explained in footnote 14, we focus on weakly undominated strategies to rule out a plethora of uninteresting equilibra. For competition with multiple designers, the restriction to pure strategies is often made in the literature, but may nevertheless have substantive consequences. As noted in Gentzkow and Kamenica (2017*b*) (page 318), when designers may use mixed strategies the information environment may no longer be Blackwell-connected, which is a key property we use to characterize equilibria under competition. Li and Norman (2018) show by examples that allowing for mixed strategies indeed changes the set of equilibria.

exists an N_m such that for all $n \geq N_m$ some censorship policy is optimal for m under monopolistic persuasion. Theorem 3 extends this observation to competition in persuasion in the following sense. For all $n \geq N_m$, it is without loss of optimality for designer m to restrict attention to censorship policies; for any pure strategy profile of other designers (which need not be censorship policies), there always exists a best response in censorship policy.

Theorem 3. *Suppose the single-crossing property holds for designer m. Then for all $n \geq N_m$, there exists a subset of censorship policies $\mathscr{P}_n^m$ such that for any pure strategy profile π_{-m} of designers other than m, there is a censorship policy in $\mathscr{P}_n^m$ that is designer m's best response to π_{-m}. This best response set $\mathscr{P}_n^m$ is explicitly given by (F.1) in Appendix F.*

For the remainder of this section we impose the following assumption:

Assumption 1. *The following conditions hold:*

1. *The single-crossing property holds for each designer $m \in M$.*
2. *For all $m \in M$ and $n \geq 0$, $\phi_n^{m\prime}(x) < 2$ holds on $[-1,1]$.*

By Lemma 3, both conditions of Assumption 1 hold for generic designer preferences when both G and $1-G$ are strictly log-concave. Under Assumption 1, the single-crossing property holds for all designers $m \in M$. Then, by Theorem 1, for any $m \in M$ there exists threshold $N_m \geq 0$ such that for all $n \geq N_m$ the monopolistically optimal information policy for designer m is a censorship policy whose revelation interval equals $[a_n^m, b_n^m]$, which contains the switching state z_n^m.[36] Following Gentzkow and Kamenica (2017*b*), we say an equilibrium is *minimally informative* if the joint information policy it induces is no more Blackwell informative than any information policy that can be induced by some other equilibrium. Theorem 4 characterizes the (unique) joint information policy induced by any minimally informative equilibria under competition in persuasion.

Theorem 4. *Suppose Assumption 1 holds and let $N := \max_{m \in M} N_m$. Then, for all $n \geq N$, the following holds:*

1. *In any minimally informative equilibrium the joint information policy induced by all designers is outcome equivalent to a censorship policy with revelation interval $\left[a_n^{\min}, b_n^{\max}\right]$, where $a_n^{\min} = \min_{m \in M}\{a_n^m\}$ and $b_n^{\max} = \max_{m \in M}\{b_n^m\}$.*

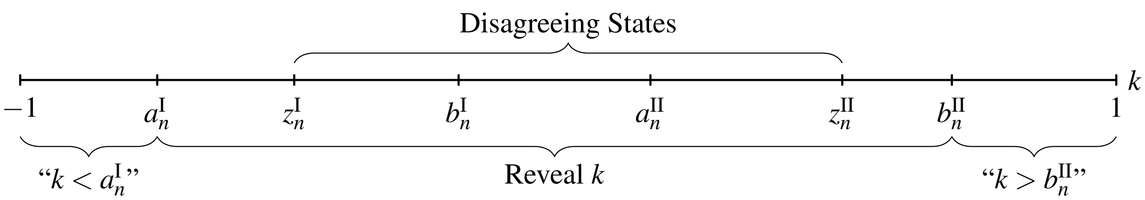
2. *If each designer $m \in M$ is restricted to use censorship policies from his best-response set $\mathscr{P}_n^m$ identified in Theorem 3, then the minimal informative equilibrium is the unique equilibrium in pure and weakly undominated strategies.*[37]

[36] z_n^m is defined according to (6) with $\phi_n(\cdot)$ replaced by $\phi_n^m(\cdot)$.

[37] In an earlier version of this paper we establish another result: If each designer is only restricted to use censorship policies, then the minimally informative equilibrium is the unique pure strategy equilibrium that survives (two rounds of) iterated eliminations of weakly dominated strategies.

Theorem 4 implies that, under a mild regularity condition (i.e., part (2) of Assumption 1), if the single-crossing property holds for all designers then for sufficiently large elections the joint information policy induced in the minimally informative equilibrium is outcome equivalent to a censorship policy whose revelation interval is the convex hull of the revelation intervals of all designers' monopolistically optimal censorship policies. Figure 6 illustrates this for the case with two designers.

Figure 6: Minimally Informative Equilibrium with Two Competing Designers



Note: a_n^m and b_n^m are cutoffs of the optimal censorship policy under monopolistic persuasion for $m \in \{\mathrm{I}, \mathrm{II}\}$.

It is well known that multiple equilibria exist under competition in public Bayesian persuasion. The literature typically focuses on minimally informative equilibria because these are Pareto-optimal for all designers (Gentzkow and Kamenica, 2017*b*).[38] Building on part (2) Theorem 4, we provide a novel argument for selecting the minimal informative equilibria outcome by restricting designers to use censorship policies from their best response sets. Under these restrictions, the minimally informative equilibrium is the unique equilibrium in pure and weakly undominated strategies. Due to this favorable equilibrium selection, all designers would indeed prefer these restrictions to be enforced. This would help them to avoid the risk of coordinating on equilibrium outcomes that are excessively informative and thereby would make all designers strictly worse off.

Finally, we discuss an interesting implication of Theorem 4. To do so we introduce the notion of '*disagreeing states*'. State k is a disagreeing state if there exist at least two designers $\mathrm{I}, \mathrm{II} \in M$ who are weakly biased towards different alternatives relative to the pivotal voter (formally, there exists designers $\mathrm{I}, \mathrm{II} \in M$ with $\phi_n^{\mathrm{I}}(k) \leq k \leq \phi_n^{\mathrm{II}}(k)$). These two designers thus have incentives to manipulate voters' beliefs in opposite directions. Disclosing more information then always benefits at least one of these designers.[39] This in the end leads to full revelation of all such states. When the single-crossing property holds for all designers $m \in M$, the set of disagreeing states is precisely the interval $[z_n^{\min}, z_n^{\max}]$, where $z_n^{\min} = \min_{m \in M}\{z_n^m\}$ and $z_n^{\max} = \max_{m \in M}\{z_n^m\}$. Because $a_n^{\min} \leq z_n^{\min}$ and $b^{\max} \geq z_n^{\max}$, Theorem 4 implies that all disagreeing states must be revealed in any equilibrium.

[38] An exception is Mylovanov and Zapechelnyuk (2021), who propose an equilibrium refinement based on a vanishing (entropy-based) cost of information disclosure.

[39] More precisely, as we show in Appendix F (Lemma F.2 therein), when condition (2) of Assumption 1 holds then in any disagreeing state k there exists at least one designer $m \in M$ whose utility function is strictly convex in the posterior expected state in a neighborhood around k. This local convexity implies positive gains from revealing more information (because it induces a mean-preserving spread on the distribution of posterior expectations about state realization).

The implication above yields the following corollary, which gives a neat sufficient condition for full information disclosure as the unique equilibrium outcome.

Corollary 4. *Suppose Assumption 1 holds. If there exists two designers $I,II \in M$ with $z_n^I = -1$ and $z_n^{II} = 1$, then full disclosure is the unique equilibrium outcome.*

This corollary says that full information disclosure the unique equilibrium outcome whenever there are two designers who are uniformly biased towards different alternatives than the pivotal voter. This condition holds, for example, in the zero-sum game where competition is between two self-interested designers who always favor opposite alternatives (i.e., $\rho_{\mathrm{I}} = \rho_{\mathrm{II}} = 0$, $\chi_{\mathrm{I}} \leq -1$ and $\chi_{\mathrm{II}} \geq 1$). Such extreme conflicts of interests are, however, not necessary to obtain full disclosure in equilibrium. This is also obtained, for example, when competition is between a 'pro-Reform' planner and an 'anti-Reform' planner under simple majority rule (cf. Examples 3 and 4 in Section 5). In this case the conflict of interests between designers are much weaker than in the previous example with opposite-minded self-interested designers; here, both planners aim at maximizing voters' payoffs and they just differ in their welfare weights.

7.1 Application: Media competition and voter welfare in large elections

Building on Corollary 4 and the asymptotic results derived in Section 6.1, we present a straightforward application to study the welfare impact of media competition.

Let $M = \{\mathrm{I}, \mathrm{II}\}$ and interpret these two designers as public mass media outlets. Suppose, for simplicity, that both outlets are partisan and opposite-minded; that is, their preferred alternatives are opposite and state-independent. We model this by assuming that they are self-interested and opposite-minded (i.e., $\rho_{\mathrm{I}} = \rho_{\mathrm{II}} = 0$, $\chi_{\mathrm{I}} \leq -1$ and $\chi_{\mathrm{II}} \geq 1$). It follows immediately from Corollary 4 that competition between these two mass media outlets will indeed lead to full information disclosure in any equilibrium.

Now we turn to the implications for voter welfare. We focus on the limiting case $n \to \infty$. Suppose a social planner with $\rho = 1$ wants to maximize voters' utilitarian welfare. The ex-ante threshold of acceptance is thus $\phi^* = \mathbb{E}_G[v]$, the expected type of the average voter.[40] According to results in Section 6.1, the asymptotic welfare under competition is W^{Full} because full disclosure is the unique equilibrium outcome. The asymptotic welfare in the second-best benchmark (i.e., the planner can implement his own optimal information policy) is given by W^* in Theorem 2. By Corollary 3, $W^* \geq W^{\mathrm{Full}}$ and the strict inequality holds whenever $v_q^* \neq \phi^*$. In other words, unless the preferences of the average and the pivotal voters are ex-ante aligned, media competition fails to

[40] To see this, recall that the weighting function is $w(x) = x$ for $x \in [0,1]$ for a Utilitarian planner. Together with $\rho = 1$, this implies $\phi^* = \int_0^1 G^{-1}(y)dw(y) = \int_{\underline{v}}^{\overline{v}} x dG(x) = \mathbb{E}_G[v]$. All analyses here apply similarly to any non-utilitarian social planner with a different weighting function $w(\cdot)$ (because it affects welfare only through ϕ^*).

maximize voters' utilitarian welfare in large elections even if it induces full information disclosure. In fact, the welfare gap $W^* - W^{\text{Full}}$ can be substantial for large difference between ϕ^* and v_q^*, which in turn depends on voting rule q (recall that v_q^* is strictly increasing while ϕ^* is invariant in q). This suggests that the welfare evaluation has to take the electoral background and institutional factors – such as the distribution of voters' preferences and voting rules – in to account because they determine the ex-ante interests misalignment between the pivotal voter and the social planner.

Despite its straightforwardness, our result complements to the debate on the effect of media competition on voter welfare, which is an important topic in the literature on political economics of mass media. Most papers in this literature take from the outset that more information is better for welfare and focus on whether media competition can improve information revelation.[41] This reasoning would lead one to conclude that media competition is ideal from the welfare perspective if it can induce full information revelation. Our result suggests that this is in general not true and a careful welfare evaluation should take electoral and institutional factors into account.

8 Conclusion

This paper studies public persuasion in elections with binary alternatives. In our model, one or multiple information designers can try to influence the election outcome by strategically providing public information about a payoff-relevant state. Compared to prior works, our paper has two distinguishing and important features. First, we allow for a wide class of designer preferences that embed both the pursuit of self-interest and maximizing any social welfare function – which can be represented as some rank-dependent weighted average of voters' payoffs – as special cases. Second, we characterize in a unified framework information provision in equilibrium under both monopolistic persuasion with a single designer and competition in persuasion with multiple designers.

Our main result identifies a sufficient condition that ensures the optimality of censorship policies, which reveal intermediate state realizations but censor extreme ones. Our sufficient condition can be intuitively interpreted as a single-crossing property over the designer's and the pivotal voter's indifference curves. This condition holds for a designer if either (i) the designer is self-interested, or (ii) the distribution of voters' preferences satisfies a mild regularity condition.

Under monopolistic persuasion by a single designer, we show that censorship policy is uniquely

[41] For example, it has been argued that media competition can benefit voters and improve political accountability by increasing the costs of media capture (Besley and Prat, 2006) that aims at suppressing disclosure of unfavorable information to the politician. Competition may also discipline media outlets to provide information that aligns better with the interests of their audiences (Gentzkow and Shapiro, 2006; Chan and Suen, 2008). These papers conclude that media competition is welfare-improving because it induces better information disclosure. On the other hand, the literature also identifies channels through which media competition can deteriorate voter welfare by reducing information disclosure. For instance, competition can drive profit-maximizing media to invest fewer resources in the provision of political news or topics of common interests (Chen and Suen, 2018; Cagé, 2019; Perego and Yuksel, 2021).

optimal in large elections if the single-crossing property holds for this monopoly designer. The boundaries of the optimal revelation interval can be determined by complementary slackness conditions with clear economic interpretations. Under competition in persuasion with multiple designers, the single-crossing property ensures that it is without loss of optimality for a designer to restrict attention to a subset of censorship policies, which always contains a best response to any pure-strategy profile of others. Moreover, when the single-crossing property holds for all designers and under a weak regularity condition, the minimally informative equilibrium outcome can be reproduced by a censorship policy whose structure can be easily deduced from the monopolistically optimal censorship policies of all designers. This outcome is the unique equilibrium outcome in pure and weakly undominated strategies if all designers commit to using censorship policies from their best-response sets only. Our analyses also produce a clean sufficient condition under which competition in persuasion can induce full information revelation as the unique equilibrium outcome.

Our results yield interesting and important normative implications. First, we stress that, perhaps surprisingly, full information disclosure is generically suboptimal even for a social planner who aims at maximizing voters' welfare. In fact, the structure of a welfare maximizing information policy can depend subtly on the planner's social preference, the electoral environment (e.g., the distributions of states and voters' preferences), and the voting rule. This observation complements the literature on media and politics by pointing out that even if media competition does induce full information revelation, it is not ideal from the welfare perspective in large elections so long as there is any ex-ante conflict of interests between the average and the pivotal voters. Second, we deliver a novel insight regarding how a prosocial designer should tailor his optimal public information policy in response to changes in voting rules. We show that for a prosocial designer who is imperfectly informed about voters' preferences, increasing the required vote share for passing an alternative can affect his optimal censorship policy through both a *stringency effect* – by making it more difficult to persuade voters to pass that alternative – and a novel *designer-preference effect* – by inducing a shift of the designer's preference towards that alternative. The latter effect is absent in environments where the planner is fully aware of voters' preferences, such as in Alonso and Câmara (2016*a*).

We conclude by discussing some limitations of our paper and suggesting some avenues for future research. First, throughout the paper we have focused on public persuasion and assumed that voters' private types are unknown to any designer. These exclude the possibilities of *targeted persuasion* (i.e., sending different information to different voters) and *eliciting voters' private information* (e.g., by offering a menu of signals for voters). It is interesting to extend our analyses to incorporate either or both possibilities.[42] The results can shed light on the strategic values of

[42] Heese and Lauermann (2021) study (in a binary-state model) targeted persuasion by a monopoly information designer whose preference is independent of voters' private types. They show that the designer can ensure his preferred alternative to win with probability one in equilibrium as the electorate size goes to infinity. It is unclear, however, whether their results continue to hold if the designer's preference can depend on voters' private types as in our model.

micro-targeting and screening for an information designer.

Second, we assume that any designer's welfare weight assigned to each voter depends on the voter's ex-post payoff ranking, but not on her identity or any other characteristics. Under our assumption that voters are ex-ante homogeneous this restriction is without loss of generality. In practice, however, voters do differ in characteristics that might be observable to designers; e.g., gender, age, region of residence, occupation, ethic group, party affiliation, etc. These characteristics often systematically influence how voters fare under policy reforms; e.g., drivers are arguably more likely to experience greater negative income shocks due to a car fuel levy. In these scenarios it is interesting to study the optimal information policy when a designer's welfare weights can depend on such observable characteristics. One possible way to do so is to enrich our model by allowing voters to be heterogeneous in both observable and unobservable dimensions.

Third, in our model the policy reform affects voters in a homogeneous way; it shifts each voter's payoff under reform by k. We impose this assumption for tractability because in this way our information design problem can be solved using established linear programming techniques. In many real-life cases, however, policy reforms affect voters in heterogeneous, and sometimes even opposite, ways. For example, citizens living in more polluted areas may benefit more from an environmental protection policy. More strikingly, in the Brexit referendum, some citizens would prefer a harder Brexit but others may instead prefer a softer one. In these examples policy reforms can affect the distribution of voters' payoffs, and sometimes may even induce preference reversal and thus polarized attitudes towards reforms. All these features are important issues in discourses of contemporary distributive politics. We therefore believe that exploring the implications of heterogeneous policy effects for information design in elections is a promising and highly relevant agenda for future research.

We hope that the theoretical framework and results of our paper can serve as a good starting point to explore the research questions mentioned above.

References

Alonso, Ricardo and Odilon Câmara. 2016*a*. "Persuading voters." *American Economic Review* 106(11):3590–3605.

Alonso, Ricardo and Odilon Câmara. 2016*b*. "Political disagreement and information in elections." *Games and Economic Behavior* 100:390–412.

Arieli, Itai, Yakov Babichenko, Rann Smorodinsky and Takuro Yamashita. 2020. "Optimal Persuasion via Bi-pooling." Working paper. Available at SSRN: https://ssrn.com/abstract=3511516 or http://dx.doi.org/10.2139/ssrn.3511516.

Ashworth, Scott and Ethan Bueno De Mesquita. 2009. "Elections with platform and valence competition." 67(1):191–216.

Au, Pak Hung and Keiichi Kawai. 2020. "Competitive information disclosure by multiple senders." *Games and Economic Behavior* 119:56–78.

Au, Pak Hung and Keiichi Kawai. 2021. "Competitive disclosure of correlated information." *Economic Theory* 72(3):767–799.

Bagnoli, Mark and Ted Bergstrom. 2005. "Log-concave probability and its applications." *Economic theory* 26(2):445–469.

Bardhi, Arjada and Yingni Guo. 2018. "Modes of persuasion toward unanimous consent." *Theoretical Economics* 13(3):1111–1149.

Bergemann, Dirk and Stephen Morris. 2016. "Bayes correlated equilibrium and the comparison of information structures in games." *Theoretical Economics* 11(2):487–522.

Bergemann, Dirk and Stephen Morris. 2019. "Information design: A unified perspective." *Journal of Economic Literature* 57(1):44–95.

Besley, Timothy and Andrea Prat. 2006. "Handcuffs for the grabbing hand? The role of the media in political accountability." *American Economic Review* 96(3):720–736.

Cagé, Julia. 2019. "Media competition, information provision and political participation: Evidence from French local newspapers and elections, 1944–2014." *Journal of Public Economics* 185:104077.

Chakraborty, Archishman and Parikshit Ghosh. 2016. "Character endorsements and electoral competition." 8(2):277–310.

Chakraborty, Archishman, Parikshit Ghosh and Jaideep Roy. 2020. "Expert-Captured Democracies." 110(6):1713–51.

Chan, Jimmy, Seher Gupta, Fei Li and Yun Wang. 2019. "Pivotal persuasion." *Journal of Economic theory* 180:178–202.

Chan, Jimmy and Wing Suen. 2008. "A spatial theory of news consumption and electoral competition." *Review of Economic Studies* 75(3):699–728.

Chen, Heng and Wing Suen. 2018. "Competition for Attention in the News Media Market." Working paper, University of Hong Kong.

Cui, Zhihan and Dilip Ravindran. 2020. "Competing persuaders in zero-sum games." Working paper. Columbia University.

Dizdar, Deniz and Eugen Kováč. 2020. "A simple proof of strong duality in the linear persuasion problem." *Games and Economic Behavior* 122:407–412.

Dworczak, Piotr and Anton Kolotilin. 2019. "The persuasion duality." Working paper.

Dworczak, Piotr and Giorgio Martini. 2019. "The simple economics of optimal persuasion." *Journal of Political Economy* 127(5):1993–2048.

Feddersen, Timothy J and Wolfgang Pesendorfer. 1996. "The Swing Voter's Curse." *American Economic Review* 86(3):408–424.

Feddersen, Timothy and Wolfgang Pesendorfer. 1997. "Voting behavior and information aggregation in elections with private information." *Econometrica* 65(5):1029–1058.

Ferguson, Sean. 2020. "How Informed Should Voters Be?".

Gentzkow, Matthew and Emir Kamenica. 2016. "A Rothschild-Stiglitz approach to Bayesian persuasion." *American Economic Review* 106(5):597–601.

Gentzkow, Matthew and Emir Kamenica. 2017*a*. "Bayesian persuasion with multiple senders and rich signal spaces." *Games and Economic Behavior* 104:411–429.

Gentzkow, Matthew and Emir Kamenica. 2017*b*. "Competition in Persuasion." *The Review of Economic Studies* 84(1):300–322.

Gentzkow, Matthew and Jesse M Shapiro. 2006. "Media bias and reputation." *Journal of Political Economy* 114(2):280–316.

Ginzburg, Boris. 2019. "Optimal Information Censorship." *Journal of Economic Behavior and Organization* 163:377–385.

Groeneveld, Richard and Glen Meeden. 1977. "The mode, median, and mean inequality." *The American Statistician* 31(3):120–121.

Groseclose, Tim. 2001. "A model of candidate location when one candidate has a valence advantage." 45(4):862–886.

Heese, Carl and Stephan Lauermann. 2021. "Persuasion and Information Aggregation in Large Elections." Working Paper.

Innocenti, Federico. 2021. Can Media Pluralism Be Harmful to News Quality? Technical report University of Mannheim.

Ivanov, Maxim. 2020. "Optimal monotone signals in Bayesian persuasion mechanisms." *Economic Theory* pp. 1–46.

Kamenica, Emir. 2019. "Bayesian persuasion and information design." *Annual Review of Economics* 11:249–272.

Kamenica, Emir and Matthew Gentzkow. 2011. "Bayesian persuasion." *American Economic Review* 101(6):2590–2615.

Kartik, Navin and Richard Van Weelden. 2019. "Informative cheap talk in elections." *The Review of Economic Studies* 86(2):755–784.

Kerman, Toygar, P Herings and Dominik Karos. 2020. "Persuading Strategic Voters." Working paper.

Kleiner, Andreas, Benny Moldovanu and Philipp Strack. 2021. "Extreme points and majorization: Economic applications." *Econometrica* 89(4):1557–1593.

Kolotilin, Anton. 2018. "Optimal information disclosure: A linear programming approach." *Theoretical Economics* 13(2):607–635.

Kolotilin, Anton, Timofiy Mylovanov and Andriy Zapechelnyuk. 2022. "Censorship as optimal persuasion." *Theoretical Economics* 17(2):561–585.

Kolotilin, Anton, Tymofiy Mylovanov, Andriy Zapechelnyuk and Ming Li. 2017. "Persuasion of a privately informed receiver." *Econometrica* 85(6):1949–1964.

Krishna, Vijay. 2009. *Auction Theory*. Academic press.

Li, Fei and Peter Norman. 2018. "On Bayesian persuasion with multiple senders." *Economics Letters* 170:66–70.

Li, Fei and Peter Norman. 2021. "Sequential persuasion." *Theoretical Economics* 16(2):639–675.

Liu, Shuo. 2019. "Voting with Public Information." *Games and Economic Behavior* 113:694–719.

Mathevet, Laurent, Jacopo Perego and Ina Taneva. 2020. "On information design in games." *Journal of Political Economy* 128(4):1370–1404.

Mylovanov, Tymofiy and Andriy Zapechelnyuk. 2021. "A model of debate: Moderation vs free speech." working paper.

Perego, Jacopo and Sevgi Yuksel. 2021. "Media Competition and Social Disagreement." forthcoming at *Econometrica*.

Quiggin, John. 1982. "A theory of anticipated utility." *Journal of Economic Behavior & Organization* 3(4):323–343.

Schnakenberg, Keith E. 2015. "Expert advice to a voting body." *Journal of Economic Theory* 160:102 – 113.

Schnakenberg, Keith E. 2017. "Informational Lobbying and Legislative Voting." *American Journal of Political Science* 61:129–145.

Sun, Junze. 2022*a*. "On the optimality of censorship in competitive Bayesian persuasion." Working paper.

Sun, Junze. 2022*b*. "A sufficient condition for full information disclosure in linear persuasion games." Working paper.

Sun, Junze, Arthur Schram and Randolph Sloof. 2021. "Why Biased Endorsements Can Manipulate Elections." Available at SSRN: https://ssrn.com/abstract=3420703 or http://dx.doi.org/10.2139/ssrn.3420703.

Taneva, Ina A. 2019. "Information Design." *American Economic Journal: Microeconomics* 11(4):151–85.

Van der Straeten, Karine and Takuro Yamashita. 2020. "On the veil-of-ignorance principle: welfare-optimal information disclosure in voting." working paper.

Van Zwet, Willem R. 1980. "A strong law for linear functions of order statistics." *The Annals of Probability* 8(5):986–990.

Wang, Yun. 2013. "Bayesian persuasion with multiple receivers." Working paper. Available at SSRN: https://ssrn.com/abstract=2625399 or http://dx.doi.org/10.2139/ssrn.2625399.

Appendices (for online publication)

A Derivations and relevant properties of $\hat{G}_n(\cdot;q)$

In this appendix we formally derive $\hat{G}_n(\cdot;q)$ and establish its relevant properties in Proposition A.1, which imply Lemma 1 in Section 3. For all $y,q \in [0,1]$, define

$$\tau_n(y;q) := \frac{(n+1)!}{\lfloor x \rfloor ! \cdot \lceil n(1-q) \rceil !} y^{\lfloor nq \rfloor}(1-y)^{\lceil n(1-q) \rceil} \tag{A.1}$$

where $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$ denote, respectively, the floor and ceiling functions.In fact, $\tau_n(\cdot;q)$ is the density function of a Beta distribution $B(\alpha,\beta)$ with parameters $\alpha = \lfloor nq \rfloor + 1$ and $\beta = \lceil n(1-q) \rceil + 1$. The following properties about $\tau_n(y;q)$ are useful.

Lemma A.1. *Suppose nq is an integer. Then the following properties hold:*

(a) $\tau_n'(y;q) = \tau_n(y;q)\frac{n(q-y)}{y(1-y)}$.

(b) $\tau_n(y;q)$ *is increasing on* $[0,q]$ *and decreasing on* $(q,1]$.

(c) $\lim_{n\to\infty} \tau_n(y;q) = \infty$ *if* $y = q$ *and* $\lim_{n\to\infty} \tau_n(y;q) = 0$ *if* $y \neq q$.

Proof of Lemma A.1. Since nq is an integer, we can drop the floor and ceiling functions in (A.1). Taking natural logarithm of $\tau_n(y;q)$ and computing its derivative then yields

$$\frac{\tau_n'(y;q)}{\tau_n(y;q)} = \frac{n(q-y)}{y(1-y)}$$

Hence, $\tau_n'(y;q) > (<) 0$ for $y < (>) q$. This proves (a) and (b). To show (c), we use Stirling's formula to approximate $n!$ for all positive integer n: $n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$.[1] With this approximation, we obtain

$$\tau_n(y;q) \approx \sqrt{\frac{n}{2\pi q(1-q)}}\left(\frac{y}{q}\right)^{nq}\left(\frac{1-y}{1-q}\right)^{n(1-q)} \tag{A.2}$$

If $y = q$, then $\tau_n(y;q) \approx \sqrt{\frac{n}{2\pi q(1-q)}} \to \infty$. If $y \neq q$, we take the natural logarithm of (A.2) and get

$$\ln \tau_n(y;q) \approx \frac{1}{2}\ln n + n\psi(y;q) - \frac{1}{2}\ln 2\pi q(1-q) \tag{A.3}$$

where

$$\psi(y;q) := q \ln \frac{y}{q} + (1-q)\ln\frac{1-y}{1-q} \tag{A.4}$$

[1] The expression $l_n \approx r_n$ denotes $\lim_{n\to\infty} \frac{l_n}{r_n} = 0$, where l_n and r_n are real number sequences.

It holds that (i) $\psi(q;q) = 0$, and (ii) $\psi'(y;q) > (<)0$ for $y < (>)q$. Therefore, if $y \neq q$, then $\psi(y;q) < 0$ and the right hand side of (A.3) converges to $-\infty$ as $n \to \infty$. This implies $\lim_{n\to\infty} \tau_n(y;q) = 0$ for $y \neq q$ and thus completes the proof for part (c). $\square$

Observe that Lemma A.1 easily extends to other values of q in which nq is not an integer. In this case, we can just replace q by $\hat{q} := \frac{\lfloor nq \rfloor}{n}$. In this way, (a) and (b) of Lemma A.1 hold with $\hat{q}$. Part (c) of Lemma A.1 also holds for q because $\hat{q}$ converges to q as $n \to \infty$. In the remainder of this appendix and all subsequent appendices we assume nq to be an integer for ease of exposure, with the understanding that this is without loss of generality.

Now we are ready to derive $\hat{G}_n(\cdot;q)$, the distribution of the pivotal voter's type $v^{(nq+1)}$. Let $\hat{g}_n(\cdot;q)$ denote the density function. Consider $x \in [\underline{v}, \overline{v}]$. For $v^{(nq+1)} = x$ to hold, there must be nq voters with $v_i \leq x$ and $n(1-q)$ others with $v_i \geq x$, with the remaining pivotal voter having $v_i = x$. Because voters' types are independently drawn from G, we have

$$\hat{g}_n(x;q) = \frac{(n+1)!}{(nq)![n(1-q)]!}(G(x))^{nq}(1-G(x))^{n(1-q)}g(x) = \tau_n(G(x);q)\,g(x) \tag{A.5}$$

and

$$\hat{G}_n(x;q) = \int_{\underline{v}}^{x} \tau_n(G(x);q)\,g(x)dx = \int_0^{G(x)} \tau_n(y;q)dy \tag{A.6}$$

Next, we prove the following proposition about $\hat{G}_n(x;q)$.

Proposition A.1. *Let $v_q^* := G^{-1}(q)$. The following properties hold:*

1. $\hat{G}_n(\cdot;q)$ is strictly increasing and $v^{(nq+1)}$ converges in probability to v_q^*.

2. $\hat{g}_n(\cdot;q)$ is single-peaked for all $q \in (0,1)$ when n is sufficiently large. In addition, if $g(\cdot)$ is strictly log-concave, then $\hat{g}_n(\cdot;q)$ is strictly log-concave for all $n \geq 0$ and q.

Statement (1) of this proposition implies Lemma 1 in Section 3. Statement (2) says that regardless the shape of G and voting rule q, for sufficiently large electorate the distribution of the pivotal voter will be single-peaked. Moreover, large n is not needed if g is already log-concave. This property will be exploited in Appendix C.2 for the proof of Lemma 5.

Proof of Proposition A.1. We first show part (1). The fact that $\hat{G}_n(\cdot;q)$ is strictly increasing follows immediately from $\hat{g}_n(x;q) = \tau_n(G(x);q)g(x) > 0$. To show that $v^{(nq+1)}$ converges in probability to v_q^*, it suffices to establish

$$\lim_{n\to\infty} \hat{G}_n(x;q) \to \begin{cases} 0, & \text{if } x < v_q^* \\ 1/2, & \text{if } x = v_q^* \\ 1, & \text{if } x > v_q^* \end{cases} \tag{A.7}$$

For $x < v_q^*$ we have $G(x) < q$ and

$$\hat{G}_n(x;q) = \int_0^{G(x)} \tau_n(y;q)dy < G(x)\tau_n(G(x);q) \to 0$$

the second and third steps of which follow from (b) and (c) of Lemma A.1, respectively. If instead $x > v_q^*$, then $G(x) > q$ and $\int_{G(x)}^1 \tau_n(y)dy < (1-G(x))\tau_n(G(x);q) \to 0$. Therefore, $\hat{G}_n(x) = 1 - \int_{G(x)}^1 \tau_n(y)dy \to 1$. Finally, if $x = v_q^*$, then $G(x) = G(v_q^*) = q$ and $\hat{G}_n(x) = \int_0^q \tau_n(y;q)dy$. Below we show $\lim_{n\to\infty} \int_0^q \tau_n(y;q)dy = 1/2$. Recall that $\tau_n(y;q)$ is the density function of a random variable Y following Beta distribution $B(\alpha,\beta)$ with parameters $\alpha = nq+1$ and $\beta = n(1-q)+1$. Let q_n denote the median of Y; that is, $\int_0^{q_n} \tau_n(y;q)dy = 1/2$. We show that the sequence of medians q_n converges to q and thus $\lim_{n\to\infty} \int_0^q \tau_n(y;q)dy = \lim_{n\to\infty} \int_0^{q_n} \tau_n(y;q)dy = 1/2$. For a Beta-distributed random variable $Y \sim Beta(\alpha,\beta)$, Groeneveld and Meeden (1977) show that its median q_n must be bounded between its mean μ_n and mode m_n. For a Beta distribution, it is well known that $\mu_n = \frac{\alpha}{\alpha+\beta}$ and $m_n = \frac{\alpha-1}{\alpha+\beta-2}$. Since $\alpha = nq+1$ and $\beta = n(1-q)+1$, both μ_n and m_n converge to q as $n \to \infty$. This implies that the median q_n must converge to q as well. This establishes part (1) of this proposition.

Next we prove part (2). By (A.5) and part (a) of Lemma A.1, we have

$$\begin{aligned}\hat{g}_n'(x;q) &= \tau_n'(G(x);q)g^2(x) + \tau_n(G(x);q)g'(x) \\ &= \hat{g}_n(x;q)\left(n\frac{g(x)}{G(x)}\frac{q-G(x)}{1-G(x)} + \frac{g'(x)}{g(x)}\right)\end{aligned} \tag{A.8}$$

Therefore,

$$\frac{\hat{g}_n'(x;q)}{\hat{g}_n(x;q)} = n\frac{g(x)}{G(x)}\frac{q-G(x)}{1-G(x)} + \frac{g'(x)}{g(x)} \tag{A.9}$$

Suppose g is strictly log-concave and thus $\frac{g'(\cdot)}{g(\cdot)}$ is strictly decreasing. By Theorem 1 of Bagnoli and Bergstrom (2005), G inherits log-concavity and $\frac{g(\cdot)}{G(\cdot)}$ is decreasing. Since $\frac{q-G(x)}{1-G(x)}$ is strictly decreasing, it follows from (A.9) that $\frac{\hat{g}_n'(x;q)}{\hat{g}_n(x;q)}$ is strictly decreasing and thus $\hat{g}_n(\cdot;q)$ is strictly log-concave for all $n \geq 0$.

Now we drop the strict log-concavity assumption of g and show that $\hat{g}_n(x;q)$ is single-peaked for sufficiently large n. By (A.9),

$$\hat{g}_n'(x;q) > 0 \Longleftrightarrow \lambda_n(x) := q - G(x) + \frac{1}{n}\frac{G(x)\,(1-G(x))}{g(x)}\frac{g'(x)}{g(x)} > 0 \tag{A.10}$$

Recall that g is strictly positive and twice-continuously differentiable on $[\underline{v},\overline{v}]$. These imply that (i) there exists some $\varepsilon > 0$ such that $g(x) > \varepsilon$ for all x, and (ii) both $\frac{G(x)(1-G(x))}{g(x)}\frac{g'(x)}{g(x)}$ and its first order derivative are uniformly bounded. Therefore, as $n \to \infty$, $\lambda_n(x)$ and $\lambda_n'(x)$ converge uniformly to $q - G(x)$ and $-g(x)$, respectively. Hence, for sufficiently large n, $\lambda_n(x)$ must be strictly decreasing and its root $\hat{x}_n$ must converge to v_q^*. This implies that $\hat{g}_n(x;q)$ is single-peaked for sufficiently large n and thus completes the proof. $\square$

B Properties of $\phi_n(\cdot)$ and the proofs of Lemmas 2 and 3

In this appendix we derive and establish some important properties for $\phi_n(\cdot)$ – the indifference curve of the information designer – and its limit as $n \to \infty$. We also prove Lemmas 2 and 3 in Section 4.

For each $j \in \{1, \cdots, n+1\}$ and $x \in [\underline{v}, \overline{v}]$, let

$$\varphi_j(x;q,n) := \mathbb{E}\left[v^{(j)} \middle| v^{(nq+1)} = x;q,n\right]$$

denote the expectation of $v^{(j)}$ conditional on event $v^{(nq+1)} = x$. By (3) in Section 4 we have

$$\phi_n(x) := \mathbb{E}\left[\varphi_n(v) \middle| v^{(nq+1)} = x\right] = \rho \sum_{j=1}^{n+1} w_j \varphi_j(x;q,n) + (1-\rho)\chi \tag{B.1}$$

If $\rho = 0$, it is obvious that $\phi_n(x) = \chi$ is a constant. If $\rho > 0$, the properties of $\phi_n(x)$ depend closely on $\varphi_j(x;q,n)$. For any $j \neq nq+1$, let $\widetilde{g}_j(\cdot|x;q,n)$ denote the density function for the distribution of $v^{(j)}$ conditional on $v^{(nq+1)} = x$ given parameters q and n. We show that

$$\widetilde{g}_j(y|x;q,n) = \begin{cases} \tau_{nq-1}\left(\frac{G(y)}{G(x)}; \frac{j-1}{nq}\right) \frac{g(y)}{G(x)}, & \text{if} j < nq+1 \\ \tau_{n(1-q)-1}\left(\frac{G(y)-G(x)}{1-G(x)}; \frac{j-nq-2}{n(1-q)}\right) \frac{g(y)}{1-G(x)}, & \text{if } j > nq+1 \end{cases}. \tag{B.2}$$

To see why, first consider $j < nq+1$. Conditional on $v^{(nq+1)} = x$, $v^{(j)}$ is the j-th lowest order statistic from nq independent random draws from a truncated distribution with cdf $\frac{G(y)}{G(x)}$ for $y \in [\underline{v}, x]$. (B.2) for $j < nq+1$ thus follows from (A.5). Now consider $j > nq+1$. Conditional on $v^{(nq+1)} = x$, $v^{(j)}$ is the $(j-nq-1)$-th lowest order statistic from $n(1-q)$ independent random draws from a truncated distribution with cdf $\frac{G(y)-G(x)}{1-G(x)}$ for $y \in [x, \overline{v}]$. This implies (B.2) for $j < nq+1$ through (A.5). Lemma B.1 explicitly characterizes $\varphi_j(x;q,n)$.

Lemma B.1. *For all $x \in [\underline{v}, \overline{v}]$,*

$$\varphi_j(x;q,n) = \begin{cases} \int_0^1 \underline{t}(x,y)\tau_{nq-1}\left(y; \frac{j-1}{nq}\right)dy, & \text{if } j < nq+1 \\ x, & \text{if } j = nq+1 \\ \int_0^1 \overline{t}(x,y)\tau_{n(1-q)-1}\left(y; \frac{j-nq-2}{n(1-q)}\right)dy, & \text{if } j > nq+1 \end{cases} \tag{B.3}$$

where

$$\underline{t}(x,y) := G^{-1}\big(yG(x)\big) \tag{B.4}$$

$$\bar{t}(x,y) := G^{-1}\big(y+(1-y)G(x)\big) \tag{B.5}$$

for all $x \in [\underline{v}, \bar{v}]$ *and* $y \in [0,1]$.

Proof of Lemma B.1. $\varphi_j(x;q,n) = x$ for $j = nq+1$ follows immediately from its definition. For $j < nq+1$, it follows from (B.2) that

$$\begin{aligned}\varphi_j(x;q,n) &= \int_{\underline{v}}^{x} y\tilde{g}_j(y|x;q,n)dy = \int_{\underline{v}}^{x} y\tau_{nq}\left(\frac{G(y)}{G(x)};\frac{j-1}{nq-1}\right)\frac{dG(y)}{G(x)} \\ &= \int_0^1 G^{-1}\big(yG(x)\big)\tau_{nq-1}\left(y;\frac{j-1}{nq}\right)dy = \int_0^1 \underline{t}(x,y)\tau_{nq-1}\left(y;\frac{j-1}{nq}\right)dy\end{aligned}$$

Finally, for all $j > nq+1$ it follows from (B.2) that

$$\begin{aligned}\varphi_j(x;q,n) &= \int_{x}^{\bar{v}} y\tilde{g}_j(y|x;q,n)dy = \int_{x}^{\bar{v}} y\tau_{n(1-q)-1}\left(\frac{G(y)-G(x)}{1-G(x)};\frac{j-nq-2}{n(1-q)}\right)\frac{dG(y)}{1-G(x)} \\ &= \int_0^1 G^{-1}\big(y+(1-y)G(x)\big)\tau_{n(1-q)-1}\left(y;\frac{j-nq-2}{n(1-q)}\right)dy \\ &= \int_0^1 \bar{t}(x,y)\tau_{n(1-q)-1}\left(y;\frac{j-nq-2}{n(1-q)}\right)dy\end{aligned}$$

This completes the proof. □

Lemma B.2 summarizes useful properties about functions $\underline{t}(x,y)$ and $\bar{t}(x,y)$ defined above.

Lemma B.2. $\underline{t}(x,y)$ *and* $\bar{t}(x,y)$ *are three times continuously differentiable and satisfy the following properties:*

1. $\underline{t}(x,y) < x < \bar{t}(x,y)$ *for all* $y \in (0,1)$.
2. *Both* $\underline{t}(x,y)$ *and* $\bar{t}(x,y)$ *are strictly increasing in* x *and* y.
3. *If* G *is strictly log-concave, then* $\underline{t}_x(x,y) < 1$ *for all* $y \in (0,1)$.
4. *If* $1-G$ *is strictly log-concave, then* $\bar{t}_x(x,y) < 1$ *for all* $y \in (0,1)$.

Proof of Lemma B.2. The fact that both $\underline{t}(x,y)$ and $\bar{t}(x,y)$ are three times continuously differentiable for all $(x,y) \in [\underline{v},\bar{v}] \times [0,1]$ follows from our assumption that G is twice continuously differentiable on $[\underline{v},\bar{v}]$. Parts (1) and (2) of this lemma follows immediately from the definitions of $\underline{t}(x,y)$ and $\bar{t}(x,y)$. To show part (3), note from (B.4) that

$$G\big(\underline{t}(x,y)\big) = yG(x)$$

Taking first order derivative with respect to x on both sides and rearranging terms yields

$$g\big(\underline{t}(x,y)\big)\underline{t}_x(x,y) = yg(y) \Longleftrightarrow \underline{t}_x(x,y) = y\frac{g(x)}{g\big(\underline{t}(x,y)\big)} = \frac{g(x)}{G(x)}\Big/\frac{g\big(\underline{t}(x,y)\big)}{G\big(\underline{t}(x,y)\big)} \tag{B.6}$$

If G is strictly log-concave, then $\frac{g(\cdot)}{G(\cdot)}$ is strictly decreasing. Since $\underline{t}_x(x,y) < x$ for $y \in (0,1)$, it follows from (B.6) that $\underline{t}_x(x,y) < 1$. To show part (4), note from (B.5) that

$$G\big(\bar{t}(x,y)\big) = y + \big(1-y\big)\big(1-G(x)\big)$$

holds for all x and y. Simple algebra reveals that

$$\bar{t}_x(x,y) = \big(1-y\big)\frac{g(x)}{g\big(\bar{t}(x,y)\big)} = \frac{g(x)}{1-G(x)}\Big/\frac{g(\bar{t}(x,y))}{1-G(\bar{t}(x,y))} \tag{B.7}$$

If $1-G$ is strictly log-concave, then $\frac{g(\cdot)}{1-G(\cdot)}$ is strictly increasing. Since $\bar{t}(x,y) > x$ for $y \in (0,1)$, it follows from (B.7) that $\bar{t}_x(x,y) < 1$. $\square$

Notice that parameters j and q affect $\varphi_j(x;q,n)$ only through their impacts on $\widetilde{g}_j(\cdot|x;q,n)$. The next lemma shows that $\widetilde{g}_j(\cdot|x;q,n)$ increases in strict monotone likelihood-ratio dominance order as j increases and q decreases. For two probability density functions $l(\cdot)$ and $r(\cdot)$, we write $l(\cdot) \succ_{LR} r(\cdot)$ if the likelihood ratio $\frac{l(\cdot)}{r(\cdot)}$ is strictly increasing.

Lemma B.3. *The following properties for $\widetilde{g}_j(\cdot|x;q,n)$ hold:*

1. *Suppose $j' > j$, then $\widetilde{g}_{j'}(\cdot|x;q,n) \succ_{LR} \widetilde{g}_j(\cdot|x;q,n)$ holds if $j > nq+1$ or $j' < nq+1$.*
2. *Suppose $q' > q$, then $\widetilde{g}_j(\cdot|x;q,n) \succ_{LR} \widetilde{g}_j(\cdot|x;q',n)$ holds if $j < nq+1$ or $j > nq'+1$.*

Proof of Lemma B.3. We first show part (1). Using (B.2) and (A.1), we obtain

$$\frac{\widetilde{g}_{j'}(y|x;q,n)}{\widetilde{g}_j(y|x;q,n)} \propto \begin{cases} \left(\frac{G(y)}{G(x)-G(y)}\right)^{j'-j} & \text{for } y \in [\underline{v},x], \quad \text{if } nq+1 > j' > j \\ \left(\frac{G(y)-G(x)}{1-G(y)}\right)^{j'-j} & \text{for } y \in [x,\bar{v}], \quad \text{if } j' > j > nq+1 \end{cases}$$

In both cases, the likelihood ratio $\frac{\widetilde{g}_{j'}(y|x;q,n)}{\widetilde{g}_j(y|x;q,n)}$ is strictly increasing in y since $j' > j$. To show part (2), suppose $q' > q$ and note that

$$\frac{\widetilde{g}_j(y|x;q,n)}{\widetilde{g}_j(y|x;q',n)} \propto \begin{cases} \left(\frac{G(x)}{G(x)-G(y)}\right)^{n(q'-q)} & \text{for } y \in [\underline{v},x], \quad \text{if } j < nq+1 \\ \left(\frac{G(y)-G(x)}{1-G(y)}\right)^{n(q'-q)} & \text{for } y \in [x,\bar{v}], \quad \text{if } j > nq'+1 \end{cases}$$

In both cases, the likelihood ratio $\frac{\widetilde{g}_j(y|x;q,n)}{\widetilde{g}_j(y|x;q',n)}$ is strictly increasing in y when $q' > q$. $\square$

With these Lemmas we are ready to estalish Proposition B.1, which collects important properties of $\varphi_j(x;q,n)$.

Proposition B.1. *Let $j \in \{1,\cdots,n+1\}$. $\varphi_j(x;q,n)$ satisfies the following properties:*

1. *$\varphi_j(x;q,n)$ is strictly increasing in index j and $\varphi_j(x;q,n) = x$ for $j = nq+1$;*
2. *$\varphi_j(x;q,n)$ is strictly increasing in x and decreasing in q for all j;*
3. *If G is strictly log-concave, then $\varphi_j'(x;q,n) < 1$ for all $j < nq+1$;*
4. *If $1-G$ is strictly log-concave, then $\varphi_j'(x;q,n) < 1$ for all $j > nq+1$.*

Proof of Proposition B.1. We start with part (1). $\varphi_{nq+1}(x;q,n) = x$ follows immediately from the definition. Moreover, (B.3) and the fact that $\underline{t}(x,y) < x < \bar{t}(x,y)$ for $y \in (0,1)$ imply $\varphi_j(x;q,n) > (<) x$ for $j > (<) nq+1$. Hence, $\varphi_{j'}(x;q,n) > \varphi_j(x;q,n)$ holds for $j' \geq nq+1 \geq j$ with at least one inequality holding strictly. Now consider $j' > j > nq+1$ or $nq+1 > j' > j$. Observe that both $\underline{t}(x,y)$ and $\bar{t}(x,y)$ are strictly increasing functions of y, and $\varphi_j(x;q,n)$ equals the expectation of $\underline{t}(x,y)$ or $\bar{t}(x,y)$ for random variable y under distribution $\tilde{g}_j(\cdot|x;q,n)$. By Lemma B.3.1, $\tilde{g}_{j'}(\cdot|x;q,n) \succ_{LR} \tilde{g}_j(\cdot|x;q,n)$ and strict likelihood ratio dominance implies $\varphi_{j'}(x;q,n) > \varphi_j(x;q,n)$ (see, for instance, Appendix B of Krishna (2009)).

To show part (2), note that both $\underline{t}(x,y)$ and $\bar{t}(x,y)$ strictly increase in x for all $y \in (0,1)$ (cf. Lemma B.2). It then follows from (B.3) that $\varphi_j(x;q,n)$ strictly increases in x. To show that $\varphi_j(x;q,n)$ decreases in q, consider two different q' and q'' with $q' < q''$. If $nq'+1 \leq j \leq nq''+1$ then by (B.3) and Lemma B.2 we have $\varphi_j(x;q',n) \leq x \leq \varphi_j(x;q'',n)$ with at least one inequality holds strictly. Now consider $j < nq'+1$ or $j > nq''+1$. In this case it follows from Lemma B.3 that $\tilde{g}_j(\cdot|x;q',n) \succ_{LR} \tilde{g}_j(\cdot|x;q'',n)$ so that $\varphi_j(x;q',n) < \varphi_j(x;q'',n)$ holds as a standard implication of likelihood ratio dominance.

To show part (3), suppose that G is strictly log-concave so that $\frac{g(\cdot)}{G(\cdot)}$ is strictly increasing. By Lemmas B.1 and B.2, for $j < nq+1$ we have

$$\varphi_j'(x;q,n) = \int_0^1 \underline{t}_x(x,y)\tau_{nq}\left(y;\frac{j-1}{nq}\right)dy < \int_0^1 \tau_{nq}\left(y;\frac{j-1}{nq}\right)dy = 1$$

The second step follows from part (3) of Lemma B.2. The proof for part (4) is analogous. □

B.1 Relevant properties of $\phi_n(\cdot)$ for finite n when $\rho > 0$

In this subsection we establish Proposition B.2, which summarizes relevant properties of $\phi_n(\cdot)$ when $\rho > 0$. The uniform Liptschitz continuity properties in statement (1) of this proposition shall play important roles in the proofs of Lemmas 2 and 5 below. Statements (2) to (4) of this proposition are consequences of the inference based on the pivotal voter's choice explained in Section 4. The second and third statements say that the indifference curve $\phi_n(\cdot)$ systematically shifts downwards –

resulting in a preference shift towards the reform – as q increases or as the weighting function $w(\cdot)$ decreases in the sense of first order stochastic dominance. These properties play crucial roles in establishing comparative static results in Section 6.2.

Proposition B.2. *Suppose $\rho > 0$. Then $\phi_n(\cdot)$ satisfies the following properties:*

1. $\phi_n(\cdot)$, $\phi_n'(\cdot)$ and $\phi_n''(\cdot)$ are L-Liptschitz continuous on $[\underline{v}, \overline{v}]$ for all $n \geq 0$ and some $L > 0$.

2. $\phi_n(x)$ is strictly increasing in x.

3. For any $x \in (\underline{v}, \overline{v})$, $\phi_n(x)$ is strictly decreasing in q.

4. For any $x \in (\underline{v}, \overline{v})$, $\phi_n(x)$ is weakly decreasing as $w(\cdot)$ shifts from $w^I(\cdot)$ to $w^{II}(\cdot)$, where $w^I(\cdot), w^{II}(\cdot) \in \Delta([-1,1])$ and $w^I(\cdot)$ is first order stochastically dominated by $w^{II}(\cdot)$.

Proof of Proposition B.2. We first show part (1). Note that $\phi_n(\cdot)$ is three times continuous differentiable. By the Mean Value Theorem, $\forall x, y \in [\underline{v}, \overline{v}]$ we have $|\phi_n(x) - \phi_n(y)| = |x-y| \cdot |\phi_n'(\xi)|$ for some ξ between x and y. Notice that

$$\left|\phi_n'(\xi)\right| = \rho \left|\sum_{j=1}^{n+1} w_j \varphi_j'(\xi; q, n)\right| \leq \max_{j \in \{1, \cdots, n+1\}} \left|\varphi_j'(\xi; q, n)\right|$$

By (B.3), each $\varphi_j'(\xi; q, n)$ is the expectation of either $\underline{t}_x(\xi, \cdot)$ or $\overline{t}_x(\xi, \cdot)$ under some distribution. Because both $\underline{t}_x(\cdot)$ and $\overline{t}_x(\cdot)$ are uniformly bounded, there exists $L > 0$ such that $L \geq \max\{\underline{t}_x(x,y), \overline{t}_x(x,y)\}$ for all $(x,y) \in [\underline{v}, \overline{v}] \times [0,1]$. These together imply

$$|\phi_n(x) - \phi_n(y)| = |x-y| \cdot \left|\phi_n'(\xi)\right| < L \cdot |x-y|$$

for all $x, y \in [\underline{v}, \overline{v}]$ and $n \geq 0$. The proofs for uniform Lipschitz continuities for $\phi_n'(\cdot)$ and $\phi_n''(\cdot)$ follow from analogous arguments by exploiting uniform boundedness of $\underline{t}_{xx}(\cdot)$, $\overline{t}_{xx}(\cdot)$, $\underline{t}_{xxx}(\cdot)$ and $\overline{t}_{xxx}(\cdot)$. Next we show parts (2) and (3). Recall that

$$\phi_n(x) = \rho \sum_{j=1}^{n+1} w_j \varphi_j(x; q, n) + (1-\rho)\chi$$

By Proposition B.1, for all $j = 1, \cdots, n+1$ it holds that $\varphi_j(x; q, n)$ is strictly increasing in x and decreasing in q. Therefore, $\phi_n(x)$ must inherit these properties whenever $\rho > 0$. This proves parts (2) and (3). To show part (4), note that

$$\begin{aligned}
\sum_{j=1}^{n+1} w_j \varphi_j(x; q, n) &= \sum_{j=2}^{n+1} \left[\sum_{l=j}^{n+1} w_l\right] \left(\varphi_j(x; q, n) - \varphi_{j-1}(x; q, n)\right) + \varphi_1(x; q, n) \\
&= \sum_{j=2}^{n+1} \left[1 - w\left(\frac{j-1}{n+1}\right)\right] \left(\varphi_j(x; q, n) - \varphi_{j-1}(x; q, n)\right) + \varphi_1(x; q, n)
\end{aligned}$$

Consider two weighting functions $w^{\mathrm{I}}(\cdot)$ and $w^{\mathrm{II}}(\cdot)$. Let $\phi_n^{\mathrm{I}}(\cdot)$ and $\phi_n^{\mathrm{II}}(\cdot)$ denote function $\phi_n(\cdot)$ when $w(\cdot)$ equals $w^{\mathrm{I}}(\cdot)$ and $w^{\mathrm{II}}(\cdot)$, respectively. Using the above equation we obtain

$$\phi_n^{\mathrm{I}}(x)-\phi_n^{\mathrm{II}}(x)=\rho\sum_{j=2}^{n+1}\left[w^{\mathrm{II}}\left(\frac{j-1}{n+1}\right)-w^{\mathrm{I}}\left(\frac{j-1}{n+1}\right)\right]\left(\varphi_j(x;q,n)-\varphi_{j-1}(x;q,n)\right)$$

By Proposition B.1, $\varphi_j(x;q,n)-\varphi_{j-1}(x;q,n)>0$ holds for all $j>1$ and $x\in(\underline{v},\overline{v})$. Suppose $w^{\mathrm{II}}(\cdot)$ first order stochastically dominates $w^{\mathrm{I}}(\cdot)$, then $w^{\mathrm{II}}(y)-w^{\mathrm{I}}(y)\leq 0$ holds for all $y\in(0,1)$. This implies $\phi_n^{\mathrm{I}}(x)-\phi_n^{\mathrm{II}}(x)\leq 0$ for all $n\geq 0$ and $x\in(\underline{v},\overline{v})$. $\square$

B.2 Asymptotic properties of $\phi_n(\cdot)$ as $n\to\infty$

In this subsection we derive asymptotic properties of $\phi_n(\cdot)$ as $n\to\infty$ and prove Lemmas 2 and 3 in Section 4. Moreover, we also establish some additional properties in Lemmas B.4 and B.5 below; these are relevant for proofs in Appendices D and E.

Given a designer's preference parameters ρ, $w(\cdot)$ and χ, we define

$$\phi(x):=\rho\left[\int_0^q \underline{t}\left(x,\frac{y}{q}\right)dw(y)+\int_q^1\bar{t}\left(x,\frac{y-q}{1-q}\right)dw(y)\right]+(1-\rho)\chi \tag{B.8}$$

for $x\in[\underline{v},\overline{v}]$, where $\underline{t}(\cdot)$ and $\bar{t}(\cdot)$ are given (B.4) and (B.5), respectively. The first order derivative of $\phi(x)$ is given by

$$\phi'(x)=\rho\left[\int_0^q \underline{t}_x\left(x,\frac{y}{q}\right)dw(y)+\int_q^1\bar{t}_x\left(x,\frac{y-1}{1-q}\right)dw(y)\right] \tag{B.9}$$

Moreover, using (B.4), (B.5) and the fact that $v_q^*=G^{-1}(q)$, we obtain

$$\underline{t}\left(v_q^*,\frac{y}{q}\right)=G^{-1}\left(\frac{y}{q}G(v_q^*)\right)=G^{-1}(y)$$

and

$$\bar{t}\left(v_q^*,\frac{y-q}{1-q}\right)=G^{-1}\left(\frac{y-q}{1-q}+(1-\frac{y-q}{1-q})G(v_q^*)\right)=G^{-1}(y)$$

These together imply

$$\phi^*:=\phi(v_q^*)=\rho\int_0^1 G^{-1}(y)dw(y)+(1-\rho)\chi \tag{B.10}$$

Lemma 2 in Section 4 is then equivalent to that $\phi_n(\cdot)$ and $\phi_n'(\cdot)$ uniformly converge to (B.8) and (B.9), respectively, and $\varphi_n(v)$ converges almost surely to (B.10).

B.2.1 Proof of Lemma 2

Consider any $z \in (0,1)$. By (B.3) in Lemma B.1 we have

$$\varphi_{\lfloor (n+1)z \rfloor}(x;q,n) = \begin{cases} \int_0^1 \underline{t}(x,y)\tau_{nq-1}\left(y; \frac{\lfloor (n+1)z\rfloor -1}{nq}\right) dy, & \text{if } z < \frac{nq+1}{n+1} \\ \int_0^1 \bar{t}(x,y)\tau_{n(1-q)-1}\left(y; \frac{\lfloor (n+1)z\rfloor -nq-2}{n(1-q)}\right) dy, & \text{if } z > \frac{nq+1}{n+1} \end{cases}$$

By Lemma A.1c, as $n \to \infty$, $\tau_{nq-1}\left(y; \frac{\lfloor (n+1)z\rfloor -1}{nq}\right)$ concentrates all its probability mass on $\frac{z}{q}$ and $\tau_{n(1-q)-1}\left(y; \frac{\lfloor (n+1)z\rfloor -nq-2}{n(1-q)}\right)$ concentrates all its mass on $\frac{z-q}{1-q}$ for all $z \neq q$. Therefore,

$$\lim_{n\to\infty} \varphi_{\lfloor (n+1)z \rfloor}(x;q,n) = \begin{cases} \underline{t}\left(x, \frac{z}{q}\right), & \text{if } z < q \\ \bar{t}\left(x, \frac{z-q}{1-q}\right), & \text{if } z > q \end{cases} \tag{B.11}$$

Using the definition of $\phi_n(x)$ and the fact that $w_j = w\left(\frac{j}{n+1}\right) - w\left(\frac{j-1}{n+1}\right)$, we get

$$\phi_n(x) = \rho \sum_{j=1}^{n+1} \left[w(z_j) - w(z_{j-1})\right] \varphi_{(n+1)z_j}(x;q,n) + (1-\rho)\chi \tag{B.12}$$

where $z_j := \frac{j}{n+1}$. Taking $n \to \infty$ and using the definition of Riemann integral, we have

$$\begin{aligned} \lim_{n\to\infty} \sum_{j=1}^{n+1} \left[w(z_j) - w(z_{j-1})\right] \varphi_{(n+1)z_j}(x;q,n) &= \int_0^1 \lim_{n\to\infty} \varphi_{\lfloor (n+1)z \rfloor}(x;q,n) dw(z) \\ &= \int_0^q \underline{t}\left(x, \frac{y}{q}\right) dw(y) + \int_q^1 \bar{t}\left(x, \frac{y-q}{1-q}\right) dw(y) \end{aligned}$$

where the last step follows from (B.11). Combining this with (B.12) yields

$$\lim_{n\to\infty} \phi_n(x) = \rho \left[\int_0^q \underline{t}\left(x, \frac{y}{q}\right) dw(y) + \int_q^1 \bar{t}\left(x, \frac{y-q}{1-q}\right) dw(y)\right] + (1-\rho)\chi \tag{B.13}$$

Therefore, $\phi_n(x)$ converges point-wise to (B.8) on $[\underline{v}, \bar{v}]$. By statement (1) of Proposition B.2, $\phi_n(x)$ is uniformly L-Liptschitz continuous on $[\underline{v}, \bar{v}]$ for some sufficient large L. Following the same argument in the proof of Proposition B.2, it can be show that $\phi(x)$ given by (B.8) is also L-Liptschitz continuous for sufficiently large L. These together imply that the convergence of $\phi_n(x)$ to (B.8) is in fact uniform.[2] The fact that $\phi_n'(x)$ converges uniformly to (B.9) can be proved analogously.

[2] This stems from the following general observation: *For any $L > 0$, let $\{f_n(\cdot)\}_{n\geq 0}$ be a sequence of L-Lipschitz continuous function on a closed interval $[a,b]$ that converges point-wise to a L-Lipschitz continuous function $f(\cdot)$. Then $f_n(\cdot)$ converges uniformly to $f(\cdot)$ on $[a,b]$.* The proof is as follows. Given any $\varepsilon > 0$, consider a pair of $\delta, \eta > 0$ such

Finally, we prove that $\varphi_n(v) := \rho \sum_{j=1}^{n+1} w_j v^{(j)} + (1-\rho)\chi$ converges in probability to ϕ^* given by (B.10). It suffices to show that $\sum_{j=1}^{n+1} w_j v^{(j)}$ converges in probability to $\int_0^1 G^{-1}(y)dw(y)$. We use a result from Van Zwet (1980), who establishes strong law for linear combinations of order statistics, to prove this. Let $U_1, U_2, \cdots, U_{n+1}$ be $n+1$ random variables drawn from a uniform distribution on $(0,1)$, and $U_{1:n+1} \le U_{2:n+1} \le \cdots \le U_{n+1:n+1}$ denote the ordered $U_1, U_2, \cdots, U_{n+1}$. We can therefore rewrite $v^{(j)}$ as $G^{-1}(U_{j:n+1})$ for each $j = 1, \cdots, n+1$. For $t \in (0,1)$, define $\gamma_n(t) := G^{-1}(U_{\lceil (n+1)t \rceil : n+1})$ and $\xi_n(t) := (n+1) \cdot \left[w\left(\frac{\lceil (n+1)t \rceil}{n+1}\right) - w\left(\frac{\lceil (n+1)t \rceil - 1}{n+1}\right) \right]$. We can then rewrite $\sum_{j=1}^{n+1} w_j v^{(j)}$ in an integral form as

$$\sum_{j=1}^{n+1} w_j v^{(j)} = \sum_{j=1}^{n+1} \left[w\left(\frac{j}{n+1}\right) - w\left(\frac{j-1}{n+1}\right) \right] G^{-1}(U_{j:n+1}) = \int_0^1 \gamma_n(t)\xi_n(t)dt \tag{B.14}$$

Our assumptions for G and $w(\cdot)$ ensure that $G^{-1}(\cdot), w(\cdot) \in L_1$, $\sup_n \|\xi_n\|_\infty < \infty$, and $\lim_{n\to\infty} \int_0^t \xi_n(x)dx = \int_0^t w'(x)dx = w(t)$ for all $t \in (0,1)$.[3] It follows from Theorem 2.1 and Corollary 2.1 of Van Zwet (1980) that the integral in (B.14) converges almost surely to $\int_0^1 G^{-1}(y)dw(y)$ as $n \to \infty$.

B.2.2 Proof of Lemma 3

In case n is finite, it follows from (B.1) that

$$\phi_n'(x) = \rho \sum_{j=1}^{n+1} w_j \varphi_j'(x; q, n) \tag{B.15}$$

Because $\varphi_j'(x;q,n)$ is uniformly bounded for all j, $\phi_n'(x)$ must converge uniformly to zero as $\rho \to 0$. Therefore, there exists some $\overline{\rho} > 0$ such that $\phi_n'(x) \le 1$ must hold for all $x \in [\underline{v}, \overline{v}]$ and $\rho \le \overline{\rho}$. Moreover, by Proposition B.1, if both G and $1-G$ are strictly log-concave, then $\varphi_j'(x;q,n) < 1$ for all $j \ne nq+1$ and $\varphi_j'(x;q,n) = 1$ for $j = nq+1$. These together implies $\phi_n{}'(x) \le \rho \le 1$ for all $x \in [\underline{v}, \overline{v}]$ by (B.15).

that $\varepsilon > 2L\delta + \eta$. Partition interval $[a,b]$ into $K+1$ intervals with cutoffs $a = x_0 < x_1 < \cdots < x_{K+1} = b$ such that $|x_i - x_{i-1}| < \delta$ for all $i \in \{1, \cdots, K+1\}$. For this finite set $\{x_i\}_{i=1}^{K+1}$ point-wise convergence implies that there exists a threshold N such that for all $n > N$ we have $|f_n(x_i) - f(x_i)| < \eta$ for all $i \in \{1, \cdots, K\}$. Now consider any $x \in [a,b]$ and let i be such that $x \in [x_{i-1}, x_i]$. We then obtain

$$\begin{aligned} |f_n(x) - f(x)| &= |f_n(x) - f_n(x_i) + f_n(x_i) - f(x_i) + f(x_i) - f(x)| \\ &\le |f_n(x) - f_n(x_i)| + |f_n(x_i) - f(x_i)| + |f(x_i) - f(x)| \\ &< 2L\delta + \eta < \varepsilon \end{aligned}$$

for all $n > N$. This proves uniform convergence on $[a,b]$.

[3] Here L_1 refers to the space of Lebesgue measurable functions $f : (0,1) \mapsto \mathbb{R}$ with finite $\|\cdot\|_1$ norm. $\|\cdot\|_\infty$ denotes the essential supremum norm. Moreover, the absolute continuity of $w(\cdot)$ ensures that its derivative $w'(\cdot)$ exists almost everywhere and $\int_0^t w'(x)dx = w(t)$ for all $t \in (0,1)$.

In what follows we show for all $x \in [\underline{v}, \overline{v}]$ that $\phi'(x) < 1$ holds under either condition (i) and (ii) of Lemma 3. Because both $\underline{t}_x\left(x, \frac{y}{q}\right)$ and $\overline{t}_x\left(x, \frac{y-1}{1-q}\right)$ are uniformly bounded, it follows from (B.9) that $\phi'(x)$ must uniformly converge to zero as $\rho \to 0$. This implies $\phi'(x) < 1$ for all $x \in [\underline{v}, \overline{v}]$ if ρ is sufficiently close to zero. If instead both G and $1-G$ are strictly log-concave, it follows from Lemma B.2 that $\underline{t}_x\left(x, \frac{y}{q}\right) < 1$ for $y < q$ and $\overline{t}_x\left(x, \frac{y-q}{1-q}\right) < 1$ for $y > q$. Therefore, by (B.9), $\phi'(x) < 1$ holds uniformly on $[\underline{v}, \overline{v}]$ if either $\rho < 1$ or $w(\cdot)$ does not put all weights on $y = q$ (i.e., $w(\cdot)$ is not a step function with threshold q). The latter condition for $w(\cdot)$ is always satisfied due to our assumption that $w(\cdot)$ is absolutely continuous.

B.2.3 Additional properties of $\phi(\cdot)$, ϕ^*, z^* and v_q^*

Here we establish several additional properties for $\phi(\cdot)$, ϕ^*, z^* and v_q^* summarized in Lemmas B.4 and B.5. The results are relevant for the proofs in Appendices D and E.

Lemma B.4. *For any $x \in [\underline{v}, \overline{v}]$, the following properties hold:*

1. If $\rho > 0$, then $\phi(x)$ is strictly decreasing in q.

2. If $\rho > 0$, then $\phi(x)$ strictly decreases as $w(\cdot)$ shifts from $w^I(\cdot)$ to $w^{II}(\cdot)$, where $w^I(\cdot), w^{II}(\cdot) \in \Delta([-1,1])$ and $w^I(\cdot) \succeq_{FOSD} w^{II}(\cdot)$.[4]

Proof of Lemma B.4. For all $y \in [0,1]$ let

$$t(x,y;q) := \begin{cases} \underline{t}\left(x, \frac{y}{q}\right), & \text{if } y \leq q \\ \overline{t}\left(x, \frac{y-q}{1-q}\right), & \text{if } y > q \end{cases} \tag{B.16}$$

where $\underline{t}(\cdot)$ and $\overline{t}(\cdot)$ are given by (B.4) and (B.5), respectively. By (B.8) we can rewrite $\phi(\cdot)$ as

$$\phi(x) = \rho \int_0^1 t(x,y;q)dw(y) + (1-\rho)\chi = \rho \mathbb{E}_w[t(x,\cdot;q)] + (1-\rho)\chi \tag{B.17}$$

Notice that $\phi(x)$ depends on x, q and $w(\cdot)$ only through the integral $\int_0^1 t(x,y;q)dw(y)$. Because both $\underline{t}(x,y)$ and $\overline{t}(x,y)$ are strictly increasing in y (cf. Lemma B.2), it follows from (B.16) that $t(x,y;q)$ is strictly decreasing in q. $\int_0^1 t(x,y;q)dw(y)$ must inherit the same property and therefore part (1) holds. Next, consider two weighting functions $w^{\mathrm{I}}(\cdot)$ and $w^{\mathrm{II}}(\cdot)$ such that $w^{\mathrm{I}}(\cdot) \succeq_{FOSD} w^{\mathrm{II}}(\cdot)$. Because $t(x,y;q)$ is strictly increasing in y, $\int_0^1 t(x,y;q)dw^{\mathrm{I}}(y) > \int_0^1 t(x,y;q)dw^{\mathrm{II}}(y)$ must hold. Therefore, $\phi(x)$ must be strictly higher under $w^{\mathrm{I}}(\cdot)$ than under $w^{\mathrm{II}}(\cdot)$. This proves part (2). $\square$

Building on Lemma B.4, we establish our next Lemma B.5, which characterizes how z^*, ϕ^* and v_q^* vary with model primitives $w(\cdot)$ and q.

[4] We use notation $\succeq_{FOSD}$ to denote the partial order implied by first order stochastic dominance.

Lemma B.5. *Suppose $\rho > 0$. The following comparative statics hold:*

1. $v_q^* = G^{-1}(q)$ is increasing in q.

2. ϕ^* is invariant in q and it strictly decreases as $w(\cdot)$ shifts from $w^I(\cdot)$ to $w^{II}(\cdot)$, where $w^I(\cdot) \succeq_{FOSD} w^{II}(\cdot)$.

3. Suppose both G and $1-G$ are strictly log-concave. Then (i) z^* weakly decreases as $w(\cdot)$ shifts from $w^I(\cdot)$ to $w^{II}(\cdot)$, where $w^I(\cdot) \succeq_{FOSD} w^{II}(\cdot)$; (ii) z^* is weakly decreasing in q; and (iii) if $\phi^* \in (-1,1)$ then $z^* = \phi^*$ if and only if $v_q^* = \phi^*$.

Proof of Lemma B.5. Part (1) is obvious from the definition of v_q^*. For part (2), recall that

$$\phi^* = \phi(v_q^*) = \rho \int_0^1 G^{-1}(y)dw(y) + (1-\rho)\chi$$

It is clear from its expression that ϕ^* is independent of q. Consider any $w^{\mathrm{I}}(\cdot)$ and $w^{\mathrm{II}}(\cdot)$ with $w^{\mathrm{I}}(\cdot) \succeq_{FOSD} w^{\mathrm{II}}(\cdot)$. Then $\int_0^1 G^{-1}(y)dw^{\mathrm{I}}(y) > \int_0^1 G^{-1}(y)dw^{\mathrm{II}}(y)$ must hold because $G^{-1}(y)$ is strictly increasing. Since $\rho > 0$, it follows that ϕ^* strictly decreases as $w(\cdot)$ shifts from $w^{\mathrm{I}}(\cdot)$ to $w^{\mathrm{II}}(\cdot)$. These together prove part (2).

Next we show part (3). By Lemma 3, strict log-concavity of G and $1-G$ ensures the single-crossing property and hence the existence of a unique z^* for all $w(\cdot)$ and q. The definition of z^* (cf. (12)) implies that it must decrease if function $\phi(\cdot)$ systematically shifts downward – i.e., $\phi(x)$ strictly decreases for all $x \in (\underline{v}, \overline{v})$ – after some shift of $w(\cdot)$ or q. The decreasing properties (i) and (ii) in part (3) then follow from Lemma B.4, which claims that $\phi(\cdot)$ shifts downwards if q increases or if $w(\cdot)$ varies from some $w^{\mathrm{I}}(\cdot)$ to $w^{\mathrm{II}}(\cdot)$ with $w^{\mathrm{I}}(\cdot) \succeq_{FOSD} w^{\mathrm{II}}(\cdot)$, when $\rho > 0$. Finally, to show (iii), recall that $\phi^* = \phi(v_q^*)$. Therefore, $\phi^* = v_q^*$ if and only if $v_q^* = \phi(v_q^*)$. Since $\phi^* \in (-1,1)$, it follows from the definition of z^* (cf. (12)) that $z^* = v_q^* = \phi^*$ must hold. □

C Omitted proofs for Section 5

C.1 Proof of Lemma 4

Our proof for Lemma 4 proceeds in two steps. In Step 1 we establish a general property (cf. Observation C.1) that a designer's utility function satisfying increasing-slope property at any interior point z implies $H \succeq_{MPS} H_{\mathscr{P}(z)}$ for any H that solves his monopolistic persuasion problem. In Step 2 we show that $W_n(\cdot)$ indeed satisfies the increasing slope property at switching point z_n whenever it is interior in $(-1,1)$. Together with Observation C.1, this implies our Lemma 4.

Step 1. Let $U(\cdot)$ be a generic utility function (of voters' posterior expected state θ) defined on

$[-1,1]$. Then, for any prior $F \in \Delta([-1,1])$ (which need not be continuous and fully supported), let

$$\mathscr{U}_\pi(U,F) := \mathbb{E}_{H_\pi}[U(\cdot)] = \int_{-1}^{1} U(\theta) dH_\pi(\theta)$$

denote the designer's expected payoff under any feasible information policy $\pi \in \Pi$.[5] Let $\underline{\pi}$ denote the *null* information policy that reveals no information. Then $H_{\underline{\pi}}$ is a degenerate distribution with all mass on prior mean $\mu_F := \mathbb{E}_F[k]$ and therefore

$$\mathscr{U}_{\underline{\pi}}(U,F) = U(\mu_F) \tag{C.1}$$

On the other hand, for a cutoff censorship policy $\mathscr{P}(z)$ with $z \in (-1,1)$, we have

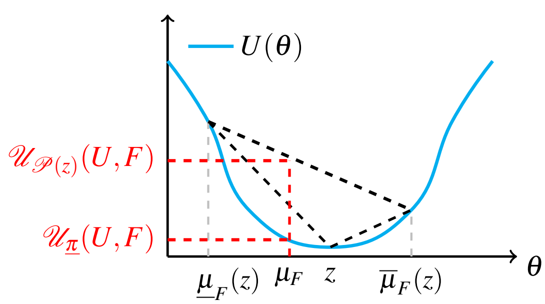
$$\mathscr{U}_{\mathscr{P}(z)}(U,F) := F^-(z)U(\underline{\mu}_F(z)) + \big(F(z) - F^-(z)\big)U(z) + \big(1 - F(z)\big)U(\overline{\mu}_F(z)) \tag{C.2}$$

where $F^-(z) := \lim_{x \uparrow z} F(z)$, $\underline{\mu}_F(z) := \mathbb{E}_F[k|k<z]$ and $\overline{\mu}_F(z) := \mathbb{E}_F[k|k>z]$.

Claim C.1. *Suppose $U(\cdot)$ satisfies the increasing slope property at some point $z \in (-1,1)$, then $\mathscr{U}_{\mathscr{P}(z)}(U,F) > \mathscr{U}_{\underline{\pi}}(U,F)$ for any $F \in \Delta([-1,1])$ that satisfies $0 < F^-(z) \le F(z) < 1$.*

Proof of Claim C.1. Figure C.1 illustrates $\mathscr{U}_{\mathscr{P}(z)}(U,F)$ and $\mathscr{U}_{\underline{\pi}}(U,F)$ for a function $U(\cdot)$ that satisfies increasing slope property at some $z \in (-1,1)$ and a prior F with no mass point at z.

Figure C.1: Graphical Illustration for the Proof of Claim C.1



By (C.1) and (C.2), we obtain

$$\begin{aligned}\mathscr{U}_{\mathscr{P}(z)}(U,F) - \mathscr{U}_{\underline{\pi}}(U,F) =& F^-(z)\left(\underline{\mu}_F(z) - \mu_F\right)\frac{U(\underline{\mu}_F(z)) - U(\mu_F)}{\underline{\mu}_F(z) - \mu_F} \\ &+ \big(F(z) - F^-(z)\big)\left(z - \mu_F\right)\frac{U(z) - U(\mu_F)}{z - \mu_F} \\ &+ \big(1 - F(z)\big)\left(\overline{\mu}_F(z) - \mu_F\right)\frac{U(\overline{\mu}_F(z)) - U(\mu_F)}{\overline{\mu}_F(z) - \mu_F}\end{aligned}$$

[5] Recall that H_π denotes the distribution of posterior expectations induced by π under prior F.

On the other hand, by law of iterated expectations, we have

$$
\begin{aligned}
& F^-(z)\underline{\mu}_F(z) + \left(F(z) - F^-(z)\right)z + \left(1 - F(z)\right)\overline{\mu}_F(z) = \mu_F \\
\Longrightarrow\ & \left(F(z) - F^-(z)\right)\left(z - \mu_F\right) = -F^-(z)\left(\underline{\mu}_F(z) - \mu_F\right) - \left(1 - F(z)\right)\left(\overline{\mu}_F(z) - \mu_F\right)
\end{aligned}
$$

These together imply

$$
\begin{aligned}
\mathscr{U}_{\mathscr{P}(z)}(U,F) - \mathscr{U}_{\underline{\pi}}(U,F) = F^-(z)\left(\mu_F - \underline{\mu}_F(z)\right)\left(\frac{U(z) - U(\mu_F)}{z - \mu_F} - \frac{U(\mu_F) - U(\underline{\mu}_F(z))}{\mu_F - \underline{\mu}_F(z)}\right) \\
+ (1 - F(z))\left(\overline{\mu}_F(z) - \mu_F\right)\left(\frac{U(\overline{\mu}_F(z)) - U(\mu_F)}{\overline{\mu}_F(z) - \mu_F} - \frac{U(z) - U(\mu_F)}{z - \mu_F}\right)
\end{aligned}
$$

Since $\underline{\mu}_F(z) < x < \overline{\mu}_F(z)$ for $x \in \{z, \mu_F\}$, increasing slope property at z implies $\frac{U(z)-U(\mu_F)}{z-\mu_F} - \frac{U(\mu_F)-U(\underline{\mu}_F(z))}{\mu_F-\underline{\mu}_F(z)} \geq 0$ and $\frac{U(\overline{\mu}_F(z))-U(\mu_F)}{\overline{\mu}_F(z)-\mu_F} - \frac{U(z)-U(\mu_F)}{z-\mu_F} \geq 0$, with at least one holds with strict inequality.[6] This implies $\mathscr{U}_{\mathscr{P}(z)}(U,F) - \mathscr{U}_{\underline{\pi}}(U,F) \geq 0$ for all F. Finally, notice that if $0 < F^-(z) \leq F(z) < 1$ holds, then both $F^-(z)\left(\mu_F - \underline{\mu}_F(z)\right)$ and $(1 - F(z))\left(\overline{\mu}_F(z) - \mu_F\right)$ are strictly positive so that $\mathscr{U}_{\mathscr{P}(z)}(U,F) - \mathscr{U}_{\underline{\pi}}(U,F) > 0$ must hold. □

We are now ready to establish the following general observation.

Observation C.1. *Suppose $U(\cdot)$ satisfies the increasing slope property at point $z \in (-1,1)$. Then $H \succeq_{MPS} H_{\mathscr{P}(z)}$ for any H (if it exists) that solves*

$$
\max_{H \in \Delta([-1,1])} \int_{-1}^{1} U(\theta)dH(\theta), \quad s.t.\ F \succeq_{MPS} H \tag{C.3}
$$

Proof of Observation C.1. Suppose $U(\cdot)$ satisfies increasing slope property at point z and let H be any solution to (C.3) (if it exists). We show that $H \succeq_{MPS} H_{\mathscr{P}(z)}$ must hold by contradiction. Suppose there exists any $H \not\succeq_{MPS} H_{\mathscr{P}(z)}$ that solves (C.3). Let $\pi = (S, \sigma)$ be an information policy that induces H. For each $s \in S$, let γ_s denote the posterior distribution induced by s and h_s denote the mean of γ_s. Finally, let $\delta \in \Delta(S)$ denote the ex-ante distribution of messages $s \in S$ induced by π. With these we obtain

$$
\int_{-1}^{1} U(\theta)dH(\theta) = \int_{s \in S} U(h_s)d\delta(s) = \int_{s \in S} \mathscr{U}_{\underline{\pi}}(U, \gamma_s)d\delta(s)
$$

Since $H \not\succeq_{MPS} H_{\mathscr{P}(z)}$, there exists $s \in S$ such that $0 < \gamma_s^-(z) \leq \gamma_s(z) < 1$ holds. Denote by $\widetilde{S} \subseteq S$

[6] In case $z = \mu_F$, we can let $\frac{U(z)-U(\mu_F)}{z-\mu_F}$ be any number between $U'_-(z)$ to $U'_+(z)$, which are the left and right derivatives of $U(\cdot)$ at point z, respectively. Notice that the increasing slope property at point z implies the existence of both $U'_-(z)$ and $U'_+(z)$ (through the monotone convergence theorem) and that $U'_-(z) \leq U'_+(z)$.

the set of all such s and $\widetilde{S}$ must have positive probability measure under δ. Consider the joint information policy $\widetilde{\pi}$ induced by π and the cutoff policy $\mathscr{P}(z)$, and let $\widetilde{H} = H_{\widetilde{\pi}}$. For all events $s \in \widetilde{S}$, it follows from Claim C.1 that $\mathscr{U}_{\mathscr{P}(z)}(U, \gamma_s) > \mathscr{U}_{\underline{\pi}}(U, \gamma_s)$. For $s \notin \widetilde{S}$, $\mathscr{U}_{\mathscr{P}(z)}(U, \gamma_s) = \mathscr{U}_{\underline{\pi}}(U, \gamma_s)$ holds trivially. Therefore, we have

$$\begin{aligned}\int_{-1}^{1} U(\theta) d\widetilde{H}(\theta) &= \int_{s \in \widetilde{S}} \mathscr{U}_{\mathscr{P}(z)}(U, \gamma_s) d\delta(s) + \int_{s \in S/\widetilde{S}} \mathscr{U}_{\mathscr{P}(z)}(U, \gamma_s) d\delta(s) \\ &= \int_{s \in \widetilde{S}} \mathscr{U}_{\mathscr{P}(z)}(U, \gamma_s) d\delta(s) + \int_{s \in S/\widetilde{S}} \mathscr{U}_{\underline{\pi}}(U, \gamma_s) d\delta(s) \\ &> \int_{s \in \widetilde{S}} \mathscr{U}_{\underline{\pi}}(U, \gamma_s) d\delta(s) + \int_{s \in S/\widetilde{S}} \mathscr{U}_{\underline{\pi}}(U, \gamma_s) d\delta(s) = \int_{-1}^{1} U(\theta) dH(\theta)\end{aligned} \tag{C.4}$$

This contradicts that H is a solution to (C.3) and thus completes the proof. □

Step 2. Now we establish that $W_n(\cdot)$ satisfies the increasing slope property at z_n when $z_n \in (-1, 1)$ holds. Recall from (9) in Section 5 that

$$W_n(\theta) := \int_{\underline{v}}^{\theta} \big(\theta - \phi_n(x)\big) \hat{g}_n(x; q) dx = \theta \hat{G}_n(\theta; q) - \int_{\underline{v}}^{\theta} \phi_n(x) \hat{g}_n(x; q) dx \tag{C.5}$$

By (C.5), for any $\theta \neq z_n$ we have

$$\begin{aligned}W_n(\theta) - W_n(z_n) &= \int_{\underline{v}}^{\theta} (\theta - \phi_n(x))\, \hat{g}_n(x; q) dx - \int_{\underline{v}}^{z_n} (z_n - \phi_n(x))\, \hat{g}_n(x; q) dx \\ &= \int_{z_n}^{\theta} (\theta - \phi_n(x))\, \hat{g}_n(x; q) dx + (\theta - z_n) \int_{\underline{v}}^{z_n} \hat{g}_n(x; q) dx\end{aligned}$$

Therefore,

$$\lambda_n(\theta; z_n) := \frac{W_n(\theta) - W_n(z_n)}{\theta - z_n} = \int_{z_n}^{\theta} \frac{\theta - \phi_n(x)}{\theta - z_n} \hat{g}_n(x; q) dx + \int_{\underline{v}}^{z_n} \hat{g}_n(x; q) dx$$

Taking derivative with respect to θ yields

$$\lambda_n{}'(\theta; z_n) = \frac{\theta - \phi_n(\theta)}{\theta - z_n} \hat{g}_n(\theta; q) + \int_{z_n}^{\theta} \frac{\phi_n(x) - z_n}{(\theta - z_n)^2} \hat{g}_n(x; q) dx \tag{C.6}$$

Recall from the premise of this lemma that $\phi_n(\cdot)$ crosses zero only once and from above at z_n. For any $\theta > z_n$, $x > \phi_n(x) \geq z_n$ holds for all $x \in (z_n, \theta]$. Therefore, the first term on the right-hand side of (C.6) must be strictly positive and the second term is non-negative. This implies $\lambda_n{}'(\theta; z_n) > 0$ for $\theta > z_n$. For any $\theta < z_n$, $x < \phi_n(x) \leq z_n$ holds for all $x \in [\theta, z_n)$. So the first term on the RHS of

(C.6) is strictly positive, and the second term equals

$$\int_{z_n}^{\theta} \frac{\phi_n(x) - z_n}{(\theta - z_n)^2} \hat{g}_n(x;q)dx = \int_{\theta}^{z_n} \frac{z_n - \phi_n(x)}{(\theta - z_n)^2} \hat{g}_n(x;q)dx$$

and is non-negative. This implies ${\lambda_n}'(\theta; z_n) > 0$ for $\theta < z_n$ as well. Taken together, ${\lambda_n}'(\theta; z_n) > 0$ holds for all $\theta \neq z_n$. Finally, since $W_n(\theta)$ is differentiable, we have that $\lambda_n(\theta; z_n)$ is continuous at $\theta = z_n$ and $\lim_{\theta \to z_n} \lambda_n(\theta; z_n) = {W_n}'(z_n)$. These together establish that $\lambda_n(\theta; z_n)$ is strictly increasing in θ, which implies the increasing slope property at point z_n. Together with Observation C.1, this implies our Lemma 4.

C.2 Proof of Lemma 5

By (9), the second order derivative of $W_n(\theta)$ is given by

$$\begin{aligned}
W_n''(\theta) &= \hat{g}_n(\theta;q)\left(2 - \phi_n'(\theta)\right) + \hat{g}_n'(\theta;q)\left(\theta - \phi_n'(\theta)\right) \\
&= \hat{g}_n(\theta;q)\left\{2 - {\phi_n}'(\theta) + (\theta - \phi_n(\theta))\frac{\hat{g}_n'(\theta;q)}{\hat{g}_n(\theta;q)}\right\} \\
&= \hat{g}_n(\theta;q)\left\{2 - \phi_n'(\theta) + \left(\theta - \phi_n(\theta)\right)\left(n\frac{g(\theta)}{G(\theta)}\frac{q - G(\theta)}{1 - G(\theta)} + \frac{g'(\theta)}{g(\theta)}\right)\right\}
\end{aligned} \tag{C.7}$$

The last step follows from (A.9). Because G is strictly positive and twice-continuously differentiable on $[\underline{v}, \bar{v}]$, $W_n''(\theta)$ is continuous. By (C.7) and the fact that $q = G(v_q^*)$, $W_n''(\theta) > 0$ if and only if

$$(\theta - \phi_n(\theta))\left(G(\theta) - G(v_q^*)\right) < \frac{G(\theta)(1 - G(\theta))}{ng(\theta)}\left(2 - {\phi_n}'(\theta) + (\theta - \phi_n(\theta))\frac{g'(\theta)}{g(\theta)}\right) \tag{C.8}$$

Because $\phi_n(\cdot)$, $\phi_n'(\cdot)$ and $\phi_n''(\cdot)$ are uniformly Lipschitz continuous (cf. Proposition B.2) and $g(\cdot)$ is positive and twice continuously differentiable on $[\underline{v}, \bar{v}]$, both the value and the first order derivative of the right-hand side of (C.8) converge to zero uniformly for all $\theta \in [\underline{v}, \bar{v}]$.[7] Let $\zeta_n(\theta) := (\theta - \phi_n(\theta))\left(G(\theta) - G(v_q^*)\right)$ denote the left-hand side of (C.8) and

$$\zeta(\theta) := \lim_{n \to \infty} \zeta_n(\theta) = (\theta - \phi(\theta))\left(G(\theta) - G(v_q^*)\right). \tag{C.9}$$

Both the value and derivative of $\zeta_n(\theta)$ converge uniformly to $\zeta(\cdot)$ and $\zeta'(\cdot)$, respectively. Therefore, $\lim_{n\to\infty} W_n''(\theta) > (<)0$ if and only if $\zeta(\theta) < (>)0$. On the one hand, $G(\theta) - G(v_q^*)$ is increasing in θ and admits a unique root v_q^* at which its derivative equals $g(v_q^*) > 0$. On the other hand, under our definition of single-crossing property (cf. Definition 1), $\theta - \phi(\theta)$ crosses zero at most once and

[7] This is because both $1 - {\phi_n}'(\theta)$ and $\frac{g'(\theta)}{G'(\theta)}$ are uniformly bounded on $[\underline{v}, \bar{v}]$ under our assumption for G.

from below on $[-1,1]$. Recall that $z^* = \lim_{n\to\infty} z_n$ (cf. (12)) and the single-crossing property requires $1-\phi'(z^*) > 0$ whenever $\phi(z^*) = z^*$ and $z^* \in [-1,1]$. Let

$$\ell^* := \max\left\{\min\left\{z^*, v_q^*\right\}, -1\right\} \quad \text{and} \quad r^* := \min\left\{\max\left\{z^*, v_q^*\right\}, 1\right\} \tag{C.10}$$

It follows from (C.9) that $\zeta(\theta) < 0$ for all $\theta \in (\ell^*, r^*)$ and $\zeta(\theta) > 0$ for all $\theta \in [-1,1]/[\ell^*, r^*]$. We distinguish between three cases.

Case 1: $z^* = -1$ and $\theta - \phi(\theta) > 0$ for all $\theta \in [-1,1]$. In this case, $\ell^* = -1$ and $\zeta(\theta) > (<)0$ for $\theta > (<)r^*$ on $[-1,1]$. It then follows that there exists some $N \geq 0$ such that for all $n \geq N$ we have (i) $\theta - \phi_n(\theta) > 0$ for all $\theta \in [-1,1]$ so that $z_n = -1$, and (ii) there is some $r_n \in [-1,1]$ with $r_n \to r^*$ such that $W_n''(\theta) > (<)0$ for $\theta < (>)r_n$. This implies that $W_n(\cdot)$ is strictly S-shape on $[z_n, 1]$ with inflection point r_n for all $n \geq N$.

Case 2: $z^* = 1$ and $\theta - \phi(\theta) < 0$ for all $\theta \in [-1,1]$. In this case, $r^* = 1$ and $\zeta(\theta) < (>)0$ for $\theta > (<)\ell^*$ on $[-1,1]$. There then exists $N \geq 0$ such that for all $n \geq N$ we have (i) $\theta - \phi_n(\theta) < 0$ for all $\theta \in [-1,1]$ so that $z_n = 1$, and (ii) there is some $\ell_n \in [-1,1]$ with $\ell_n \to \ell^*$ such that $W_n''(\theta) > (<)0$ for $\theta > (<)\ell_n$. This implies that $W_n(\cdot)$ is strictly inverse S-shape on $[-1, z_n]$ with inflection point ℓ_n for all $n \geq N$.

Case 3: $z^* \in [-1,1]$ and $z^* = \phi(z^*)$. Recall that the single-crossing property requires $1-\phi'(z^*) > 0$ whenever $\phi(z^*) = z^* \in [-1,1]$. It follows that there exists some $N \geq 0$ and $\varepsilon > 0$ such that for all $n \geq N$ we have

(i) there exists a unique $\tilde{z}_n \in \delta(z^*;\varepsilon) := (z^*-\varepsilon, z^*+\varepsilon)$ such that both $\tilde{z}_n = \phi_n(\tilde{z}_n)$ and $1-\phi_n'(\tilde{z}_n) > 0$ hold, and

(ii) there are $\tilde{\ell}_n, \tilde{r}_n \in \mathbb{I} := [-1,1] \cup \delta(z^*;\varepsilon)$ such that $W_n''(\theta) > 0$ if $\theta \in \left(\tilde{\ell}_n, \tilde{r}_n\right)$ and $W_n''(\theta) < 0$ if $\theta \in \mathbb{I}/\left[\tilde{\ell}_n, \tilde{r}_n\right]$, where $\tilde{\ell}_n$ and $\tilde{r}_n$ satisfy $\tilde{\ell}_n \leq \tilde{r}_n$, $\tilde{\ell}_n \to \max\left\{\min\left\{z^*, v_q^*\right\}, \min\left\{-1, z^*-\varepsilon\right\}\right\}$, and $\tilde{r}_n \to \min\left\{\max\left\{z^*, v_q^*\right\}, \max\left\{1, z^*+\varepsilon\right\}\right\}$.

Let $\ell_n = \max\left\{\tilde{\ell}_n, -1\right\}$ and $r_n = \min\left\{\tilde{r}_n, 1\right\}$. It follows from (ii) that $W_n''(\theta) > 0$ if $\theta \in (\ell_n, r_n)$ and $W_n''(\theta) < 0$ if $\theta \in [-1,1]/[\ell_n, r_n]$. Moreover, $\ell_n \to \ell^*$ and $r_n \to r^*$. Finally, (i) implies

$$\begin{aligned} W_n''(\tilde{z}_n) &= \hat{g}_n(\tilde{z}_n;q)\left(2-\phi_n'(\tilde{z}_n)\right) + \hat{g}_n'(\tilde{z}_n;q)\left(\tilde{z}_n - \phi_n(\tilde{z}_n)\right) \\ &= \hat{g}_n(\tilde{z}_n;q)\left(2-\phi_n'(\tilde{z}_n)\right) > \hat{g}_n(\tilde{z}_n;q) > 0 \end{aligned}$$

Therefore $\tilde{z}_n \in \left(\tilde{\ell}_n, \tilde{r}_n\right)$ must hold. If $\tilde{z}_n \in [-1,1]$, then $\tilde{z}_n = z_n$and we obtain $\ell_n \leq z_n \leq r_n$ with at least one inequality holds strictly. If $\tilde{z}_n < -1$ (resp. $\tilde{z}_n > 1$) then $-1 = z_n = \ell_n \leq r_n$ (resp. $1 = z_n = r_n \geq \ell_n$). Taken together, for all $n \geq N$, $W_n(\theta)$ is S-shaped on $[z_n, 1]$ with inflection point $r_n \geq z_n$ and is inverse S-shaped on $[-1, z_n]$ with inflection point $\ell_n \leq z_n$.

These three cases together imply that there exists $N \geq 0$ such that for all $n \geq N$ it holds that $W_n(\cdot)$ is strictly S-shaped on $[z_n, 1]$ with inflection point $r_n \in [z_n, 1]$ while strictly inverse S-shaped on $[-1, z_n]$ with inflection point $\ell_n \in [-1, z_n]$. Moreover, in the limit we have

$$\lim_{n\to\infty} \ell_n = \ell^* \quad \text{and} \quad \lim_{n\to\infty} r_n = r^* \; . \tag{C.11}$$

Finally, we show that $N = 0$ if $g(\cdot)$ is log-concave and ρ is equal or sufficiently close to zero. By (C.7) we have

$$W_n''(\theta) = \hat{g}_n(\theta;q)\,(\theta - \phi_n(\theta)) \left\{ \frac{2 - \phi_n{}'(\theta)}{\theta - \phi_n(\theta)} + \frac{\hat{g}_n'(\theta;q)}{\hat{g}_n(\theta;q)} \right\} \tag{C.12}$$

We only consider interval $[z_n, 1]$ and show that $W_n(\theta)$ is strictly S-shaped on it for all $n \geq 0$.[8] Observe that $\theta - \phi_n(\theta) > 0$ holds for all $\theta > z_n$ due to the single-crossing property. It therefore suffices to show that term in the last curly bracket in (C.12) is strictly decreasing. When $g(\cdot)$ is log-concave, it follows from Proposition A.1 in Appendix A that $\hat{g}_n(\cdot;q)$ is strictly log-concave, and hence $\frac{\hat{g}_n'(\theta;q)}{\hat{g}_n(\theta;q)}$ is strictly decreasing, for all $n \geq 0$. Next we show that $\frac{2-\phi_n{}'(\theta)}{\theta-\phi_n(\theta)}$ is also strictly decreasing in θ on $(z_n, 1]$ for all $n \geq 0$. Let $\xi_n(\theta) := \frac{2-\phi_n{}'(\theta)}{\theta-\phi_n(\theta)}$ and simple algebra shows that $\xi_n'(\theta) < 0$ if and only if

$$\phi_n{}''(\theta)\,(\theta - \phi_n(\theta)) + \left(2 - \phi_n{}'(\theta)\right)\left(1 - \phi_n{}'(\theta)\right) \geq 0 \tag{C.13}$$

Recall from the definition of $\phi_n(\theta)$ (cf. (B.1)) that $\phi_n{}'(\theta) = \rho \sum_{j=1}^{n+1} w_j \cdot \varphi_j'(x;q,n)$ and $\phi_n{}''(\theta) = \rho \sum_{j=1}^{n+1} w_j \cdot \varphi_j''(x;q,n)$. As $\rho \to 0$ we have $\phi_n{}'(\theta) \to 0$ and $\phi_n{}''(\theta) \to 0$ uniformly for all $\theta \in [\underline{v}, \overline{v}]$. Therefore, the left-hand side of (C.13) converges to 2 for all $\theta \in [-1, 1]$ as $\rho \to 0$. This implies $\xi_n'(\theta) < 0$ for all $\theta \in [-1, 1]$ and $n \geq 0$ if ρ is sufficiently close to zero. This completes the proof.

D Omitted Proofs for Section 6.1: Asymptotic Results

D.1 Proof of Theorem 2

To prove Theorem 2 we introduce two Lemmas D.1 and D.2, which respectively characterize the asymptotically optimal designer payoff and the set of censorship policies that generate this payoff as $n \to \infty$.

Lemma D.1. *Suppose $v_q^*, \phi^* \in [-1, 1]$ and let W_n be the value of problem (MP) for any given $n \geq 0$.*

[8] The proof for the inverse S-shape property on $[-1, z_n]$ is analogous and hence omitted.

Then

$$W^* := \lim_{n\to\infty} W_n = \begin{cases} \int_{\underline{\phi}(v_q^*)}^1 (k-\phi^*)dF(k), & \textit{if } \phi^* < \underline{\phi}(v_q^*) \\ \int_{\phi^*}^1 (k-\phi^*)dF(k), & \textit{if } \underline{\phi}(v_q^*) \le \phi^* \le \overline{\phi}(v_q^*) \\ \int_{\overline{\phi}(v_q^*)}^1 (k-\phi^*)dF(k), & \textit{if } \phi^* > \overline{\phi}(v_q^*) \end{cases} \tag{D.1}$$

Proof of Lemma D.1. We start by presenting the monopoly designer's asymptotic persuasion problem. Recall that $W_n(\cdot)$ is the designer's expected utility function and let $W(\theta) := \lim_{n\to\infty} W_n(\theta)$ for all $\theta \in [-1,1]$. Then we have

$$W(\theta) := \begin{cases} \theta - \phi^*, & \text{if } \theta > v_q^* \\ 0, & \text{if } \theta < v_q^* \end{cases}.$$

This is because $v^{(nq+1)} \xrightarrow{p} v_q^*$ (cf. Lemma 1) and $\varphi_n(v) \xrightarrow{p} \phi^*$ (cf. Lemma 2). If $\theta < v_q^*$, then $v^{(nq+1)} > \theta$ almost surely for large n so that status quo is maintained with probability one as $n \to \infty$. The designer's payoff thus converges to 0, the normalized payoff under status quo. Conversely, if $\theta > v_q^*$, then reform is passed with probability one as $n \to \infty$ so that the designer's payoff converges to $\lim_{n\to\infty} (\theta - \varphi_n(v)) = \theta - \phi^*$. Let

$$\widetilde{W}(\theta) = \begin{cases} W(\theta), & \text{if } \theta \ne v_q^* \\ \max\{v_q^* - \phi^*, 0\}, & \text{if } \theta = v_q^* \end{cases}$$

The designer's asymptotic persuasion problem is then

$$\max_{H\in\Delta([-1,1])} \int_{-1}^1 \widetilde{W}(\theta)dH(\theta), \text{ s.t. } F \succeq_{MPS} H \tag{D.2}$$

Because $\widetilde{W}(\theta)$ is upper semi-continuous, problem (D.2) always admits a solution. We denote the value to (D.2) by W^* and characterize it using Theorem 1 of Dworczak and Martini (2019). For ease of reference we restate their theorem applied to our problem in the following observation.

Observation D.2. *(Dworczak and Martini, 2019) If there exists some $H \in \Delta([-1,1])$ and a convex function $p(\cdot)$ on $[-1,1]$ with $p(\cdot) \ge \widetilde{W}(\cdot)$ that satisfy*

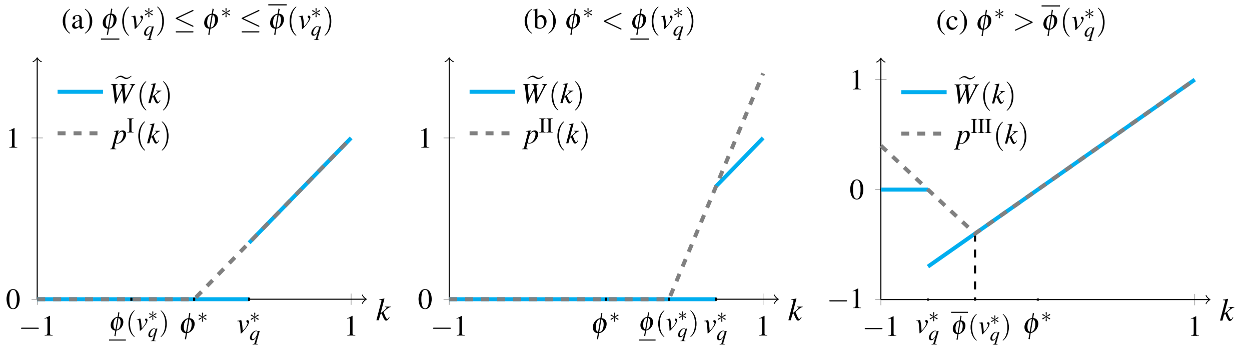
$$supp(H) \subseteq \{\theta : p(\theta) = \widetilde{W}(\theta)\}, \textit{ and} \tag{D.3}$$

$$\int_{-1}^1 p(\theta)dH(\theta) = \int_{-1}^1 p(\theta)dH(\theta), \textit{ and} \tag{D.4}$$

$$F \succeq_{MPS} H, \tag{D.5}$$

then H is a solution to (D.2), and the value of (D.2) equals $\int_{-1}^1 p(\theta)dH(\theta)$.

Figure D.1: Illustration for the proof of Lemma D.1



Note: In all three panels, the blue solid lines denote $\widetilde{W}(\cdot)$ and the gray dashed lines denote the auxiliary functions $p(\cdot)$.

We distinguish between three cases.

Case 1. $\underline{\phi}(v_q^*) \leq \phi^* \leq \overline{\phi}(v_q^*)$. In this case we have $\mathbb{E}_F[k|k \geq \phi^*] \geq v_q^*$ and $\mathbb{E}_F[k|k \leq \phi^*] \leq v_q^*$. Let $p^{\mathrm{I}}(\theta) := \max\{\theta - \phi^*, 0\}$ for $\theta \in [-1,1]$. $p^{\mathrm{I}}(\cdot)$ is illustrated in Figure D.1a. Consider the cutoff policy $\mathscr{P}(\phi^*)$ and let $\underline{H}^{\mathrm{I}} = H_{\mathscr{P}(\phi^*)}$. The following conditions are straightforward to verify: (i) $p^{\mathrm{I}}(\cdot)$ is convex and $p^{\mathrm{I}}(\cdot) \geq \widetilde{W}(\cdot)$ on $[-1,1]$; (ii) $F \succeq_{MPS} \underline{H}^{\mathrm{I}}$ and $\int_{-1}^{1} p^{\mathrm{I}}(\theta) d\underline{H}^{\mathrm{I}}(\theta) = \int_{-1}^{1} p^{\mathrm{I}}(\theta) dF(\theta)$, and (iii)

$$\begin{aligned}
\operatorname{supp}(\underline{H}^{\mathrm{I}}) &= \{\mathbb{E}_F[k|k \leq \phi^*], \mathbb{E}_F[k|k \geq \phi^*]\} \\
&\subset \left\{\theta \middle| p^{\mathrm{I}}(\theta) = \widetilde{W}(\theta)\right\} = \begin{cases} [-1,1], & \text{if } v_q^* = \phi^* \\ [-1, v_q^*] \cup [\phi^*, 1], & \text{if } v_q^* < \phi^* \\ [-1, \phi^*] \cup [v_q^*, 1], & \text{if } v_q^* > \phi^* \end{cases}
\end{aligned}$$

Therefore, by Observation D.2, $\underline{H}^{\mathrm{I}}$ solves problem (D.2) and the value of (D.2) equals

$$\int_{-1}^{1} p^{\mathrm{I}}(k) dF(k) = \int_{-1}^{1} \max\{k - \phi^*, 0\} dF(k) = \int_{\phi^*}^{1} (k - \phi^*) dF(k) \tag{D.6}$$

Case 2. $\phi^* < \underline{\phi}(v_q^*)$. For this case we use the definition of $\underline{\phi}(v_q^*)$ (cf. (14)) and obtain $\underline{\phi}(v_q^*) \in (-1,1)$ and $\mathbb{E}_F[k|k \geq \underline{\phi}(v_q^*)] = v_q^*$. Let

$$p^{\mathrm{II}}(\theta) := \begin{cases} \frac{v_q^* - \phi^*}{v_q^* - \underline{\phi}(v_q^*)} \left(\theta - \underline{\phi}(v_q^*)\right), & \text{if } \theta \in \left[\underline{\phi}(v_q^*), 1\right] \\ 0, & \text{if } \theta \in \left[-1, \underline{\phi}(v_q^*)\right) \end{cases}.$$

$p^{\mathrm{II}}(\cdot)$ is illustrated in Figure D.1b. Consider cutoff policy $\mathscr{P}\left(\underline{\phi}(v_q^*)\right)$ and let $\underline{H}^{\mathrm{II}} = H_{\mathscr{P}(\underline{\phi}(v_q^*))}$. The following conditions are easy to verify: (i) $p^{\mathrm{II}}(\cdot)$ is convex and $p^{\mathrm{II}}(\cdot) \geq \widetilde{W}(\cdot)$ on $[-1,1]$;

(ii) $F \succeq_{MPS} \underline{H}^{\text{II}}$ and $\int_{-1}^{1} p^{\text{II}}(\theta)d\underline{H}^{\text{II}}(\theta) = \int_{-1}^{1} p^{\text{II}}(\theta)dF(\theta)$; and (iii)

$$\text{supp}(\underline{H}^{\text{II}}) = \left\{\mathbb{E}_F\left[k|k<\underline{\phi}(v_q^*)\right], v_q^*\right\} \subset \left\{\theta \middle| p^{\text{II}}(\theta) = \widetilde{W}(\theta)\right\} = \left[-1, \underline{\phi}(v_q^*)\right] \cup \{v_q^*\}.$$

Hence, $\underline{H}^{\text{II}}$ solves problem (D.2) and the value of (D.2) equals

$$\begin{aligned}\int_{-1}^{1} p^{\text{II}}(k)dF(k) &= \left(1-F\left(\underline{\phi}(v_q^*)\right)\right)\frac{v_q^*-\phi^*}{v_q^*-\underline{\phi}(v_q^*)}\left(\mathbb{E}_F\left[k|k>\underline{\phi}(v_q^*)\right]-\underline{\phi}(v_q^*)\right)\\ &= \left(1-F\left(\underline{\phi}(v_q^*)\right)\right)\left(v_q^*-\phi^*\right) = \int_{\underline{\phi}(v_q^*)}^{1}(k-\phi^*)dF(k)\end{aligned} \tag{D.7}$$

Case 3. $\phi^* > \overline{\phi}(v_q^*)$. For this case we use the definition of $\overline{\phi}(v_q^*)$ (cf. (13)) and obtain $\overline{\phi}(v_q^*) \in (-1,1)$ and $\mathbb{E}_F\left[k|k \le \overline{\phi}(v_q^*)\right] = v_q^*$. Let

$$p^{\text{III}}(\theta) = \begin{cases}\theta-\phi^*, & \text{if } \theta \in \left[\overline{\phi}(v_q^*), 1\right]\\ \frac{\overline{\phi}(v_q^*)-\phi^*}{\overline{\phi}(v_q^*)-v_q^*}\left(\theta-v_q^*\right), & \text{if } \theta \in \left[-1, \overline{\phi}(v_q^*)\right)\end{cases}.$$

$p^{\text{III}}(\cdot)$ is illustrated in Figure D.1c. Consider cutoff policy $\mathscr{P}\left(\overline{\phi}(v_q^*)\right)$ and let $\underline{H}^{\text{III}} = H_{\mathscr{P}(\overline{\phi}(v_q^*))}$. The following conditions are again easy to verify: (i) $p^{\text{III}}(\cdot)$ is convex and $p^{\text{III}}(\cdot) \ge \widetilde{W}(\cdot)$; (ii) $F \succeq_{MPS} \underline{H}^{\text{III}}$ and $\int_{-1}^{1} p^{\text{III}}(\theta)d\underline{H}^{\text{III}}(\theta) = \int_{-1}^{1} p^{\text{III}}(\theta)dF(\theta)$; and (iii)

$$\text{supp}(\underline{H}^{\text{III}}) = \left\{\mathbb{E}_F\left[k|k>\overline{\phi}(v_q^*)\right], v_q^*\right\} \subset \left\{\theta \middle| p^{\text{III}}(\theta) = \widetilde{W}(\theta)\right\} = \{v_q^*\} \cup \left[\overline{\phi}(v_q^*), 1\right]$$

Following analogous arguments as in previous cases, we can establish that $\underline{H}^{\text{III}}$ solves problem (D.2) and the value of (D.2) equals

$$\begin{aligned}\int_{-1}^{1} p^{\text{III}}(k)dF(k) &= F\left(\overline{\phi}(v_q^*)\right)\frac{\overline{\phi}(v_q^*)-\phi^*}{\overline{\phi}(v_q^*)-v_q^*}\left(\mathbb{E}\left[k|k<\overline{\phi}(v_q^*)\right]-v_q^*\right) + \int_{\overline{\phi}(v_q^*)}^{1}(k-\phi^*)dF(k)\\ &= \int_{\overline{\phi}(v_q^*)}^{1}(k-\phi^*)dF(\theta)\end{aligned} \tag{D.8}$$

Taken together, (D.6) to (D.8) imply (D.1). □

Lemma D.2. *Suppose $v_q^*, \phi^* \in [-1,1]$ and let $\mathscr{P}^*$ denote the set of censorship policies that generate asymptotically optimal payoff W^* given by (D.1). Then $\mathscr{P}^*$ is characterized as follows.*

1. *If $\phi^* = v_q^*$ then $\mathscr{P}^* = \{\mathscr{P}(a,b):\ -1 \le a \le \phi^* \le b \le 1\}$.*
2. *If $\phi^* < v_q^*$ the $\mathscr{P}^* = \left\{\mathscr{P}(a,b):\ -1 \le a \le b = \min\{\phi^*, \overline{\phi}(v_q^*)\}\right\}$.*
3. *If $\phi^* > v_q^*$ the $\mathscr{P}^* = \left\{\mathscr{P}(a,b):\ \max\{\phi^*, \underline{\phi}(v_q^*)\} = a \le b \le 1\right\}$.*

Proof of Lemma D.2. Observe that (D.1) can be rewritten as $W^* = \int_{t^*}^1 (k-\phi^*)dF(k)$ where $t^* = \text{median}\left\{\overline{\phi}(v_q^*), \phi^*, \underline{\phi}(v_q^*)\right\}$. We distinguish between three cases:

Case 1. $\overline{\phi}(v_q^*) \geq \phi^* \geq \underline{\phi}(v_q^*)$ and therefore $W^* = \int_{\phi^*}^1 (k-\phi^*)dF(k)$. If $v_q^* = \phi^*$, then all $\mathscr{P}(a,b)$ with $a \leq \phi^* \leq b$ yield the same optimal asymptotic payoff W^*. If $\phi^* < v_q^*$, then $a \leq b = \phi^*$ is necessary. If $\phi^* > v_q^*$ then $\phi^* = a \leq b$ is necessary.

Case 2: $\phi^* > \overline{\phi}(v_q^*)$ and therefore $W^* = \int_{\overline{\phi}(v_q^*)}^1 (k-\phi^*)dF(k)$. In this case it must hold that $\phi^* > v_q^*$. A censorship policy $\mathscr{P}(a,b)$ can implement the same outcome if and only if $\overline{\phi}(v_q^*) = a \leq b$.

Case 3: $\phi^* < \underline{\phi}(v_q^*)$ and therefore $W^* = \int_{\underline{\phi}(v_q^*)}^1 (k-\phi^*)dF(k)$. In this case there must be $\phi^* < v_q^*$. A censorship policy $\mathscr{P}(a,b)$ can implement the same outcome if and only if $a \geq b = \underline{\phi}(v_q^*)$.

These together complete the proof of Lemma D.2. □

Now we prove Theorem 2.

Proof of Theorem 2. The statements in Theorem 2 about W^* follow directly from Lemma D.1. It remains to characterize the limits of a_n and b_n as $n \to \infty$. Consider any pair of sequences $\{b_n\}_{n \geq N}$ and $\{a_n\}_{n \geq N}$ of optimal thresholds. Because both sequences are bounded on a closed interval, by the Bolzano–Weierstrass Theorem they must contain at least one convergent subsequence each. Let b^* and a^* denote the limits of these convergent subsequences. In what follows we shall explicitly characterize b^* and a^*, and then show that all sub-sequences of $\{a_n\}_{n \geq N}$ and $\{b_n\}_{n \geq N}$ converge to them so that $\{a_n\}_{n \geq N}$ and $\{b_n\}_{n \geq N}$ indeed converge.[9]

On the one hand, asymptotic optimality requires that $\mathscr{P}(a^*,b^*) \in \mathscr{P}^*$ must hold, where $\mathscr{P}^*$ is characterized in Lemma D.2. On the other hand, by Lemma 5, the single-crossing property implies for sufficiently large n that there exists $z_n \in [-1,1]$ and $-1 \leq \ell_n \leq z_n \leq r_n \leq 1$ such that the following conditions must hold:

$$\ell_n \leq a_n \leq z_n \leq b_n \leq r_n$$

and $z_n \to z^*$, $\ell_n \to \min\left\{z^*, v_q^*\right\}$ and $r_n \to \max\left\{z^*, v_q^*\right\}$ for $v_q^* \in (-1,1)$.[10] We now distinguish between three cases. For all these cases recall that $\phi(v_q^*) = \phi^*$ and $\phi(\cdot)$ is non-decreasing.

Case 1. $\phi^* = v_q^*$. In this case $z^* = v_q^*$ because $\phi(v_q^*) = \phi^* = v_q^*$. Therefore, both ℓ_n and r_n converge to ϕ^*. By the squeeze theorem both a_n and b_n must also converge to ϕ^* and thus $a^* = b^* = \phi^*$.

Case 2. $\phi^* < v_q^*$. In this case $z^* \leq \phi^* < v_q^*$ so that $\ell_n \to z^*$ and hence $a^* = z^* < v_q^*$. Moreover, by part (2) of Lemma D.2, $b^* = \phi^*$if $\phi^* \in [\underline{\phi}(v_q^*), v_q^*)$ and $b^* = \underline{\phi}(v_q^*)$ if $\phi^* < \underline{\phi}(v_q^*)$.

[9] Here we explore the following observation: let $\{x_n\}$ be a sequence on a bounded closed interval and suppose all its convergent subsequences have the same limit x^*, then x_n converges to x^*. To see this, suppose instead that x_n does not converge to x^*. Then there exists some $\varepsilon > 0$ and a subsequence $\left\{x_{n_j}\right\}$ indexed by $j = 1,2,\cdots$ such that $\left|x_{n_j} - x^*\right| > \varepsilon$ holds for all n_j. Since x_{n_j} is bounded in a closed interval, by Bolzano-Weierstrass Theorem it must contain a convergent subsequence. Yet this subsequence does not converge to x^*, leading to a contradiction.

[10] The limiting results for ℓ_n and r_n follow from (C.10) and (C.11) in Appendix C.2.

Case 3. $\phi^* > v_q^*$. In this case $z^* \geq \phi^* > v_q^*$ so that $r_n \to z^*$ and hence $b^* = z^* > v_q^*$. Moreover, by part (3) of Lemma D.2 we have $a^* = \phi^*$if $\phi^* \in (v_q^*, \overline{\phi}(v_q^*)]$ and $a^* = \overline{\phi}(v_q^*)$ if $\phi^* > \overline{\phi}(v_q^*)$.

These complete the characterizations of $a^*$and b^* and these apply for any convergent subsequences of a_n and b_n. Therefore, the limits of $\{a_n\}$ and $\{b_n\}$ exist and are equal to a^* and b^*, respectively. □

D.2 Proofs of Corollaries 1 to 3 in Section 6.1

Proof of Corollary 1. It is immediate from (15). □

Proof of Corollary 2. The proof follows from (15) and the discussion after this corollary in the main text. □

Proof of Corollary 3. It is straightforward from the definitions of $\overline{W}$ and W^{Full} that

$$\overline{W} = \int_{\phi^*}^{1} (k - \phi^*) f(k) dk \tag{D.9}$$

$$W^{\text{Full}} = \int_{v_q^*}^{1} (k - \phi^*) f(k) dk \tag{D.10}$$

This is because, as $n \to \infty$, in the omniscient scenario the designer chooses reform (status quo) if $k > (<)\phi^*$ (cf. Lemma 2), while under full information the pivotal voter chooses reform (status quo) if $k > (<)v_q^*$ (cf. Lemma 1).

Let $\gamma(x) := \int_x^1 (k - \phi^*) dF(k)$ for $x \in [-1, 1]$. By (D.9) and (D.10) we have $\overline{W} = \gamma(\phi^*)$ and $W^{\text{Full}} = \gamma(v_q^*)$. Note that $\gamma'(x) = (\phi^* - x) f(x) > (<)0$ for $x < (>)\phi^*$. This suggests that $\gamma(x)$ is strictly increasing on $[-1, \phi^*]$ and strictly decreasing on $[\phi^*, 1]$. Therefore, $\overline{W} \geq W^{\text{Full}}$ and equality holds if and only if $\phi^* = v_q^*$ whenever $v_q^* \in (-1, 1)$. Moreover, by Lemma D.1, $W^* = \gamma(\phi^*) = \overline{W}$ if $\underline{\phi}(v_q^*) \leq \phi^* \leq \overline{\phi}(v_q^*)$. These together establish statements (1) and (2). To show statement (3), consider $\phi^* > \overline{\phi}(v_q^*)$ first. In this case, it follows from Lemma D.1 that $W^* = \gamma(\overline{\phi}(v_q^*))$ with $\overline{\phi}(v_q^*) \in (\phi^*, v_q^*)$. Since $\gamma(x)$ is strictly decreasing for $x \geq \phi^*$, it holds that $\gamma(\phi^*) > \gamma(\overline{\phi}(v_q^*)) > \gamma(v_q^*)$, or equivalently $\overline{W} > W^* > W^{\text{Full}}$. The proof for the case $\phi^* < \underline{\phi}(v_q^*)$ is analogous. □

E Omitted Proofs for Section 6.2: Comparative Statics

In this appendix we prove the comparative static results presented in Section 6.2. We combine two different approaches to establish these results.

E.1 First-order approach and proofs of Propositions 1 and 2

In this subsection we use the first-order approach to prove Propositions 1 and 2. We only prove the statements concerning threshold b_n; the proofs for claims concerning a_n are similar and thus omitted.

Recall from (FOC: b_n) that the optimality condition for b_n is given by

$$\left(\tilde{b}_n - b_n\right) W_n'(\tilde{b}_n) \leq W_n(\tilde{b}_n) - W_n(b_n) \tag{E.1}$$

where $\tilde{b}_n = \mathbb{E}_F[k|k \geq b_n]$ and this condition is binding whenever $b_n \in (-1,1)$. Recall from (9) that $W_n(\theta) = \int_{\underline{v}}^{\theta} (\theta - \phi_n(x))\, \hat{g}_n(x;q)dx$ and its derivative is given by

$$W_n{}'(\theta) = \hat{G}_n(\theta;q) + (\theta - \phi_n(\theta))\, \hat{g}_n(\theta;q)$$

With these we have

$$\begin{aligned}
W_n(\tilde{b}_n) - W_n(b_n) &= \int_{\underline{v}}^{\tilde{b}_n} \left(\tilde{b}_n - \phi_n(x)\right) \hat{g}_n(x;q)dx - \int_{\underline{v}}^{b_n} (b_n - \phi_n(x))\, \hat{g}_n(x;q)dx \\
&= \int_{b_n}^{\tilde{b}_n} \left(\tilde{b}_n - \phi_n(x)\right) \hat{g}_n(x;q)dx + (\tilde{b}_n - b_n) \int_{\underline{v}}^{b_n} \hat{g}_n(x;q)dx \\
\left(\tilde{b}_n - b_n\right) W_n'(\tilde{b}_n) &= \left(\tilde{b}_n - b_n\right) \left[\left(\tilde{b}_n - \phi_n^m(\tilde{b}_n)\right) \hat{g}_n(\tilde{b}_n;q) + \int_{\underline{v}}^{\tilde{b}_n} \hat{g}_n(x;q)dx \right]
\end{aligned}$$

Plugging these into (E.1), we obtain that (E.1) is equivalent to

$$\tilde{b}_n - b_n \leq \int_{b_n}^{\tilde{b}_n} \frac{b_n - \phi_n(x)}{\tilde{b}_n - \phi_n(\tilde{b}_n)} \frac{\hat{g}_n(x;q)}{\hat{g}_n(\tilde{b}_n;q)} dx \tag{E.2}$$

By Lemma 5, for sufficiently large n it holds that $W_n(\cdot)$ is strictly S-shaped on $[z_n, 1]$ with some inflection point $r_n \in [z_n, 1]$. This implies that

$$\left(\mathbb{E}_F\left[k|k \geq x\right] - x\right) W_n'\left(\mathbb{E}_F\left[k|k \geq x\right]\right) - \left[W_n\left(\mathbb{E}_F\left[k|k \geq x\right]\right) - W_n\left(x\right)\right]$$

can cross zero at most once and from above as x increases from z_n to 1. In particular, suppose b_n satisfies (E.2) with equality and hold it fixed, then if any parameter change increases the value of the right-hand side of (E.2), then $\left(\tilde{b}_n - b_n\right) W_n'(\tilde{b}_n) - \left[W_n(\tilde{b}_n) - W_n(b_n)\right]$ will be negative following this parameter change. b_n must therefore decrease to regain equality. Comparative static analyses thus can done with the right-hand side of (E.2) alone. With this we can prove Propositions 1 and 2.

Proof of Proposition 1. Let $\gamma_n(x) := \sum_{j=1}^{n+1} w_j \varphi_j(x;q,n)$. If $\rho < 1$, it follows from (3) that

$$\phi_n(\cdot) = \rho\gamma_n(x) + (1-\rho)\chi$$

As is explained in the proof of Lemma 3 in Appendix B, under either condition (i) or (ii) of Lemma 3 it holds that $1-\phi'(x) > 0$ for all $x \in [\underline{v}, \overline{v}]$. Since $\phi_n(\cdot)$ converges uniformly to $\phi'(\cdot)$ (cf. Lemma 2), $1-\phi_n'(\cdot) > 0$ on $[\underline{v}, \overline{v}]$ must hold for sufficiently large n. This implies that $x - \phi_n(x)$ is strictly increasing. Moreover, because $\tilde{b}_n = \mathbb{E}_F[k|k \geq b_n] > b_n$ and $\phi_n(x)$ is non-decreasing (cf. Proposition B.2), for all $x \in (b_n, \tilde{b}_n)$ we have

$$1 > \frac{b_n - \phi_n(b_n)}{\tilde{b}_n - \phi_n(\tilde{b}_n)} \geq \frac{b_n - \phi_n(x)}{\tilde{b}_n - \phi_n(\tilde{b}_n)} = \frac{b_n - \rho\gamma_n(x) - (1-\rho)\chi}{\tilde{b}_n - \rho\gamma_n(\tilde{b}_n) - (1-\rho)\chi}$$

Consider any $\chi_{\mathrm{I}} > \chi_{\mathrm{II}}$. Observe that a decrease of χ from χ_{I} to χ_{II} induces a common increase on both the nominator and the denominator of $\frac{b_n - \phi_n(x)}{\tilde{b}_n - \phi_n(\tilde{b}_n)}$, which is smaller than one for all $x \in \left(b_n, \tilde{b}_n\right)$. This shift of χ therefore strictly increases the value of $\frac{b_n - \phi_n(x)}{\tilde{b}_n - \phi_n(\tilde{b}_n)}$ for all $x \in \left(b_n, \tilde{b}_n\right)$.[11] On the other hand, the term $\frac{\hat{g}_n(x;q)}{\hat{g}_n(\tilde{b}_n;q)}$ is independent of χ for all $x \in \left(b_n, \tilde{b}_n\right)$. These together implies that a shift of χ from χ_{I} to χ_{II} strictly increases the right-hand side of (E.2). Therefore, if $b_n \in (-1,1)$ under χ_{I} so that (E.2) is binding, such a shift of χ will make the right-hand side of (E.2) strictly higher than the left-hand side. b_n must strictly decrease to make (E.2) binding again or drop to -1. If $b_n = -1$ under χ_{I} so that (E.2) holds with '$\leq$', then this must remain to be the case after the shift of χ so that the optimal b_n remains to be -1. These together show that b_n is non-increasing as χ decreases and thus prove the claim for b_n in Proposition 1. □

Next we prove Proposition 2. To do so we need to introduce an auxiliary result.

Lemma E.1. *For any $0 < y < z < 1$, $\int_y^z \frac{\tau_n(x;q)}{\tau_n(z;q)} dx$ is strictly decreasing in q, where $\tau_n(x;q)$ is defined by (A.1) in Appendix A.*[12]

Proof of Lemma E.1. For any pair of $(x,y) \in (0,1)^2$ and $q \in (0,1)$, define

$$\Delta\psi(x,y;q) := q\ln\frac{x}{y} + (1-q)\ln\frac{1-x}{1-y} = \ln\frac{1-x}{1-y} + q\left(\ln\frac{x}{1-x} - \ln\frac{y}{1-y}\right) \tag{E.3}$$

It then follows from the definition of $\tau_n(\cdot;q)$ that

$$\ln\frac{\tau_n(x;q)}{\tau_n(y;q)} = n\left(q\ln\frac{x}{y} + (1-q)\ln\frac{1-x}{1-y}\right) = n\Delta\psi(x,y;q)$$

[11] This follows from the fact that $\frac{a+c}{b+c} > \frac{a}{b}$ for all $b,c > 0$ and $b > a$.

[12] Here we implicitly assume nq is an integer for ease of exposure. If this is not the case, then just replace q with $\hat{q} = \lfloor nq \rfloor / n$ and the all arguments hold for $\hat{q}$.

We can thus rewrite $\int_y^z \frac{\tau_n(x;q)}{\tau_n(z;q)}dx$ as

$$\int_y^z \frac{\tau_n(x;q)}{\tau_n(z;q)}dx = \int_y^z e^{n\Delta\psi(x,z;q)}dx$$

Using (E.3) and the fact that $\ln\frac{x}{1-x}$ is strictly increasing in x, we obtain for all $y<z$ that

$$\frac{\partial}{\partial q}\int_y^z \frac{\tau_n(x;q)}{\tau_n(z;q)}dx = n\int_y^z e^{n\Delta\psi(x,z;q)}\left(\ln\frac{x}{1-x}-\ln\frac{z}{1-z}\right)dx < 0$$

This implies the strict decreasing property stated in this lemma. □

Proof of Proposition 2. Recall from (A.5) that $\hat{g}_n(x;q) = \tau_n(G(x);q)\,g(x)$ for all $x\in[\underline{v},\overline{v}]$. Plugging this to (E.2), we obtain

$$\begin{aligned}\int_{b_n}^{\tilde{b}_n} \frac{b_n-\phi_n(x)}{\tilde{b}_n-\phi_n(\tilde{b}_n)}\frac{\hat{g}_n(x;q)}{\hat{g}_n(\tilde{b}_n;q)}dx &= \int_{b_n}^{\tilde{b}_n} \frac{b_n-\phi_n(x)}{\tilde{b}_n-\phi_n(\tilde{b}_n)}\frac{\tau_n(G(x);q)\,g(x)}{\tau_n(G(\tilde{b}_n);q)\,g(\tilde{b}_n)}dx \\ &= \frac{1}{g(\tilde{b}_n)}\int_{G(b_n)}^{G(\tilde{b}_n)} \frac{b_n-\phi_n\left(G^{-1}(y)\right)}{\tilde{b}_n-\phi_n(\tilde{b}_n)}\frac{\tau_n(y;q)}{\tau_n\left(G(\tilde{b}_n);q\right)}dy\end{aligned} \tag{E.4}$$

For $\rho=0$ we have $\phi_n(x)=\chi$ for all $x\in[\underline{v},\overline{v}]$ and therefore $W_n(\theta)=(\theta-\chi)\hat{G}_n(\theta;q)$ (cf. (3) and (9)). Plugging $W_n(\theta)=(\theta-\chi)\hat{G}_n(\theta;q)$ into (E.4), we obtain

$$\int_{b_n}^{\tilde{b}_n} \frac{b_n-\phi_n(x)}{\tilde{b}_n-\phi_n(\tilde{b}_n)}\frac{\hat{g}_n(x;q)}{\hat{g}_n(\tilde{b}_n;q)}dx = \frac{1}{g(\tilde{b}_n)}\left(\frac{b_n-\chi}{\tilde{b}_n-\chi}\right)\int_{G(b_n)}^{G(\tilde{b}_n)}\frac{\tau_n(x)}{\tau_n(G(\tilde{b}_n)}dx \tag{E.5}$$

Because $\tilde{b}_n > b_n > \chi$ and $G(\tilde{b}_n) > G(b_n)$, Lemma E.1 implies that (E.5) is strictly decreasing in q. Therefore, the right-hand side of (E.2) strictly increases as q rises from q_{I} to q_{II} for all $q_{\mathrm{I}} < q_{\mathrm{II}}$. If $b_n\in(-1,1)$ under q_{I} so that (E.2) is binding, then such a shift of q will make (E.2) hold with '$>$' so that b_n must strictly increase to regain equality or up to 1. If $b_n=-1$ under q_{I}, then (E.2) holds with '$\leq$'. The shift of q either (i) retains (E.2) with '$\leq$' so that the optimal b_n is still -1, or it shifts '$\leq$' to '$>$' so that the optimal $b_n>-1$. These together show that b_n is non-decreasing as q increases and thus prove the claim for b_n in Proposition 2. □

E.2 Limiting approach for comparative statics

In this subsection we use the limiting approach to prove Proposition 3 and establish Proposition E.1 below, which is an analog of Proposition 1 for the welfare weighting function $w(\cdot)$ when $\rho>0$.

Suppose the single-crossing property holds and let $a^* := \lim_{n\to\infty} a_n$ and $b^* := \lim_{n\to\infty} b_n$ be the limits of optimal thresholds characterized in Theorem 2. Our comparative statics primarily concern how

a^* and b^* vary with voting rule q and a pro-social designer's welfare weighting function $w(\cdot)$. By Theorem 2, this boils down to understanding how these factors affect ϕ^*, v_q^* and z^*. These are already summarized in Lemma B.5 in Appendix B. With these we are ready to prove Proposition 3 and establish Proposition E.1.

Proof of Proposition 3. We prove Proposition 3 by construction. For any pair of q_{I} and q_{II}, we use a_i^* and b_i^* to denote the thresholds of the asymptotically optimal censorship policy under $q = q_i$ for $i \in \{\mathrm{I}, \mathrm{II}\}$. We assume $\phi^* \in (-1,1)$ and let $\widehat{q} := G(\phi^*)$; under $q = \widehat{q}$ we have $\phi^* = G^{-1}(q) = v_q^*$.

First, suppose q_{I} and q_{II} satisfy (i) $\widehat{q} \leq q_{\mathrm{I}} < q_{\mathrm{II}}$, (ii) $\phi^* \in \left[\underline{\phi}(v_{q_{\mathrm{I}}}^*), \overline{\phi}(v_{q_{\mathrm{I}}}^*)\right]$, and (iii) $\phi^* < \underline{\phi}(v_{q_{\mathrm{II}}}^*)$. Then, by Theorem 2, we have $(a_{\mathrm{I}}^*, b_{\mathrm{I}}^*) = (z_{\mathrm{I}}^*, \phi^*)$ and $(a_{\mathrm{II}}^*, b_{\mathrm{II}}^*) = \left(z_{\mathrm{II}}^*, \underline{\phi}(v_{q_{\mathrm{II}}}^*)\right)$. Since $\underline{\phi}(v_{q_{\mathrm{II}}}^*) > \phi^*$ and $z_{\mathrm{II}}^* < z_{\mathrm{I}}^*$ (cf. part (3) of Lemma B.5), we get $a_{\mathrm{II}}^* < a_{\mathrm{I}}^* \leq b_{\mathrm{I}}^* < b_{\mathrm{II}}^*$. This implies for sufficiently large n that a_n decreases and b_n increases as q shifts from q_{I} to q_{II}. This shows case 1 of Proposition 3 is possible.

To show that case 2 of Proposition 3 is also possible, consider any q_{I} and q_{II} that satisfy (i) $q_{\mathrm{I}} < q_{\mathrm{II}} \leq \widehat{q}$, (ii) $\phi^* > \overline{\phi}(v_{q_{\mathrm{I}}}^*)$, and (iii) $\phi^* \in \left[\underline{\phi}(v_{q_{\mathrm{II}}}^*), \overline{\phi}(v_{q_{\mathrm{II}}}^*)\right]$. By Theorem 2, $(a_{\mathrm{I}}^*, b_{\mathrm{I}}^*) = \left(\overline{\phi}(v_{q_{\mathrm{I}}}^*), z_{\mathrm{I}}^*\right)$ and $(a_{\mathrm{II}}^*, b_{\mathrm{II}}^*) = (\phi^*, z_{\mathrm{II}}^*)$. In this case we have $a_{\mathrm{I}}^* < a_{\mathrm{II}}^* \leq b_{\mathrm{II}}^* < b_{\mathrm{I}}^*$. So, for sufficiently large n, a_n increases while b_n decreases as q varies from q_{I} to q_{II}.

To show that case 3 of Proposition 3 is also possible, consider any q_{I} and q_{II} that satisfy (i) $q_{\mathrm{I}} < \widehat{q} < q_{\mathrm{II}}$, (ii) $\phi^* \in \left[\underline{\phi}(v_{q_{\mathrm{I}}}^*), \overline{\phi}(v_{q_{\mathrm{I}}}^*)\right]$, and (iii) $\phi^* \in \left[\underline{\phi}(v_{q_{\mathrm{II}}}^*), \overline{\phi}(v_{q_{\mathrm{II}}}^*)\right]$. By Theorem 2, $(a_{\mathrm{I}}^*, b_{\mathrm{I}}^*) = (\phi^*, z_{\mathrm{I}}^*)$ and $(a_{\mathrm{II}}^*, b_{\mathrm{II}}^*) = (z_{\mathrm{II}}^*, \phi^*)$. In this case we have $a_{\mathrm{I}}^* > a_{\mathrm{II}}^*$ and $b_{\mathrm{I}}^* > b_{\mathrm{II}}^*$. So, for sufficiently large n, both a_n and b_n decrease as q varies from q_{I} to q_{II}.

Finally, to show that case 4 of Proposition 3 is also possible, consider any q_{I} and q_{II} that satisfy (i) $q_{\mathrm{I}} < \widehat{q} < q_{\mathrm{II}}$, (ii) $\overline{\phi}(v_{q_{\mathrm{I}}}^*) < \phi^* < \underline{\phi}(v_{q_{\mathrm{II}}}^*)$. Then, by Theorem 2, we have $(a_{\mathrm{I}}^*, b_{\mathrm{I}}^*) = \left(\overline{\phi}(v_{q_{\mathrm{I}}}^*), z_{\mathrm{I}}^*\right)$ and $(a_{\mathrm{II}}^*, b_{\mathrm{II}}^*) = \left(z_{\mathrm{II}}^*, \underline{\phi}(v_{q_{\mathrm{II}}}^*)\right)$. By Lemma B.5, $z_{q_I}^* < \phi^* < z_{q_{II}}^*$ must hold for all $\rho > 0$. If, however, $\rho \to 0$, then $\phi(\cdot)$ must be close to a flat line so that both $z_{q_I}^*$ and $z_{q_{II}}^*$ shall be arbitrarily close to ϕ^*. This implies that $\overline{\phi}\left(v_{q_I}^*\right) < z_{q_{II}}^*$ and $z_{q_I}^* < \underline{\phi}\left(v_{q_{II}}^*\right)$ must hold for ρ sufficiently close to 0. In such case we indeed have $a_I^* < a_{II}^*$ and $b_I^* < b_{II}^*$. □

Proposition E.1. *Suppose $\rho > 0$ and $v_q^* \in (-1,1)$. Let $w^I(\cdot)$ and $w^{II}(\cdot)$ be two absolutely continuous cdfs on $[-1,1]$ that satisfy (i) $w^I(\cdot) \succeq_{FOSD} w^{II}(\cdot)$, and (ii) for both $w^I(\cdot)$ and $w^{II}(\cdot)$ there are $\phi^* \in (-1,1)$ and $\phi^* \in \left(\underline{\phi}(v_q^*), \overline{\phi}(v_q^*)\right)$. Then, for sufficiently large n, both a_n and b_n decrease as $w(\cdot)$ shifts from $w^I(\cdot)$ to $w^{II}(\cdot)$.*

Proof of Proposition E.1. We use a_i^* and b_i^* to denote the thresholds of the asymptotically optimal censorship policy under $w(\cdot) = w^i(\cdot)$ for $i \in \{\mathrm{I}, \mathrm{II}\}$. For ease of exposure we focus on the case where $\phi^* \in (-1,1)$ and $\phi^* \in \left(\underline{\phi}(v_q^*), \overline{\phi}(v_q^*)\right)$ hold for both $w^{\mathrm{I}}(\cdot)$ and $w^{\mathrm{II}}(\cdot)$.[13] In this case, Theorem

[13] The proof is almost identical for the case where only one weighting function satisfies this condition.

2 implies $a_i^* = \min\{z_i^*, \phi_i^*\}$ and $b_i^* = \max\{z_i^*, \phi_i^*\}$ for both $i \in \{\mathrm{I}, \mathrm{II}\}$. By Lemma B.5, we have $\phi_{\mathrm{I}}^* > \phi_{\mathrm{II}}$ and $z_{\mathrm{I}}^* > z_{\mathrm{II}}^*$ (equality holds only if both values are -1). These together imply (i) $b_{\mathrm{I}}^* > b_{\mathrm{II}}^*$ and (ii) $a_{\mathrm{I}}^* \geq a_{\mathrm{II}}^*$ (equality holds only if both values are -1). Hence, for sufficiently large n, both a_n and b_n decrease as q shifts from $w^{\mathrm{I}}(\cdot)$ to $w^{\mathrm{II}}(\cdot)$. □

F Omitted Equilibrium Derivations and Proofs for Section 7

In this Appendix we solve for the equilibria of competitive persuasion model in Section 7 and prove Theorems 3 and 4.

Equilibrium. When there are $|M| \geq 2$ designers, let $\pi = \langle \pi_m \rangle_{m \in M}$ be any joint information policy induced by all designers and let H_π denote the distribution of the posterior means induced by π. We say that H_π is *unimprovable for designer $m \in M$* if he has no incentive to reveal more information. For $m \in M$, let $\mathscr{H}_m$ denote the set of all unimprovable distributions. The set of distributions H that are *unimprovable for all designers* is then $\mathscr{H} = \cap_{m \in M} \mathscr{H}_m$. By Proposition 2 of Gentzkow and Kamenica (2017*b*), π can be sustained in equilibrium if and only if $H_\pi \in \mathscr{H}$.

To further solve for the unimprovable H's, we introduce some useful observations about properties of solutions to a general linear persuasion problem of the following kind:

$$\max_{H \in \Delta([\underline{\kappa}, \overline{\kappa}])} \int_{\underline{\kappa}}^{\overline{\kappa}} U(\theta) dF(\theta), \quad \text{s.t. } F\big|_{[\underline{\kappa}, \overline{\kappa}]} \succeq_{MPS} H \tag{MP'}$$

where $U(\cdot)$ is a designer's utility function defined on some closed interval $[\underline{\kappa}, \overline{\kappa}] \subseteq [-1, 1]$ and $F\big|_{[\underline{\kappa}, \overline{\kappa}]}$ is the cdf of prior F truncated on interval $[\underline{\kappa}, \overline{\kappa}]$.[14] We assume that $U(\cdot)$ is twice continuously differentiable and can be partitioned into finitely many intervals on which $U(\cdot)$ is either strictly concave, strictly convex, or affine.

The first observation, due to Theorem 4 of Dworczak and Martini (2019), provides a convenient way to verify whether an induced distribution of posterior means H is unimprovable for a designer with utility function $U(\cdot)$ by solving a monopolistic persuasion problem with his utility function modified by its convex translations.

Observation F.3. *(Theorem 4 of Dworczak and Martini (2019)) H is unimprovable for a designer with utility function $U(\cdot)$ if H is a solution to (MP') with objective function $U(\cdot)$ replaced by $\widehat{U}(\cdot) = U(\cdot) + \omega(\cdot)$, where $\omega(\cdot)$ is some convex function.*

Using Observation F.3, we establish the following Lemma F.1, which implies that any unimprovable H must induce cutoff partitions at z_n^m for all $m \in M$. As a result, any $H \in \mathscr{H}$ must be unimprovable separately on each segment of interval $[-1, 1]$ partitioned by z_n^m's.

[14] That is, $F\big|_{[\underline{\kappa}, \overline{\kappa}]}(k) = \frac{F(k) - F(\underline{\kappa})}{F(\overline{\kappa}) - F(\underline{\kappa})}$ for $k \in [\underline{\kappa}, \overline{\kappa}]$ and it equals 1 (resp. 0) for $k > \overline{\kappa}$ (resp. $k < \underline{\kappa}$).

Lemma F.1. *Suppose that the single-crossing property holds for a designer $m \in M$ and $\phi_n^m(x) - x$ crosses zero only once and from above at $z_n^m \in (-1,1)$. Then $H \succeq_{MPS} H_{\mathscr{P}(z_n^m)}$ must hold for any H that is unimprovable for m.*

Proof of Lemma F.1. By Observation F.3, H is unimprovable for designer m if and only if H is a solution to (MP) with utility function $W_n^m(\cdot)$ replaced by $\widehat{W}_n^m(\cdot) = W_n^m(\cdot) + \omega_m(\cdot)$ for some convex $\omega_m(\cdot)$. By Lemma 4, $W_n^m(\cdot)$ satisfies the increasing slope property at z_n^m. We show that $\widehat{W}_n^m(\cdot)$ must also satisfy this property and hence $H \succeq_{MPS} H_{\mathscr{P}(z_n^m)}$ must hold following Lemma 4. To see why, observe that

$$\frac{\widehat{W}_n^m(x) - \widehat{W}_n^m(z_n^m)}{x - z_n^m} = \frac{W_n^m(x) - W_n^m(z_n^m)}{x - z_n^m} + \frac{\omega_m(x) - \omega_m(z_n^m)}{x - z_n^m}$$

and the latter term is non-decreasing in x because $\omega_m(\cdot)$ is convex. $\widehat{W}_n^m(x)$ thus satisfies the increasing slope property whenever $W_n^m(\cdot)$ does. □

The next two observations establish, for strictly S-shaped and inverse-S-shaped utility functions, the unimprovability and best-response properties of censorship policies under competitive persuasion.

Observation F.4. *Let $U(\cdot)$ be designer m's utility function and suppose that $U(\cdot)$ is strictly S-shaped on $[\underline{\kappa}, \overline{\kappa}]$ with inflection point r. The following properties hold:*

1. *(Kolotilin, Mylovanov and Zapechelnyuk, 2022) The unique solution H to problem (MP')is induced by an upper censorship policy $\mathscr{P}(\underline{\kappa}, b)$ with $b \geq \underline{\kappa}$.*

2. *(Sun, 2022a) Let $\mathscr{H}_m$ denote the set of unimprovable outcomes for designer m, then (i) $H \succeq_{MPS} H_{\mathscr{P}(\underline{\kappa},b)}$ for all $H \in \mathscr{H}_m$, and (ii) $H_{\mathscr{P}(\underline{\kappa},d)} \in \mathscr{H}_m$ for all $d \in [b, \overline{\kappa}]$.*

3. *(Sun, 2022a) Given any pure strategy profile of other designers π_{-m}, there exists some $d \in [b, r]$ such that the upper censorship policy $\mathscr{P}(\underline{\kappa}, d)$ is designer m's best response to π_{-m}.*

Observation F.5. *Let $U(\cdot)$ be designer m's utility function and suppose that $U(\cdot)$ is strictly inverse S-shaped on $[\underline{\kappa}, \overline{\kappa}]$ with inflection point ℓ. The following properties hold:*

1. *Kolotilin, Mylovanov and Zapechelnyuk (2022) The unique solution H to problem (MP') is induced by an lower censorship policy $\mathscr{P}(a, \overline{\kappa})$ with $a \leq \overline{\kappa}$.*

2. *(Sun, 2022a) Let $\mathscr{H}_m$ denote the set of unimprovable outcomes for designer m, then (i) $H \succeq_{MPS} H_{\mathscr{P}(a,\overline{\kappa})}$ for all $H \in \mathscr{H}_m$, and (ii) $H_{\mathscr{P}(c,\overline{\kappa})} \in \mathscr{H}_m$ for all $c \in [\underline{\kappa}, a]$.*

3. *(Sun, 2022a) Given any pure strategy profile of other designers π_{-m}, there exists some $c \in [a, \ell]$ such that the lower censorship policy $\mathscr{P}(c, \overline{\kappa})$ is designer m's best response to π_{-m}.*

The first statements of the two observations above suggest that under monopolistic persuasion *upper* (resp. *lower*) censorship policies are uniquely optimal for a designer whose utility function $U(\cdot)$ is strictly S-shaped (resp. inverse S-shaped). These are proved by Kolotilin, Mylovanov and Zapechelnyuk (2022). The remaining statements of these observations, established by Sun (2022*a*), extend this insight to competition in persuasion. For a designer whose utility function is either strictly S-shaped or inverse S-shaped, any censorship policy that is no less informative than the monopolistic optimal one is unimprovable for him. Moreover, it is without loss of optimality for him to restrict attention to a proper subset of censorship policies in the following sense: given any pure strategy profile of other designers, he can always find a best response from this subset of censorship policies. Notice that all information policies in this subset are no less informative than his monopolistically optimal one.

Finally, we introduce a general and easy-to-check sufficient condition for full disclosure to be the unique equilibrium outcome under competition. For each designer $m \in M$ let $U_m(\cdot)$ denote his utility function. Our last observation states an easily verifiable sufficient condition for full disclosure to be the unique equilibrium outcome. Given $\{U_m(\cdot)\}_{m\in M}$, we say that *strictly convex finite open cover property* holds on interval $[x,y]$ if there exists a finite collection of open intervals $\{I_j\}_{j=1}^{J}$ such that (i) $[x,y] \subset \cup_{j=1}^{J} I_j$, and (ii) on each I_j there exists some $m \in M$ such that $U_m(\cdot)$ is strictly convex. Sun (2022*b*) establishes the following observation.

Observation F.6. *(Sun, 2022b) Let $\{U_m(\cdot)\}_{m\in M}$ be a profile of utility functions defined on $[\underline{\kappa},\overline{\kappa}]$. If strictly convex finite open cover property holds on $[\underline{\kappa},\overline{\kappa}]$, then full disclosure is the unique equilibrium outcome.*

With these ingredients we are now ready to prove Theorems 3 and 4.

F.1 Proof of Theorem 3

For all $n \geq N_m$ we define

$$\mathscr{P}_n^m := \{\mathscr{P}(c,d) : [a_n^m, b_n^m] \subseteq [c,d] \subseteq [\ell_n^m, r_n^m]\} \tag{F.1}$$

where a_n^m and b_n^m are the thresholds of the monopolistically optimal censorship policy. Clearly, $\mathscr{P}_n^m$ is a subset of censorship policies. We show below that for any pure strategy profile π_{-m} of other designers, there exists a $\pi_m \in \mathscr{P}_n^m$ such that π_m is designer m's best response to π_{-m}. Again, we distinguish between three cases depending on the value of z_n^m.

If $z_n^m = -1$, then $\ell_n^m = a_n^m = 0$ and $W_n^m(\cdot)$ is strictly S-shaped on $[-1,1]$ with inflection point $r_n^m \geq b_n^m$. By Observation F.4, for any π_{-m} there exists $d \in [b_n^m, r_n^m]$ such that $\pi_m = \mathscr{P}(-1,d)$ is designer m's best response to π_{-m}. Similarly, if $z_n^m = 1$ then $r_n^m = b_n^m = 1$ and $W_n^m(\cdot)$ is strictly inverse S-shaped on $[-1,1]$ with inflection point $\ell_n^m \leq a_n^m$. Observation F.5 implies that for any π_{-m} there exists $c \in [\ell_n^m, a_n^m]$ such that $\pi_m = \mathscr{P}(c,1)$ is designer m's best response to π_{-m}. In both cases $\pi_m \in \mathscr{P}_n^m$ holds.

Now we consider the case $z_n^m \in (-1,1)$ and let π_m be any best response to π_{-m} for designer m. Because the information environment is Blackwell-connected, the induced joint information policy $\langle \pi_m, \pi_{-m} \rangle$ must be unimprovable for designer m. Recall that $W_n^m(\cdot)$ satisfies increasing slope property at point z_n^m (cf. Lemma 4), it follows from Lemma F.1 that $\langle \pi_m, \pi_{-m} \rangle$ must be Blackwell more informative than the cutoff policy $\mathscr{P}(z_n^m)$. This implies that there always exists a best response π_m that is Blackwell more informative than $\mathscr{P}(z_n^m)$ (i.e., $H_{\pi_m} \succeq_{MPS} H_{\mathscr{P}(z_n^m)}$).[15] For such π_m, it must be a best response to π_{-m} on both $[-1, z_n^m]$ and $[z_n^m, 1]$ separately. By Lemma 5, $W_n^m(\cdot)$ is strictly inverse S-shaped on $[-1, z_n^m]$ with inflection point $\ell_n^m < z_n^m$ and strictly S-shaped on $[z_n^m, 1]$ with inflection point $r_n^m > z_n^m$. It follows that there exists $c \in [\ell_n^m, a_n^m]$ and $d \in [b_n^m, r_n^m]$ such that $\mathscr{P}(c, z_n^m)$ is a best response to π_{-m} on $[-1, z_n^m]$ and $\mathscr{P}(z_n^m, d)$ is a best response on $[z_n^m, 1]$. These together produce a censorship policy $\pi_m = \mathscr{P}(c,d)$, which belongs to $\mathscr{P}_n^m$, that is a best response to π_{-m}. This completes the proof.

F.2 Proof of Theorem 4

We start by establishing the following lemma.

Lemma F.2. *Suppose Assumption 1 holds. Then any unimprovable outcome $H \in \mathscr{H}$ must satisfy $H \succeq_{MPS} H_{\mathscr{P}(z_n^{\min}, z_n^{\max})}$, where $z_n^{\min} = \min_{m \in M}\{z_n^m\}$ and $z_n^{\max} = \max_{m \in M}\{z_n^m\}$.*

Proof. By Lemma F.1, any $H \in \mathscr{H}$ must satisfy $H \succeq_{MPS} H_{\mathscr{P}(z_n^{\min})}$, $H \succeq_{MPS} H_{\mathscr{P}(z_n^{\max})}$, and be unimprovable on $[z_n^{\min}, z_n^{\max}]$. Following Observation F.6, we only need to establish that strictly finite open cover property holds on $[z_n^{\min}, z_n^{\max}]$ to complete the proof. By (9) we have

$$W_n^{m\prime\prime}(k) = \left(2 - \phi_n^{m\prime}(k)\right)\hat{g}_n(k;q) + \left(k - \phi_n^m(k)\right)\hat{g}_n'(k;q)$$

The first term is strictly positive for all $m \in M$ because part (2) of Assumption 1 ensures that $\phi_n^{m\prime}(k) < 2$ for all $k \in [z_n^{\min}, z_n^{\max}]$. Let I (resp. II) denote the index of the designer for whom $z_n^{\mathrm{I}} = z_n^{\min}$ (resp. $z_n^{\mathrm{II}} = z_n^{\max}$). Then for all $k \in [z_n^{\min}, z_n^{\max}]$ we have

$$\phi_n^{\mathrm{I}}(k) \leq k \leq \phi_n^{\mathrm{II}}(k)$$

holds. So, no matter what the sign of $\hat{g}_n'(k;q)$ is, $\left(k - \phi_n^m(k)\right)\hat{g}_n'(k;q)$ must be non-negative for at

[15] To see why, let $\pi = \langle \pi_m, \pi_{-m} \rangle$ and $\pi' = \langle \pi_m, \pi_{-m}, \mathscr{P}(z_n^m) \rangle$. The best response property of π_m ensures that $H_\pi \succeq_{MPS} H_{\mathscr{P}(z_n^m)}$. Therefore, $H_\pi = H_{\pi'}$. Consider $\pi_m' = \langle \pi_m, \mathscr{P}(z_n^m) \rangle$ (which is always feasible) and observe that $H_{\pi_m'} \succeq_{MPS} H_{\mathscr{P}(z_n^m)}$ and $\pi' = \langle \pi_m', \pi_{-m} \rangle$. Then π_m' must also be a best response to π_{-m} because $H_\pi = H_{\pi'}$.

least one $m \in \{\mathrm{I}, \mathrm{II}\}$. Hence, for any $k \in [-1,1]$, there exists some $m \in M$ for whom $W_n^{m\prime\prime}(k) > 0$ holds. By continuity of $W_n^{m\prime\prime}(\cdot)$, $W_n^m(\cdot)$ must be strictly convex on an open interval I_k that contains k. $\{I_k\}_{k\in[-1,1]}$ is then an collection of open intervals that covers $[-1,1]$ and by Heine-Borel Theorem there exists a finite subcover. This implies strictly convex finite open cover property on $[-1,1]$. $\square$

Combining Lemma F.1, Observations F.4 to F.5, and the curvature properties of $W_n^m(\cdot)$ summarized in Lemma 5, we obtain

Lemma F.3. *Suppose the single-crossing property holds for each designer $m \in M$. Then for any $n \geq N_m$ we have (i) $H \succeq_{MPS} H_{\mathscr{P}(a_n^m, b_n^m)}$ for all $H \in \mathscr{H}_m$, and (ii) $\mathscr{P}(a,b) \in \mathscr{H}_m$ for all $a \in [-1, a_n^m]$ and $b \in [b_n^m, 1]$.*

In words, Lemma F.3 says that under the single-crossing property and sufficiently large n all unimprovable outcomes for designer m must be no less informative than his monopolistically optimal censorship policy. Moreover, all censorship policies that are more informative than the monopolistically optimal one are unimprovable for designer m.

We now use Lemmas F.2 and F.3 to establish part (1) of Theorem 4. Consider any $n \geq N$. By Lemma F.2, $H \succeq_{MPS} H_{\mathscr{P}(z_n^{\min}, z_n^{\max})}$ must hold for all $H \in \mathscr{H} = \cap_{m\in M} \mathscr{H}_m$. By Lemma F.3, $H \succeq_{MPS} H_{\mathscr{P}(a_n^m, b_n^m)}$ must hold for all $H \in \mathscr{H}_m$. Moreover, for each $m \in M$, it holds that $H_{\mathscr{P}(c,d)} \in \mathscr{H}_m$ for all $c \in [-1, a_n^m]$ and $d \in [b_n^m, 1]$. Therefore, $\mathscr{P}(a_n^{\min}, b_n^{\max})$ is unimprovable for all designers and hence $H_{\mathscr{P}(a_n^{\min}, b_n^{\max})} \in \mathscr{H}$. Next we show that any $H \in \mathscr{H}$ must be weakly more informative than $\mathscr{P}(a_n^{\min}, b_n^{\max})$, that is $H \succeq_{MPS} H_{\mathscr{P}(a_n^{\min}, b_n^{\max})}$. Let $\tilde{i}$ (resp. $\tilde{j}$) denote the identity of the designer with $a_n^{\tilde{i}} = a_n^{\min}$ (resp. $b_n^{\tilde{j}} = b_n^{\max}$). Recall that any $H \in \mathscr{H}$ must satisfy $H \succeq_{MPS} H_{\mathscr{P}(a_n^m, b_n^m)}$ with $a_n^m \leq z_n^m \leq b_n^m$ for all $m \in M$. The choices of $\tilde{i}$ and $\tilde{j}$ imply that $\left[a_n^{\tilde{i}}, b_n^{\tilde{i}}\right]$, $\left[z_n^{\min}, z_n^{\max}\right]$ and $\left[a_n^{\tilde{j}}, b_n^{\tilde{j}}\right]$ are overlapping and $\left[a_n^{\tilde{i}}, b_n^{\tilde{i}}\right] \cup \left[z_n^{\min}, z_n^{\max}\right] \cup \left[a_n^{\tilde{j}}, b_n^{\tilde{j}}\right] = \left[a_n^{\min}, b_n^{\max}\right]$. Therefore, $H \succeq_{MPS} H_{\mathscr{P}(a_n^{\min}, b_n^{\max})}$ must hold for all $H \in \mathscr{H}$ and this completes the proof for statement (1) of Theorem 4.

Next we prove statement (2) of Theorem 4. Given any censorship policy $\mathscr{P}(c,d)$ with revelation interval $[c,d] \subseteq [-1,1]$, each designer m's expected payoff under this policy is

$$\mathbb{E}_{\mathscr{P}(c,d)}\left[W_n^m(\cdot)\right] = F(c) W_n^m\left(\underline{\mu}_F(c)\right) + \int_c^d W_n^m(k)dF(k)\,(1-F(d))\,W_n^m\left(\overline{\mu}_F(d)\right) \tag{F.2}$$

where $\underline{\mu}_F(c) := \mathbb{E}_F[k|k \leq c]$ and $\overline{\mu}_F(d) := \mathbb{E}_F[k|k \geq d]$. Lemma F.4 shows that $\mathbb{E}_{\mathscr{P}(c,d)}\left[W_n^m(\cdot)\right]$ is single-peaked in both thresholds c and d.

Lemma F.4. *The following properties hold:*

(i) $\frac{\partial \mathbb{E}_{\mathscr{P}(c,d)}[W_n^m(\cdot)]}{\partial d} > (<) 0$ *for* $d < (>) b_n^m$, *and*

(ii) $\frac{\partial \mathbb{E}_{\mathscr{P}(c,d)}[W_n^m(\cdot)]}{\partial c} > (<) 0$ *for* $c > (<) a_n^m$.

Proof. Taking derivatives of (F.2) with respect to c and d yield[16]

$$\begin{aligned}
\frac{\partial \mathbb{E}_{\mathscr{P}(c,d)}\left[W_n^m(\cdot)\right]}{\partial d} &= f(d)\left(W_n^m(d) - W_n^m\left(\overline{\mu}_F(d)\right)\right) + (1-F(d))\,W_n^{m\prime}\left(\overline{\mu}_F(d)\right)\overline{\mu}_F'(d) \\
&= f(d)\cdot\left(\overline{\mu}_F(d) - d\right)\cdot\left[\frac{W_n^m\left(\overline{\mu}_F(d)\right) - W_n^m(d)}{\overline{\mu}_F(d) - d} - W_n^{m\prime}\left(\overline{\mu}_F(d)\right)\right] \\
\frac{\partial \mathbb{E}_{\mathscr{P}(c,d)}\left[W_n^m(\cdot)\right]}{\partial c} &= f(c)\left(W_n^m\left(\underline{\mu}_F(c)\right) - W_n^m(c)\right) + F(c)\cdot W_n^{m\prime}\left(\underline{\mu}_F(c)\right)\underline{\mu}_F'(c) \\
&= f(c)\cdot\left(c - \underline{\mu}_F(c)\right)\cdot\left[W_n^{m\prime}\left(\underline{\mu}_F(c)\right) - \frac{W_n^m(c) - W_n^m\left(\underline{\mu}_F(c)\right)}{c - \underline{\mu}_F(c)}\right]
\end{aligned}$$

Because both $\tilde{f}(d)$ and $\overline{\mu}_F(d) - d$ are positive, $\frac{\partial \mathbb{E}_{\mathscr{P}(c,d)}\left[W_n^m(\cdot)\right]}{\partial d}$ is sign-equivalent to

$$\frac{W_n^m\left(\overline{\mu}_F(d)\right) - W_n^m(d)}{\overline{\mu}_F(d) - d} - W_n^{m\prime}\left(\overline{\mu}_F(d)\right) \tag{F.3}$$

By Lemma 5, $W_n^m(\cdot)$ is strictly S-shaped on $[z_n^m, 1]$ and hence $\frac{W_n^m(\overline{\mu}_F(d)) - W_n^m(d)}{\overline{\mu}_F(d) - d} - W_n^{m\prime}\left(\overline{\mu}_F(d)\right)$ crosses zero at most once and above at b_n^m, which is pinned down by condition (FOC: b_n). This proves part (i). The proof for part (ii) is similar; it exploits the inverse S-shape property of $W_n^m(\cdot)$ on $[-1, z_n^m]$ and the definition of a_n^m. □

We establish below that any equilibrium outcome in pure and weakly undominated strategies must be both no more and no less informative than $\mathscr{P}(a_n^{\min}, b_n^{\max})$. These together imply the uniqueness of $\mathscr{P}(a_n^{\min}, b_n^{\max})$ as the induced outcome of any pure strategy equilibrium in weakly undominated strategies.

We first show for any $m \in M$ that all $\mathscr{P}(c,d)$ with $d > b_n^{\max}$ is weakly dominated by $\mathscr{P}(c, b_n^{\max})$, provided that all other designers $i \neq m$ choose strategies from $\mathscr{P}_n^i$. Let π_{-m} denote a strategy profile by other designers and under π_{-m} let $\eta \in [b_n^{\max}, 1]$ be the threshold such that $k \in [b_n^{\max}, \eta]$ are revealed, while $k > \eta$ are pooled together. Replacing $\mathscr{P}(c,d)$ with $\mathscr{P}(c, b_n^{\max})$ can only make a difference in states $k \in [b_n^{\max}, 1]$. If $d \leq \eta$, then such replacement has no effect on the joint information policy so designer m is indifferent with it. If instead $d > \eta$, then such replacement lowers the threshold of upper pooling interval and it reduces the informativeness of the joint policy. By Lemma F.4, for each $m \in M$, $\mathbb{E}_{\mathscr{P}(c,d)}\left[W_n^m(\cdot)\right]$ is single-peaked in d at with a peak at b_n^m. Since $\eta \geq b_n^{\max} > b_n^m$, it follows that any designer m's expected payoff would increase were $\mathscr{P}(c,d)$ replaced by $\mathscr{P}(c, b_n^{\max})$. Hence, any $\mathscr{P}(c,d)$ with $d > b_n^{\max}$ is weakly dominated by $\mathscr{P}(c, b_n^{\max})$. Using analogous argument we can also show that any $\mathscr{P}(c,d)$ with $c < a_n^{\min}$ is weakly dominated by

[16] In the derivation we exploit the fact that $\overline{\mu}_F'(d) = \frac{f(d)}{1-F(d)}\left(\overline{\mu}_F(d) - d\right)$ and $\underline{\mu}_F'(c) = \frac{f(c)}{F(c)}\left(c - \underline{\mu}_F(c)\right)$.

$\mathscr{P}(a_n^{\min}, d)$. Together these imply that any $\mathscr{P}(c,d)$ with $d > b_n^{\max}$ or $c < a_n^{\min}$ is weakly dominated. This shows that any outcome induced by a pure-strategy equilibrium with undominated strategies must be weakly less informative than $\mathscr{P}(a_n^{\min}, b_n^{\max})$.

Next we show that no equilibrium outcome can be strictly less informative than censorship policy $\mathscr{P}(a_n^{\min}, b_n^{\max})$.[17] Observe that the structure of $\mathscr{P}_n^m$ implies that any feasible outcome must be weakly more informative than $\mathscr{P}(a_n^m, b_n^m)$ for all $m \in M$. Therefore, if $\cup_{m \in M} [a_n^m, b_n^m] = \left[a_n^{\min}, b_n^{\max}\right]$ the result holds trivially. In what follows we assume that $\cup_{m \in M} [a_n^m, b_n^m]$ is a proper subset of $\left[a_n^{\min}, b_n^{\max}\right]$. In this case, there must be at least one pair of designers $l, r \in M$ such that (i) $b_n^l < a_n^r$, and (ii) for all $m \in M \backslash \{l, r\}$ there are $[a_n^m, b_n^m] \cap \left(b_n^l, a_n^r\right) = \emptyset$.[18] By the construction of $\mathscr{P}_n^m$ for all $m \in M$, there could be at most one nontrivial pooling interval $[x,y] \subseteq \left[b_n^l, a_n^r\right]$. Given $[x,y]$ and let $\mu(x,y) := \mathbb{E}_F[k | k \in [x,y]]$, the expected utility of designer $m = \{l, r\}$ conditional on event $k \in \left[b_n^l, a_n^r\right]$ is given by

$$V_m = \int_{b_n^l}^{x} W_n^m(k) d\widetilde{F}(k) + \left(\widetilde{F}(y) - \widetilde{F}(x)\right) W_n^m(\mu(x,y)) + \int_{y}^{a_n^r} W_n^m(k) d\widetilde{F}(k)$$

where $\widetilde{F}(\cdot)$ denote the cdf of the distribution of k conditional on $k \in \left[b_n^l, a_n^r\right]$. Taking derivatives of V_m with respect to x and y yields[19]

$$\begin{aligned}\frac{\partial V_m}{\partial x} &= \widetilde{f}(x)\left(W_n^m(x) - W_n^m(\mu(x,y))\right) + \left(\widetilde{F}(y) - \widetilde{F}(x)\right) W_n^{m\prime}(\mu(x,y))\, \mu_x(x,y) \\ &= \widetilde{f}(x)\left(\mu(x,y) - x\right)\left[W_n^{m\prime}(\mu(x,y)) - \frac{W_n^m(\mu(x,y)) - W_n^m(x)}{\mu(x,y) - x}\right]\end{aligned}$$

and

$$\begin{aligned}\frac{\partial V_m}{\partial y} &= \widetilde{f}(y)\left(W_n^m(\mu(x,y)) - W_n^m(y)\right) + \left(\widetilde{F}(y) - \widetilde{F}(x)\right) W_n^{m\prime}(\mu(x,y))\, \mu_y(x,y) \\ &= \widetilde{f}(x)\left(y - \mu(x,y)\right)\left[W_n^{m\prime}(\mu(x,y)) - \frac{W_n^m(y) - W_n^m(\mu(x,y))}{y - \mu(x,y)}\right]\end{aligned}$$

For both l and r to have no incentive to reveal any extra information, it is necessary that $\frac{\partial V_l}{\partial x} \leq 0$ and

[17] Notice that the information environment is no longer Blackwell-connected when each designer m is restricted to choose information policies from $\mathscr{P}_n^m$. Therefore, Proposition 2 of Gentzkow and Kamenica (2017*b*) no longer applies (i.e., a feasible outcome being unimprovable to all designers is no longer necessary for that outcome to be an equilibrium).

[18] In fact, there could be at most $|M| - 1$ such pairs. The argument presented below holds for any such pair.

[19] Here we use the fact that $\mu_x(x,y) := \frac{\partial \mu}{\partial x} = \frac{\widetilde{f}(x)(\mu(x,y) - x)}{\widetilde{F}(y) - \widetilde{F}(x)}$ and $\mu_y(x,y) := \frac{\partial \mu}{\partial y} = \frac{\widetilde{f}(y)(y - \mu(x,y))}{\widetilde{F}(y) - \widetilde{F}(x)}$.

$\frac{\partial V_r}{\partial y} \geq 0$, or equivalently[20]

$$W_n^{l\prime}(\mu(x,y)) \leq \frac{W_n^l(\mu(x,y)) - W_n^l(x)}{\mu(x,y) - x} \tag{F.4}$$

and

$$W_n^{r\prime}(\mu(x,y)) \geq \frac{W_n^r(y) - W_n^l(\mu(x,y))}{y - \mu(x,y)} \tag{F.5}$$

Because $z_n^l \leq b_n^l \leq x \leq y \leq a_n^r \leq z_n^r$, both $[z_n^l, 1]$ and $[-1, z_n^r]$ must contain $[x,y]$ in their interior. For (F.4) to hold, $\mu(x,y) > r_n^l$ must be true so that $\mu(x,y)$ falls into the concave region of $W_n^l(\cdot)$. Similarly, for (F.5) to hold, $\mu(x,y) < \ell_n$ must hold for $\mu(x,y)$ to fall into the concave region of $W_n^r(\cdot)$. These together imply that both $W_n^l(\cdot)$ and $W_n^r(\cdot)$ are strictly concave at $\mu(x,y)$. This, however, is impossible because strictly convex open cover property holds for $\{W_n^m(\cdot)\}_{m\in\{l,r\}}$ on $[z_n^l, z_n^r]$, which contains $[x,y]$.[21] Therefore, the incentive compatibility conditions for designers l and r cannot be simultaneously satisfied and hence it is impossible to have any non-trivial pooling interval $[x,y]$ in equilibrium. This implies that no equilibrium outcome can be strictly less informative than $\mathscr{P}(a_n^{\min}, b_n^{\max})$ and thus completes the proof for statement (2) of Theorem 4.

[20] This is because each $m \in \{l,r\}$ can only choose censorship policies from $\mathscr{P}_n^m$, whose revelation interval must contain $[a_n^m, b_n^m]$. Therefore, since $[x,y] \subset \left(b_n^l, a_n^r\right)$, only designer l can marginally increase x while only designer r can marginally decrease y. These two inequalities are necessary to insure that such marginal deviations are not profitable for either designer.

[21] This is an application of Lemma F.2 with $M = \{l,r\}$.