Firm Heterogeneity and the Transmission of Central Bank Credit Policy*

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Abstract

I study the role of firm heterogeneity for the transmission of unconventional monetary policy intended to ease firms' borrowing constraints, often referred to as "credit policy". To this end, I lay out a Two-Agent New-Keynesian model with financially constrained and unconstrained firms and financial intermediaries with endogenous leverage constraints. I find two main results: First, when firms are heterogeneous, the effectiveness of credit policy in stimulating aggregate investment is significantly reduced compared to a representative firm setting. Second, a credit policy shock leads to a persistent reallocation of capital between firms even if the policy is evenly directed towards all firms. I also use my model to compare the effects of credit policy targeted at different firms and the effectiveness of credit policy in stabilising economic activity in a financial crisis.

Keywords: Credit Policy, Firm Heterogeneity, Investment, Financial Frictions

JEL Classification: E50, E52, E58

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1 Introduction

Since the Great Financial Crisis, central banks around the world have engaged in unprecedented measures intended to ease borrowing constraints for firms and thereby stimulate investment. These policies, often referred to as "credit policy", have also been at the centre of efforts to mitigate deteriorations in firm financing conditions caused by the Covid-19 shock from March 2020 onwards.[1]

To the extent that borrowing constraints differ across firms, policy actions which affect these constraints might lead to heterogeneous firm responses. In turn, these heterogeneous responses potentially influence the macroeconomic effects of central bank credit policy. Motivated by this reasoning, this paper addresses the following research question: What is the role of firm heterogeneity for the transmission of central bank credit policy?

My analysis builds on two main observations: First, central bank credit policy has usually taken the form of purchases of specific asset classes. To the extent that there are limits to arbitrage between these asset classes, this would give rise to heterogeneous effects of the policy across different firms depending on the debt instruments they use (see, e.g., Papoutsi et al., 2021; Kurtzman and Zeke, 2020). Second, Ottonello and Winberry (2020) document how, in the context of conventional monetary policy, differences in the degree to which firms are affected by default risk-related borrowing constraints induce heterogeneous sensitivities to general equilibrium increases in the price of capital. These heterogeneous sensitivities should also play a role in the context of credit policy which raises the price of capital by stimulating capital demand.

To the best of my knowledge, this is the first paper to simultaneously shed light on two layers of firm heterogeneity and their respective role for the transmission of credit policy: heterogeneity in (i) sources of debt financing and (ii) default risk and the associated borrowing constraints. In particular, it is the first to consider the effects of heterogeneity in default risk in the context of credit policy as opposed to conventional monetary policy. In doing so, it uncovers a degree of state dependence of credit policy in the sense that the composition of the production sector affects the policy's effectiveness in stimulating aggregate investment. Furthermore, my analysis shows how the presence of default risk heterogeneity may affect the relative effectiveness of credit policy directed at different types of firms.

To address the research question, I introduce credit policy into a New-Keynesian model which incorporates important aspects of firm heterogeneity. In particular, I divide the production sector into two ex-ante heterogeneous subsectors, which I refer to as "constrained" and "unconstrained". This allows for a tractable illustration of the relevant channels in the transmission of credit policy. In principle, firms in both subsectors can fund investment using internal and external financing from specialised intermediaries where the latter is subject to two forms of financial frictions.

[1] Woodford (2022) provides a theoretical framework to think about the differences between the Covid-19 shock and previous crises. This theoretical framework is used to compare the efficacy of conventional monetary policy, fiscal transfers and credit policy.

The first friction is related to firm specific default risk and originates from a costly state verification problem as in Bernanke et al. (1999). Under the baseline calibration, only firms in the constrained subsector are affected by this friction whereas default risk of firms in the unconstrained subsector is sufficiently low for them to be unrestricted in this sense. In the following, I refer to the constraint associated with this friction as a firm's "financial constraint".

The second friction originates from a costly enforcement problem which limits the amount of funds intermediaries can obtain from depositors as in Gertler and Karadi (2011). This imposes an endogenous leverage constraint on their lending operations. By assumption, intermediaries are specialised in the sense that their lending is confined to one of the two firm subsectors. In steady state, the costly enforcement constraint applies equally to both subsectors although its relative tightness may change in response to shocks.

Following Gertler and Karadi (2011), credit policy is modelled as an increase in the scope of financial intermediation by the central bank. This eases the endogenous leverage constraint of financial intermediaries and lowers firm financing costs.

I consider the transmission mechanism of credit policy as well as the role of implementation and the ability of credit policy to stabilise economic activity in a financial crisis. For the latter I consider a shock to the net worth of financial intermediaries. I find that credit policy is generally effective in stimulating investment by lowering the external finance premium schedule which governs the investment decision of firms.

In my analysis, the presence of firm heterogeneity gives rise to two main results. First, credit policy is less effective in stimulating investment on impact when there is heterogeneity between subsectors. The reason for this result is that a *crowding out effect* via the price of capital induces unconstrained firms to demand less capital, partly offsetting the increased capital demand by constrained firms and dampening the aggregate investment response. In particular, the initial increase in the demand for capital implied by an improvement in financing conditions leads to an increase in the price of capital. This increase in the price of capital raises firms' net worth and further relaxes their financial constraint which leads to an additional increase in the demand for capital. This is the well known financial accelerator mechanism in Bernanke et al. (1999). However, the increase in the capital price also directly raises the marginal cost of capital, reducing capital demand. Unconstrained firms are not affected by the financial accelerator effect but strongly affected by the increase in the price of capital and therefore react to the policy intervention by decreasing their demand for capital.[2] The crowding out effect also implies that credit policy is less effective in responding to a financial crisis.

Second, I find that credit policy leads to a persistent *reallocation of capital* from unconstrained to constrained firms. In my model, the production technology is identical across

[2] This crowding out effect has also been identified to play a role in the transmission of conventional monetary policy in Ottonello and Winberry (2020). Crowding out in the context of credit policy is also a feature in Kurtzman and Zeke (2020), following directly from assumptions on the intermediation friction. In my setting, it arises endogenously from the default risk related financial friction as discussed below.

firms in both subsectors such that relative capital holdings are entirely determined by the relative degree to which firms are affected by financial constraints. The credit policy induced increase in constrained firm net worth changes these relative financial constraints in a way to induce relatively higher capital holdings by constrained firms.

Qualitatively, these effects do not crucially depend on whether credit policy is directed at one subsector or the other. This is due to the fact that the responses to a credit policy shock are driven by sensitivities to general equilibrium changes in the price of capital and the financial accelerator mechanism. Differences in the implementation of credit policy only affect the strength of these effects.

In general, my analysis suggests that the effectiveness of credit policy in inducing a stimulative effect on aggregate investment depends crucially on the degree of default risk heterogeneity between constrained and unconstrained firms. Moreover, credit policy effectiveness is substantially affected by the choice of the subsector targeted by the central bank's intervention.

This paper mainly relates to three strands of the literature. First, it contributes to the literature on unconventional monetary policy in the form of "credit policy". Like this paper, a substantial part of the existing literature builds on Gertler and Karadi (2011, 2013). Another contribution in this area is Cúrdia and Woodford (2011) who take a broader perspective in evaluating the role of the central bank balance sheet as a policy instrument. A more recent approach, focussing on the intermediearies' risk tolerance is Caballero and Simsek (2021). I complement the approach by Gertler and Karadi (2011, 2013) with an explicit formulation of default risk and thereby allow for financial heterogeneity to have a substantial effect on the transmission of credit policy.

The presence of firm heterogeneity also allows me to capture effects of a specific implementation of asset purchases. Investigating credit policy under different kinds of firm heterogeneity, Papoutsi et al. (2021) and Kurtzman and Zeke (2020) show that the decision on the types of assets a central bank purchases may play a role for aggregate effects of credit policy. Adding to these observations, my approach including default risk shows that firm heterogeneity may play a role for the aggregate effects of credit policy even when the central bank purchases debt in equal proportions, i.e., "across the board".

Second, my analysis builds on the literature on financial frictions related to default risk and costly state verification. This strand of the literature can be traced back to seminal contributions like Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997). Bernanke et al. (1999) embed this kind of financial friction in a New-Keynesian DSGE model which provides the basic framework around which my model is constructed. While these approaches relate borrowing constraints to the liquidation value of physical assets, recent contributions have highlighted the relevance of earnings based borrowing constraints (see, e.g., Lian and Ma, 2021; Drechsel, 2021; Greenwald, 2019). By adopting the approach by Bernanke et al. (1999) I am able to directly relate my analysis to existing work on firm heterogeneity in the transmission of monetary policy.

Third, I build more broadly on the literature on firm heterogeneity with financial frictions (see, e.g., Khan and Thomas, 2013; Arellano et al., 2019). My approach particularly relates to contributions which explore the role of these aspects for conventional monetary policy. Empirical contributions include Jeenas (2019), Cloyne et al. (2019) and Anderson and Cesa-Bianchi (2021). Formulating a theoretical model to rationalise their own empirical findings, Ottonello and Winberry (2020) provide insights which are particularly relevant for my analysis. In contrast to these contributions, my model features limited heterogeneity across firms by allowing for perfect aggregation within firm subsectors.

My model setup is closely related to Rannenberg (2016) and Kühl (2018) who also combine the frictions of the kind found in Bernanke et al. (1999) and Gertler and Karadi (2011, 2013) in one model. My framework extends these models by incorporating firm heterogeneity and analysing the effects of private sector credit policy instead of government bond purchases as in Kühl (2018).

The rest of the paper is structured as follows: Section two provides a detailed description of the model with a particular focus on the relevant financial frictions. Section three then presents the results of the model analysis with respect to the transmission of credit policy. Section four sheds light on the role of implementation and the ability of the central bank to counteract the effects of and adverse financial shock by engaging in credit policy. Section five concludes and the appendix provides derivations as well as additional results.

2 Model

Figure 1 provides an overview of my model building on Bernanke et al. (1999) and Gertler and Karadi (2011). In particular, I extend the framework in Bernanke et al. (1999) in two significant ways as indicated by the light blue coloured boxes in figure 1. First, I split the continuum of intermediate goods firms into two groups which I refer to as constrained and unconstrained firms. I calibrate my model such that constrained firms are limited in their investment decision by a costly state verification (CSV) friction whereas unconstrained firms are effectively unrestricted in this regard. This distinction between constrained and unconstrained firms lies at the heart of my analysis and allows my model to capture important mechanisms with respect to firm heterogeneity.

As a second extension, I allow for frictions in intermediation and credit policy as a tool to mitigate this friction as assumed in Gertler and Karadi (2011). In particular, there is a costly enforcement problem which limits the amount of funds specialised intermediaries can obtain from households to lend to intermediate good firms from a specific subsector. In my model, there may be differences in the degree to which this friction limits the amount of funds extended to the two subsectors, which introduces another dimension of firm heterogeneity.

As I will explain in more detail below, the two frictions in the process of channelling funds from households to firms imply that borrowing is subject to an external finance premium which consists of two components. A default risk premium associated with the CSV friction á la Bernanke et al. (1999), and an excess bond premium associated with the costly enforcement problem in Gertler and Karadi (2011).[3]

As in Gertler and Karadi (2011), the central bank can bypass the costly enforcement friction and take over a certain share of intermediation by issuing riskless short-term debt to households and extending funds to intermediate goods firms. As intermediation by the central bank is not subject to the same financial friction as private intermediation, this eases financing conditions for intermediate goods firms and thereby encourages investment.

The following model description focuses on financial intermediaries, intermediate goods firms and the financial frictions governing their investment decision, and how intermediation is affected by credit policy. The standard elements of the model, i.e., households, retailers, capital producers and conventional monetary policy are described at the end of the section.

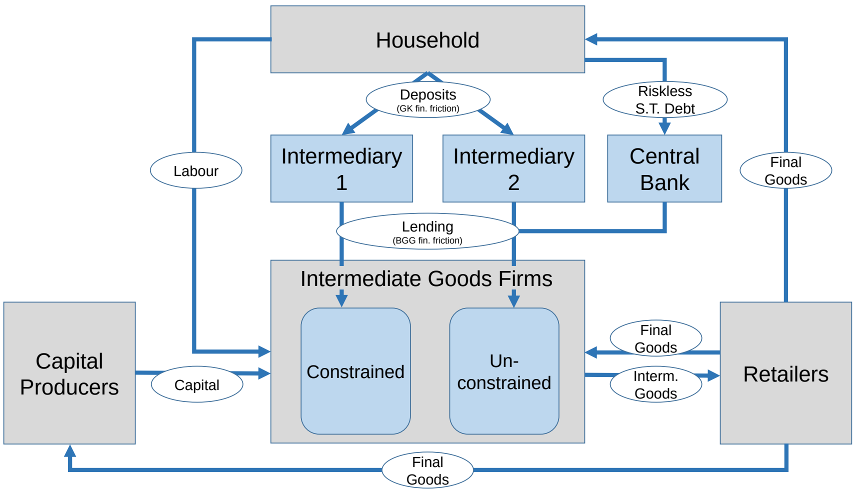


Figure 1: Model overview. Light blue boxes represent extensions to the basic framework of Bernanke et al. (1999). Financial intermediation and the role of the central bank are based on Gertler and Karadi (2011).

2.1 Intermediate goods firms

The intermediate goods firms sector consists of a continuum of risk-neutral firms which combine labour and capital to produce intermediate goods which they sell to retailers in exchange for final goods. Each firm belongs to a subsector $i \in \{c, u\}$. The share of firms belonging to subsector i is denoted by s_i. In what follows, I first describe how the two subsectors aggregate into the overall intermediate goods production sector. Subsequently, I characterise a generic subsector i intermediate goods firm $j \in [0, s_i]$ and the financial contract

[3] Rannenberg (2016) refers to these premia as "entrepreneurial quasi-profit margin" and "bank profit margin" respectively while Kühl (2018) uses the terms "finance premium" and "bank profit margin". The term excess bond premium is taken from Gilchrist and Zakrajšek (2012) where it broadly represents the component of credit spreads not accounted for by idiosyncratic default risk (see section 2.2 below).

governing its investment decision. I then describe how the mass of individual firms within each subsector can be aggregated into a representative intermediate goods firm. Finally, I lay out the determinants of default risk heterogeneity between the two subsectors.

2.1.1 Aggregation across subsectors

I assume that the quantities of intermediate goods produced in the two subsectors aggregate into a single aggregate intermediate good according to the following CES aggregator

$$Y_t = \left(s_c Y_{c,t}^{\rho} + s_u Y_{u,t}^{\rho}\right)^{\frac{1}{\rho}},$$

where ρ is the elasticity of substitution between the two subsectors' intermediate goods. This implies that the relative price of the respective sectors' intermediate good output in terms of the final good is given by

$$P_{i,t}^{f} \equiv \frac{1}{X_t} s_i \left(\frac{Y_{i,t}}{Y_t}\right)^{\rho-1},$$

where $\frac{1}{X_t}$ is the relative price of the (aggregate) intermediate good in terms of the final good.[4]

Firms in both subsectors are assumed to use the same type of capital such that aggregate capital is composed of capital employed in each of the two subsectors according to

$$K_t = s_c K_{c,t} + s_u K_{u,t}.$$

By abstracting from sector specific capital, I am able to isolate the effects of financial heterogeneity from other potential dimensions of firm heterogeneity. Perfect substitutability of capital across sectors is also crucial for a single equilibrium price of capital to affect decisions of all firms. This way, capital demand of one subsector affects the price of capital faced by the other.

Firms in both subsectors employ the same type of labour. Aggregate labour is thus allocated to the two subsectors according to

$$H_t = s_c H_{c,t} + s_u H_{u,t}.$$

As Bernanke et al. (1999), my model also features entrepreneurial labour. This is required for new firms without retained earnings from previous periods to enter with a certain amount of initial net worth (see section 2.1.3 below). Aggregate entrepreneurial labour is allocated to the two subsectors according to

$$H_t^e = s_c H_{c,t}^e + s_u H_{u,t}^e.$$

My assumptions regarding the aggregation across subsectors have important implica-

[4]Equivalently, X_t is the gross markup of the retail good's price over the intermediate good's price.

tions for the relationship of factor prices between subsectors. First, households are indifferent between supplying labour to either of the subsectors. Hence, the real wage is equalised between the two subsectors. In contrast, the supply schedule of capital differs between subsectors. As described below, firms in the two subsectors potentially have different equilibrium levels of net worth and leverage which, given the financial friction, induces different levels of expected cost of default. This in turn implies that a marginal cost of capital in one subsector is not the same as in the other subsector. In equilibrium, where the return on capital equals its marginal cost, the returns in the respective subsectors are therefore not equalised.

2.1.2 Financial contract and individual firm behaviour

Each individual firm j in subsector i uses net worth and borrowing from intermediaries to finance expenditures on capital. Its balance sheet at the end of period t is thus given by

$$Q_t K^j_{t+1} = N^j_{t+1} + B^j_{t+1}, \tag{1}$$

where Q_t is the economy-wide price of capital, N^j_{t+1} is the net worth of firm j at the end of period t (i.e., the net worth it carries over to period $t+1$) and B^j_{t+1} is its borrowing due in period $t+1$. K^j_{t+1} denotes the amount of capital goods of firm j purchased at the end of period t and used for production in the subsequent period.[5] To ease notation, I do not include a subsector i subscript when characterising the behaviour of the individual firm. In the following exposition of the financial contract, I assume that the firm takes as given the price Q_t and the return on its capital, denoted by R^k_{t+1}. Later, these terms will be endogenously determined.

A given firm's return on capital is subject to an idiosyncratic shock ω^j, i.e., the individual return on capital of firm j is given by $\omega^j R^k_{t+1}$. I assume ω^j to be log-normally distributed, i.e., $ln(\omega^j) \sim N(-0.5\sigma^2, \sigma^2)$ such that $E_j[\omega^j] = 1$.

As in Bernanke et al. (1999) lenders must pay a fixed auditing cost μ to be able to observe the idiosyncratic returns of potential borrowers. Under this costly state verification (CSV) assumption, firm optimisation yields representations of the expected discounted return on capital and leverage in terms of a default threshold $\bar{\omega}^j_{t+1}$ (see appendix A.1 for details).

Ultimately, these imply the following relationship between the expected return on capital discounted by the riskless lending rate R^b_{t+1} and leverage defined as the ratio of capital expenditure to net worth

$$\frac{E_t\{R^k_{t+1}\}}{R^b_{t+1}} = s\left(\frac{Q_t K^j_{t+1}}{N^j_{t+1}}\right), \tag{2}$$

with $s'(x) > 0$ if $x > 1$. Firm j will always accumulate capital up to the point where the marginal return on capital is equal to its marginal cost. The right hand side of equation (2) can thus be interpreted as the marginal cost of capital expenditure discounted by the

[5] As in Bernanke et al. (1999), firms are modelled to repurchase capital goods in each period to ensure financial constraint applies to the whole firm instead of marginal investment.

riskless lending rate, i.e., the default risk premium. Higher leverage implies a higher risk of costly default and a higher default risk premium (see appendix A.2 for details).

2.1.3 Aggregation within subsectors

Following Bernanke et al. (1999), intermediate goods are produced in subsector i according to the following production function

$$Y_{i,t}^{j} = A_{i,t}\left(K_{i,t}^{j}\right)^{\alpha}\left(L_{i,t}^{j}\right)^{1-\alpha},$$

where $K_{i,t}^{j}$ is effective capital, the labour input $L_{i,t}^{j}$ is a composite of household and entrepreneurial labour by the firm owner and $A_{i,t}$ is an exogenous technology shifter. The assumption of constant returns to scale implies linearity of capital in net worth (i.e., identical leverage of all firms) which allows for perfect aggregation across firms with different levels of net worth into a single representative intermediate goods firm. For a given subsector i, condition (2) characterising the financial contract, can thus be written as a condition determining the relationship between the representative firm's leverage and its expected discounted return on capital expenditure

$$\frac{E_t\{R_{i,t+1}^{k}\}}{R_{i,t+1}^{b}} = s\left(\frac{Q_t K_{i,t+1}}{N_{i,t+1}}\right), \tag{3}$$

where the function $s(.)$ exhibits the same properties as above.

This aggregation result allows for the determination of the equilibrium default risk premium and aggregate leverage without having to track the distribution of firms with respect to net worth. Aggregating over firms in subsector i, the production function for subsector i intermediate good output is given by

$$Y_{i,t} = A_{i,t}K_{i,t}^{\alpha}L_{i,t}^{1-\alpha},$$

where $L_{i,t} = H_{i,t}^{\Omega}\left(H_{i,t}^{e}\right)^{1-\Omega}$ is a composite of household labour $H_{i,t}$ and labour provided inelastically by entrepreneurs running the intermediate goods firms, $H_{i,t}^{e}$.

Given this production function, the expected return on capital expenditure in subsector i obtains as

$$E_t\{R_{i,t+1}^{k}\} = E_t\left\{\frac{s_i^{-1}P_{i,t+1}^{f}\frac{\alpha Y_{i,t+1}}{K_{i,t+1}} + (1-\delta)Q_{t+1}}{Q_t}\right\}. \tag{4}$$

In this specification, the return on capital consists of two parts: the term $s_i^{-1}P_{i,t+1}^{f}\frac{\alpha Y_{i,t+1}}{K_{i,t+1}}$ is the marginal product of capital employed in subsector i in terms of retail goods.[6] The term $(1-\delta)Q_{t+1}$ is the marginal resale value of capital where δ denotes the capital depreciation

[6]The marginal product of capital features the term $s_i^{-1}P_{i,t+1}^{f}$ to take account of the across subsector aggregation of output and factor inputs as explained in appendix A.3.

rate.

The equilibrium leverage of the representative intermediate goods firm in subsector i is determined by substituting (4) into (3). The corresponding equilibrium in the market for capital for a given level of net worth is depicted in figure 2. The marginal cost of capital is given by the right hand side of equation (3) multiplied by Q_t.[7] It is horizontal as long as capital is entirely financed using net worth, i.e., leverage is below one. Levels of capital expenditure which exceed net worth incur default risk and therefore a positive default risk premium. The higher the amount of capital relative to net worth, the higher the risk of costly default and the higher the risk premium. The existence of the possibility of costly default therefore leads to a marginal cost curve which is upward sloping in capital. Due to diminishing returns to capital, the marginal benefit represented by the expected return on capital in equation (4) discounted by the riskless lending rate is downward sloping.

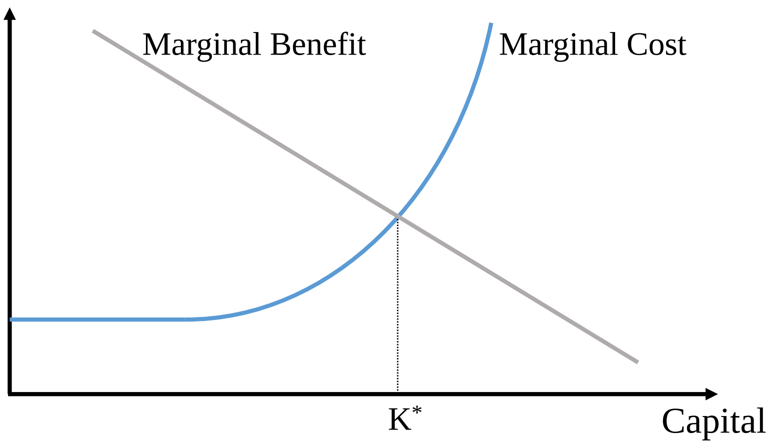


Figure 2: Equilibrium in the market for capital. The marginal cost curve is determined by the financial contract; the marginal benefit curve plots the discounted marginal return on capital

Inferring the choice of capital in general equilibrium (that is, from the equilibrium leverage determined by the financial contract) requires a formulation of the evolution of firm net worth. In order to prevent firms from accumulating large levels of net worth and thereby escape the financing constraint, I follow Bernanke et al. (1999) in introducing a finite lifetime of individual firms. In particular, a fraction $(1-\gamma_i)$ of firms exit the economy in each period and exiting firms simply consume their equity value. To keep the mass of intermediate goods firms constant, I assume that the fraction of exiting firms is replaced by an equal number of new entrants in each period. New entrants' net worth is equal to their income from entrepreneurial labour. Given this assumption, the aggregate net worth which subsector i firms carry over to the next period (end of period t net worth) is composed of

[7]Note that this is the marginal cost of capital only in the absence of further frictions in intermediation. As I will explain below, the introduction of a costly enforcement problem on the part of the intermediaries will introduce an additional marginal cost component which will shift up the marginal cost curve.

the equity value of non-exiting firms and total entrepreneurial labour income

$$N_{i,t+1} = \gamma_i V_{i,t} + H^e_{i,t} W^e_{i,t}, \tag{5}$$

where $W^e_{i,t}$ is the real wage on entrepreneurial labour and $V_{i,t}$ is the equity of firms in subsector i. Specifically, equity consists of realised earnings on capital net of the cost of servicing debt

$$V_{i,t} = R^k_{i,t} Q_{t-1} K_{i,t} - \left(R_t + \frac{\mu \int_0^{\bar{\omega}} \omega R^k_{i,t} Q_{t-1} K_{i,t} dF(\omega)}{Q_{t-1} K_{i,t} - N_{i,t}} \right) (Q_{t-1} K_{i,t} - N_{i,t}) \,.$$

The term $\frac{\mu \int_0^{\bar{\omega}} \omega R^k_{i,t} Q_{t-1} K_{i,t} dF(\omega)}{Q_{t-1} K_{i,t} - N_{i,t}}$ represents the ratio of default costs to the amount of credit extended which can equally be interpreted as the credit spread.

Finally, the intermediate goods firms' demand for household and entrepreneurial labour satisfies

$$\begin{aligned} W_{i,t} &= (1-\alpha)\Omega \frac{Y_{i,t}}{H_{i,t}} s_i^{-1} P^f_{i,t} \\ W^e_{i,t} &= (1-\alpha)(1-\Omega) \frac{Y_{i,t}}{H^e_{i,t}} s_i^{-1} P^f_{i,t}, \end{aligned}$$

i.e., the marginal product of the two labour inputs (in terms of final goods) must equal the respective real wage. Again, these terms include $s_i^{-1} P^f_{i,t}$ to take account of the across subsector aggregation of output and factor inputs (see appendix A.3).

2.1.4 Default risk heterogeneity between subsectors

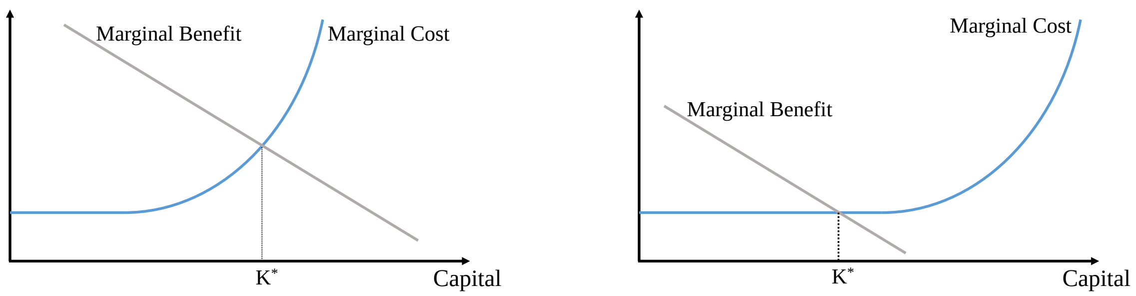
As in Ottonello and Winberry (2020), the key dimension along which the two subsectors differ in my model is the degree to which they are affected by the CSV friction. My limited heterogeneity setup is intended to capture key aspects of the distribution of firms therein.

Broadly speaking, the universe of firms in Ottonello and Winberry (2020) can be divided into two categories depending on their level of net worth: High net worth firms who optimally choose a level of capital which implies zero probability of default in the next period, and low net worth firms who choose a level of capital which may result in default depending on the realisation of idiosyncratic shocks. Visually, firms in the former category are located on the horizontal segment of the marginal cost curve whereas firms in the latter category are located on the upward sloping segment (see Ottonello and Winberry, 2020, pp. 2487-2488).

My objective is to analyse differences in the behaviour of firms in these two broad categories, i.e., between relatively high net worth firms with zero default risk premium and lower net worth firms with a positive default risk premium. To this end, I assign two different values to the survival rates γ_i of the two subsectors where $\gamma_c < \gamma_u$. As a result, subsector c firms have on average a lower steady state level of net worth according to equation (5).

In my calibration, I set γ_u sufficiently high for firms in the corresponding subsector to be effectively unconstrained, i.e., located on the flat segment of the marginal cost curve in figure 2 whereas firms in subsector c are constrained. The steady state equilibrium capital choice in the two subsectors is visualised in figure 3.

An additional implication of the heterogeneous values for γ_i is that unconstrained firms are on average older and larger (in terms of net worth) than constrained firms. This is in line with the observation that younger and smaller firms are more severely affected by funding constraints than their older and larger counterparts (see, e.g., Cloyne et al., 2019).



(a) Constrained firm

(b) Unconstrained firm

Figure 3: Equilibrium in the market for capital - default risk heterogeneity between subsectors. The position of firms on the marginal cost curves in the respective subsectors is determined by the calibration of survival rates γ_i.

2.2 Financial intermediation

Financial intermediation is conducted by two types of representative financial intermediaries which are modelled in the spirit of Gertler and Karadi (2011). The two intermediaries are assumed to be specialised in the sense that they exclusively lend to firms in either of the two subsectors. In particular, the intermediary specialised in subsector i lending obtains funds from households by issuing riskless deposits and lends to intermediate goods firms in subsector i.

The assumption of complete segmentation between the markets for subsector debt follows Anderson and Cesa-Bianchi (2021). It is motivated by the observation, that in practice, investors face various relevant restrictions (e.g. regulartoy constraints) with respect to the riskiness of assets they can acquire as shown by Chernenko and Sunderam (2012). The formulation can be viewed as a simple way to introduce local supply effects on asset prices where a change in the supply of a specific asset (e.g., due to credit policy) leads to a change in the price of this specific asset. Local supply effects of credit policy have been documented by e.g., D'Amico and King (2013) and Krishnamurthy and Vissing-Jorgensen (2013). The assumed extreme form of segmentation allows for a clear representation of the transmission mechanisms of credit policy. In a setting with partial market segmentation, the observed effects would be qualitatively similar but quantitatively different depending on the degree of market segmentation.

The balance sheet of the intermediary specialised in subsector i lending is given by

$$B^b_{i,t+1} = N^b_{i,t} + D_{i,t},$$

where $B^b_{i,t+1}$ denotes total lending of the subsector i intermediary to intermediate goods firms in this subsector, $N^b_{i,t}$ is the intermediary's net worth and $D_{i,t}$ denotes household deposits with the intermediary. Lending is subject to a costly enforcement problem. In particular, the intermediary can decide to divert a certain fraction λ^b_i of funds. Upon diversion, the depositors force the intermediary into bankruptcy and recover the remaining assets while the intermediary keeps the fraction λ^b_i of assets. In equilibrium, households will only be willing to provide an amount of funds which ensures that the intermediary will never divert funds, i.e., up to the point where the continuation value of the intermediary is higher than the value of diverting funds.

Specifically, the incentive constraint of the intermediary is given by.

$$V^b_{i,t} \geq \lambda^b_i B^b_{i,t+1}$$

where $V^b_{i,t}$ denotes the continuation value of the intermediary. This incentive constraint gives rise to an endogenous intermediary leverage multiple ϕ_t such that

$$B^b_{i,t+1} = \frac{\eta^b_{i,t}}{\lambda^b - \nu^b_{i,t}} N^b_{i,t} = \phi_{i,t} N^b_{i,t} \tag{6}$$

where $\eta^b_{i,t}$ is the marginal gain to the intermediary of net worth and $\nu^b_{i,t}$ is the marginal gain of expanding assets (see appendix A.4 for details). According to leverage constraint (6), variations in net worth of intermediary i will induce changes in the supply of funds to intermediate goods firms in subsector i. In order to prevent intermediaries to escape the financing constraint, I assume that a certain fraction of intermediaries exits the economy in each period. Aggregate intermediary net worth then evolves as the sum of net worth by existing intermediaries and net worth of new entrants.

$$N^b_{i,t} = \theta^b_i \left[(R^b_{i,t} - R_t)\phi_{i,t-1} + R_t \right] N^b_{i,t-1} exp(\varepsilon^b_t) + \omega^b_i B^b_{i,t-1},$$

where $R^b_{i,t}$ denotes the interest rate on default risk-free lending to firms in subsector i, θ^b_i is the share of surviving intermediaries and ω^b_i governs the start up funds of entering intermediaries. Following Rannenberg (2016), I include ε^b_t as an exogenous i.i.d. shock to intermediary net worth. This shock will later be used to induce a tightening of firm financing conditions to assess the effects of credit policy in a financial crisis.

By imposing a limit on the intermediary's ability to provide funding for firms, the costly enforcement problem gives rise to a positive interest rate differential between the default risk-less lending rate and the deposit rate, i.e., $\frac{R^b_{i,t+1}}{R_{t+1}} > 0$.[8] As this premium does not reflect compensation for idiosyncratic firm default risk, it can be interpreted as the excess bond premium in Gilchrist and Zakrajšek (2012).[9] In my model, the total external finance premium is the product of the subsector-specific default risk premium, determined by expression (3), and the excess bond premium determined by the costly enforcement friction in intermediation.

$$\frac{E_t\{R^k_{i,t+1}\}}{R_{t+1}} = \frac{E_t\{R^k_{i,t+1}\}}{R^b_{i,t+1}} \frac{R^b_{i,t+1}}{R_{t+1}}$$

As part of the external finance premium, the excess bond premium enters the marginal cost of capital for intermediate goods firms. The marginal cost curve as depicted in figure 2 above can thus be thought of as a special case where the incentive constraint of the intermediary does not bind, i.e., when $\frac{R^b_{i,t+1}}{R_{t+1}} = 1$.

2.3 Credit policy

As in Gertler and Karadi (2011), credit policy takes the form of financial intermediation by the central bank. In particular, the central bank may issue riskless short-term debt to the household at the deposit rate R_{t+1} and lend to intermediate goods firms in either subsector at the equilibrium riskless lending rate $R^b_{i,t+1}$.[10] Total borrowing by intermediate goods firms in each subsector i is thus composed of privately and publicly intermediated funds, $B^b_{i,t+1}$ and $B^c_{i,t+1}$ respectively such that

$$B_{i,t+1} = B^b_{i,t+1} + B^c_{i,t+1}.$$

When conducting credit policy, the central bank decides on the fraction of total lending it wishes to intermediate. Specifically, the central bank chooses $\psi_{i,t}$ in $B^c_{i,t+1} = \psi_{i,t} B_{i,t+1}$.

Importantly, there is no costly enforcement problem with respect to the ability of the central bank to acquire funds from households. The fact that central bank and private intermediation are imperfect substitutes implies that the additional demand for assets by the central bank does not completely crowd out asset demand by private intermediaries such that overall asset demand rises. This increase in asset demand leads to a lower excess bond premium for firms. Assuming that the central bank is willing to intermediate a fraction $\psi_{i,t}$ of total assets, the effective leverage ratio for total intermediated funds is given by

$$\phi^{cp}_{i,t} = \frac{1}{1-\psi_{i,t}} \phi_{i,t}.$$

[8] In the absence of any lending limit, the intermediary would extend funds up to the point where the riskless lending rate equals the deposit rate

[9] In their words, the excess bond premium reflects the "price of bearing exposure to corporate credit risk above and beyond the compensation for expected defaults" (Gilchrist and Zakrajšek, 2012, p. 1700)

[10] As highlighted in Gertler and Karadi (2011), credit policy could equivalently be formulated as the central bank financing its lending operations by issuing debt to the financial intermediary.

By raising the amount of debt it intermediates, the central bank relaxes the leverage constraint of private intermediaries and allows for more lending to the production sector (see appendix A.4).

Visually, an increase in the fraction of funds intermediated by the central bank lowers the marginal cost of capital schedule and thereby induces higher levels of capital, i.e., it stimulates investment by relaxing funding constraints.

Note that due to the assumption of specialised intermediaries, the effective leverage multiple is subsector specific. An increase in central bank intermediation of debt in one of the two subsectors will thus have no effect on the effective leverage ratio and excess bond premium in the other subsector.

In the model analysis below, I assume $\psi_{i,t}$ to follow a log AR(1) process, i.e.

$$\frac{\psi_{i,t}}{\psi_i} = \left(\frac{\psi_{i,t-1}}{\psi_i}\right)^{\rho_\psi} exp(\varepsilon^{\psi}_{i,t}) \tag{7}$$

where $\varepsilon^{\psi}_{i,t}$ is the credit policy shock and $\varepsilon^{\psi}_{i,t} \sim N(0, \sigma^2_\psi)$. To illustrate the transmission of credit policy, I will first consider a credit policy shock in the form of an exogenous increase in $\varepsilon^{\psi}_{i,t}$. Subsequently, I investigate the effects of an increase in $\psi_{i,t}$ in response to an adverse shock to firm financing conditions.

2.4 Standard model components

This section describes the remaining model components, namely capital producers, retailers and households as well as the conduct of conventional monetary policy. The formulation of these elements of the model largely follows Bernanke et al. (1999).

Capital producers: Capital producers combine aggregate installed capital with the amount of final goods allocated to investment to produce new aggregate capital according to the following production function

$$K^n_{t+1} = \Phi\left(\frac{I_t}{K_t}\right) K_t,$$

where K^n_{t+1} is the amount of newly produced capital and $\Phi'(.) > 0$, $\Phi''(.) < 0, \Phi(0) = 0$.[11]

The newly produced capital is sold to intermediate goods producers at the market price Q_t. The first order condition of profit maximisation with respect to investment then yields

$$Q_t = \left[\Phi'\left(\frac{I_t}{K_t}\right)\right]^{-1}.$$

As the effectiveness in converting investment into new capital is decreasing in I_t, higher

[11] In particular, I assume $\Phi\left(\frac{I_t}{K_t}\right) = \frac{1}{1-\eta}\left(\frac{I}{K}\right)^{\eta}\left(\frac{I_t}{K_t}\right)^{1-\eta} - \frac{\eta}{1-\eta}\left(\frac{I}{K}\right)$ such that in steady state, $\frac{I}{K} = \delta$, $\Phi\left(\frac{I}{K}\right) = \delta$ and $\Phi'\left(\frac{I}{K}\right) = 1$ as in the calibration of Bernanke et al. (1999)

investment implies a higher marginal adjustment cost of capital and a higher capital price.

The stock of aggregate capital evolves as

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right)K_t + (1-\delta)K_t,$$

that is, end of period capital is the sum of new capital produced from investment activity and non-depreciated capital from the beginning of the period.

Retailers: The introduction of a separate retail sector separates the elements required to introduce a role for conventional monetary policy, i.e., monopolistic competition and rigid prices, from the investment decision.

There is a continuum of retail firms, each transforming (aggregate) intermediate goods into differentiated retail goods using a one to one technology. The aggregate quantity of the final retail good is given by the following composite of these individual retail goods

$$Y_t^f = \left[\int_0^1 Y_t(z)^{(\epsilon-1)/\epsilon}dz\right]^{\epsilon/(\epsilon-1)},$$

where $Y_t(z)$ is the output sold by retailer z. The corresponding price index is

$$P_t = \left[\int_0^1 P_t(z)^{1-\epsilon}dz\right]^{1/(1-\epsilon)},$$

where $P_t(z)$ is the nominal price of a unit of retail output charged by retailer z.

Final goods are then allocated to the sectors of the economy according to the following resource constraint

$$Y_t^f = C_t + s_c C_{c,t}^e + s_u C_{u,t}^e + I_t + s_c\mu\int_0^{\bar{\omega}_c}\omega dF(\omega)R_{c,t}^k Q_{t-1}K_{c,t} + s_u\mu\int_0^{\bar{\omega}_u}\omega dF(\omega)R_{u,t}^k Q_{t-1}K_{u,t},$$

where C_t is household consumption, $s_c C_{c,t}^e + s_u C_{u,t}^e$ entrepreneurial consumption and the last two terms represent monitoring expenses caused by firm defaults in the two subsectors. As in Bernanke et al. (1999), staggered price setting as in Calvo (1983) yields a standard New-Keynesian Phillips curve relating output to inflation.

Households: The household sector consists of a continuum of identical households. Each household derives utility from consumption of the final retail good and leisure. They receive labour and dividend income to finance consumption and save in real riskless deposits with intermediaries and debt securities issued by the central bank. The household problem is given by

$$\begin{aligned} \max_{\{C_t,H_t,D_t\}_{t=0}^{\infty}} \quad & E_t\sum_{t=0}^{\infty}\beta^t\left[ln(C_t)+\xi ln(1-H_t)\right] \\ \text{s.t.} \quad & C_t = W_tH_t + \Theta_t^r + \Theta_t^b + R_tD_t - D_{t+1} \quad \forall t, \end{aligned} \tag{8}$$

where Θ_t^r are dividends from retail firms and Θ_t^b are profits from financial intermediation.[12] D_t is the sum of deposits with financial intermediaries and central bank debt securities.[13]

The first order conditions yield conventional Euler and labour supply equations

$$\frac{1}{C_t} = E_t\left\{\beta R_{t+1}\frac{1}{C_{t+1}}\right\}$$
$$W_t\frac{1}{C_t} = \frac{\xi}{1-H_t}.$$

Monetary Policy: As in Bernanke et al. (1999), the central bank sets the gross nominal risk-free interest rate R_t^n according to the following policy rule

$$\frac{R_t^n}{R^n} = \left(\frac{R_{t-1}^n}{R^n}\right)^{\rho_m}\left(\frac{\Pi_{t-1}}{\Pi}\right)^{\zeta} exp(\varepsilon_t^m),$$

where Π_t denotes gross inflation and ε_t^m the monetary policy shock with $\varepsilon_t^m \sim N(0, \sigma_m^2)$.

3 Model Analysis

3.1 Calibration

The calibration for the standard parts of my model, i.e., households, capital producers and retailers as well as the parameters governing firms' production technology are directly taken from Bernanke et al. (1999).

Parameters associated with the costly enforcement friction in intermediation are chosen in accordance with Gertler and Karadi (2011). The elasticity of substitution between the intermediate goods of the two subsectors, ρ_y is set to 0.9 as in Kurtzman and Zeke (2020).

The severity of the CSV friction for firms in the two subsectors is governed by the monitoring cost μ, the standard deviation of the idiosyncratic return to capital shock ω^j, and the survival rates, γ_c and γ_u. These determine steady state values of default probabilities, leverage ratios and default risk premia in the two subsectors. In my calibration of these parameters, I target two key statistics from Bernanke et al. (1999), namely an average leverage ratio of two and the annualised fraction of defaulting intermediate goods firms of three per cent.

To induce these values for leverage and the default rate under identical subsectors, I set the parameters relating to the CSV friction as follows: The standard deviation of the

[12]Note that Θ_t^b includes profits from private and central bank intermediation alike. Essentially , the consolidated government's only role is that of an additional intermediary which rebates any profits directly to the household. To the household, it makes no difference, whether it receives profits from private intermediaries or the central bank which is why intermediation profits enter the households budget constraint via a single term.

[13]In Bernanke et al. (1999), monetary balances also enter household utility. These are included to allow for seigniorage profits of the government and thereby government spending shocks. I abstract from government purchases and therefore also from a utility role of monetary balances.

idiosyncratic return to capital shock ω^j is set to $\sigma = 0.06$ and the auditing cost to $\mu = 0.45$, close to the value in Ottonello and Winberry (2020). The survival rates in the two subsectors are set to $\gamma_c = \gamma_u = 0.981$. In a setting where the intermediate goods firms are identical, the production sector of my model economy resembles the one in Bernanke et al. (1999). My model then corresponds to a combination of the representative firm economies of Gertler and Karadi (2011) and Bernanke et al. (1999) due to perfect aggregation across all firms.[14]

For my heterogeneous subsector setting, I adjust parameters such that one subsector is unconstrained while the mean economy-wide leverage ratio remains at around two.[15] I set the share of firms in the constrained subsector, s_c to 0.5 which is close to the steady state share of constrained firms in Ottonello and Winberry (2020). The survival rates in the two subsectors are set to $\gamma_c = 0.973$ and $\gamma_u = 0.986$. These values imply that constrained firms are on average approximately half as old as unconstrained firms.

Moreover, I set the steady state value of ψ_t to 0.05, which implies a moderate amount of central bank intermediation in steady state. A complete overview of all parameter values is provided in appendix B.1.

3.2 Credit policy transmission

This subsection considers the transmission of an expansionary credit policy shock, i.e., an exogenous increase in the share of credit intermediated by the central bank, $\psi_{i,t}$. I analyse a relatively persistent credit policy shock and set $\rho_\psi = 0.8$. Crucially, I also assume for now that credit policy takes the form of "across the board" intermediation, i.e., $\varepsilon^\psi_{c,t} = \varepsilon^\psi_{u,t}$ at all times. This means that the central bank intermediates the same fraction of each subsector's debt. Later on, I will relax this assumption to assess the effects of different credit policy implementation regimes.

3.2.1 Transmission under identical subsectors

To illustrate how a credit policy shock transmits through the economy, I first present the impulse responses in a setting where both firms are identical.

The magnitude of the credit policy intervention is set to just below 1.3 percentage points to induce an impact aggregate investment response identical to the one implied by a 25 basis point monetary policy shock as considered in Bernanke et al. (1999).[16] Figure 4 shows the corresponding impulse responses of key model variables.[17]

[14]The setup is thus similar to Rannenberg (2016) and qualitatively mimics the impulse responses to monetary policy and technology shocks therein.

[15]My objective is to construct a heterogeneous intermediate goods firm economy which is subject to the same mean level of "constrainedness" as in the identical subsector setting. In my setting, the degree to which firms are affected by the costly state verification friction is governed by their leverage. Alternatively, I could target the mean level of the survival rates or capital and output.

[16]The dynamic responses to a monetary policy shock in the identical subsector economy are presented in appendix B.2 and largely resemble those in Bernanke et al. (1999).

[17]Impulse response functions of other model aggregates are presented in appendix B.3

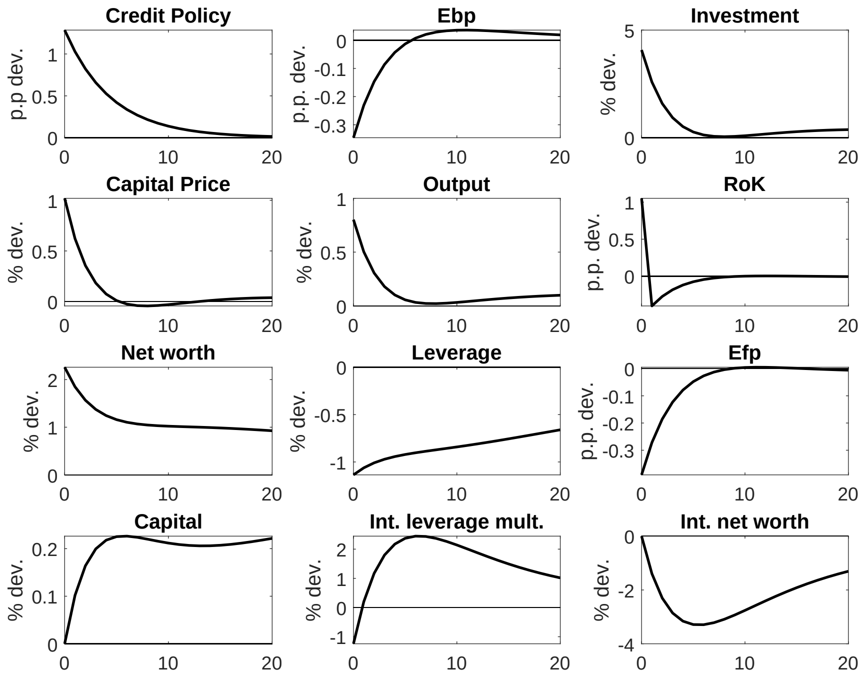


Figure 4: Dynamic consequences of a credit policy shock when firms are identical. Ebp = Excess bond premium, RoK = Return on Capital, Drp = Default risk premium, Efp = External finance premium, Int. = Intermediary.

The increase in central bank intermediation lowers the excess bond premium and thereby raises investment. This increases the price of capital, the ex-post realised return on capital and firm net worth. This increase in net worth further relaxes financing constraints of firms which has an amplifying effect on the demand for and the price of capital. This is the well-known financial accelerator mechanism. Ultimately, the credit policy shock leads to a reduction in leverage and the default risk premium.[18] The external finance premium falls while the amount of capital increases persistently. The lower return on capital leads to a gradual decrease in net worth and an increase in leverage.

The policy also affects the intermediary. Initially, the intermediary leverage multiple ϕ_t falls as a part of the credit is intermediated by the central bank. However, the fall in the excess bond premium, interpretable as the intermediary profit margin, reduces intermediary net worth. Combined with the gradual increase in lending as the credit policy shock subsides, this leads to an increase in the intermediary leverage ratio from the second period onwards. Overall, these effects are similar to the effects of government bond purchases in Kühl (2018).

3.2.2 Transmission under heterogeneous subsectors

I now consider the effects of the same 1.3 percentage point credit policy shock as in the previous subsection in a setting with heterogeneous subsectors. The corresponding impulse

[18] The reduction in default risk premia in response to large scale asset purchases has been empirically documented by Gilchrist and Zakrajšek (2013).

responses are depicted in figure 5.

While the overall transmission mechanism is similar, there are two important differences in the response of investment and capital compared to the setting with identical subsectors. First, the response of aggregate investment is much lower at less than one half of the response under identical subsectors. Second, there is a persistent reallocation of capital from unconstrained to constrained firms. While constrained subsector capital increases, capital by the unconstrained firms is persistently lower.

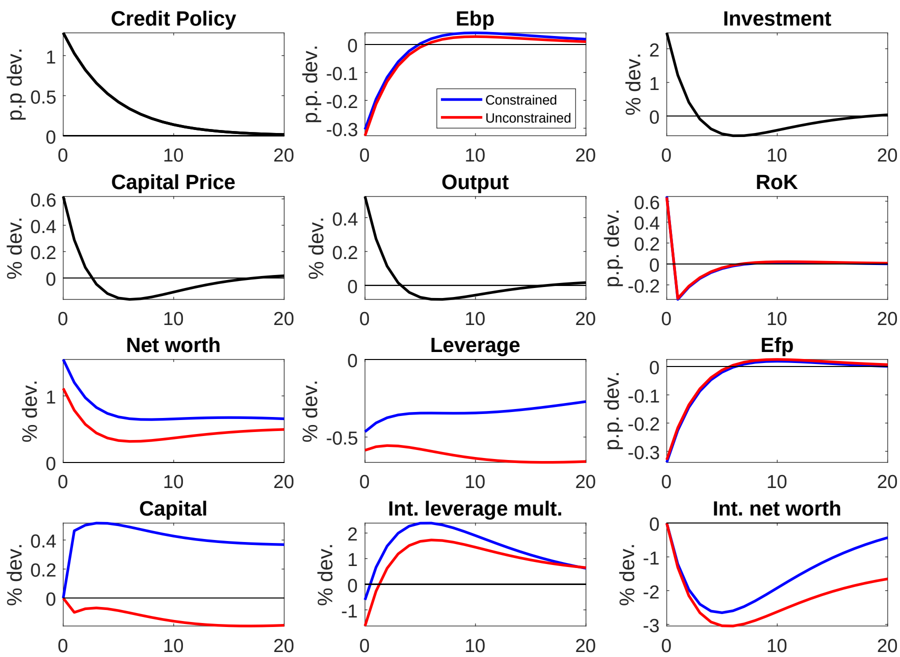
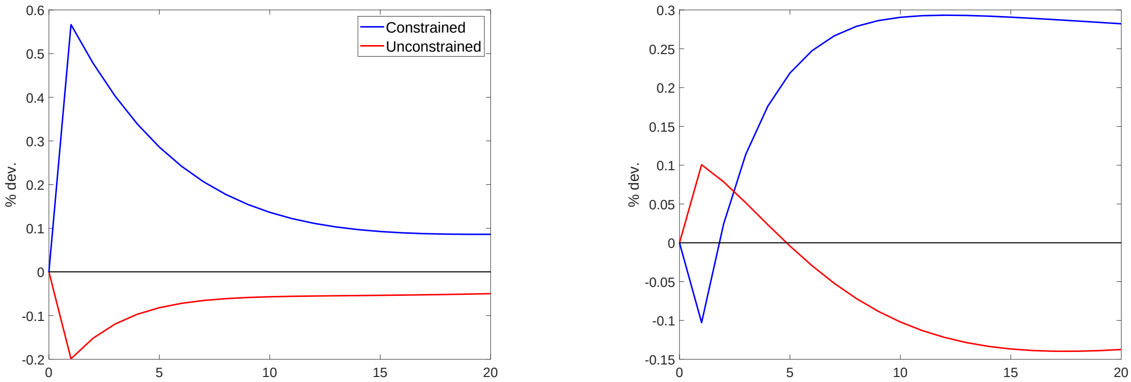


Figure 5: Dynamic consequences of a credit policy shock when firms are heterogeneous. Ebp = Excess bond premium, RoK = Return on Capital, Drp = Default risk premium, Efp = External finance premium, Int. = Intermediary.

To understand the mechanisms giving rise to these results, it is instructive to decompose the responses of the "across the board" credit policy intervention considered above into the parts induced by central bank intermediation of constrained subsector debt and unconstrained subsector debt respectively. The left panel of figure 6 shows the effect of a 1.3 per cent increase in central bank intermediation of constrained subsector debt and the right panel the effects of an equal increase in the intermediation of unconstrained subsector debt. Adding the impulse responses in the two panels yields the total capital response for each subsector shown in figure 5.

The transmission of credit policy can be separated into two parts: (i) the immediate effect of a temporary exogenous relaxation in funding conditions and (ii) the long-term effects via improvements in firm balance sheets.



(a) Constrained firm intermediation

(b) Unconstrained firm intermediation

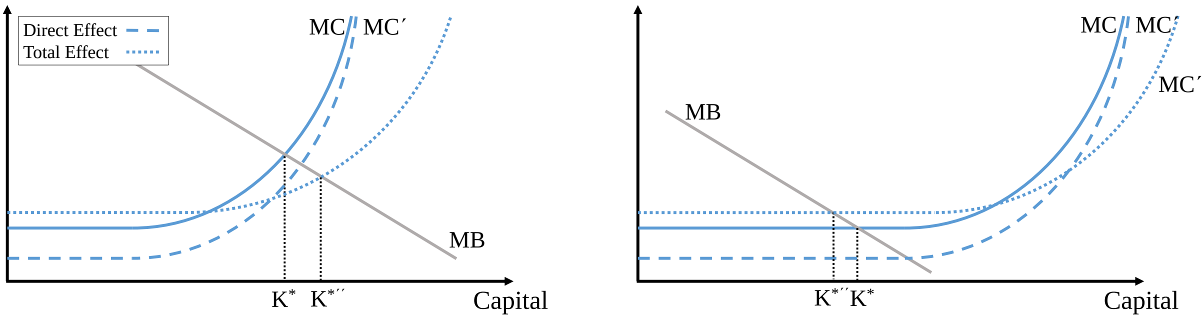
Figure 6: Dynamic consequences of a credit policy shock - decomposition of responses. Left panel: effect of intermediation in the constrained subsector; right panel: effect of intermediation in the unconstrained subsector. Note: Adding impulse responses yields response of intermediation “across the board” as shown in figure 5.

(i) Impact response: crowding out. On impact, the increase in central bank intermediation of constrained firms' debt (left panel in figure 6) substantially lowers the excess bond premium in this subsector. The ensuing increase in demand for capital leads to an increase in the price of capital. which induces unconstrained firms not affected directly by the policy to lower their capital.

By a similar reasoning, an increase in central bank intermediation in the unconstrained subsector (right panel in figure 6) leads to an increase in capital in that subsector and a decrease in capital in the constrained subsector. Therefore, in each case, there is an impact crowding out effect via the price of capital, leading to an opposing response of the two subsectors.

However, the magnitude of these opposing responses is different. The reason for this is the following: In the case of central bank intermediation in the constrained subsector, the crowding out effect is amplified by the financial accelerator which relaxes financing constraints, further raising capital demand by constrained firms. On the other hand, the same financial accelerator effect dampens the crowding out effect resulting from unconstrained subsector intermediation. On balance, the credit policy shock therefore leads to an expansion of the capital stock by constrained firms and a reduction by unconstrained firms on impact.

The intuition behind the differential reaction of capital between the two subsectors to an across the board credit policy shock is illustrated more explicitly in figure 7. Here I consider the effects of the increase in ψ_t (dashed lines) as well as the general equilibrium rise in the price of capital and net worth (dotted lines) for the choice of capital of generic firms in each of the two subsectors.



(a) Constrained firm (b) Unconstrained firm

Figure 7: Intuition behind the dynamic consequences of a credit policy shock. Dashed lines represent the immediate effect of an increase in central bank intermediation ψ_t; dotted lines represent the effects from general equilibrium increases in the price of capital and net worth.

The credit policy shock initially shifts down the marginal cost curve, which would induce an increase in the demand for capital by both subsectors.

In general equilibrium, the increase in capital demand raises the price of capital which affects the marginal cost curve in two ways. First, it shifts the marginal cost curve upwards, reducing the initial increase in capital demand by constrained and unconstrained firms. Second, the increase in the price of capital raises firm net worth and shifts the marginal cost curve to the right. This improvement in firm balance sheets amplifies the effect of an improvement in financing conditions following central bank intervention for constrained firms (more crowding out in the left panel of figure 6) and dampens the negative effect of the capital price increase for constrained firms (less crowding out in the right panel of figure 6). As unconstrained firms are already on the horizontal part of their marginal cost curve, the rightward shift in the marginal cost curve does not affect their demand for capital. For these firms, the overall effect of the increase in the price of capital is purely negative.

On balance, the general equilibrium increases in the price of capital and firm net worth imply that constrained firms expand their capital stock while unconstrained firms reduce their demand for capital as reflected in the impact response of capital depicted in figure 5.

(ii) Long-term response: reallocation. In equilibrium, the relative degree to which firms in the two subsectors are financially constrained determines the relative scale of production and capital employed.[19]

Central bank intermediation in either of the two subsectors leads to a differential increase in net worth. By the costly state verification friction, this alters the relative degree to which firms in the two subsectors are financially constrained. In particular, as their net worth rises, constrained firms become relatively less constrained compared to unconstrained firms and there is a reallocation of capital from the latter towards the former.

[19]See appendix A.5 for details. Intuitively, given constant returns to scale in production, the different degree to which firms are affected by financial constraints prevents the marginal products of capital to equalise across the subsectors.

As the crowding out effect of changes in the excess bond premium and the price of capital fades, this effect dominates such that, after some time, constrained capital is above steady state for an extended period of time while unconstrained capital is below steady state. Because this effect originates in the relative response of net worth, it equally emerges in response central bank intervention in either of the two firm subsectors (see appendix A.5 for details).

The effects described above play a crucial role for the aggregate consequences of credit policy. The crowding out effect leads to a muted impact response of investment compared to the identical subsector benchmark. The reallocation effect implies that more capital is in the hand of constrained firms compared to unconstrained firms. As the marginal product of capital is higher for constrained firms, this reallocation raises the average marginal product of capital and output (see Kurtzman and Zeke, 2020).

3.3 Discussion

Both the crowding out and the reallocation effects described above are also a feature of the model by Kurtzman and Zeke (2020). In their setting, crowding out and reallocation of capital across heterogeneous firm subsectors are a consequence of changes in relative credit spreads which are purely determined by the allocation of asset purchases towards particular subsectors. For example, purchases of securities issued by large (relatively less constrained) corporations lower credit spreads for these borrowers and raise their capital holdings at the expense of those of smaller corporations.

On the other hand, government bond purchases which leave relative credit spreads intact do not produce any reallocation of capital. Presumably, the same would be true in their model for firm asset purchases “across the board”.[20]

In contrast, by allowing for financial heterogeneity, my setting produces a reallocation of capital across subsectors, even if credit policy is conducted “across the board”. The reason is that the change in relative constraints giving rise to reallocation arises endogenously via the non-linearities in the degree to which firms are affected by financial frictions and financial accelerator effects on net worth. This leads to a potential divide between the direction of the immediate crowding out effect and the longer-term reallocation of capital.

Reallocation arises independently from the particular implementation of credit policy and can therefore be viewed as separate form the initial crowding out which may well depend on the implementation of credit policy: The crowding out effect always favours the subsector in which central bank intermediation predominantly takes place as in Kurtzman and Zeke (2020). In contrast, any implementation regime leads to a relative change in financing

[20] Abstracting from financial heterogeneity (as in the identical subsector setting above), my model produces very similar results (see figure 15 in appendix B.4). Targeting only one of the two subsectors leads to reallocation towards the affected subsector similar to Kurtzman and Zeke (2020). However, for purchases “across the board”, the effects of targeted purchases offset each other so that no reallocation takes place.

constraints in favour of constrained firms and thus to a reallocation of capital towards these firms.

The endogenous reallocation of capital in my model may also be relevant for the welfare consequences of the policy. In contrast to Kurtzman and Zeke (2020), capital is in my model always reallocated towards the more constrained subsector with a higher marginal product of capital, even if credit policy is targeted at unconstrained firms. As a result, credit policy targeting unconstrained firms does not necessarily lead to an efficiency loss as in Kurtzman and Zeke (2020).

3.4 The role of the financial accelerator

The strength of the crowding out effect and the presence of the reallocation effect originate in the fact that the financial accelerator affects the capital choice of firms in the two subsectors to a different degree. This subsection sheds more light on this finding by comparing my baseline results to a counterfactual setting in which the effect of the financial accelerator is shut off.[21]

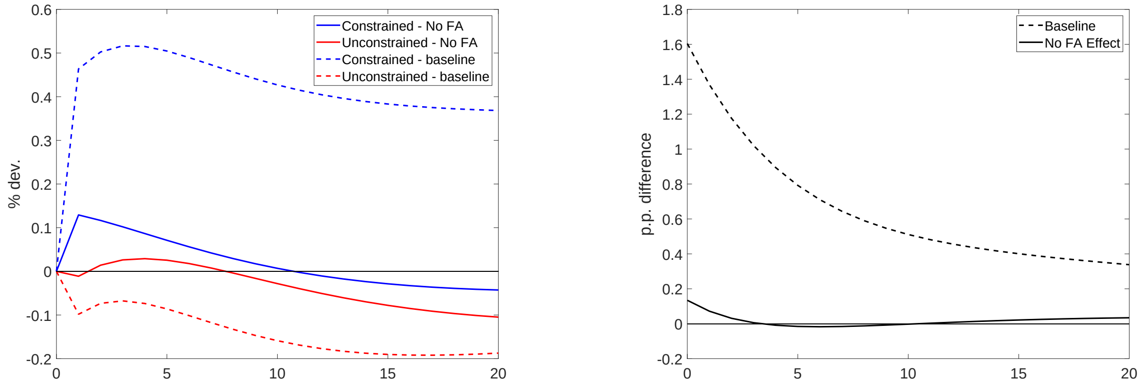
The left panel of figure 8 compares the capital response in the two subsectors when firm net worth is fixed at its steady state value to the baseline. Fixing net worth in this way breaks the feedback loop between firm net worth, financing conditions and the price of capital, which characterises the financial accelerator.

On impact, there is still a crowding out effect on the choice of capital by unconstrained firms. However, this crowding out effect is less pronounced as the absence of the financial accelerator limits the increase in the price of capital. Because there is no change in relative financial constraints when firm net worth is fixed, effectively no reallocation takes place in the long run.

The absence of the financial accelerator and the resulting limited crowding out effect also eliminates most of the difference between the investment response in the heterogeneous and identical subsector settings. The right panel of figure 8 compares the reduction in effectiveness caused by firm heterogeneity in a setting with and without the financial accelerator. In particular, it shows the difference between the investment responses to the same credit policy under identical and heterogeneous subsectors when the financial accelerator is active (dashed line) and when it is shut off (solid line).

In the baseline setting, there is a substantial difference in the investment response as explained in section 3.2 above. In contrast, the absence of the financial accelerator, leading

[21]The analysis in this subection potentially also affects the extent to which the results in my limited heterogeneity setup carry over to a more elaborate setting allowing for full heterogeneity across firms. As highlighted by Ottonello and Winberry (2020), the assumption of constant returns to scale implies a relatively stronger financial accelerator compared to diminishing returns as in their full heterogeneity setup. The reason is that the marginal propensity to invest out of net worth is larger under constant returns to scale. Part of the effects of credit policy in my model might therefore be caused by the relatively strong financial accelerator effect due to the assumption of constant returns to scale and therefore not necessarily arise in a full heterogeneity setup with diminishing returns.



(a) Capital response

(b) Diff. in Investment Response

Figure 8: Credit policy - role of the financial accelerator for capital and investment. Left panel: firm capital response in the baseline (dashed lines) and when the financial accelerator is inactive (solid lines); right panel: differential between aggregate investment responses in the baseline (dashed line) and when the financial accelerator is inactive (solid line)

to a muted crowding out effect, implies that the difference between the two responses is effectively eliminated.

3.5 Aggregate policy effectiveness

The previous analysis indicates that the aggregate investment response to a credit policy shock is shaped by general equilibrium increases in the price of capital and the financial accelerator. In this subsection, I provide a more detailed assessment of how these mechanisms quantitatively affect the stimulus provided by a given policy.

Using expressions for the marginal cost and marginal benefit of capital and linearising around steady state, the deviation of capital from steady state in the period after the impact of the shock can be written as:

$$\hat{K}_{t+1} = \Xi\left(\chi_{Q_l}\frac{\eta}{\delta K}E_t\{\hat{K}_{t+2}\} + \chi_S\hat{S}_{t+1} + \Psi(X_t, X, \theta)\right) \tag{9}$$

where

$$\Xi \equiv \frac{1}{1 - \frac{\eta}{\delta K}\left(\chi_Q + \chi_{N_c}h_{Q_c} + \chi_{N_u}h_{Q_u} - \chi_{Q_l}\right)}.$$

Here, $\hat{K}_{t+1}$ (and other variables analogously) denotes the absolute deviation in period $t+1$ of variable K from steady state. The χ-terms denote slope coefficients determined by the contributions of the respective variables to the marginal cost and marginal benefit of capital. Similarly, h_X denotes the derivative of net worth with respect to some variable X evaluated at steady state. The variable S_{t+1} stands for the excess bond premium and the term $\Psi(X_t, X, \theta)$ captures the effects of general equilibrium changes of any remaining endogenous variables unrelated to changes in the price of capital. Details of the derivations are provided in appendix A.6.

The derivation of equation (9) uses two main properties: First, firm net worth is a function of the price of capital via the resale value of capital. Second, the price of capital is endogenously determined by changes in the demand for capital. Expression (9) therefore endogenises all general equilbrium effects of the price of capital on the determination of capital.

In particular, the term Ξ captures the three ways in which the price of capital affects the demand for capital in general equilibrium:

1. Directly via the marginal cost of capital (with slope χ_Q).
2. Indirectly via net worth, leverage and the default risk premium ($\chi_{N_c} h_{Q_c} + \chi_{N_u} h_{Q_u}$).
3. Via the expected resale price and the marginal benefit of capital ($-\chi_{Q_l}$).

Given that general equilibrium increases in the price of capital are the dominant part of the effects of a change in the excess bond premium on capital, the magnitude of Ξ is the key determinant of the strength of the aggregate capital response. It is entirely determined by the slopes of the marginal cost and marginal benefit curves at steady state which, for a given subsector i, crucially depend on the value of γ_i. As such, the term Ξ is determined by the calibration of this parameter in the two subsectors.

Figure 9 displays the relationship between the calibration of γ_i and the value of Ξ. In particular, it visualises different magnitudes of Ξ relative to its value in the representative agent baseline in section 3.2.1 for combinations of different mean values of γ_i, given by $\bar{\gamma} = s_c\gamma_c + s_u\gamma_u$ and differences of the values of γ between the subsectors, $(\gamma_u - \gamma_c)$. While the former represents the degree to which firms are financially constrained on average, the latter represents the degree of financial heterogeneity between firms in the two subsectors.

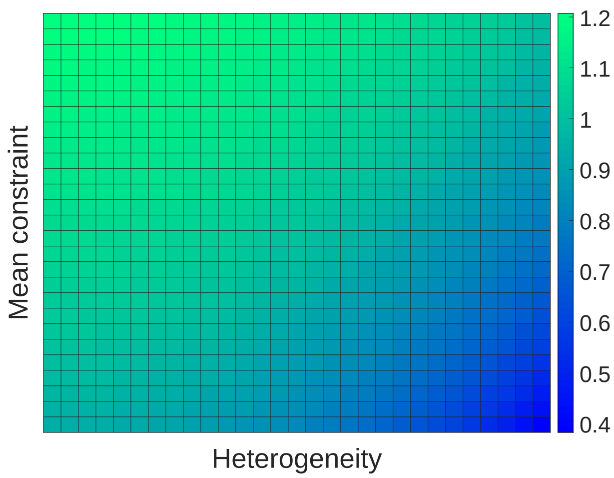


Figure 9: Magnitude of Ξ relative to representative firm baseline by mean financial constraint and degree of subsector heterogeneity. The baseline representative firm calibration subsection from 3.2.1 is in the bottom left corner. The baseline heterogeneous firm calibration from subsection 3.2.2 is in the bottom right corner.

Starting from the representative firms baseline calibration in the bottom left corner, the magnitude of Ξ increases in the mean degree to which firms are financially constrained and decreases in the degree of heterogeneity. The reason lies in the relative magnitudes of the effects of shifts in the marginal cost and marginal benefit curves for different slopes of these curves and the contribution of the financial accelerator (see figure 7).

Importantly, neither of the two metrics can on its own explain whether Ξ increases or decreases relative to the representative firm baseline. In assessing how effective a given credit policy is in stimulating investment, the central bank therefore needs to take both of these metrics into account. Ultimately, the interaction between the mean level of financial constraints and the degree of heterogeneity determines the effectiveness of credit policy.

More generally, my analysis highlights the importance of indirect general equilibrium effects associated with firm default risk in the transmission of central bank credit policy. This contrasts with the findings by Ottonello and Winberry (2020) where such general equilibrium effects are dominated by direct effects in the transmission of conventional monetary policy.

4 Applications

This section provides two applications of the insights on the transmission of credit policy from the previous section. First, it looks at the effects of credit policy under different implementation regimes. Second, it considers the ability of the central bank to respond to a financial crisis using credit policy.

4.1 Effects of credit policy implementation

When engaging in credit policy in practice, central banks have often imposed strict limits on the types of assets to be purchased. For example, both the Federal Reserve and the European Central Bank primarily engaged in purchases of investment-grade bonds to counteract increases in credit spreads following the Covid-19 shock in 2020. This raises the question whether the results of central bank intervention would have been different had they chosen to allocate their intermediation differently across firms.

In this section, I address this question by considering targeted central bank intermediation and its aggregate effects. Specifically, I compare the effects of three different implementation regimes. The baseline "across the board" implementation as well as the two limiting cases of "constrained only" and "unconstrained only" credit policy.

Due to differences in steady state borrowing, given intermediation shares correspond to different central bank lending volumes in the two subsectors. To allow for a sensible comparison, I adjust the shock size such that the *volume* of intermediation is identical to the baseline volume under all three implementation regimes.

Figure 10 depicts the investment responses to credit policy under the three implementation regimes when the total volume of intermediation is identical. Although not directly

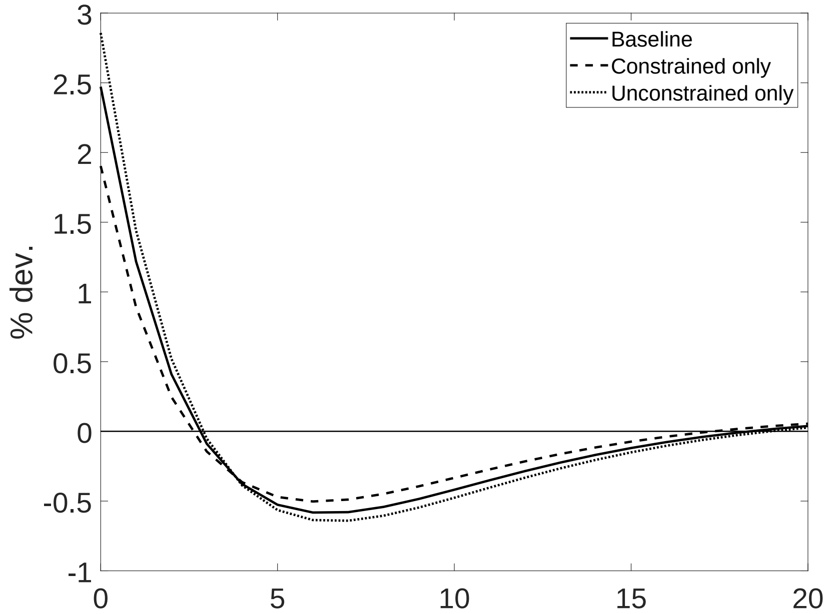


Figure 10: Aggregate investment response to a credit policy shock under different implementation regimes. Intermediation shares are adjusted to induce identical volumes of central bank intermediation across intermediation regimes.

affecting firms which are financially constrained, the impact response is strongest under the "unconstrained only" regime. This is due to the fact that the financial accelerator is dampening the crowding out effect when credit policy targets the unconstrained subsector whereas it amplifies it when the constrained subsector is targeted.

As a key insight, the implementation of credit policy is therefore important in my model. This is in line with other contributions on the effects of credit policy with heterogeneous firms such as Kurtzman and Zeke (2020) and Ferrari and Nispi Landi (2020). However, these setups abstract from default risk such that credit spreads and heterogeneity in financing conditions are entirely determined by credit policy induced changes in the tightness of intermediation frictions. In my setting, the implementation of credit policy matters even if intermediation frictions are the same for all firms. Differential effects across implementation regimes emerge purely from the financial heterogeneity between subsectors.[22]

Besides its role for aggregate investment, the implementation of credit policy also has distributional consequences. In the "constrained only" regime, credit policy helps constrained firms in the short run and raises their relative capital holdings in the long run. On the other hand, this policy is less effective overall. At the same time, constrained firms in the long run always benefit from the strengthening of balance sheets in response to an increase in net worth. As this strengthening of balance sheets is more pronounced under "unconstrained" only policy, constrained firms may benefit more under this implementation even if they are

[22]Gilchrist and Zakrajšek (2012) argue that individual default risk accounts for the largest part of credit spreads, justifying the inclusion of the CSV friction as a dominant factor in determining the external finance premium in my model.

not directly affected by the policy.

4.2 Credit policy in a financial crisis

Credit policy has typically been used to facilitate the extension of credit to firms at times when the provision of funds by private lenders is deemed inadequate. To take account of this observation, I now assess the ability of credit policy to stabilise disruptions caused by tightening financial conditions for firms.

Specifically, I consider the effectiveness of credit policy in counteracting the consequences of an adverse shock to the net worth of financial intermediaries. When assessing the effectiveness of credit policy, Gertler and Karadi (2011) consider shocks to the return on capital. As the lending contract is state contingent, this is equivalent to a shock to the return on intermediary assets inducing a drop in their net worth. In my setting, where debt is not state contingent, such a capital quality shock would not have an effect on intermediary net worth. To induce a tightening of lending standards as in Gertler and Karadi (2011), I therefore use an exogenous shock which directly affects intermediary net worth as proposed by Rannenberg (2016).[23]

As in the previous section, I illustrate the overall mechanism through which credit policy mitigates the consequences of an adverse shock in a setting where subsectors are identical. Subsequently, I compare this to the outcome under heterogeneous firm subsectors under different implementation regimes. In all cases, I consider a net worth shock which equally affects both types of intermediaries.

The comparability of credit policy effectiveness in the two settings is complicated by the fact that the shock itself affects the economy differently when the subsectors are heterogeneous. In the following, I consider intermediary net worth shocks leading to the same increase in the average external finance premium in the two settings. The rationale behind the focus on the external finance premium is that the availability of credit and firms financing costs appear to be main objectives when it comes to credit policy intervention (see e.g., BIS, 2020). In line with this reasoning, Gertler and Karadi (2011, 2013) formulate feedback rules where the central bank reacts to deviations of the external finance premium from steady state. The net worth shock necessary to induce the same increase in the average external finance premium is larger when subsectors are heterogeneous.

4.2.1 Identical subsectors

Figure 11 shows the dynamic consequences of a shock to intermediary net worth when there is no heterogeneity between intermediate goods firms in the two subsectors. In particular, firm survival rates are set to $\gamma_u = \gamma_c = 0.981$ as in subsection 3.2.1. Following Rannenberg

[23] Denoted by ε_t^b in equation (2.2) above

(2016), I consider the consequences of a purely transitory 5 per cent shock to intermediary net worth.

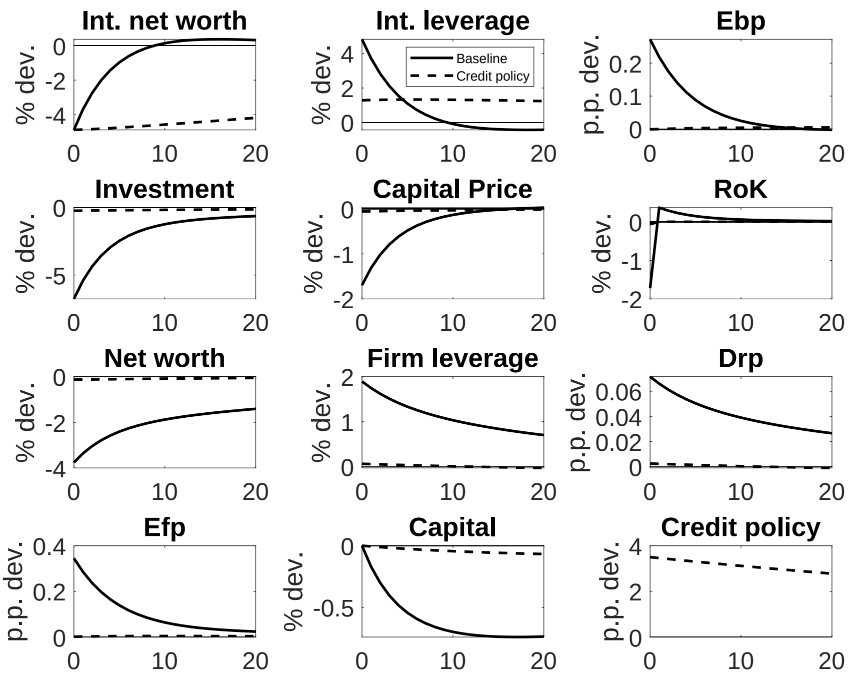


Figure 11: Dynamic consequences of a shock to intermediary net worth when firms are identical. Ebp = Excess bond premium, RoK = Return on Capital, Drp = Default risk premium, Efp = External finance premium, Int. = Intermediary.

Under the baseline of passive credit policy (solid lines), the reduction in intermediary net worth leads to an increase in intermediary leverage and the excess bond premium. The ensuing increase in firm financing costs reduces the demand for capital and therefore investment. This lowers the capital price, the return on capital and consequently firm net worth. As a result, firm leverage increases, leading to an increase in the default risk premium, which amplifies the drop in investment. Overall, the external finance premium increases and capital is persistently lower.

By engaging in credit policy, the central bank can alleviate the increase in the external finance premium. The increase in central bank intermediation lowers intermediary leverage sufficiently to fully stabilise the excess bond premium. As a result, all the other variables also remain largely unchanged. As shown in the bottom right panel of figure 11, complete stabilisation requires a relatively persistent increase in the fraction of funds intermediated by the central bank. The reason is the following: full stabilisation of the excess bond premium (which is equivalent to the intermediary profit margin) keeps the intermediary from rebuilding its net worth. To limit the increase in the excess bond premium despite this persistently low level of intermediary net worth requires a prolonged credit policy intervention.

4.2.2 Heterogeneous subsectors and the role of implementation

I now consider the ability of the central bank to stabilise the external finance premium when subsectors are heterogeneous. The size of the intermediary net worth shock is chosen to induce the same average impact response of the external finance premium as in the identical subsector setting above. As in section 4.1, I consider the possibility that credit policy may be directed towards different subsectors.

The responses to the intermediary net worth shock when subsectors are heterogeneous are generally comparable to those in the identical subsector specification. Moreover, when the central bank responds to the increase in the external finance premium by raising its intermediation share to the same extent as in the identical subsector setting, intermediary net worth and leverage respond very similarly to the credit policy intervention and the reactions of all the key variables are weakened (see figure 16 in appendix B.5).

However, the increase in central bank intermediation does not lead to a full stabilisation of the external finance premium and the other variables. In particular, as depicted in figure 12, the net worth shock still leads to a substantial reduction in investment despite the reaction of the central bank.

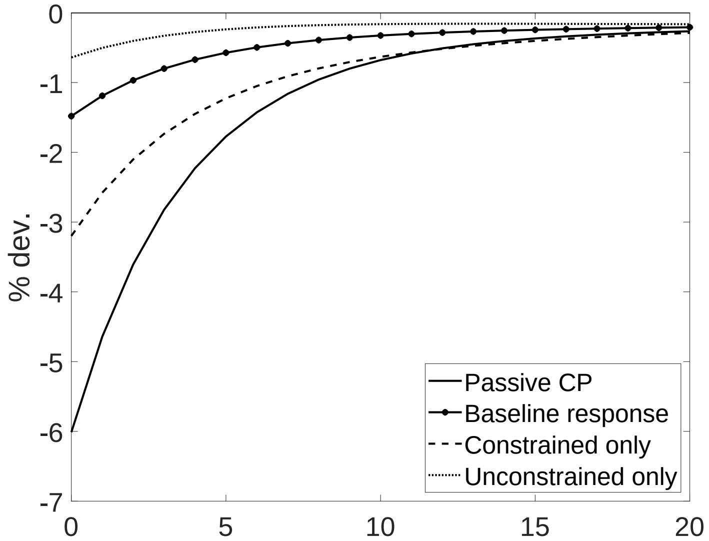


Figure 12: Investment response in a crisis with heterogeneous subsectors under different credit policy implementation regimes. The magnitude of the intervention is chosen to induce the same volume of central bank intermediation across regimes.

The reason lies in the reduced effectiveness of credit policy as established in section 3.2. Although the policy benefits some firms, the crowding out effect in general equilibrium reduces the effects this has on aggregate investment. The key result that credit policy is less effective when firms are heterogeneous therefore also holds when it comes to mitigating the consequences of a financial crisis.

The same is true for the finding that credit policy effectiveness depends on the subsector towards which it is targeted. Figure 12 also displays the response of investment to an active policy directed exclusively at constrained firms or unconstrained firms.[24]

Directing credit policy at the constrained subsector only, the central bank achieves a stabilisation to only half of the response in the absence of credit policy instead of a quarter under "across the board" intervention. On the other hand, by targeting the unconstrained subsector, the central bank can achieve close to full stabilisation despite the presence of heterogeneity between subsectors.

To summarise, the central bank is in principle able to counteract an increase in the external finance premium and stabilise investment in response to an exogenous shock to intermediary net worth. However, in the presence of heterogeneity, the effects of the credit policy are muted such that the full stabilisation policy under identical subsectors is insufficient to achieve full stabilisation under heterogeneous subsectors.

A central bank relying on a representative agent framework to assess the scope of its policy may therefore underestimate the volume of credit policy intervention necessary to achieve its goals. More generally, the ability of the central bank to react effectively in a crisis and the assessment of the desirability of such a policy may crucially depend on the composition of the firm sector.

Furthermore, the analysis shows that the central bank can achieve the highest effect of its policy when it targets unconstrained firms. The strategy often chosen by central banks in response to crises may therefore not only be justified on political grounds but also with regard to maximising policy effectiveness.

5 Conclusion

This paper considers the role of firm heterogeneity for the transmission of central bank credit policy in a New-Keynesian DSGE model with two kinds of financial frictions where firms are allocated to two ex-ante heterogeneous subsectors. My analysis shows that credit policy is in principle able to stimulate investment by improving financing conditions of firms.

However, in contrast to a setting with identical subsectors, a general equilibrium increase in the price of capital implies that the ability of credit policy to stimulate aggregate investment is significantly reduced. Moreover, capital is reallocated from unconstrained to constrained firms in response to the shock. The financial accelerator effect on firm balance sheets is crucial for these effects to emerge. My results are qualitatively independent of whether credit policy is targeted at the debt of constrained or unconstrained firms although the magnitude of the effects differs substantially.

[24] To ensure comparability, the magnitude of the intervention is again chosen to induce the same path of the volume of central bank intermediation as in section 4.1.

Furthermore, I find that firm heterogeneity reduces the effectiveness of credit policy in counteracting a financial crisis. At the same time, a strategy of credit policy targeted at unconstrained firms is most effective to mitigate the negative consequences of a financial crisis and provides substantial benefits also for constrained firms not directly affected by the policy.

My analysis suggests that financial heterogeneity matters for the aggregate effects of credit policy. Looking ahead, a quantitative assessment of the effectiveness of credit policy and how it depends on the distribution of firms along their financial position will require the analysis of credit policy in a full heterogeneity setup. This would also allow me to assess how additional dimensions of heterogeneity, i.e., movements across subsectors or within subsector heterogeneity, affect the results.

In the heterogeneous subsector framework presented in this paper, credit policy has considerable effects on the allocation of capital across the two subsectors. A promising complement to the positive analysis in this paper would therefore be to determine the conditions under which credit policy is on balance welfare improving and to investigate how these conditions depend on the firm distribution.

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Appendix

A Model derivations and intuition

A.1 Financial contract

In the following, I provide derivations to characterise the financial contract and show how it implies relationship (2) in the main text. The derivations closely follow those provided in appendix A in Bernanke et al. (1999).

Following Townsend (1979), lenders must pay a fixed auditing cost μ to be able to observe the idiosyncratic returns of potential borrowers. Under this "costly state verification" (CSV) assumption, the financial contract is characterised by a borrower-specific loan rate Z^j_{t+1} and a threshold value of the idiosyncratic shock $\bar{\omega}^j_{t+1}$. The threshold value is determined by:

$$\bar{\omega}^j_{t+1} R^k_{t+1} Q_t K^j_{t+1} = Z^j_{t+1} B^j_{t+1} \tag{A.1.10}$$

If $\omega^j \geq \bar{\omega}^j_{t+1}$, the firm honours its debt, repays $Z^j_{t+1} B^j_{t+1}$ and keeps the difference $\omega^j R^k_{t+1} Q_t K^j_{t+1} - Z^j_{t+1} B^j_{t+1}$. However, if $\omega^j < \bar{\omega}^j_{t+1}$, the firm cannot repay its debt using the proceeds of pledged capital and is forced to declare bankruptcy. In this case, the lender pays the auditing cost and seizes the borrower's return. In the case of bankruptcy, the lender thus receives $(1-\mu)\omega^j R^k_{t+1} Q_t K^j_{t+1}$ from firm j.

Lenders are modelled as competitive entities which collect funds from households and lend to intermediate goods firms. The costly enforcement problem in intermediation requires lenders to achieve a certain profit margin between a (hypothetical) riskless lending rate and the interest rate promised to depositors (see appendix A.4 below). In order for intermediaries to make zero profits in excess of this in expectation, the expected proceeds from the loan contract must equal those associated with the riskless lending rate to household depositors,R^b_{t+1}, i.e.,

$$[1 - F(\bar{\omega}^j_{t+1})] Z^j_{t+1} B^j_{t+1} + (1-\mu) \int_0^{\bar{\omega}^j_{t+1}} \omega R^k_{t+1} Q_t K^j_{t+1} dF(\omega) = R^b_{t+1} B^j_{t+1} \tag{A.1.11}$$

The first term on the left hand side of the equation (A.1.11) is the proceed in case of repayment weighted by the respective probability and the second term is the expected proceed in the case of bankruptcy. The right hand side of the equation is the opportunity cost of lending to the risky firm, i.e. the proceeds from riskless lending. Combining equation (A.1.11) with

(1) in the main text and (A.1.10) yields the following intermediary zero profit-condition:

$$\left[(1-F(\bar{\omega}^j_{t+1}))\bar{\omega}^j_{t+1}+(1-\mu)\int_0^{\bar{\omega}^j_{t+1}}\omega dF(\omega)\right]R^k_{t+1}Q_tK^j_{t+1}=R_{t+1}\left(Q_tK^j_{t+1}-N^j_{t+1}\right) \tag{A.1.12}$$

Given the financial contract, the expected return for the firm can be expressed in the following way:

$$E_t\left\{\int_{\bar{\omega}^j_{t+1}}^{\infty}\omega R^k_{t+1}Q_tK^j_{t+1}dF(\omega)-\left(1-F(\bar{\omega}^j_{t+1})\right)\bar{\omega}^jR^k_{t+1}Q_tK^j_{t+1}\right\} \tag{A.1.13}$$

In particular, the expected return consists of the expected gross return net of payments to intermediaries in non-defaulting states. The firm then chooses a specific debt contract characterised by an amount of capital and the associated default threshold $\left(K^j_{t+1},\bar{\omega}^j_{t+1}\right)$ to maximise (A.1.13) subject to (A.1.12).

To ease notation, it is convenient to reformulate the firm problem the following way:

$$\begin{aligned}\max_{\bar{\omega}^j,K^j_{t+1}} \quad & \left(E[\omega]-\Gamma(\bar{\omega}^j)\right)R^k_{t+1}Q_tK^j_{t+1}\\ \text{s.t.} \quad & \left(\Gamma(\bar{\omega}^j)-\mu G(\bar{\omega}^j)\right)R^k_{t+1}Q_tK^j_{t+1}=R^b_{t+1}\left(Q_tK^j_{t+1}-N^j_{t+1}\right)\end{aligned} \tag{A.1.14}$$

where

$$\begin{aligned}\Gamma(\bar{\omega}^j)&=\int_0^{\bar{\omega}^j}\omega dF(\omega)+\bar{\omega}^j\int_{\bar{\omega}^j}^{\infty}dF(\omega)\\ G(\bar{\omega}^j)&=\int_0^{\bar{\omega}^j}\omega dF(\omega)\end{aligned}$$

Here, $\Gamma(\bar{\omega}^j)$ can be interpreted as the expected gross share of the return going to the lender and $\mu G(\bar{\omega}^j)$ as the expected monitoring costs. Therefore, $\Gamma(\bar{\omega}^j)-\mu G(\bar{\omega}^j)$ denotes the return share accruing to the lender. According to the optimisation problem (A.1.14), the intermediate goods firm maximises its profits, equal to a share $(E[\omega]-\Gamma(\bar{\omega}^j))$ of the overall return on capital, subject to the zero profit condition of the intermediary. The latter prescribes that the part of profits going to the lender net of monitoring costs as implied by the financial contract must equal the opportunity cost of risky lending.

To see why the objective function in (A.1.14) is equivalent to (A.1.13), use the definitions of $\Gamma(\bar{\omega}^j)$ and $G(\bar{\omega}^j)$ to write:

$$\begin{aligned}
E[\omega]-\Gamma(\bar{\omega}^j) &= \int_0^\infty \omega dF(\omega) - \int_0^{\bar{\omega}^j} \omega dF(\omega) - \bar{\omega}^j \int_{\bar{\omega}^j}^\infty dF(\omega) \\
&= \int_{\bar{\omega}^j}^\infty \omega dF(\omega) - \bar{\omega}^j \int_{\bar{\omega}^j}^\infty dF(\omega) \\
&= \int_{\bar{\omega}^j}^\infty \omega dF(\omega) - \left(1-F(\bar{\omega}^j)\right)\bar{\omega}^j
\end{aligned}$$

Multiplying the last expression by $R^k_{t+1} Q_t K^j_{t+1}$ yields the objective (A.1.13). Similarly, the constraint in (A.1.14) can be rewritten as follows to show it is equivalent to the zero-profit-condition (A.1.12):

$$\begin{aligned}
\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j) &= \int_0^{\bar{\omega}^j} \omega dF(\omega) + \bar{\omega}^j \int_{\bar{\omega}^j}^\infty dF(\omega) - \int_0^{\bar{\omega}^j} \omega dF(\omega) \\
&= \bar{\omega}^j \int_{\bar{\omega}^j}^\infty dF(\omega) + (1-\mu) \int_0^{\bar{\omega}^j} \omega dF(\omega) \\
&= \left(1-F(\bar{\omega}^j)\right)\bar{\omega}^j + (1-\mu)\int_0^{\bar{\omega}^j} \omega dF(\omega)
\end{aligned}$$

The last expression is equivalent to the term inside the squared bracket on the left hand side of the zero profit condition (A.1.12).

The first order conditions of the intermediate goods firm optimisation problem (A.1.14) are:

$$\bar{\omega}^j: \quad \lambda\left(\Gamma'(\bar{\omega}^j) - \mu G'(\bar{\omega}^j)\right) = \Gamma'(\bar{\omega}^j) \tag{A.1.15}$$

$$K^j_{t+1}: \quad \left(E[\omega] - \Gamma(\bar{\omega}^j) + \lambda\left(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right)\right)\frac{R^k_{t+1}}{R^b_{t+1}} - \lambda = 0 \tag{A.1.16}$$

$$\lambda: \quad \left(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right)\frac{R^k_{t+1}}{R^b_{t+1}}\frac{Q_t K^j_{t+1}}{N^j_{t+1}} - \left(\frac{Q_t K^j_{t+1}}{N^j_{t+1}} - 1\right) = 0 \tag{A.1.17}$$

These first order conditions allow for a representation of the discounted return on capital $\frac{R^k_{t+1}}{R^b_{t+1}}$ and the leverage ratio $\frac{Q_t K^j_{t+1}}{N^j_{t+1}}$ in terms of the threshold productivity realisation $\bar{\omega}^j$. In particular, using FOC2 yields:

$$\frac{R^k_{t+1}}{R^b_{t+1}} = \frac{\lambda(\bar{\omega}^j)}{\Upsilon(\bar{\omega}^j)} \equiv \rho(\bar{\omega}^j) \tag{A.1.18}$$

where

$$\lambda(\bar{\omega}^j) = \frac{\Gamma'(\bar{\omega}^j)}{\Gamma'(\bar{\omega}^j) - \mu G'(\bar{\omega}^j)}$$

from FOC1 and

$$\Upsilon(\bar{\omega}^j) \equiv E[\omega] - \Gamma(\bar{\omega}^j) + \lambda\left(\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j)\right)$$

FOC3, using the expression for $\frac{R^k_{t+1}}{R^b_{t+1}}$ above, can be rearranged to:

$$\frac{Q_t K^j_{t+1}}{N^j_{t+1}} = \frac{\Upsilon(\bar{\omega}^j)}{E[\omega] - \Gamma(\bar{\omega}^j)} \equiv \psi(\bar{\omega}^j) \tag{A.1.19}$$

Inverting (A.1.19) to get $\bar{\omega}^j = \psi^{-1}\left(\frac{Q_t K^j_{t+1}}{N^j_{t+1}}\right)$ and substituting into (A.1.18) finally yields:

$$\frac{R^k_{t+1}}{R^b_{t+1}} = \rho\left(\psi^{-1}\left(\frac{Q_t K^j_{t+1}}{N^j_{t+1}}\right)\right) \equiv s\left(\frac{Q_t K^j_{t+1}}{N^j_{t+1}}\right) \tag{A.1.20}$$

Under the assumption of a log-normal distribution for ω^j, i.e., $ln(\omega) \sim N(-m\sigma^2, \sigma^2)$, $E[\omega] = e^{(0.5-m)\sigma^2}$. Also, defining $z \equiv \frac{ln(\bar{\omega}^j) - m\sigma^2}{\sigma}$,

$$\Gamma(\bar{\omega}^j) = \Phi(z-\sigma) + \bar{\omega}^j[1-\Phi(z)]$$
$$\Gamma(\bar{\omega}^j) - \mu G(\bar{\omega}^j) = (1-\mu)\Phi(z-\sigma) + \bar{\omega}^j[1-\Phi(z)]$$

where Φ is the c.d.f. of the standard normal (see Bernanke et al., 1999). Moreover, it generally holds that

$$\Gamma'(\bar{\omega}^j) = 1 - F(\bar{\omega}^j)$$
$$G'(\bar{\omega}^j) = \bar{\omega}^j f(\bar{\omega}^j)$$

This provides me with all the necessary parts to pin down the relationship in equation (A.1.20).

A.2 Marginal benefit and marginal cost of capital

This section provides more intuition on the relationship between leverage and the default risk premium, i.e., on the marginal cost relationship depicted in figure 2 in the main text. In the notation used so far, R^k_{t+1} is defined as the rate of return on a unit of capital *expenditure*. The rate of return on a unit of physical capital is thus $R^k_{t+1}Q_t$. In equilibrium, the normalised marginal cost of a unit of physical capital as is thus given by $s\left(\frac{Q_t K^j_{t+1}}{N^j_{t+1}}\right)Q_t$. For capital levels which can be entirely financed using net worth, i.e., internal finance, the marginal cost of capital *expenditure* is simply the opportunity cost of purchasing capital, i.e., the safe rate R^b_{t+1}. The normalised marginal cost of capital is therefore constant at Q_t. If the firm desires to purchase capital in excess of what it can finance internally, it needs to raise external finance. Rearranging (A.1.10) by inserting (1) and assuming $Z_{t+1} = R^k_{t+1}$ yields:

$$\bar{\omega}^j = 1 - \frac{N^j_{t+1}}{Q_t K^j_{t+1}}$$

For $N^j_{t+1} < Q_t K^j_{t+1}$, i.e., if the firm raises external financing to fund capital expenditure, $\bar{\omega}^j$ increases above zero, leading to a non-zero probability of default. The borrower must compensate the lender for the auditing cost associated with the possible default in order for the intermediary to still make zero (excess) profit. This required compensation implies that the marginal cost of raising external finance is higher than the safe rate R^b_{t+1} and there is a positive default risk premium. The normalised marginal cost of capital is now higher than the price of capital. The above expression for $\bar{\omega}^j$ shows that the larger the amount of desired capital expenditure relative to net worth, the larger is the default threshold and the higher is the probability of costly default. Higher levels of leverage are therefore associated with a higher required compensation, i.e., a higher default risk premium. Consequently, the normalised marginal cost of capital increases with capital in the region where additional capital is associated with more borrowing. This rationalises the positive slope of the normalised marginal cost curve for sufficiently high levels of capital.

A.3 Return on production inputs

This section provides the rationale for including subsector shares and relative prices of subsector output in the specification of factor prices, in particular the return on capital. The expression for $R^k_{i,t+1}$ presented in the main text, reflects the return of *aggregate* capital employed in a specific subsector. It features two elements: the resale price of non-depreciated aggregate capital and the marginal product of aggregate capital employed in subsector i. To obtain the marginal product of aggregate capital employed in subsector i, one must multiply the partial derivative of subsector i output with respect to subsector i capital by the partial derivative of subsector i capital with respect to aggregate capital (and by $\frac{1}{X_t}$ to obtain the marginal product in terms of final goods). The marginal product of aggregate capital employed in subsector i therefore obtains as

$$MPK_{i,t} = \frac{1}{X_t}\frac{\partial Y_t}{\partial K_t(i)} = \frac{\partial K_{i,t}}{\partial K_t}\frac{1}{X_t}\frac{\partial Y_t}{\partial Y_{i,t}}\frac{\partial Y_{i,t}}{\partial K_{i,t}} = s_i^{-1} P^f_{i,t}\frac{\partial Y_{i,t}}{\partial K_{i,t}}$$

where $K_t(i)$ denotes aggregate capital employed in sector i. The final term is the one which enters the expected return on capital in equation (4) below. This slightly elaborate specification is necessary because I want the two subsectors to have unequal weight. If the subsectors could be of equal size, then I could simply specify $K_t = K_{1,t} + K_{2,t}$ in which case the exposition would become somewhat simpler. By a similar reasoning, on the basis of the definitions of aggregate household and entrepreneurial labour, the term $s_i^{-i} P^f_{1,t}$ also appears in the specification of the household wage and the entrepreneurial wage.

A.4 Financial intermediation and credit policy

This section provides derivations relating to the costly enforcement problem outlined in section 2.2 in the main text. It characterises the leverage multiple for an intermediary specialised in subsector i, $\phi_{i,t}$, and shows how the friction intermediation induces the excess bond premium. The structure and notation closely follow Gertler and Karadi (2011).

Intermediary Leverage Multiple: The derivation of the intermediary leverage multiple starts from the intermediary's incentive constraint provided in the main text. Given that the marginal value of lending is positive, the intermediary will expand its assets up to the point where the incentive constraint A.4.21 holds with equality.

$$V_{i,t}^b = \lambda_i^b B_{i,t+1}^b \tag{A.4.21}$$

where $V_{i,t}$ is the continuation value of the intermediary. This continuation value can be written in the following way

$$V_{i,t}^b = \nu_{i,t}^b B_{i,t+1}^b + \eta_{i,t}^b N_{i,t}^b \tag{A.4.22}$$

where $\eta_{i,t}^b$ is the marginal gain to the intermediary of net worth and $\nu_{i,t}^b$ is the marginal gain of expanding assets. These will be specified in more detail below.

Combining the incentive constraint (A.4.21) with the expression for the continuation value (A.4.22) yields the following expression:

$$\lambda_i^b B_{i,t+1}^b = \nu_{i,t}^b B_{i,t+1}^b + \eta_{i,t}^b N_{i,t}^b$$

Rearranging this finally yields

$$B_{i,t+1}^b = \frac{\eta_{i,t}^b}{\lambda^b - \nu_{i,t}^b} N_{i,t}^b = \phi_{i,t} N_{i,t}^b \tag{A.4.23}$$

The marginal value of assets and net worth, $\nu_{i,t}^b$ and $\eta_{i,t}^b$ can be written recursively as:

$$\nu_{i,t}^b = E_t\left\{(1-\theta^b)\beta\Lambda_{t,t+1}(R_{i,t+1}^b - R_{t+1}) + \theta^b\beta\Lambda_{t,t+1}x_{i,t+1}\nu_{i,t+1}\right\}$$
$$\eta_{i,t}^b = E_t\left\{(1-\theta^b) + \theta^b\beta\Lambda_{t,t+1}z_{i,t+1}\eta_{i,t+1}\right\}$$

where $x_{i,t+1} \equiv \frac{B_{i,t+2}^b}{B_{i,t+1}^b}$ is the growth rate of intermediary assets (lending due in the subsequent period) and $z_{i,t+1} \equiv \frac{N_{i,t+1}^b}{N_{i,t}^b}$ is the growth rate of intermediary net worth from the current to the next period.

Both of these expressions contain two terms accruing to exiting and continuing bankers.

The marginal value of a unit of lending (assets) to the exiting banker is the difference between the interest payments expected to be received from borrowers $R^b_{i,t+1}$ and the risk-free interest rate representing the interest payment promised to depositors. The marginal value to the continuing banker is the expected discounted marginal value of (grown) assets in the subsequent period.

As bankers keep their net worth in full upon exit, the marginal value of a unit of net worth the exiting banker is one. The marginal value to the continuing banker is the expected discounted marginal value of (grown) net worth in the subsequent period.

The growth rates of assets and net worth follow directly from the evolution of net worth for exiting bankers and the formulation of assets in terms of the leverage multiple:

$$z_{i,t+1} = \frac{N^b_{i,t+1}}{N^b_{i,t}} = \frac{((R^b_{i,t+1} - R_{t+1})\phi_{i,t} + R_{t+1})N^b_{i,t}}{N^b_{i,t}} = (R^b_{i,t+1} - R_{t+1})\phi_{i,t} + R_{t+1}$$

$$x_{i,t+1} = \frac{B^b_{i,t+2}}{B^b_{i,t+1}} = \frac{\phi_{i,t+1} N^b_{i,t+1}}{\phi_{i,t} N^b_{i,t}} = \frac{\phi_{i,t+1}}{\phi_{i,t}} z_{i,t+1}$$

Credit policy takes the form of central bank intermediation of firm debt. In particular, total lending to intermediate good firms in subsector i is composed of funds from private intermediaries and the central bank:

$$B_{i,t+1} = B^b_{i,t+1} + B^c_{i,t+1} \tag{A.4.24}$$

The central bank issues government debt to households at the deposit rate R_{t+1} to extend credit to intermediate goods firms at the market (riskless) lending rate $R^b_{i,t+1}$. Unlike private intermediation, central bank intermediation is not subject to a costly enforcement problem and has thus perfect discretion over the amount of funds it wishes to lend to firms.

Assuming that the central bank chooses to intermediate a given share $\psi_{i,t}$ of total debt of subsector i firms, and reformulating private intermediation using the endogenous leverage condition (A.4.23), quation (A.4.24) can be rewritten as:

$$B_{i,t+1} = \phi_{i,t} N^b_{i,t} + \psi_{i,t} B_{i,t+1} = \phi^{cp}_{i,t} N^b_{i,t} \tag{A.4.25}$$

where $\phi^{cp}_{i,t} = \frac{1}{1-\psi_{i,t}}\phi_{i,t}$. The larger the share of subsector i lending the central bank is willing to intermediate, the higher the effective leverage multiple.

A.5 Allocation of capital across subsectors

This section establishes the determinants of the relative allocation of capital between the two subsectors. In particular, I consider the choice of capital of the respective representative firms in the two subsectors as determined by the the marginal cost and marginal benefit of capital in equations (A.6.27) and (A.6.28). The choice of capital in subsector i is characterised by

$$s\left(\frac{Q_t K_{i,t+1}}{N_{i,t+1}}\right) Q_t S_{t+1} = E_t\left\{\frac{1}{R_{t+1}}\left[s_i^{-1} P^f_{i,t+1}\frac{\alpha Y_{i,t+1}}{K_{i,t+1}} + (1-\delta)Q_{t+1}\right]\right\}.$$

Dividing the respective expression for the constrained firm by the same expression for the unconstrained firm yields

$$\frac{s\left(\frac{Q_t K_{c,t+1}}{N_{c,t+1}}\right)}{s\left(\frac{Q_t K_{u,t+1}}{N_{u,t+1}}\right)} = \frac{E_t\left\{s_c^{-1} P^f_{c,t+1}\frac{\alpha Y_{c,t+1}}{K_{c,t+1}} + (1-\delta)Q_{t+1}\right\}}{E_t\left\{s_u^{-1} P^f_{u,t+1}\frac{\alpha Y_{u,t+1}}{K_{u,t+1}} + (1-\delta)Q_{t+1}\right\}}. \tag{A.5.26}$$

The left hand side can be interpreted as the relative degree of to which the two firms are affected by the CSV friction, in other words the degree to which they are financially constrained. The right hand side is the return on capital for the constrained firm relative to unconstrained firm. As the resale price of capital is identical across firms, this is governed by the marginal product of capital employed in the constrained subsector relative to the marginal return on capital employed in the unconstrained subsector. As the production technology is identical across firms, this in turn is purely determined by the amount of capital in the constrained subsector relative to the unconstrained subsector, $K_{c,t+1}/K_{u,t+1}$ (given that the shares of the two subsectors are identical).

In a calibration where $N_{c,t+1} = N_{u,t+1}$ and, consequently, the marginal cost schedules for the two representative firms are identical, an identical capital choice across firms, i.e. $K_{c,t+1} = K_{u,t+1}$ clearly satisfies equation (A.5.26).

However, in my baseline calibration, $N_{c,t+1} < N_{u,t+1}$, such that the marginal cost schedule of the constrained firm is strictly below the one of the unconstrained firm for each level of capital. Equivalently, the left hand side of equation (A.5.26) is always below one. As a result, satisfying equation (A.5.26) requires $K_{c,t+1} < K_{u,t+1}$.[25]

How does the allocation of capital in response to a change in the two firms' relative net worth? For a given capital ration between the two subsectors, an increase in the relative net worth (e.g., following a credit policy shock) lowers the degree to which constrained firms are financially constrained relative to unconstrained firms. This decreases the left hand side of equation, (A.5.26) putting downward pressure on the relative return on capital and therefore upward pressure on the relative amount of constrained subsector

[25] Note that this requires that $P^f_{i,t+1}$ moves in same direction as $K_{i,t+1}$, which is always the case given that the marginal product of capital is positive.

capital, ${}^{K_{c,t+1}}\!/\!{}_{K_{u,t+1}}$.

Therefore, in response to a relative improvement of the constrained firm's balance sheet (net worth and leverage), the relative amount of capital of this firm will increase.[26] In the long-run, model aggregates revert back to steady state, while the relative net worth remains persistently distorted. Therefore, any remaining deviation of the optimal choice of capital in the two subsectors from steady state is due to the different relative degree of financial constraints. As these imply an increase in the relative capital choice of constrained firms, capital is above steady state for these firms and below steady state for unconstrained firms.

[26]The increase in net worth implies that the constrained firm moves closer to the identical firm case, driving the capital ratio closer to one.

A.6 Aggregate capital response

In what follows, I drop expectation operators for variables determined in the subsequent period to ease notation. I start from the following expressions for the marginal cost and marginal benefit of capital:

$$MC_t = s\left(\frac{Q_t K^j_{t+1}}{N^j_{t+1}}\right) Q_t S_{t+1} \tag{A.6.27}$$

$$MB_t = E_t\left\{\frac{1}{R_{t+1}}\left[s_i^{-1} P^f_{t+1}\frac{\alpha Y^j_{t+1}}{K^j_{t+1}} + (1-\delta)Q_{t+1}\right]\right\}, \tag{A.6.28}$$

Setting equations (A.6.28) and (A.6.27) equal and linearising around the steady state yields:

$$f_{Si}\hat{S}_{t+1} + f_{Ki}\hat{K}_{i,t+1} + f_{Qi}\hat{Q}_t + f_{Ni}\hat{N}_{i,t+1} = g_{Ki}\hat{K}_{i,t+1} + g_{Ri}\hat{R}_{t+1} + g_{Pi}\hat{P}^f_{i,t+1} + g_{Yi}\hat{Y}_{i,t+1} + g_{Qi}\hat{Q}_{t+1}$$

Solving for the deviation of subsector i capital from steady state yields:

$$\hat{K}_{i,t+1} = \frac{1}{f_{Ki}-g_{Ki}}\left(g_{Ri}\hat{R}_{t+1} + g_{Pi}\hat{P}^f_{i,t+1} + g_{Yi}\hat{Y}_{i,t+1} + g_{Qi}\hat{Q}_{t+1} - f_{Si}\hat{S}_{t+1} - f_{Qi}\hat{Q}_t - f_{Ni}\hat{N}_{i,t+1}\right)$$

Combining these two, I get for the response of aggregate capital:

$$\begin{aligned}
\hat{K}_{t+1} = &\left(\frac{s_c g_{Rc}}{f_{Kc}-g_{Kc}} + \frac{s_u g_{Ru}}{f_{Ku}-g_{Ku}}\right)\hat{R}_{t+1} \\
&+ \left(\frac{s_c g_{Qc}}{f_{Kc}-g_{Kc}} + \frac{s_u g_{Qu}}{f_{Ku}-g_{Ku}}\right)\hat{Q}_{t+1} \\
&- \left(\frac{s_c f_{Sc}}{f_{Kc}-g_{Kc}} + \frac{s_u f_{Su}}{f_{Ku}-g_{Ku}}\right)\hat{S}_{t+1} \\
&- \left(\frac{s_c f_{Qc}}{f_{Kc}-g_{Kc}} + \frac{s_u f_{Qu}}{f_{Ku}-g_{Ku}}\right)\hat{Q}_t \\
&+ \frac{s_c g_{Pc}}{f_{Kc}-g_{Kc}}\hat{P}^f_{c,t+1} + \frac{s_c g_{Yc}}{f_{Kc}-g_{Kc}}\hat{Y}_{c,t+1} - \frac{s_c f_{Nc}}{f_{Kc}-g_{Kc}}\hat{N}_{c,t+1} \\
&+ \frac{s_u g_{Pu}}{f_{Ku}-g_{Ku}}\hat{P}^f_{u,t+1} + \frac{s_u g_{Yu}}{f_{Ku}-g_{Ku}}\hat{Y}_{u,t+1} - \frac{s_u f_{Nu}}{f_{Ku}-g_{Ku}}\hat{N}_{u,t+1}
\end{aligned}$$

Now let the coefficients in front of the deviations of a given variable X from steady state on the right hand side of this equation be denoted by χ_X. Then, the expression can be written more compactly as:

$$\hat{K}_{t+1} = \chi_R\hat{R}_{t+1} + \chi_{Q_l}\hat{Q}_{t+1} + \chi_S\hat{S}_{t+1} + \chi_Q\hat{Q}_t \tag{A.6.29}$$

$$+ \chi_{P_c}\hat{P}^f_{c,t+1} + \chi_{Y_c}\hat{Y}_{c,t+1} - \chi_{N_c}\hat{N}_{c,t+1} + \chi_{P_u}\hat{P}^f_{u,t+1} + \chi_{Y_u}\hat{Y}_{u,t+1} - \chi_{N_u}\hat{N}_{u,t+1} \tag{A.6.30}$$

This expression illustrates the different effects contributing to changes in aggregate capital: The direct effect of a change in the excess bond premium ($\hat{S}_{t+1}$) via the marginal cost of capital, the indirect effects of changes in the price of capital ($\hat{Q}_t$) and the financial accelerator ($\hat{N}_{i,t+1}$) via the marginal cost and the indirect effects via the expected marginal benefit of capital. The most important component of this expected marginal benefit of capital is the expected resale price of capital in the next period $\hat{Q}_{t+1}$. In general equilibrium, firm net worth is an endogenous object which depends crucially on the price of capital. To take accout of this fact, I now express $\hat{N}_{i,t+1}$ as a function of $\hat{Q}_t$.

Determination of $\hat{N}_{i,t+1}$: As highlihted by BGG, the evolution of net worth is dominated by variation in firm equity V_t. Defining $\Gamma(\bar{\omega}_i)$ as the gross share of the return on capital going to the intermediary (see appendix A.1 above), the evolution of firm equity can be written as

$$V_{i,t} = R^k_{i,t} Q_{t-1} K_{i,t} (1 - \Gamma(\bar{\omega}_{i,t-1}))$$

Inserting expression (4) for the return on capital into this equation then yields

$$V_{i,t} = \left(s_i^{-1} P^f_{i,t} \frac{\alpha Y_{i,t}}{K_{i,t}} + (1-\delta) Q_t \right) K_{i,t} (1 - \Gamma(\bar{\omega}_{i,t-1}))$$

As also highlighted by BGG, unexpected changes in the price of capital ("asset prices") are a key driver of unexpected changes in firm equity and therefore net worth. Writing $N_{i,t+1} = h(Q_t, K_{i,t}, P^f_{i,t}, Y_{i,t}, \bar{\omega}_{i,t-1}, W^e_{i,t}, H^e_{i,t})$, the deviation of firm net worth from steady state is then given by:

$$\hat{N}_{i,t+1} = h_{Qi}\hat{Q}_t + h_{Ki}\hat{K}_{i,t} + h_{Pi}\hat{P}^f_{i,t} + h_{Yi}\hat{Y}_{i,t} + h_{\bar{\omega}i}\hat{\bar{\omega}}_{i,t-1} + h_{Wi}\hat{W}_{i,t} + h_{Hi}\hat{H}_{i,t}$$

Inserting the expression for $\hat{N}_{i,t+1}$ into equation A.6.30 and collecting terms associated with changes in the price of capital yields:

$$\begin{aligned}\hat{K}_{t+1} = &\left(\chi_Q + \chi_{N_c} h_{Q_c} + \chi_{N_u} h_{Q_u}\right)\hat{Q}_t + \chi_{Q_l}\hat{Q}_{t+1} + \chi_S \hat{S}_{t+1} + \chi_R \hat{R}_{t+1} \\ &+ \chi_{P_c}\hat{P}^f_{c,t+1} + \chi_{Y_c}\hat{Y}_{c,t+1} + \chi_{N_c}\left(h_{Kc}\hat{K}_{c,t} + h_{Pc}\hat{P}^f_{c,t} + h_{Yc}\hat{Y}_{c,t} + h_{\bar{\omega}c}\hat{\bar{\omega}}_{c,t-1} + h_{Wc}\hat{W}_{c,t} + h_{Hc}\hat{H}_{c,t}\right) \\ &+ \chi_{P_u}\hat{P}^f_{u,t+1} + \chi_{Y_u}\hat{Y}_{u,t+1} + \chi_{N_u}\left(h_{Ku}\hat{K}_{u,t} + h_{Pu}\hat{P}^f_{u,t} + h_{Yu}\hat{Y}_{u,t} + h_{\bar{\omega}u}\hat{\bar{\omega}}_{u,t-1} + h_{Wu}\hat{W}_{u,t} + h_{Hu}\hat{H}_{u,t}\right)\end{aligned}$$

For tractability, I will write this more compactly as:

$$\hat{K}_{t+1} = \chi_S \hat{S}_{t+1} + \left(\chi_Q + \chi_{N_c} h_{Q_c} + \chi_{N_u} h_{Q_u}\right)\hat{Q}_t + \chi_{Q_l}\hat{Q}_{t+1} + \Psi(X_t, X, \theta)$$

where $X_t = (R_{t+1}, P^f_{i,t+1}, Y_{i,t+1}, K_{i,t}, P^f_{i,t}, Y_{i,t}, \bar{\omega}_{i,t-1}, W_{i,t}, H_{i,t})$, X denotes the steady state values of variables in X_t and θ is a vector of parameters. Specifically,

$$\begin{aligned}\Psi(X_t, X, \theta) = {} & \chi_R \hat{R}_{t+1} \\ & + \chi_{P_c} \hat{P}^f_{c,t+1} + \chi_{Y_c} \hat{Y}_{c,t+1} + \chi_{N_c} \left(h_{Kc} \hat{K}_{c,t} + h_{Pc} \hat{P}^f_{c,t} + h_{Yc} \hat{Y}_{c,t} + h_{\bar{\omega}c} \hat{\bar{\omega}}_{c,t-1} + h_{Wc} \hat{W}_{c,t} + h_{Hc} \hat{H}_{c,t} \right) \\ & + \chi_{P_u} \hat{P}^f_{u,t+1} + \chi_{Y_u} \hat{Y}_{u,t+1} + \chi_{N_u} \left(h_{Ku} \hat{K}_{u,t} + h_{Pu} \hat{P}^f_{u,t} + h_{Yu} \hat{Y}_{u,t} + h_{\bar{\omega}u} \hat{\bar{\omega}}_{u,t-1} + h_{Wu} \hat{W}_{u,t} + h_{Hu} \hat{H}_u\end{aligned}$$

The part $\chi_S \hat{S}_{t+1}$ captures direct effect of a reduction in the excess bond premium on the demand for capital. The part $\left(\chi_Q + \chi_{N_c} h_{Q_c} + \chi_{N_u} h_{Q_u}\right) \hat{Q}_t + \chi_{Q_l} \hat{Q}_{t+1}$ captures the three ways in which the price of capital affects the quantity of capital in general equilibrium:

1. Directly via the marginal cost of capital (with slope χ_Q).
2. Indirectly via net worth, leverage and the default risk premium ($\chi_{N_c} h_{Q_c} + \chi_{N_u} h_{Q_u}$).
3. Via the expected resale price and the marginal benefit of capital ($-\chi_{Q_l}$).

The term $\Psi(X_t, X, \theta)$ captures the general equilibrium effects of any remaining variables unrelated to changes in the price of capital. As it turns out, the direct excess bond premium and these three general equilibrium effects via the price of capital dominate the overall response of capital to a change in the excess bond premium. Importantly, in general equilibrium, the price of capital is not an exogenous object but endogenously determined by the amount of capital. It obtains as follows:

Determination of $\hat{Q}_t$: From the specification of Q_t,

$$\begin{aligned}\hat{Q}_t &= -\left[\Phi'\left(\frac{I}{K}\right)\right]^{-2} \Phi''\left(\frac{I}{K}\right) \widehat{\left(\frac{I_t}{K_t}\right)} \\ &= \frac{\eta}{\delta} \widehat{\left(\frac{I_t}{K_t}\right)}\end{aligned}$$

To express this in terms of $\hat{K}_{t+1}$, I use the specification of the evolution of capital to write:

$$\begin{aligned}\hat{K}_{t+1} &= \Phi'\left(\frac{I}{K}\right) K \widehat{\left(\frac{I_t}{K_t}\right)} + \left(\Phi\left(\frac{I}{K}\right) + (1-\delta)\right) \hat{K}_t \\ &= K \widehat{\left(\frac{I_t}{K_t}\right)} + \hat{K}_t\end{aligned}$$

Solving for $\widehat{(I_t/K_t)}$ and inserting into the expression for $\hat{Q}_t$ then yields

$$\hat{Q}_t = \frac{\eta}{\delta} \left(\frac{\hat{K}_{t+1} - \hat{K}_t}{K}\right)$$

Applying this implies that capital can be expressed a function of parameters, the (here exogenous) variation in the excess bond premium and general equilibrium responses of remaining variables. The latter turn out to be negligible for reasonable calibrations.

$$\hat{K}_{t+1} = \chi_S \hat{S}_{t+1} + \chi_{Q_l} \frac{\eta}{\delta}\left(\frac{\hat{K}_{t+2} - \hat{K}_{t+1}}{K}\right) + \left(\chi_Q + \chi_{N_c} h_{Q_c} + \chi_{N_u} h_{Q_u}\right)\frac{\eta}{\delta}\left(\frac{\hat{K}_{t+1} - \hat{K}_t}{K}\right) + \Psi(X_t, X, \theta)$$

this can be rewritten as:

$$\begin{aligned}\hat{K}_{t+1} = {} & \chi_S \hat{S}_{t+1} + \Psi(X_t, X, \theta) \\ & + \chi_{Q_l} \frac{\eta}{\delta K} \hat{K}_{t+2} \\ & + \left(\chi_Q + \chi_{N_c} h_{Q_c} + \chi_{N_u} h_{Q_u} - \chi_{Q_l}\right) \frac{\eta}{\delta K} \hat{K}_{t+1} \\ & - \left(\chi_Q + \chi_{N_c} h_{Q_c} + \chi_{N_u} h_{Q_u}\right) \frac{\eta}{\delta K} \hat{K}_t\end{aligned}$$

In the initial period, $K_{i,t} = K_i$ and therefore $\hat{K}_{i,t} = 0$. I therefore arrive at the expression presented in the main text:

$$\hat{K}_{t+1} = \Xi\left(\chi_{Q_l} \frac{\eta}{\delta K} \hat{K}_{t+2} + \chi_S \hat{S}_{t+1} + \Psi(X_t, X, \theta)\right)$$

where

$$\Xi \equiv \frac{1}{1 - \frac{\eta}{\delta K}\left(\chi_Q + \chi_{N_c} h_{Q_c} + \chi_{N_u} h_{Q_u} - \chi_{Q_l}\right)}$$

This expresses the deviation of capital from steady state recursively as a function of next periods deviation of capital, the current deviation of the excess bond premium and further endogenous variables.

B Model Analysis

B.1 Calibration

Parameters calibrated directly

Parameter	Explanation	Value	Source
ξ	Disutility of labour	10.6	BGG
β	Discount factor	0.99	BGG
ϵ	Elasticity of substitution between retail goods	8	BGG
θ	Fraction of retailers unable to adjust prices	0.75	BGG
δ	Capital depreciation rate	0.025	BGG
η	Elasticity of capital price w.r.t. investment capital ratio	0.25	BGG
α	Capital share in production	0.35	BGG
Ω	Household labour share in total labour	0.99	BGG
ζ	Taylor rule weight on inflation	0.18	–
ρ_m	Taylor rule smoothing parameter	0.9	–
a	Share of constrained firms	0.528	OW
ρ_y	Elasticity of substitution between intermediate goods	0.8	KuZ
λ_b	Fraction of divertible intermediary assets	0.381	GK11
ω_b	Fraction of assets transferred to entering bankers	0.002	GK11
θ^b	Survival rate of bankers	0.972	GK11

Targeted steady states and corresponding parameter values

Metric/Parameter	Explanation	Value	Source
Targeted values			
$F(\omega)$	Default probability (quarterly)	0.0075	BGG
$\frac{QK}{N}$	Average leverage ratio of firms	2	BGG
$\frac{R_u^k}{R_u^b}$	Def. risk premium of unconstrained firms	1	-
Implied parameter values			
σ	Parameter governing distribution of ω	0.06	-
γ_c	Survival rate of constrained firms	0.973	-
γ_u	Survival rate of unconstrained firms	0.986	-
s_c	Share of constrained firms	0.5	-
μ	Monitoring cost to intermediary	0.45	-

Note: BGG = Bernanke et al. (1999), OW = Ottonello and Winberry (2020), KuZ = Kurtzman and Zeke (2020), GK = Gertler and Karadi (2011)

B.2 Monetary policy under identical subsectors

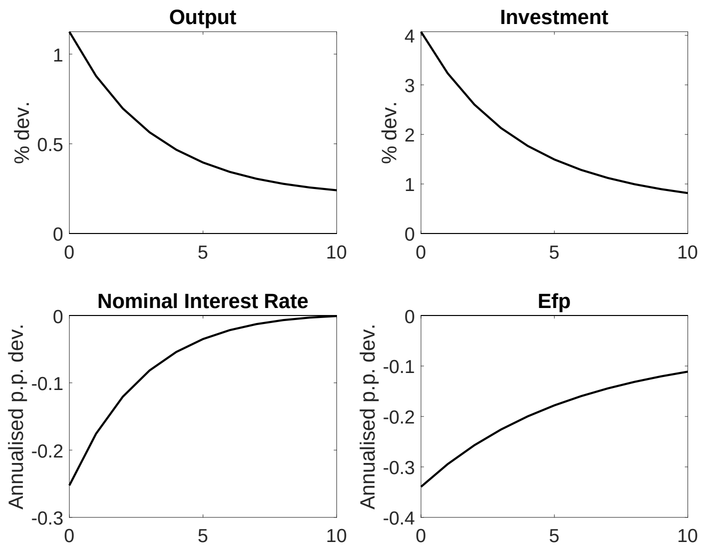

Figure 13: Dynamic consequences of a monetary policy shock - identical subsector benchmark

B.3 Credit policy - response of model aggregates

Figure 14 depicts the dynamic consequences of model aggregates to a 2.5 percentage point cent credit policy shock under identical (first row panels) and heterogeneous subsectors (second row panels).

Under identical firms, the stimulative effect of the credit policy shock is reduced after a few periods but remains positive for an extended period of time. The same is true for net output and consumption. In contrast, under heterogeneous firms, the price of capital rises much less in the first period, incentivising constrained firms to borrow more for investment in new capital. The ensuing deleveraging implies that net worth and thus capital is below steady state from the second period onwards as described above. As a consequence, aggregate investment, aggregate net output and consumption turn negative relatively shortly after the shock and remain below steady state for an extended period of time.

The above analysis indicates that firm heterogeneity not only reduces the initial stimulating effect of credit policy but also implies that the policy might have adverse effects in subsequent time periods. These potentially unintended consequences of credit policy cannot be observed when evaluating the policy based on a representative agent framework.

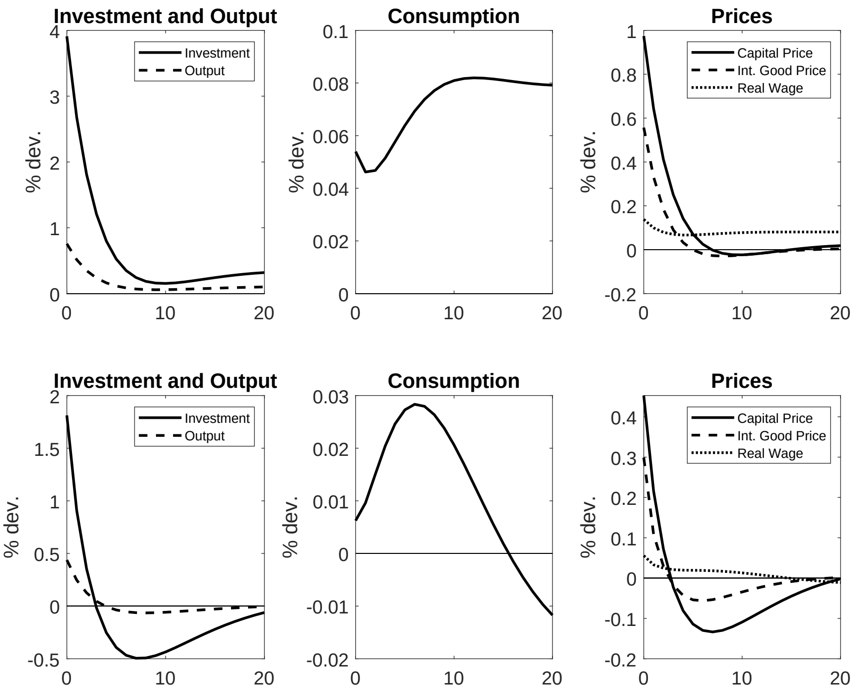

Figure 14: Dynamic consequences of a credit policy shock - model aggregates. Top row: identical firms; bottom row: heterogeneous firms

B.4 Credit policy under identical subsectors: decomposition of effects

Figure 15 shows a decomposition of the effects of “across the board” central bank intermediation of debt when intermediate good firms are identical analogous to figure 6 in the main text.

As in the heterogeneous subsector case, the crowding out mechanism goes in favour of the affected firms. However, in contrast to before, long term reallocation goes towards the unaffected subsector in each case. The reason is that affected firms aim to deleverage following the initial expansion of their capital stock. Also contrasting with the results under heterogeneous subsectors depicted in figure 6, the reallocation effects following purchases in each subsector offset each other perfectly such that “across the board” purchases do not lead to any reallocation. This is in line with the finding of Kurtzman and Zeke (2020) that government bond purchases which leave the relative credit costs unchanged do not lead to reallocation of capital across firms.

B.5 Credit policy in a crisis: heterogeneous firms

The consequences of a shock to intermediary net worth are presented in figure 16. Qualitatively, the responses to the intermediary net worth shock are comparable to those in the identical subsector specification. While the drop in the return on capital following the capital

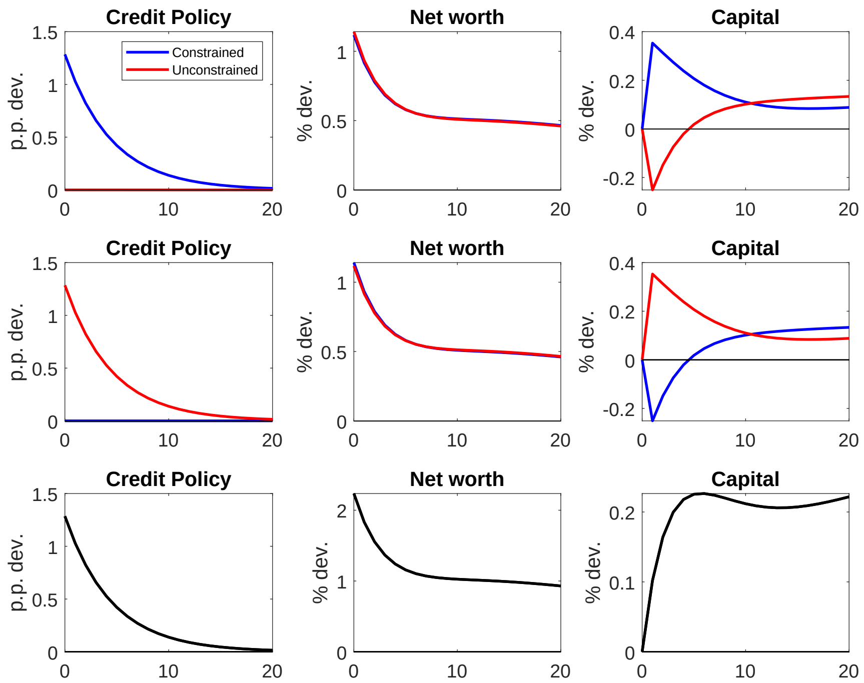

Figure 15: Dynamic consequences of a credit policy shock under identical subsectors - decomposition of responses. First row: effect of intermediation in the constrained subsector. Second row: effect of intermediation in the unconstrained subsector. Thrid row: response of intermediation "across the board" as shown in figure 4.

price decrease is identical for both subsectors, the ensuing fall in net worth is relatively lower for unconstrained firms as their steady state net worth is larger. Leverage increases in both subsectors but only lead to an increase in default risk of constrained firms (unconstrained firms remain on the horizontal segment of their marginal cost curve). The external finance premium rises in both subsectors and output falls. Despite the higher magnitude of the net worth shock, the response of model aggregates is quantitatively comparable between the two settings.

I now assume that the central bank responds to the increase in the external finance premium by raising its intermediation share to the same extent as in the identical subsector setting. Intermediary net worth and leverage respond very similarly to the credit policy intervention and the reactions of all the key variables displayed in are weakened.

However, the increase in central bank intermediation does not lead to a full stabilisation of the external finance premium and the other variables depicted in figure 16. In particular, the net worth shock still leads to a substantial reduction in investment and output, despite the reaction of the central bank.

The reason lies in the reduced effectiveness of policy as shown in the previous subsection. Although the policy benefits constrained firms, the crowding out effect on unconstrained firms in general equilibrium reduces the effects this has on aggregate investment. After a

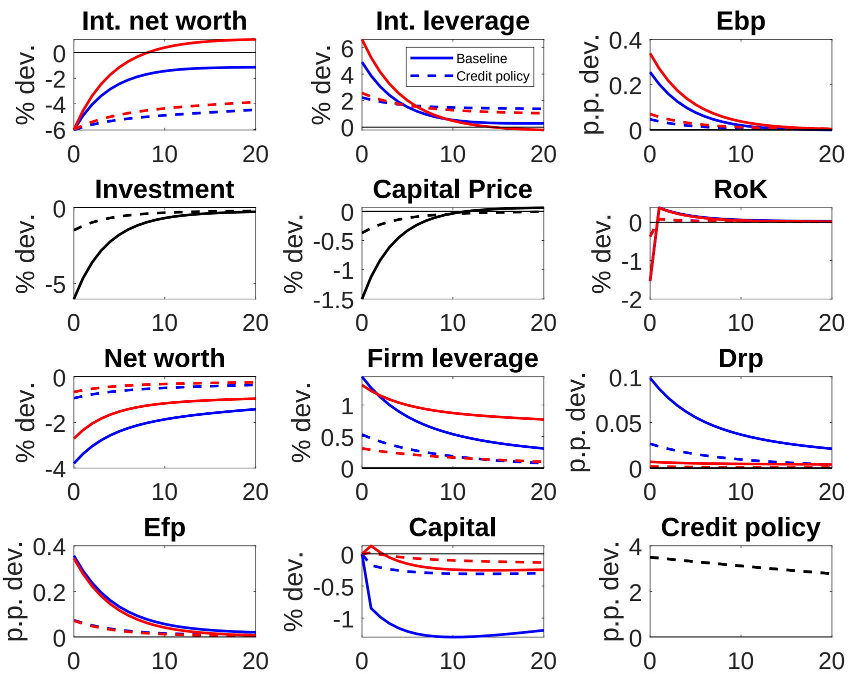

Figure 16: Dynamic consequences of a shock to intermediary net worth - heterogeneous firms

certain number of time periods, the investment and capital price response under active credit policy even lies below the one without credit policy. This reflects the fact that expansionary credit policy has an adverse effect on these two variables after some time as highlighted in section 3.2.2 above.[27]

[27]The negative effect occurs later here than in section 3.2.2 above because the credit policy is more persistent.