# Firm Revenue Elasticity and Business Cycle Behaviour

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**Abstract**

Recent literature on market power in macroeconomics notes the limitations of using revenue elasticities to proxy output elasticities when estimating markups. Although revenue elasticities may not unlock markups, we can – to some extent – circumvent markup estimation and use revenue elasticities to study business cycle dynamics. Using U.S. firm-level data, we measure revenue elasticities and estimate impulse responses using local projections. We present theory to show higher revenue elasticity firms generate greater business cycle amplification, and thus they are more cyclically sensitive than others. We find empirical evidence consistent with this theory.

**JEL**: E32, D2, L1.
**Keywords**: Amplification, Business Cycles, Markups, Revenue Elasticity, Output Elasticity.

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# 1 Introduction

Aggregate measures of price markups are informative for various macroeconomic topics. However, estimating markups requires *output elasticity* measures which are difficult to acquire across many industries because datasets with broad industry coverage include revenue but not price and output separately. Consequently, for aggregate analysis researchers often use *revenue elasticity* measures to proxy output elasticities. Bond, Hashemi, Kaplan, and Zoch (2021) explain the limitations of substituting output elasticity with revenue elasticity when estimating markups. Given this, we take a step back and ask: *What can revenue elasticities tell us about business cycle behaviour?* In other words, we circumvent the tricky issue of markup estimation and instead investigate this related and easy-to-acquire measure.

Our main focus is the role of firm-level revenue elasticities in propagating macroeconomic shocks. We present theory to relate output fluctuations to demand and supply shocks, conditional on revenue elasticity and markups. We use data on U.S. firms to test the relationship. In the theory section, our flexible model framework of firm production shows revenue elasticities are sufficient to infer firm-level revenue responses to economic shocks, which implies a firm's cyclical sensitivity depends on its revenue elasticity. The empirical section documents firms' revenue elasticities since the 1980s and test our theoretical relationship. We show i) Decreasing revenue elasticity on average, ii) Increasing dispersion in revenue elasticities across firms, and iii) Higher revenue elasticity firms are more sensitive to productivity shocks and aggregate fluctuations and thus cause greater amplification, which is consistent with the theory. We conclude that trends towards lower revenue elasticity implies firm-level amplification mechanisms are declining over the period.

The core relationship in our paper is between a firm's revenue elasticity, output elasticity and markup. Revenue elasticity is the response of firm revenue to a change in input. Output elasticity is the response of firm output to a change in input. Markup is the ratio of price to marginal cost. Later, we show they are related as follows:

$$\text{revenue elasticity } = \frac{\text{output elasticity}}{\text{markup}}.$$

The above relationship indicates that firms' revenue changes come from the supply and demand sides (output elasticity and markup, respectively). The relative value is important in shaping their revenue changes regardless of the absolute levels. Furthermore, since the markup is the ratio of price to marginal cost and the output elasticity is the ratio of average variable cost to marginal cost, we can write

$$\text{revenue elasticity } = \frac{\text{variable cost}}{\text{revenue}}.$$

The second relationship provides a simple accounting method to measure revenue elasticity at the firm level, whilst the first relationship emphasizes the difficulty in identifying output elasticity and/or markup given only revenue elasticity. Consequently, our paper will construct these simple measures of revenue elasticity and show what revenue elasticity *alone* can tell us about systematic differences in sensitivity of firms to the business cycle.

From firm-level data, this paper constructs revenue elasticity estimates at the firm-level as opposed to using production function estimation which leads to industry-level revenue elasticities. This allows us to classify firms as high-revenue elasticity or low-revenue elasticity. We show that when our firm-level measures of revenue elasticity are averaged to an industry level, they are strongly correlated with directly estimated industry-level elasticity from revenue production functions. Then, we document a decreasing trend in revenue elasticity in the U.S. since 1984. Firms' revenue is less responsive to changes in inputs than it was in the past. Firms with low revenue elasticities experience a sharper decline in revenue elasticity than firms with high revenue elasticity.

We test the relationship between revenue elasticities and responses to the business cycle and shocks. In particular, we use local projection estimation to acquire impulse responses to productivity shocks and aggregate business cycles. Our main finding is that firms with higher revenue elasticities are more sensitive to i) firm-level labour (revenue) productivity shocks, ii) aggregate-level total productivity shocks, and iii) the aggregate business cycle (GDP changes). This means that shocks are amplified when revenue elasticities are higher and dampened when they are lower.

Our findings show that despite the limitations of revenue elasticities in estimating markups, they are informative for business cycle research. In addition to this theoretical point, they also exhibit long-run trends similar to other measures in the market power literature. This is notable because our construction of revenue elasticity does not use production function estimation used in recent literature.

De Loecker, Eeckhout, and Unger (2020) document rising markups in the U.S. over recent decades using a production function estimation approach. We use the same dataset and cleaning procedures as them. However, our interest is constructing revenue elasticities at the firm level with our accounting approach, rather than constructing revenue elasticities at the industry level with production function estimation. Their paper has caused a surge in research on market power in the macroeconomy. Syverson (2019) reviews the literature, and Basu (2019) discusses the role of returns to scale. Bond, Hashemi, Kaplan, and Zoch (2021) urge caution when proxying output elasticity by revenue elasticity in the production function approach to markup estimation, and more generally there is a line of papers that uncovers biases in production function estimation when using this proxy (Marschak and Andrews 1944;

Klette and Griliches 1996). These papers provide the inspiration for our work, which focuses on what we can learn from revenue elasticities. Revenue elasticities are attractive because they are uncontroversial to measure and simple to acquire across many industries. Related to us, Hashemi, Kirov, and Traina (2021) also investigate what we can learn from revenue elasticities. They offer a general, non-parametric, conclusion that the production approach to markup estimation, does not uncover markups, but teaches us about input wedges.

Our paper is related to literature that studies the effect of output elasticity (returns to scale) and markups on amplification and propagation over the business cycle (e.g., Hall 1986; Hornstein 1993; Rotemberg and Woodford 1993; Devereux, Head, and Lapham 1996; Basu and Fernald 2001). Kaplan and Zoch (2020) show that output elasticity to labour (returns to scale in their framework) determines whether a change in markups accrues income to labour or profits. In addition to the propagation mechanism we study, existing literature shows that the ratio of output elasticity to markups (our revenue elasticity) affects other variables in the economy. For example, Hopenhayn (2014) shows that the ratio affects allocation across firms with heterogeneous productivity, and Atkeson and Kehoe (2005) propose a widely-used calibration of the ratio in an analysis of organizational capital. Lastly, our paper is related to literature on heterogeneous firm behaviour over the business cycle. It is well-documented that large and small firms make different financing decisions over the business cycle (Covas and Den Haan 2011; Begenau and Salomao 2018), and recently Burstein, Carvalho, and Grassi (2020) show that they also make different pricing decisions: large firms have procyclical markups, but small firms have counter-cyclical markups. Crouzet and Mehrotra (2020) find that large firms' sales are less sensitive to the business cycle than small firms. According to our paper, less sensitive sales for large firms implies lower revenue elasticities for large firms. This is consistent with declining trends in average revenue elasticity if large firms are gaining market share. There is widespread evidence of growing market shares of large firms in the U.S. economy (Grullon, Larkin, and Michaely 2019).

Section 2 presents a theoretical framework with revenue and output elasticity as well as markups. Then, Section 3 derives a theoretical relationship between revenue elasticity and a firm's cyclical sensitivity —revenue responses shocks. Section 4 covers data and revenue elasticity measurement. It presents descriptive analysis of revenue elasticity behaviour. Section 5 presents an empirical model that tests the theory. The last section concludes.

# 2 Theoretical Framework

We present a standard theory of production similar to Bond, Hashemi, Kaplan, and Zoch (2021). Firms have product market power and production has returns to scale. Alternatively to a two-stage problem, we could express the problem as a single-stage profit maximization problem subject to a demand and production constraints.[1] However, we intend to distinguish that cost minimization, not profit maximization, is necessary for the production approach to markup estimation and our results. This feature of the framework, and resulting scalability, that makes it adaptable to macroeconomic settings. Additionally, the relevant demand elasticity is the firm's 'perceived' demand elasticity, not 'proportional' demand elasticity. A firm's perceived demand curve follows from the assumption that when a firm changes its price other firms do not respond by charging the same price, whereas a proportional demand curve follows from the assumption that competitors also change their price to the firm's proposed new price. Lastly, our results are not restricted to monopolistic competition (Dixit and Stiglitz 1977) such that price elasticity of demand and markup are constant. Our framework applies to more general models of competition such as generalizations of monopolistic competition (Zhelobodko, Kokovin, Parenti, and Thisse 2012; Bertoletti and Etro 2017), translog preferences (Feenstra 2003; Bilbiie, Ghironi, and Melitz 2012) and Cournot or Bertrand oligopolistic competition (Atkeson and Burstein 2008; Etro and Colciago 2010).

## 2.1 Environment

Firms solve a two-stage problem in each period. First, a firm indexed by $j = 1, \cdots, N$ chooses inputs to minimize variable costs subject to a production function. There is no factor market power (monoposony power), so cost-minimizing firms take input costs as given. Firm $j$'s production function is given by

$$Q_j = \mathcal{F}_j(A_j X_j). \tag{1}$$

The variables $Q_j$, $X_j$, and $A_j$ denote firm $j$'s output, variable input, and factor-augmenting productivity.[2] The production function $\mathcal{F}_j(\cdot)$ is twice continuously differentiable and

[1] In other words, we could express the problem as follows:

$$\max_{Q_j, X_j, P_j} \quad P_j Q_j - W X_j - \mathrm{FC}_j \qquad \text{s.t.} \qquad Q_j = \mathcal{D}_j(P_j) \quad \text{and} \quad Q_j = \mathcal{F}_j(A_j X_j).$$

[2] Here, we implicitly assume one input. With $M$ inputs, we should introduce an aggregating function $\mathcal{G}_j : \mathbb{R}_+^M \to \mathbb{R}_+$ satisfying $Q_j = \mathcal{F}(\mathcal{G}(X_j^1, \cdots, X_j^M))$ where $\mathcal{G}_j(\cdot)$ is homogeneous of degree one.

strictly concave. The output elasticity is

$$\frac{\partial \mathcal{F}_j}{\partial X_j}\frac{X_j}{Q_j} = \frac{\partial \mathcal{F}_j}{\partial (A_j X_j)}\frac{A_j X_j}{Q_j}. \tag{2}$$

Second, given their cost-minimizing input choices, the firm chooses output to maximize profits subject to a demand constraint. The demand is represented as the maximizing the following aggregator of products

$$U = \max_{\{Q_j\}_{j=1}^N} \mathcal{U}(Q_1, \cdots, Q_N) \tag{3}$$

subject to a budget constraint. The first-order condition yields that the firm faces the following inverse demand:

$$P_j = \frac{1}{\lambda_j}\frac{\partial \mathcal{U}}{\partial Q_j} \tag{4}$$

where $P_j$ is the price, and $\lambda_j$ is the budget constraint's Lagrangian multiplier. The firm's price (inverse demand) depends on its output. At this point, we introduce a demand, or preference, shock $\xi_j$ that affects the marginal utility directly. Hence, we re-express the first-order condition as

$$P_j = \xi_j \mathcal{P}_j(Q_j). \tag{5}$$

The corresponding demand function is $Q_j = \mathcal{D}_j(P_j/\xi_j)$ where $\mathcal{D}_j(\cdot)$ is an inverse function of $\mathcal{P}_j(\cdot)$ that is strictly deceasing and twice continuously differentiable. The price elasticity of demand is defined by

$$-\frac{\partial \mathcal{D}_j}{\partial P_j}\frac{P_j}{Q_j} = -\left[\frac{\partial \mathcal{P}_j}{\partial Q_j}\frac{Q_j}{P_j}\right]^{-1}, \tag{6}$$

where we assume its value is larger than one. From the output and inverse demand functions, the revenue function is given by

$$P_j Q_j = \xi \mathcal{P}_j(Q_j) Q_j = \xi \mathcal{P}_j(\mathcal{F}_j(A_j X_j))\mathcal{F}_j(A_j X_j) = \mathcal{R}_j(A_j X_j; \xi_j). \tag{7}$$

Since the inverse demand curve depends on output, a firm's revenue elasticity includes an indirect effect of changing input on demand and also a direct effect of changing input. Therefore, revenue elasticity is given by

$$\frac{\partial \mathcal{R}_j}{\partial X_j}\frac{X_j}{P_j Q_j} = \left[\frac{\partial \mathcal{P}_j}{\partial Q_j}\frac{\partial \mathcal{F}_j}{\partial X_j} + P_j \frac{\partial \mathcal{F}_j}{\partial X_j}\right]\frac{X_j}{P_j Q_j} = \left[-\left(\frac{\partial \mathcal{D}}{\partial P_j}\frac{P_j}{Q_j}\right)^{-1} + 1\right]\frac{\partial \mathcal{F}_j}{\partial X_j}\frac{X_j}{Q_j}. \tag{8}$$

The left-hand side is our definition of revenue elasticity and the right-hand side shows that it is the product of the price elasticity of demand (6) and output elasticity (2). Revenue elasticity and output elasticity are equivalent when price is independent of output such that there is a perfectly elastic demand curve, $(\partial\mathcal{D}/\partial P_j)P_j/Q_j = \infty$.

### 2.2 Cost Minimization

Firms choose inputs to minimize (variable) costs subject to the production constraints. Then, the cost function becomes a function of $Q$ and $W/A$ when $A$ is a factor-augmenting productivity.

$$\mathcal{C}_j(Q_j; W/A_j) := \min_{X_j} \quad WX_j \qquad \text{s.t.} \quad Q_j = \mathcal{F}_j(A_j X_j) \tag{9}$$

The first-order condition ($W = \lambda_j \partial\mathcal{F}_j/\partial X_j$) implies that the variable cost function at optimality is

$$WX_j = \lambda_j \frac{\partial\mathcal{F}_j}{\partial X_j} X_j = \frac{\partial\mathcal{C}_j}{\partial Q_j} \times Q_j \times \frac{\partial\mathcal{F}_j}{\partial X_j}\frac{X_j}{Q_j}, \tag{10}$$

where the Lagrange multiplier $\lambda_j$ is equivalent to marginal cost: $\lambda_j = \partial\mathcal{C}_j/\partial Q_j \equiv \mathrm{MC}_j$. The cost function (10) implies that the inverse variable cost elasticity is equal to the output elasticity. The inverse variable cost elasticity is the ratio of average variable cost to marginal cost, where $\mathrm{AVC}_j = WX_j/Q_j$, hence

$$\left(\frac{\partial\mathcal{C}_j}{\partial Q_j}\frac{Q_j}{WX_j}\right)^{-1} = \frac{\mathrm{AVC}_j}{\mathrm{MC}_j} = \frac{\partial\mathcal{F}_j}{\partial X_j}\frac{X_j}{Q_j}. \tag{11}$$

Using Shephard's lemma that factor demand equals the derivative of the cost function with respect to factor price $X_j = \partial\mathcal{C}_j/\partial W_j = A_j^{-1}\partial\mathcal{C}_j/\partial(W/A_j)$, we can show that the elasticity of variable costs with respect to an efficiency unit factor $(W/A_j)$ is

$$\frac{\partial\mathcal{C}_j}{\partial(W/A_j)}\frac{(W/A_j)}{WX_j} = A_j X_j \frac{1}{A_j X_j} = 1. \tag{12}$$

This well-established result shows that the cost function is homogeneous of degree one in efficiency-unit factor prices. Equations (10) and (12) later inform our log-linearization.

## 2.3 Profit Maximization

Firms maximize profits by choosing price and output subject to the inverse demand function. $\mathrm{FC}_j$ is a fixed cost.

$$\max_{P_j, Q_j} \quad P_j Q_j - \mathcal{C}_j(Q_j; W/A_j) - \mathrm{FC}_j \qquad \text{s.t.} \quad P_j = \xi_j \mathcal{P}_j(Q_j). \tag{13}$$

The first-order condition rearranges to give the following optimality condition of operating firms:

$$P_j \left(1 + \frac{\partial P_j}{\partial Q_j} \frac{Q_j}{P_j}\right) = \mathrm{MC}_j. \tag{14}$$

Defining the markup as $\mu_j \equiv P_j/\mathrm{MC}_j$ and using equations (6) and (14) yields the familiar expression for the markup as a function of price elasticity of demand:

$$\mu_j = \left(-\frac{\partial \mathcal{D}_j}{\partial P_j} \frac{P_j}{Q_j}\right)\left(-\frac{\partial \mathcal{D}_j}{\partial P_j} \frac{P_j}{Q_j} - 1\right)^{-1}. \tag{15}$$

## 2.4 Revenue Elasticity

Next we present two results about revenue elasticity. Denote the revenue and output elasticity by

$$\zeta_j \equiv \frac{\partial \mathcal{R}_j}{\partial X_j} \frac{X_j}{P_j Q_j} \quad \text{and} \quad \gamma_j \equiv \frac{\partial \mathcal{F}_j}{\partial X_j} \frac{X_j}{Q_j}. \tag{16}$$

Equations (2), (6), (8), (11), and (15) combine to give expressions for revenue elasticity.

First, revenue elasticity is the ratio of output elasticity to the markup. Using the definitions of output and price elasticities in equations (2) and (6), equations (8) and (15) yield the first result:

$$\zeta_j = \frac{\gamma_j}{\mu_j}. \tag{17}$$

The first result (17) implies that revenue elasticity and output elasticity are not equivalent in the presence of markups.

Second, revenue elasticity is equal to the ratio of firm variable costs to firm revenue. The second result shows that we can construct a firm-level measure of revenue elasticity. Firm-level variable cost and revenue data are easily available, so the result implies revenue elasticities are simple to construct. In equation (17), replace the output elasticity ($\gamma_j$) by the average variable and marginal costs ($\mathrm{AVC}_j$ and $\mathrm{MC}_j$) using equation (11). Since $\mu_j = P_j/\mathrm{MC}_j$, we obtain the following equation:

$$\zeta_j = \frac{W X_j}{P_j Q_j}. \tag{18}$$

The second result also gives a helpful interpretation of revenue elasticity. Revenue elasticity is the share of revenue paid to variable factors of production.[3]

We can also interpret revenue elasticity in terms of profits. Define profits ($\Pi$) as revenue ($P_jQ_j$) less variable costs ($WX_j$) less fixed costs ($FC_j$), so $\Pi_j = P_jQ_j - WX_j - FC_j$. Divide profits by revenue which gives the profit share in revenue as a function of variable cost and fixed cost shares in revenue: $s_{\Pi,j} = 1 - s_{\mathcal{C},j} - s_{FC,j}$. Since revenue elasticity is the variable cost share in revenue, then revenue elasticity is equal to the profit and fixed cost share remainder: $\zeta_j = 1 - s_{\Pi,j} - s_{FC,j}$. Therefore a decline in revenue elasticity implies a rise in profit and/or fixed cost share in revenue.

The main point of this section is that revenue elasticity is simple to construct, but given revenue elasticity we cannot identify output elasticity and markups separately. Therefore in the next section, we ask: what can we learn from revenue elasticity without decomposing between output elasticity and markup?

# 3 Theoretical Results

This section shows that a higher revenue elasticity firm has greater cyclical sensitivity. That means worsening economic conditions are associated with systematically larger declines in revenue.

## 3.1 Business Cycle Framework

We begin by deriving a revenue function in terms of log changes and elasticities from the static revenue function. The differential form of equation (7) is

$$\mathrm{d}\ln P_jQ_j \approx \left(\frac{\partial \mathcal{R}_j}{\partial X_j}\frac{X_j}{P_jQ_j}\right)(\mathrm{d}\ln A_j + \mathrm{d}\ln X_j) + \mathrm{d}\ln\xi_j = \zeta_j(\mathrm{d}\ln A_j + \mathrm{d}\ln X_j) + \mathrm{d}\ln\xi_j. \quad (19)$$

The equation is similar to Decker, Haltiwanger, Jarmin, and Miranda (2020) equation (3), though the functional form differs because of the shock formulation. We will discuss this issue at the end of this section. From equation (19), define the total factor revenue productivity (TFPR) by

$$\mathrm{d}\ln \mathrm{TFPR}_j = \zeta_j\,\mathrm{d}\ln A_j + \mathrm{d}\,\xi_j \approx \mathrm{d}\ln P_jQ_j - \zeta_j\,\mathrm{d}\ln X_j. \quad (20)$$

We construct TFPR to satisfy that its change leads to the (approximately) same amount of revenue change when there is no factor change.

[3] Cost minimization is necessary to achieve this result, but profit maximization is not (De Loecker, Eeckhout, and Unger 2020). Section 3 requires profit maximization to achieve our results on the relationship between revenue elasticity and firm responses to shocks.

The cost function $\mathcal{C}_j(Q_j; W/A_j) = WX_j$ of the firm can be expressed as

$$\mathrm{d}\ln WX_j \approx \left(\frac{\partial \mathcal{C}_j}{\partial W/A_j}\frac{W/A_j}{WX_j}\right)\mathrm{d}\ln\frac{W}{A_j} + \left(\frac{\partial \mathcal{C}_j}{\partial Q_j}\frac{Q_j}{WX_j}\right)\mathrm{d}\ln Q_j = \mathrm{d}\ln\frac{W}{A_j} + \frac{1}{\gamma_j}\Delta\ln Q_j, \quad (21)$$

where we have taken the implicit derivative of $\mathcal{C}(Q; W/A)$ and used equations (11) and (12). Similarly, the demand function $Q_j = \mathcal{D}_j(P_j/\xi_j)$ of the firm can be expressed as

$$\mathrm{d}\ln Q_j \approx \left(\frac{\partial \mathcal{D}_j}{\partial P_j}\frac{P_j}{Q_j}\right)(\mathrm{d}\ln P_j - \mathrm{d}\ln\xi_j) = -\left(\frac{\mu_j}{\mu_j - 1}\right)(\mathrm{d}\ln P_j - \mathrm{d}\ln\xi_j), \quad (22)$$

where we use equation (15) to characterise the demand elasticity. Additionally, from the definition of markup and equation (11), we obtain that

$$\mathrm{d}\ln P_j = \mathrm{d}\ln\mu_j + \mathrm{d}\ln \mathrm{MC}_j, \quad (23)$$

$$\mathrm{d}\ln \mathrm{MC}_j = \mathrm{d}\ln WX_j - \mathrm{d}\ln Q_j - \mathrm{d}\ln\gamma_j. \quad (24)$$

### 3.2 Propagation Mechanism

Aggregating individual firms' real revenue responses indicate aggregate business cycle fluctuations. This is because, by definition, real GDP (aggregate output) is equal to the sum of firms' real value-added, which is revenues less material costs. From equations (22) – (24), we can approximate firm revenue responses as a function of economic conditions:

$$\mathrm{d}\ln P_jQ_j \approx \frac{\zeta_j}{1-\zeta_j}(\mathrm{d}\ln A_j - \mathrm{d}\ln W + \mathrm{d}\ln\zeta_j) + \frac{1}{1-\zeta_j}\mathrm{d}\ln\xi_j. \quad (25)$$

We provide full derivations in Appendix A. Four shocks affect a firm's revenue change: productivity, factor price, revenue elasticity (inversely related to operating profitability), and demand changes. Revenue elasticity $\zeta_j$ determines the coefficient which captures the responsiveness of firm revenue to different shocks. A higher revenue elasticity firm has a greater increase in revenue in response to (1) positive productivity and demand shocks, (2) factor price decrease, and (3) operating profit decreases (i.e. revenue elasticity increases).

We can re-write equation (25) in terms of revenue productivity as define in equation (20):

$$\mathrm{d}\ln P_jQ_j \approx \frac{1}{1-\zeta_j}(\mathrm{d}\ln \mathrm{TFPR}_j - \zeta_j\,\mathrm{d}\ln W + \zeta_j\,\mathrm{d}\ln\zeta_j). \quad (26)$$

This shows that revenue elasticity is helpful to understand the response of firm revenue to revenue productivity shocks $\mathrm{d}\ln \mathrm{TFPR}_j$. A high revenue elasticity firm's revenue increases more than a low revenue elasticity firm when TFPR increases or/and factor prices decrease. Here, the factor price change component $\mathrm{d}\ln W$ can account for general equilibrium channels in reacting to shocks. Additionally, the revenue elasticity change component $\mathrm{d}\ln\zeta_j$ can represent market structure changes, such as factor market power, as well as inverse of the operating profitability changes $\mathrm{d}\ln[1-(WX_j)/(P_jQ_j)]$. This can be seen through equation (18) which equates revenue elasticity to factor shares in revenue.

In summary, we conclude that a higher revenue elasticity firm has greater cyclical sensitivity to economic conditions. The key point is that revenue elasticity is useful to understand a firm's business cycle behaviour. We do not need to decompose it further into markups and output elasticity, which are more controversial to identify.

## 3.3 Model Discussion

Our results show that the revenue elasticity is the ratio of output elasticity to the markup and that this ratio determines the revenue response to shocks. Low revenue elasticities imply weaker revenue responses to (revenue) productivity shocks. In other words, a large markup relative to output elasticity or a low output elasticity relative to markup. Consider a decrease in revenue productivity. Firms with low revenue elasticity are cushioned as their scale falls. Low revenue elasticity occurs because output elasticity (production side) is small relative to markup (demand side). On the production side, low output elasticity dampens the negative shock because firms produce more efficiently at a lower scale, so the output component of revenue is cushioned. On the demand side, a high markup dampens the negative shock because the price component of revenue dominates the output component of revenue, hence revenues are less sensitive to output change.

The main insight is that rather than having to identify the markup or output elasticity separately, we can study their ratio – the revenue elasticity – in order to make macroeconomic inference.

## 3.4 Further Issue: Alternative Shock Formula

Even if we do not specify the production and (inverse) demand functions, we assume the factor augmenting productivity and the marginal utility shifting demand shocks. Here, we compare between our previous results and results from an alternative shock formula. In particular, we use the Hicks neutral productivity and the Marshallian demand shocks as supply and demand shocks, respectively.

Denote the alternative functional forms by tilde as follows. The production and variable cost functions are

$$Q_j = \tilde{A}_j \tilde{\mathcal{F}}_j(X_j) \quad \text{and} \quad W X_j = \tilde{\mathcal{C}}_j(Q_j/\tilde{A}_j; W). \tag{27}$$

Similarly, the demand and inverse demand functions are

$$Q_j = \tilde{\xi}_j \tilde{\mathcal{D}}_j(P_j) \quad \text{and} \quad P_j = \tilde{\mathcal{P}}_j(Q_j/\tilde{\xi}_j). \tag{28}$$

Then, the revenue function with changes is given by

$$\mathrm{d}\ln P_j Q_j \approx \zeta\left(\frac{1}{\mu_j}\mathrm{d}\ln \tilde{A}_j + \mathrm{d}\ln X_j\right) + \left(1 - \frac{1}{\mu_j}\right)\mathrm{d}\ln \tilde{\xi}_j. \tag{29}$$

As in equation (20), define TFPR as

$$\mathrm{d}\ln \widetilde{\mathrm{TFPR}}_j = \frac{\zeta_j}{\mu_j}\mathrm{d}\ln \tilde{A}_j + \left(1 - \frac{1}{\mu_j}\right)\mathrm{d}\,\tilde{\xi}_j \approx \mathrm{d}\ln P_j Q_j - \zeta_j \,\mathrm{d}\ln X_j, \tag{30}$$

in which the measurement of TFPR ($\mathrm{d}\ln P_j Q_j - \zeta_j \,\mathrm{d}\ln X_j$) is the same as TFPR with our benchmark shock formula in equation (20).

As in the previous subsection, we can approximate firm revenue responses as a function of economic conditions.

$$\mathrm{d}\ln P_j Q_j \approx \frac{\zeta_j}{1-\zeta_j}\left(\frac{1}{\mu_j}\mathrm{d}\ln \tilde{A}_j - \mathrm{d}\ln W + \mathrm{d}\ln \zeta_j\right) + \left(\frac{1}{1-\zeta_j}\right)\left(1 - \frac{1}{\mu_j}\right)\mathrm{d}\ln \tilde{\xi}_j. \tag{31}$$

Equation (31) shows that the revenue elasticity is not sufficient to understand the response of revenues to Hicks neutral productivity and Marshallian demand shocks described in equations (27) and (28), respectively. we need the markup and revenue elasticity to understand the response of revenues to supply and demand shocks $d\tilde{A}_j$ and $d\tilde{\xi}_j$. More specifically, we need any two of the three variables related in equation (17) as $\zeta_j = \gamma_j/\mu_j$. In contrast, revenue elasticity was enough in equation (25) with the factor augmenting productivity and marginal utility shifting demand shocks, $dA_j$ and $\mathrm{d}\,\xi_j$, respectively.

Despite of requirement of separating the output elasticity and markups in equation (31), the revenue elasticity is enough to investigate the revenue response to TFPR shocks.

$$\mathrm{d}\ln P_j Q_j \approx \frac{1}{1-\zeta_j}(\mathrm{d}\ln \widetilde{\mathrm{TFPR}}_j - \zeta_j \,\mathrm{d}\ln W + \zeta_j \,\mathrm{d}\ln \zeta_j), \tag{32}$$

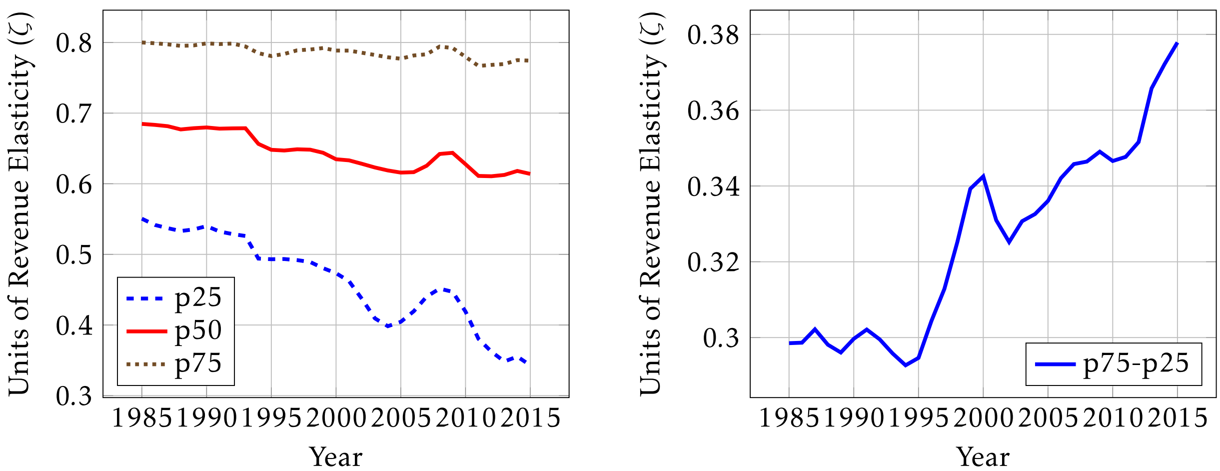
which is identical to equation (26) except for $\mathrm{d}\ln \widetilde{\mathrm{TFPR}}_j$. Equation (32) shows that a higher revenue elasticity leads a firm to greater changes in revenue in reacting to economic conditions such as TFPR, factor price, and profitability.

# 4 Data and Measurement

We measure firm-level revenue elasticities using the *Compustat North America Fundamentals Annual* database. We use data from 1984 to 2016. The database covers publicly listed firms in the US. We follow De Loecker, Eeckhout, and Unger (2020) for data cleaning and variable construction. From the database, we use firm-level data on sales growth, productivity growth, revenue elasticity, market share, employment, cash holdings, short-term debt, long-term debt and working-capital ratio. We present summary statistics in Appendix B.

## 4.1 Revenue Elasticity Measurement and Trends

We construct revenue elasticity as the ratio of firm costs to firm revenue, respectively $WX_j$ and $P_jQ_j$ in equation (18). We use *cost of goods sold* (COGS) for costs and *net-sales/turnover* (SALE) for revenue. This yields a firm-specific measure of revenue elasticity. We replace the ratio by 1.5 when it exceeds 1.5. That upper-bound is close to the top 1% in our sample. A ratio exceeding 1 implies negative operating profits, and capping at 1.5 means we adjust observations that have an operating profit share of below –50%. Then we calculate the three-year moving average, hence the plots are from 1985–2015. *Firm-level* revenue elasticity is important for us because our interest is the relationship in equation (25). That is, we estimate a model of firm-level revenue responses conditional on firm-level revenue elasticities.




(a) Quartiles

(b) Inter-Quartile Range

Figure 1: Revenue Elasticity Quartile Trends

Figure 1 shows the trends in revenue elasticity based on our cost-share approach.

The trends show decreasing revenue elasticity over the sample period. As shown in the previous section, decreasing revenue elasticity is equivalent to an increasing profit share. Barkai (2020) documents a rising profit share in the U.S. over the same period. Similarly, as described above, decreasing revenue elasticity is consistent with increasing markups or decreasing output elasticity. De Loecker, Eeckhout, and Unger (2020) document rising markups. Revenue elasticity is decreasing for high, low and medium revenue elasticity firms. That is, the upper quartile, lower quartile and median revenue elasticity firms all observe declining revenue elasticity as shown in Figure 1. However, the decline among the high revenue elasticity firms is weaker than the decline among the low revenue elasticity firms. This causes an increase in the interquartile range which indicates an increase in *revenue elasticity dispersion*. In addition to the long-run trends, Figure 1 indicates that revenue elasticity is countercyclical in the short run. That is, revenue elasticity and GDP are negatively correlated. In recession average revenue elasticity increases. In Appendix C, we plot the cyclical behaviour of revenue elasticity relative to GDP.

## 4.2 Other Measures of Revenue Elasticity

### 4.2.1 Cost Share and Estimation Approaches

There are two methods to measure revenue elasticity:

(i) *Cost-share approach:* Construct revenue elasticity as the ratio of variable costs to revenue, as in equation (18).

(ii) *Revenue function estimation approach:* Estimate elasticity from a revenue function, as in De Loecker, Eeckhout, and Unger (2020).

The cost-share approach is firm-specific. It is the ratio of firm costs to firm revenue, respectively $WX_j$ and $P_jQ_j$ in equation (18). We use *cost of goods sold* (COGS) for costs and *net-sales/turnover* (SALE) for revenue. We take both directly from Compustat. The revenue function estimation approach is industry level. One should estimate a panel regression on all firms in an industry in order to get a coefficient corresponding to revenue elasticity for that industry. In this paper, we prefer to use the cost-share approach because it is independent of estimation methods and functional forms of production function.

Figure 2 shows that the cost-share approach and production function approach are highly correlated. We plot the De Loecker, Eeckhout, and Unger (2020) estimates of industry-level revenue elasticities against the industry averages from our cost-share approach. The data is pooled across years (1985 – 2015). Each scatter point represents a two-digit industry in a year. We provide plots by year and industry in Appendix C.

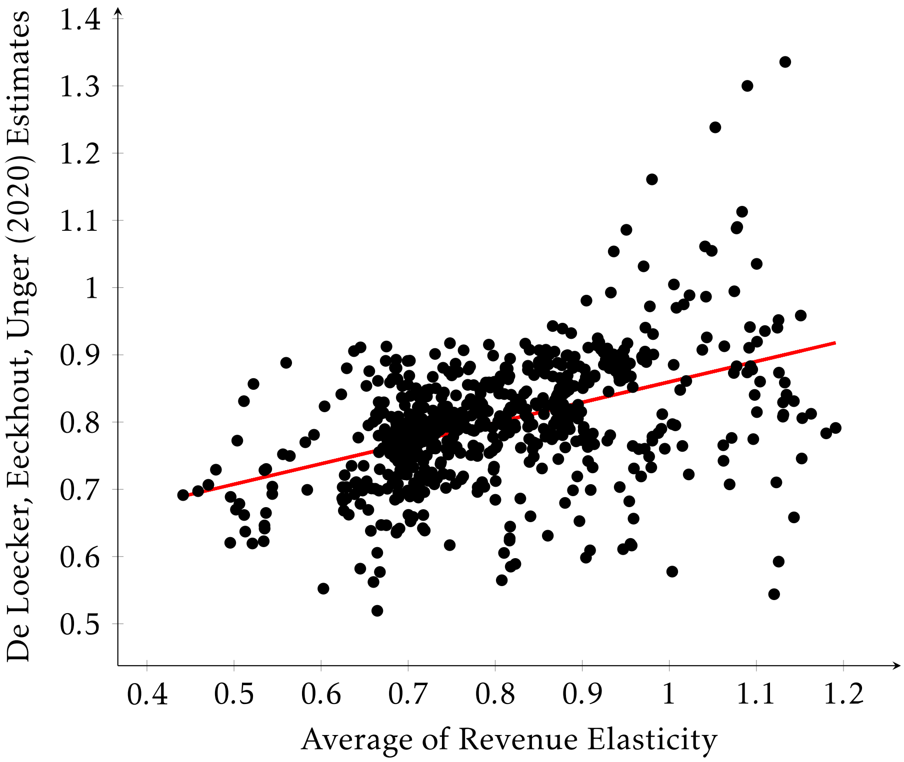



Figure 2: Revenue Elasticities and Revenue Function Estimates: Two-digit NAICS

Notes: The x-axis is the average of our firm-level revenue elasticity for each industry in each year. The y-axis is the De Loecker, Eeckhout, and Unger (Online Appendix pp. 18 2020, Figure 12.2) estimated coefficient (labelled PF2) on inputs (variable input, capital, and overhead inputs). The results without overhead inputs (PF1) yield similar results.

#### 4.2.2 Alternative Cost-Share Measures of Revenue Elasticity

Our measure of revenue elasticity is costs divided by revenue as in equation (18). We present a *benchmark* and two *alternative* measures of revenue elasticity depending on our measure of variable cost. For the alternative measures, we perform the same cleaning of revenue elasticity greater than 1.5 as in the benchmark case. The cases we consider are as follows:

(i) Benchmark: We measure variable costs as *cost of goods sold* (COGS).

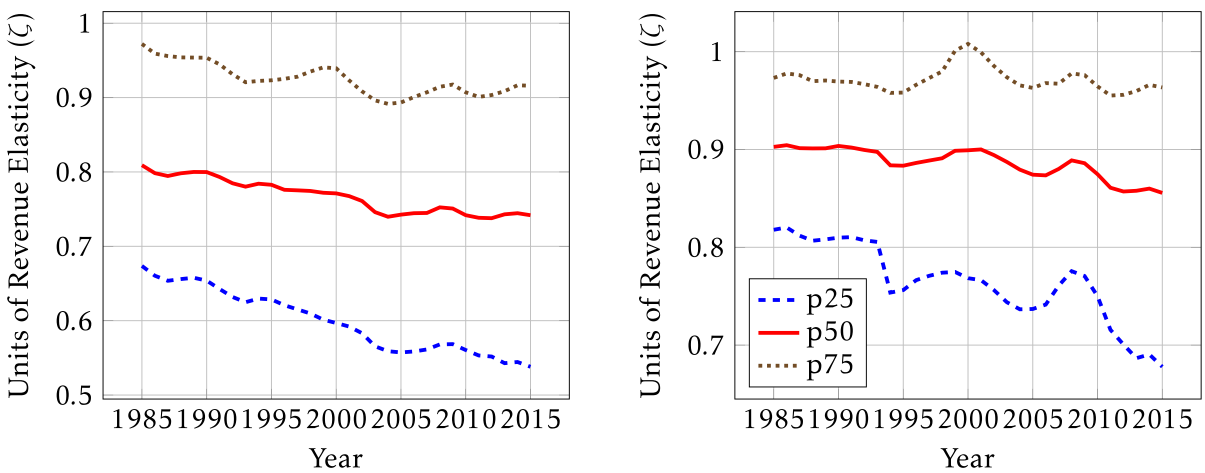
(ii) Alternative I: We measure variable costs as *cost of goods sold* (COGS) plus capital costs.

(iii) Alternative II: We measure variable costs as operating expenses, i.e., *cost of goods sold* (COGS) plus *selling, general, and administrative expense* (SGA).

Benchmark corresponds to theory that treats capital as pre-determined. That is, capital is not a variable input for a firm in the short run. This is more common in industrial organization literature (e.g. De Loecker, Eeckhout, and Unger 2020).

Alternative I corresponds to theory that treats capital as a variable input. This is common in macroeconomics, particularly business cycle literature. Capital is not fixed in the firm's profit maximization problem. We measure capital costs using the same method as De Loecker, Eeckhout, and Unger (2020, p.8). The capital rental rate is $(I - \Pi) + \delta$ where $I$, $\Pi$, and $\delta$ are the nominal interest rate, the inflation rate, and a depreciation rate. Capital is measured by gross capital (PPEGT) adjusted by the Relative Price of Investment Goods from FRED.

Alternative II considers the Traina (2018) criticism of De Loecker, Eeckhout, and Unger's (2020) measurement of variable cost. Traina explains that operating expenses (OPEX), which are costs of goods sold (COGS) plus selling, general, and administrative expenses (SGA), are a better measure of variable costs than COGS alone. COGS measure direct inputs in production, such as materials and most labour, whereas SGA measures indirect inputs in production, most commonly marketing and management expenses. Traina explains that over time firms have reallocated expenditure to SGA away from COGS. Traina shows that using OPEX instead of COGS leads to stable markups and operating profits since 1980. This result could imply that our average revenue elasticity measure has not decreased over time. However, it is debatable that variable costs should include SGA because SGA can represent fixed costs. According to Compustat, SGA accounts for all operating expenses (other than those directly related to production) incurred in the regular course of business. We can interpret OPEX and the alternative II measure by the long run variable costs and revenue elasticity because there are no fixed costs in the long run.



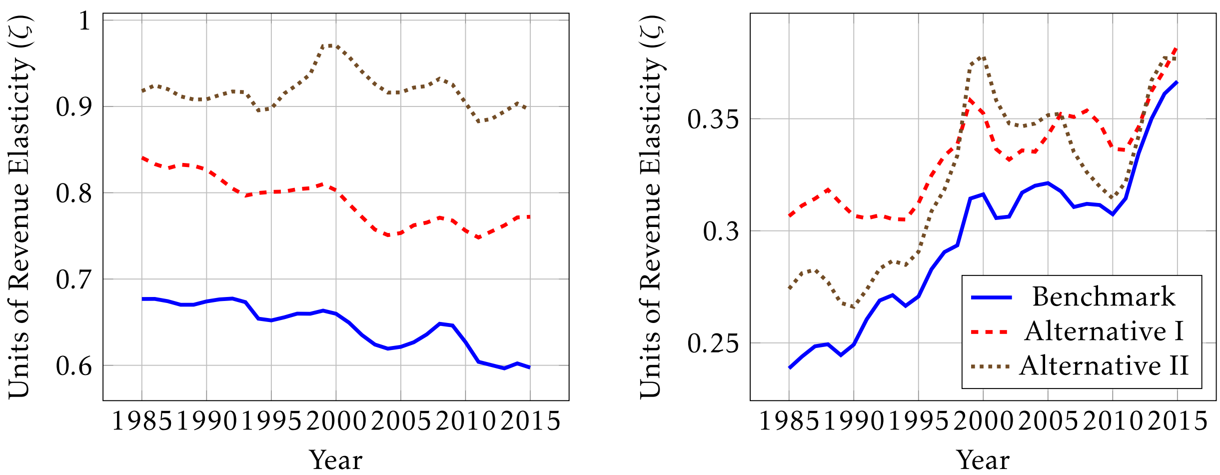

(a) Alternative I

(b) Alternative II

Figure 3: Alternative Revenue Elasticity Quartile Trends

Figure 3 shows that the alternative measures of revenue elasticity have similar distributional trends to the benchmark measure in Figure 1. For all measures the interquartile range (p75–p25) increases over time and median (p50) revenue elasticity decreases over time. The increasing IQR (p75–p25), represents greater cross-sectional heterogeneity, and is caused by low elasticity firms. There is a strong downward trend in the lower quartile (p25) for all measures of revenue elasticity, whereas the upper quartile (p75) is more stable.

Figure 4 shows that the benchmark and alternative measures of revenue elasticity have similar trends in mean revenue elasticity and similar trends in dispersion. The first panel shows that the mean level of revenue elasticity is decreasing in the long run, and all measures are inversely related to the business cycle in the short run. That is, revenue elasticity is counter-cyclical. The second panel shows that the standard deviation of revenue elasticity across firms is increasing in the long run. This implies a greater dispersion in firm-level revenue elasticity. In the short-run, dispersion in revenue elasticity across firms is procyclical. The standard deviation of revenue elasticities across firms decreases in recession, implying that on average firms have revenue elasticities closer to the mean than in normal times.




(a) Mean

(b) Standard Deviation

Figure 4: Revenue Elasticity Mean and Standard Deviation Trends

As a final robustness test, we compare our accounting-approach measures of revenue elasticity to common measures in the literature. Our levels of revenue elasticity are all centered around 0.85, which is a common *implied* revenue elasticity in macroeconomic models, and widely used in calibration exercises. For example, Atkeson and Kehoe (2005) specify the markup as $\mu \approx 1.11$, based on estimates of the underlying elasticity of substitution, and they specify output elasticity of variable factors as $\gamma = 0.95$ based on surveying production function estimation literature. Therefore the implied revenue elasticity is $\zeta = \gamma/\mu \approx 0.85$ which is consistent with earlier work by Atkeson, Khan, and Ohanian (1996). A number of subsequent papers adopt this calibration (e.g. Restuccia and Rogerson 2008; Barseghyan and DiCecio 2011). Notably the calibration is used regardless of the presence of a markup, so it is interpreted in the broader sense of the factor revenue share, whilst being agnostic on the division between markups and output elasticity. Recent work by Ruzic and Ho (2021) finds that in U.S. manufacturing weighted, industry-level, revenue elasticities have declined from 0.84 in 1982 to 0.64 in 2007. They use an accounting approach to gain revenue elasticities for labour and a GMM estimation approach to get revenue elasticities for capital, then they sum the two elasticites to get revenue elasticity. Lastly, a revenue elasticity $\zeta = 0.85$ implies that the economic profit share in revenue is 15%, which is consistent with recent evidence (Barkai 2020).

To conclude our discussion of alternative measures, all measures show a significant downward trend of in revenue elasticity among low elasticity firms, a downward trend in average elasticity, and an upward trend in inter-quartile range (cross-sectional heterogeneity). The mean and median levels of revenue elasticity are within a range of existing estimates acquired by econometric estimation.

## 4.3 Shocks

We investigate the effect of firm-level revenue elasticity on cyclical sensitivity, in particular revenue amplification for firm-level revenue productivity shocks, aggregate productivity shocks, and GDP shocks.

### 4.3.1 Firm-level Revenue Productivity Shocks

From equation (25), we enables our hypothesis: high revenue elasticity firms more strongly react to the supply and demand shocks than low revenue elasticity firms. Also, equation (26) allows us to test the modified hypothesis in terms of total factor revenue productivity (TFPR) shocks. First, we measure a *firm-level* revenue productivity shock series. We define the series as firm-level labour productivity annual growth rate denoted by $\Delta^1 LP_{j,t} \equiv \ln LP_{j,t+1} - \ln LP_{j,t}$ where $LP_{j,t} = \text{sales}_{j,t}/\text{employees}_{j,t}$ represents labour productivity, and the delta operator $\Delta^h$ represents the difference between $t+h$ and $t$. This measure is a proxy for the difference between revenue and factor growth rates (i.e., $\Delta^1 P_{j,t}Q_{j,t} - \Delta^1 X_{j,t}$). When the revenue elasticity equals one (zero operating profit), the difference measures the productivity changes $\Delta^1 \ln A_{j,t} + \Delta^1 \ln \xi_{j,t}$ well. However, the simple difference is biased with non-unit elasticity. According to equation (20), we obtain that

$$\Delta^1 \ln \text{TFPR}_{j,t} \approx \Delta^1 \ln P_j Q_{j,t} - \Delta^1 \ln X_{j,t} + (1 - \zeta_{j,t})\Delta^1 \ln X_{j,t}, \tag{33}$$

where we use $\Delta^1 \text{employees}_{j,t}$ as a proxy for $\Delta^1 \ln X_{j,t}$. For a firm with positive factor growths, the difference between revenue and factor growths underestimates TFPR growths, and the bias is larger when revenue elasticity is low. Thus, we consider labour productivity growth $\Delta^1 LP_{j,t}$ as well as corrected labour productivity growth $\Delta^1 LP_{j,t} + (1 - \zeta_{j,t})\Delta^1 \ln X_{j,t}$ representing TFPR growth. To minimize the impact of outliers, we winsorize the shock measurements at the 1% and 99% levels. Finally, we de-mean shocks using 3-digit NAICS industry average growth rate.

#### 4.3.2 Aggregate Productivity and GDP Shocks

In addition to revenue responses to firm-level productivity shocks, we measure the sensitivity of firm-level revenue to aggregate conditions. We introduce an *aggregate-level* productivity shock that directly affects $\Delta^1 A_{j,t}$ and $\Delta^1 \xi_{j,t}$. We construct the aggregate productivity shock by aggregate total factor productivity growth from Penn World Table 9.1 (RTFPNA: TFP at constant national prices, 2011 = 1), which is stationary. This aggregate productivity shock series also allow us to test our hypothesis from equation (25). Lastly, we study aggregate GDP growth rates similar to Crouzet and Mehrotra (2020). In this case, a firm's revenue response to aggregate GDP represents its cyclical sensitivity to the business cycle. If firms' responses to firm- and aggregate-level shocks are larger, then their cyclical sensitivity would be greater than others.

# 5 Empirical Analysis

## 5.1 Empirical Methodology

This section outlines a reduced-form model to quantify the effect of shocks on firm revenues conditional on firm revenue elasticity. In order to estimate the dynamics of differential responses across firms, we use local projection estimation following Jordà (2005).

#### 5.1.1 Specification with Continuous Measure of Revenue Elasticity

To test our hypothesis from equation (25), we interact productivity shocks with a firm's pre-existing traits as follows.[4]

$$\begin{aligned}\Delta^h \ln P_{j,t} Q_{j,t} = \beta_0^h \text{shock}_{j,t} + \beta_{1,\zeta}^h (\text{shock}_{j,t} \times \ln \zeta_{j,t}) + (\text{shock}_{j,t} \times \mathbf{traits}_{j,t}^\intercal)\mathbf{b}_1^h \\ + \beta_2^h \ln \zeta_{j,t} + \mathbf{traits}_{j,t}^\intercal \mathbf{b}_2^h + \delta_{j,t}^h + \varepsilon_{j,t}^h. \end{aligned} \tag{34}$$

[4]Our specification is similar to Morlacco and Zeke (2021) without lagged variables. But, Jordà (2005)'s local projection allows lagged exogenous control variables (pre-existing traits, in our framework). Adding the lagged variable in the regression equations (34) and (35) does not affect our main results.

We index a firm with $j$ and $h \geq 1$ represents the forecast horizon. Vectors are in bold font and $\mathbf{X}^\intercal$ represents the vector transpose. The delta operator $\Delta^h$ represents the difference between $t+h$ and $t$, such that $\Delta^h \ln P_{j,t}Q_{j,t} \equiv \ln P_{j,t+h}Q_{j,t+h} - \ln P_{j,t}Q_{j,t}$ for $h = 1,2,3,4$. Hence, the dependent variable is the difference between log revenue in period $t+h$ and log revenue in the current period $t$. As an example, $\beta_0^2$ represents the effect of a shock in period $t$ on revenue growth after two periods. The variable $\zeta_{j,t}$ represents firm $j$'s revenue elasticity at $t$. The variable 'shock$_{j,t}$' represents a shock. The variable $\textbf{traits}_{j,t}$ is a vector of controls. The controls are sales share in industry, employment, cash holding to asset ratio, short-term debt to asset ratio, long-term debt to asset ratio, and working capital ratio all in logs. The vector $\mathbf{b}_1^h$ contains coefficients that represent the effect of the shock on firm revenue conditional on firm traits. The vector $\mathbf{b}_2^h$ contains coefficients that represent the effect of firm traits on firm revenue. We control for lagged revenue elasticity $\beta_2^h \ln \zeta_{j,t}$ and firm-level fixed effects $\delta_{j,t}^h$. The firm-level fixed effects control for (i) time-invariant firm characteristics that generate firm-specific trends in revenue growth, and (ii) time-varying and time-invariant industry differences that might affect firms' reactions to the shocks, for example general equilibrium effects. Standard errors are clustered at the firm-level.

The main coefficients of interest are $\beta_{1,\zeta}^h$ for $h = 1,2,3,4$. The coefficient $\beta_{1,\zeta}^h$ represents a firm's percentage change (log difference) in revenue after $1,2,3$ and $4$ years following a shock in $t$ relative to a firm with a (log) unit lower revenue elasticity. Hence a positive coefficient means a shock has a greater effect on revenue for firms with higher revenue elasticity.

### 5.1.2 Specification with Discrete Measure of Revenue Elasticity

As an alternative to regression equation (34), we consider a discrete measure of revenue elasticity. We classify a firm-year revenue elasticity observation as in the upper quartile or lower quartile of the revenue elasticity distribution. The dummy variable $UQ_{j,t}$ is 1 if firm $j$ is in the upper quartile of revenue elasticities and is 0 otherwise. The dummy variable $LQ_{j,t}$ is 1 if firm $j$ is in the lower quartile of revenue elasticities and is 0 otherwise. Our re-specified equation is

$$\begin{aligned}\Delta^h \ln P_{j,t}Q_{j,t} = {} & \beta_0^h \text{shock}_{j,t} + \beta_{1,UQ}^h(\text{shock}_{j,t} \times UQ_{j,t}) + \beta_{1,LQ}^h(\text{shock}_{j,t} \times LQ_{j,t}) \\ & + \beta_{2,UQ}^h UQ_{j,t} + \beta_{2,LQ}^h LQ_{j,t} + \textbf{traits}_{j,t}\mathbf{b}_2^h + \delta_{j,t}^h + \varepsilon_{j,t}^h. \end{aligned} \tag{35}$$

The coefficient $\beta_{1,UQ}^h$ captures the effect of a shock on revenue conditional on being a high revenue elasticity firm, and $\beta_{1,LQ}^h$ captures the effect of a shock on revenue conditional on being a low revenue elasticity firm. The difference between the upper and lower quartile coefficients, $\beta_{1,UQ}^h - \beta_{1,LQ}^h$ for $h = 1,2,3,4$, represents the difference in revenue response of high and low revenue elasticity firms to shocks. When the difference is positive, it implies that high revenue elasticity firms respond more to shocks than low revenue elasticity firms.

## 5.2 Empirical Results

We find evidence that a firm's revenue elasticity increases its revenue response to firm- and aggregate-level shocks. In this section, we report results with our benchmark revenue elasticities. Results with alternative elasticity measures are in Appendix B.

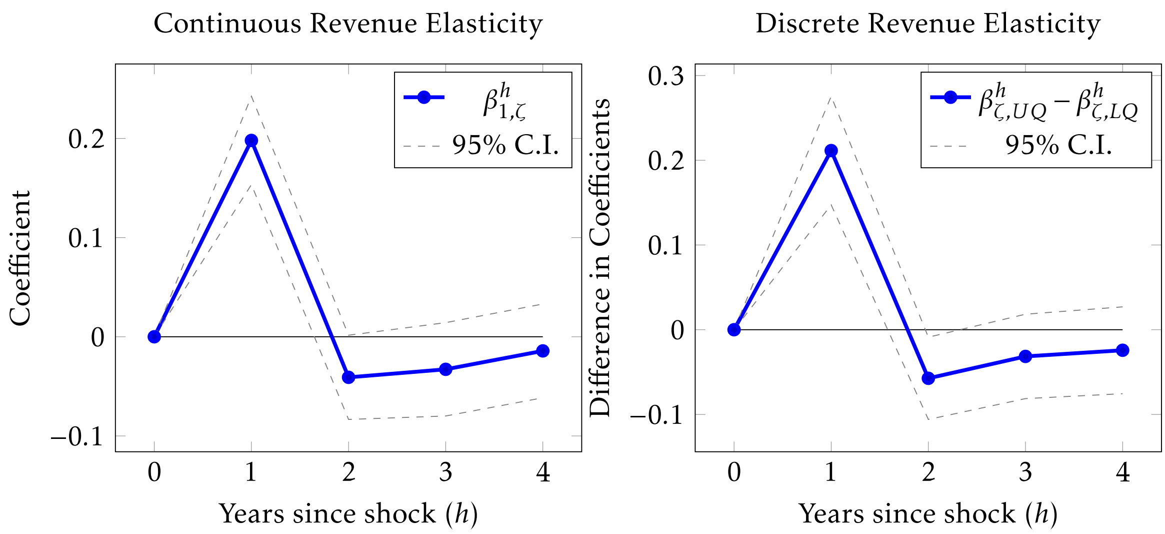
### 5.2.1 Firm-Level Revenue Productivity Shocks

Figures 5 and 6 represent how revenue elasticity shapes the impulse response function (IRF) following a productivity change. All results with firm's labour revenue productivity shocks in Figure 5 are robust after considering productivity biases arising from non-unit revenue elasticities. Figure 6 plots the IRF to a labour productivity shocks with corrections described in equation (33).

In both figures, panel (a) shows the effect of the shock on impact ($h = 1$) and after one, two and three years ($h = 2, 3, 4$). More specifically, the plots capture the effect of a productivity shock on revenue conditional on a firm's revenue elasticity. All the plots show that firms with higher revenue elasticity adjust revenues more in response to a productivity shock than firms with lower revenue elasticity. The effect is large on impact, but dissipates after one year.

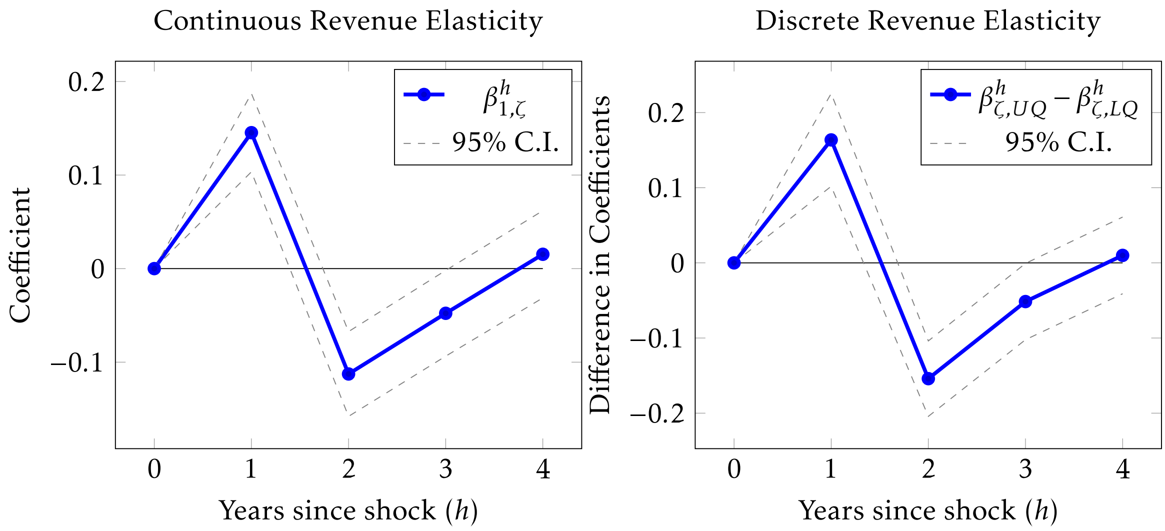
Panel (a) displays the estimate of $\beta^h_{1,\zeta}$ on the y-axis for x-axis values $h = 1, 2, 3, 4$. These follow from the regression equation (34) where we use a continuous measure of revenue elasticity. The coefficient $\beta^h_{1,\zeta}$ represents a firm's percentage change (log difference) in revenue following a technology shock compared to a firm with a (log) unit lower revenue elasticity. For example, if we take the first point $\beta^0_{1,\zeta} \approx 0.15$, this implies that following a productivity shock, on impact, a firm increases revenue by 15% more than a firm with one (log) unit lower revenue elasticity

Panel (b) plots the differential response of productivity shocks on firms in the upper and lower quartiles $\beta^h_{1,UQ} - \beta^h_{1,LQ}$ on the y-axis for x-axis values $h = 1, 2, 3, 4$ year since shocks. These follow from the regression equation (35) where we use a discrete measure of revenue elasticity. The value $\beta^h_{1,UQ} - \beta^h_{1,LQ}$ represents a firm's percentage change (log difference) in revenue following a technology shock for a high revenue elasticity firm (upper quartile) compared to a low revenue elasticity firm (lower quartile). For example Panel (a) Figure B, if we take the first point $\beta^0_{1,UQ} - \beta^0_{1,LQ} \approx 0.2$ implies that the upper quartile of revenue elasticity firms increase their revenue 0.2% more than the lower quartile of revenue elasticity firms following a one percent productivity shock.




(a) Regression Equation (34)  (b) Regression Equation (35)

Figure 5: Impulse Response Functions to Firm-Level Labour Productivity Shock

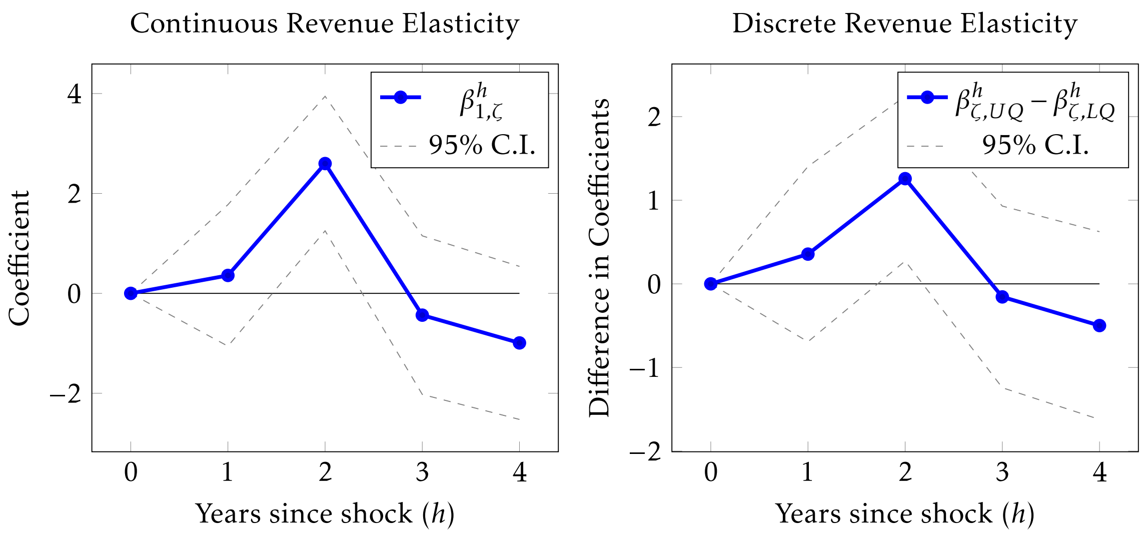



(a) Regression Equation (34)  (b) Regression Equation (35)

Figure 6: Impulse Response Functions to Firm-Level Corrected Labour Productivity Shock

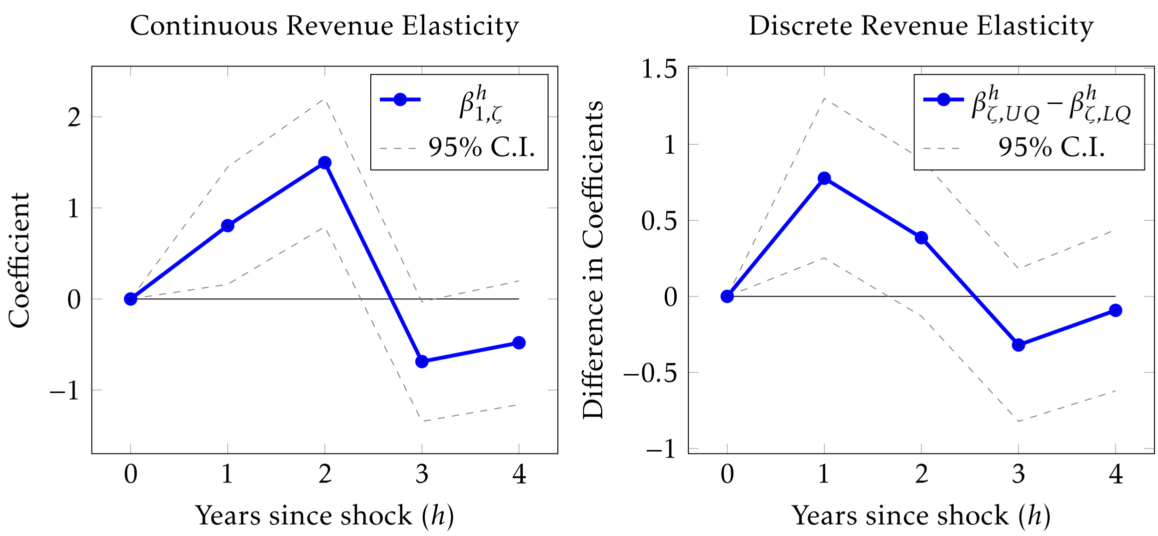
#### 5.2.2 Cyclical Sensitivity: Aggregate Total Factor Productivity and GDP Shocks

Figure 7 displays the impulse response functions (IRFs) to the aggregate productivity shocks from estimating regression equations (34) and (35) with a continuous and discrete measure of revenue elasticity, respectively. Consistent with our hypothesis, high revenue elasticity firms' sales respond more to aggregate productivity shocks than lower revenue elasticity firms. Notably, the coefficients in Figure 7 are higher than the coefficients for the firm-level shocks in Figures 5 and 6. In equation (25), each firm's de-meaned labour productivity shock has a negligible effect on factor prices $\Delta W$ because we present a partial equilibrium framework and an individual firm has




(a) Regression Equation (34) (b) Regression Equation (35)

Figure 7: Impulse Response Functions to Aggregate TFP Shock




(a) Regression Equation (34) (b) Regression Equation (35)

Figure 8: Impulse Response Functions to Aggregate GDP Shock

no factor market power. However, aggregate productivity changes will have general equilibrium effects on factor prices that are not present with firm-level idiosyncratic shocks. This general equilibrium channel could account for the higher firm responses to aggregate shocks compared with its responses to idiosyncratic firm-level shocks.

Figure 8 shows the IRFs following a change in GDP. This conveys how revenue elasticity shapes a firm's sensitivity to aggregate GDP (business cycle) fluctuations. If a high revenue elasticity firm more responses to productivity changes, the firm would be more procyclical than low elasticity firms. The estimated coefficients in equations (34) and (35) represent the elasticity of firm-level revenue to aggregate GDP. When using a GDP shock, we capture whether a firms' revenue growth is procylical or countercyclical in response to aggregate GDP growth. The results show that high revenue elasticity firms are more strongly procyclical.

# 6 Conclusion

We analyse the effect of firm-level revenue elasticities on business cycle dynamics. We focus on revenue elasticities because they are simple to obtain at the firm level, but are understudied relative to the related concepts of price markups and output elasticities. We present theory which shows that higher revenue elasticity are more responsive to business cycle shocks than low revenue elasticity firms. We test this theoretical relationship on U.S. data and find evidence in support of the theory. Furthermore, we present empirical results on the behaviour of revenue elasticities of U.S. firms over the last three decades. In particular, we show a secular decline in average revenue elasticity. According to our theory this implies declining amplification of business cycle shocks. Overall, our paper stresses that the complexities of identifying markups from revenue data need not be an obstacle to making macroeconomic inference, and empirical trends in revenue data are consistent with existing analyses of markups and market power.

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# Appendix

## A Derivations

### A.1 Derivation of Equation (25)

Denote price elasticity of demand as $\theta \equiv -\partial \ln \mathcal{D}/\partial \ln P$. Then, a first-order log-linearization of the demand function $Q = \mathcal{D}(P)$ yields the approximation

$$\mathrm{d}\ln Q_j \approx -\theta_j(\mathrm{d}\ln P_j - \mathrm{d}\ln \xi_j). \tag{A.1}$$

Similarly, we can approximate the variable cost change by

$$\mathrm{d}\ln WX_j \approx \mathrm{d}\ln W - \mathrm{d}\ln A_j + (1/\gamma_j)\,\mathrm{d}\ln Q_j, \tag{A.2}$$

where we note that $1/\gamma_j = \left(\frac{\partial \mathcal{F}_j}{\partial X_j}\frac{X_j}{Q_j}\right)^{-1}$. We have shown that the firm cost function under optimality is given by $\gamma = (WX_j/Q_j)/\mathrm{MC}_j$. Therefore, log-linearizing gives

$$\begin{aligned}\mathrm{d}\ln \mathrm{MC}_j &= -\mathrm{d}\ln\gamma_j - \mathrm{d}\ln Q_j + \mathrm{d}\ln \mathcal{C}_j & \text{(A.3)}\\ &\approx -\mathrm{d}\ln\gamma_j + \mathrm{d}\ln W - \ln A_j + \left(\frac{1}{\gamma_j} - 1\right)\mathrm{d}\ln Q_j. & \text{(A.4)}\end{aligned}$$

In the first line we use that $\mathcal{C}_j(W/A_j; Q_j) = WX_j$, and in the second line we use (A.2). Our markup definition is $\mu_j \equiv P_j/\mathrm{MC}_j$, therefore log-linearizing yields

$$\mathrm{d}\ln P_j = \mathrm{d}\ln\mu_j + \mathrm{d}\ln \mathrm{MC}_j. \tag{A.5}$$

Combining the approximated marginal cost function (A.4) with the approximated markup function (A.5), we obtain the price change as follows

$$\mathrm{d}\ln P_j \approx -\mathrm{d}\ln\zeta_j + \mathrm{d}\ln W - \mathrm{d}\ln A_j + \left(\frac{1}{\gamma_j} - 1\right)\mathrm{d}\ln Q_j, \tag{A.6}$$

where $\mathrm{d}\ln\zeta_j = \mathrm{d}\ln\gamma_j - \mathrm{d}\ln\mu_j$. Combining equations (A.1) and (A.6), the price change is given by $\mathrm{d}\ln P_j \approx -\mathrm{d}\ln\zeta_j + \mathrm{d}\ln W - \mathrm{d}\ln A_j - \theta(1/\gamma_j - 1)(\mathrm{d}\ln P_j - \mathrm{d}\ln\xi_j)$. Rearraning this expression yields

$$(1-\theta)\,\mathrm{d}\ln P_j \approx \frac{\gamma_j}{\mu_j - \gamma_j}(\mathrm{d}\ln\zeta_j - \mathrm{d}\ln W + \mathrm{d}\ln A_j) + \frac{\gamma_j - 1}{\mu_j - \gamma_j}\theta_j\,\mathrm{d}\ln\xi_j, \tag{A.7}$$

where we use the optimal pricing rule $\mu_j = \theta_j/(\theta_j - 1)$ on the right-hand side.

Using the demand function equation (A.1), we can rewrite the revenue change in terms of price:

$$d\ln P_j Q_j = d\ln P_j + d\ln Q_j \approx (1-\theta_j)\, d\ln P_j + \theta\, d\ln \xi_j. \tag{A.8}$$

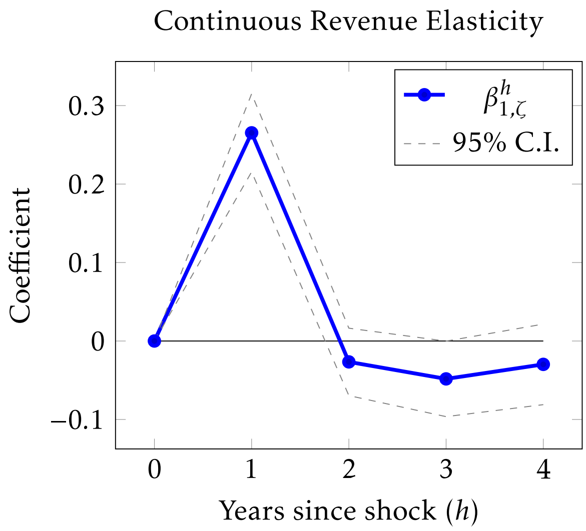
Putting equation (A.7) into the above equation, we obtain the main theoretical result of our paper:

$$d\ln P_j Q_j \approx \frac{\gamma_j}{\mu_j - \gamma_j}(d\ln A_j - d\ln W + d\ln \zeta_j) + \frac{\mu_j}{\mu_j - \gamma_j}\, d\ln \xi_j \tag{A.9}$$

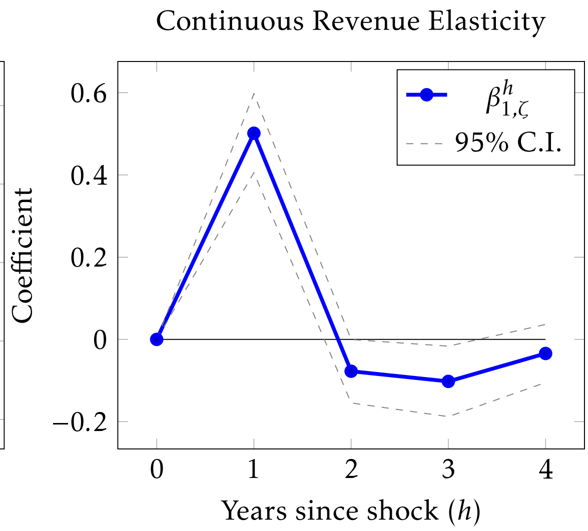
$$= \frac{\zeta}{1-\zeta_j}(d\ln A_j - d\ln W + d\ln \zeta_j) + \frac{1}{1-\zeta_j}\, d\ln \xi_j \tag{A.10}$$

$$= \frac{1}{1-\zeta_j}(d\ln \mathrm{TFPR}_j - \zeta\, d\ln W + \zeta_j\, d\ln \zeta_j). \tag{A.11}$$

# B Empirical Results with Alternative Elasticity Measures

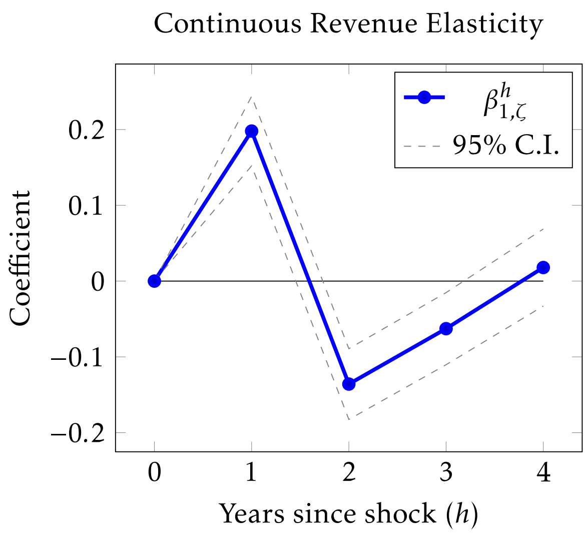



(a) Alternative Elasticity I

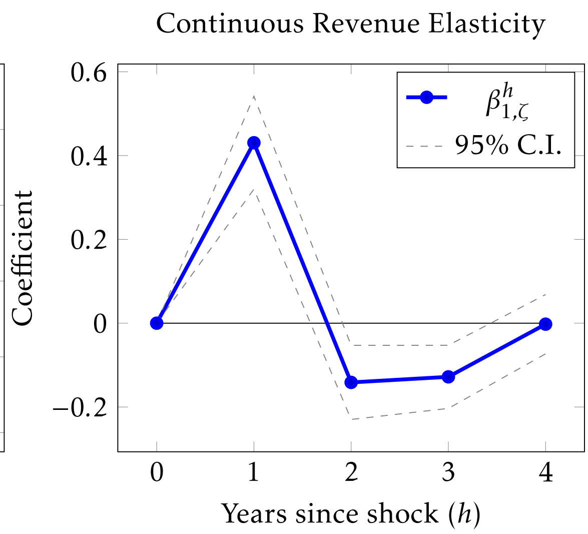



(b) Alternative Elasticity II

Figure A.1: Impulse Response Functions to Firm-Level Labor Productivity Shock with Alternative Elasticities

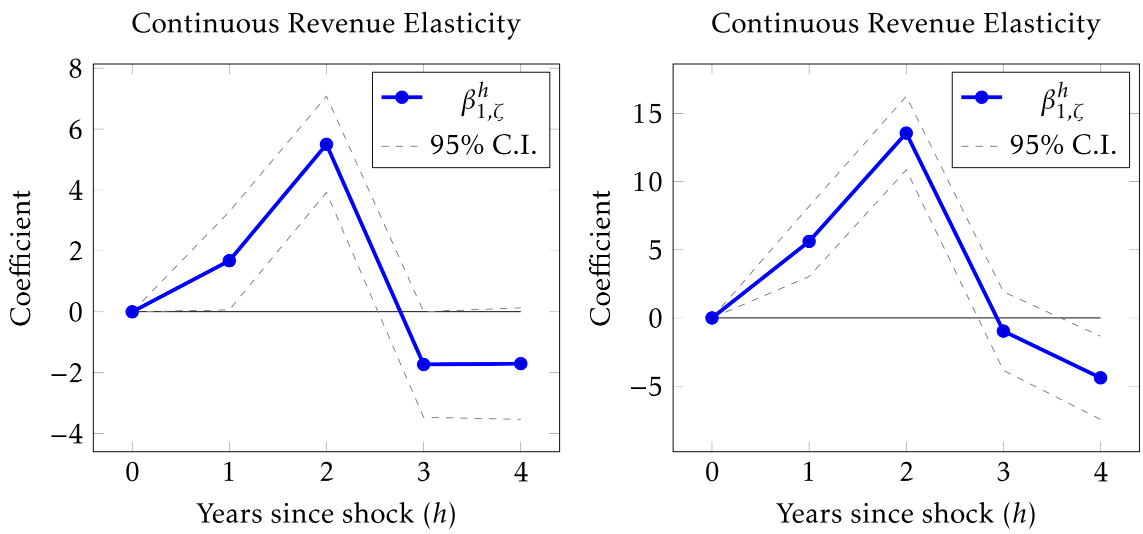



(a) Alternative Elasticity I




(b) Alternative Elasticity II

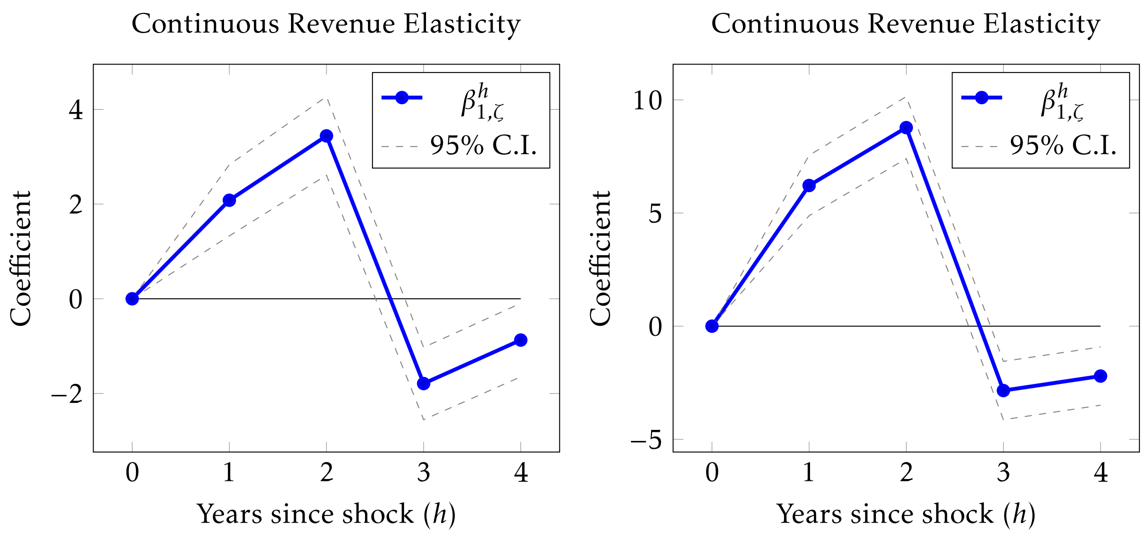
Figure A.2: Impulse Response Functions to Firm-Level Corrected Labour Productivity Shock with Alternative Elasticities




(a) Alternative Elasticity I

(b) Alternative Elasticity II

Figure A.3: Impulse Response Functions to Aggregate TFP with Alternative Elasticities Shock




(a) Alternative Elasticity I

(b) Alternative Elasticity II

Figure A.4: Impulse Response Functions to Aggregate GDP with Alternative Elasticities Shock

# C Additional Figures and Tables

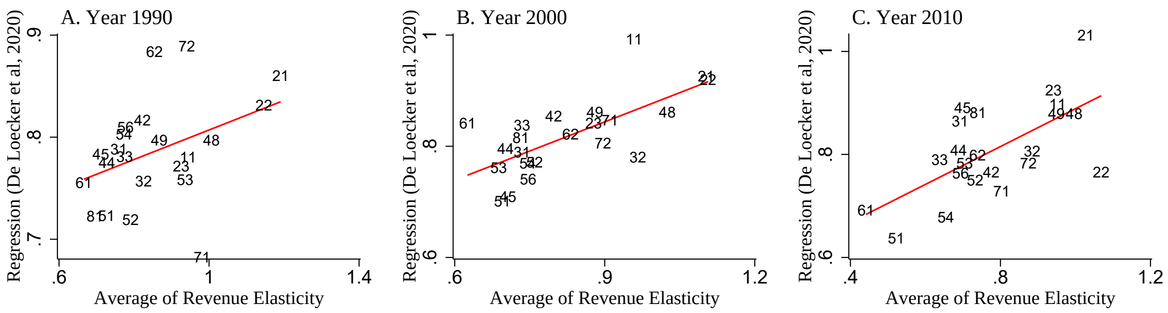



Figure A.5: 1990, 2000, 2010 Revenue Elasticities and Revenue Function Estimates: Two-digit NAICS

Notes: We use De Loecker, Eeckhout, and Unger (Online Appendix pp. 18 2020, Figure 12.2)'s estimated coefficients (labeled PF2) of variable input bundle where there are three inputs (variable input, capital, and overhead inputs). The results with the other specification (PF1) without overhead inputs yield similar results.

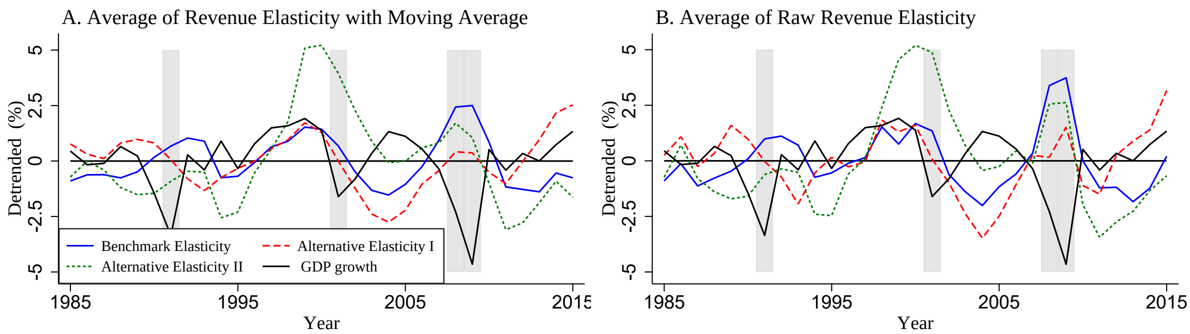



Figure A.6: Detrended Average Revenue Elasticities and GDP Growths

Notes: The detrend series are residuals of the regression of each variable on constant and time. In Panel A, we calculate an individual firm's revenue elasticity with three years moving average within firms. Then, we detrend their cross-sectional average. In Panel B, we use the detrended cross-sectional average of contemporaneous firms' revenue elasticities. The shaded areas are NBER recession years.

Table A.1: Summary Statistics

| | Count | Mean | SD | p10 | p25 | p50 | p75 | p90 |
|---|---|---|---|---|---|---|---|---|
| **Panel A. Period: 1985–1995** | | | | | | | | |
| Sales growth (%) | 64343 | 8.58 | 43.74 | -21.62 | -4.94 | 5.40 | 19.24 | 42.94 |
| Productivity growth (%) | 49236 | 0.15 | 31.30 | -26.58 | -9.97 | -0.29 | 9.58 | 27.16 |
| Revenue elasticity | | | | | | | | |
| Benchmark | 64343 | 0.66 | 0.23 | 0.38 | 0.53 | 0.67 | 0.79 | 0.89 |
| Alternative I | 59517 | 0.80 | 0.26 | 0.49 | 0.64 | 0.79 | 0.94 | 1.12 |
| Alternative II | 64342 | 0.89 | 0.22 | 0.65 | 0.80 | 0.90 | 0.97 | 1.12 |
| Market share (%) | 64343 | 1.52 | 6.04 | 0.00 | 0.02 | 0.12 | 0.69 | 2.97 |
| Employment (thousand, log) | 56616 | -0.51 | 2.37 | -3.51 | -2.06 | -0.40 | 1.14 | 2.45 |
| Cash holding/Asset | 64126 | 0.13 | 0.16 | 0.01 | 0.02 | 0.07 | 0.17 | 0.34 |
| Short-term debt/Asset | 63788 | 0.23 | 34.54 | 0.00 | 0.01 | 0.04 | 0.10 | 0.23 |
| Long-term debt/Asset | 64120 | 0.20 | 0.50 | 0.00 | 0.03 | 0.15 | 0.30 | 0.45 |
| Working capital ratio | 54111 | 2.84 | 8.27 | 0.79 | 1.22 | 1.89 | 2.94 | 4.79 |
| **Panel B. Period: 1996–2005** | | | | | | | | |
| Sales growth (%) | 66235 | 12.52 | 48.65 | -20.77 | -3.62 | 7.25 | 23.04 | 52.20 |
| Productivity growth (%) | 55900 | 1.60 | 35.25 | -28.01 | -9.91 | 0.62 | 12.14 | 32.78 |
| Revenue elasticity | | | | | | | | |
| Benchmark | 66235 | 0.63 | 0.26 | 0.31 | 0.46 | 0.64 | 0.78 | 0.90 |
| Alternative I | 56706 | 0.77 | 0.28 | 0.41 | 0.59 | 0.76 | 0.92 | 1.10 |
| Alternative II | 66235 | 0.90 | 0.26 | 0.62 | 0.76 | 0.89 | 0.98 | 1.30 |
| Market share (%) | 66235 | 1.25 | 5.24 | 0.00 | 0.01 | 0.08 | 0.53 | 2.29 |
| Employment (thousand, log) | 57624 | -0.43 | 2.27 | -3.19 | -1.96 | -0.46 | 1.17 | 2.48 |
| Cash holding/Asset | 66136 | 0.16 | 0.20 | 0.01 | 0.03 | 0.07 | 0.23 | 0.48 |
| Short-term debt/Asset | 65971 | 0.11 | 1.59 | 0.00 | 0.00 | 0.03 | 0.08 | 0.18 |
| Long-term debt/Asset | 65920 | 0.21 | 0.43 | 0.00 | 0.02 | 0.12 | 0.30 | 0.49 |
| Working capital ratio | 52838 | 2.87 | 8.54 | 0.78 | 1.20 | 1.91 | 3.14 | 5.30 |
| **Panel C. Period: 2006–2015** | | | | | | | | |
| Sales growth (%) | 47754 | 6.18 | 41.83 | -21.23 | -5.97 | 3.61 | 15.24 | 35.33 |
| Productivity growth (%) | 42710 | 1.88 | 30.38 | -21.70 | -7.76 | 0.95 | 10.17 | 26.03 |
| Revenue elasticity | | | | | | | | |
| Benchmark | 47754 | 0.60 | 0.28 | 0.22 | 0.41 | 0.62 | 0.78 | 0.89 |
| Alternative I | 39574 | 0.74 | 0.30 | 0.35 | 0.56 | 0.74 | 0.91 | 1.09 |
| Alternative II | 47754 | 0.87 | 0.26 | 0.58 | 0.73 | 0.87 | 0.96 | 1.21 |
| Market share (%) | 47754 | 1.65 | 6.18 | 0.00 | 0.01 | 0.11 | 0.75 | 3.27 |
| Employment (thousand, log) | 43594 | -0.27 | 2.48 | -3.32 | -1.89 | -0.17 | 1.51 | 2.82 |
| Cash holding/Asset | 47717 | 0.17 | 0.20 | 0.01 | 0.03 | 0.09 | 0.24 | 0.48 |
| Short-term debt/Asset | 47688 | 0.12 | 2.16 | 0.00 | 0.00 | 0.02 | 0.05 | 0.13 |
| Long-term debt/Asset | 47449 | 0.22 | 0.59 | 0.00 | 0.02 | 0.13 | 0.31 | 0.51 |
| Working capital ratio | 36577 | 3.37 | 56.85 | 0.79 | 1.21 | 1.90 | 3.08 | 5.23 |

Notes: The market share is in the three-digit NAICS industry. We replace employment by one for zero. We replace replace revenue elasticity by two for higher than two. We keep the following observations in the Compustat database. (a) No major mergers flag: Comparability status (COMPST) does not equal to AB, (b) Country ISO 3 digit code (LOC) equals to USA, (c) Currency ISO 3 digit code (CURCD) equals USD, (d) Real sales are higher than 0.1M.