A dynamic equilibrium model of commuting, residential and work location choices

Christian Langholz Carstensen [1] Fedor Iskhakov [2] Maria Juul Hansen [1]
John Rust [3] Bertel Schjerning [1]

EEA, Milan, August 2022

[1]University of Copenhagen

[2]Australian National University

[3]Georgetown University

Motivation

- A large group of countries are undergoing a process of strong urbanization and spatial concentration of economic activity
- $\rightarrow$ increased productivity through agglomeration but...
    - ...rural areas have been declining in terms of population, labor demand and house prices
    - ...urban areas experience large increases in traffic congestion and house prices
    - $\rightarrow$ demographic composition of locations change and increased regional inequality
- Various policies are considered to ameliorate downsides of this development
- Dynamic effects of such spatial policies not well understood due to the complexity of households' joint choice of employment, work and residential locations and commuting

Motivation

- A large group of countries are undergoing a process of strong urbanization and spatial concentration of economic activity
- $\rightarrow$ increased productivity through agglomeration but...
  - ...rural areas have been declining in terms of population, labor demand and house prices
  - ...urban areas experience large increases in traffic congestion and house prices
  - $\rightarrow$ demographic composition of locations change and increased regional inequality
- Various policies are considered to ameliorate downsides of this development
- Dynamic effects of such spatial policies not well understood due to the complexity of households' joint choice of employment, work and residential locations and commuting

Motivation

- A large group of countries are undergoing a process of strong urbanization and spatial concentration of economic activity
- $\rightarrow$ increased productivity through agglomeration but...
  - ...rural areas have been declining in terms of population, labor demand and house prices
  - ...urban areas experience large increases in traffic congestion and house prices
  - $\rightarrow$ demographic composition of locations change and increased regional inequality
- Various policies are considered to ameliorate downsides of this development
- Dynamic effects of such spatial policies not well understood due to the complexity of households' joint choice of employment, work and residential locations and commuting

Motivation

- A large group of countries are undergoing a process of strong urbanization and spatial concentration of economic activity
- $\rightarrow$ increased productivity through agglomeration but...
    - ...rural areas have been declining in terms of population, labor demand and house prices
    - ...urban areas experience large increases in traffic congestion and house prices
    - $\rightarrow$ demographic composition of locations change and increased regional inequality
- Various policies are considered to ameliorate downsides of this development
- Dynamic effects of such spatial policies not well understood due to the complexity of households' joint choice of employment, work and residential locations and commuting

Motivation

- A large group of countries are undergoing a process of strong urbanization and spatial concentration of economic activity
- $\rightarrow$ increased productivity through agglomeration but...
  - ...rural areas have been declining in terms of population, labor demand and house prices
  - ...urban areas experience large increases in traffic congestion and house prices
  - $\rightarrow$ demographic composition of locations change and increased regional inequality
- Various policies are considered to ameliorate downsides of this development
- Dynamic effects of such spatial policies not well understood due to the complexity of households' joint choice of employment, work and residential locations and commuting

Motivation

- A large group of countries are undergoing a process of strong urbanization and spatial concentration of economic activity
- $\rightarrow$ increased productivity through agglomeration but...
  - ...rural areas have been declining in terms of population, labor demand and house prices
  - ...urban areas experience large increases in traffic congestion and house prices
  - $\rightarrow$ demographic composition of locations change and increased regional inequality
- Various policies are considered to ameliorate downsides of this development
- Dynamic effects of such spatial policies not well understood due to the complexity of households' joint choice of employment, work and residential locations and commuting

This paper

We develop a dynamic equilibrium model that simulatenously tracks the following mechanisms:

- Choice of residence, work location, housing demand and commuting...
- ...while allowing life cycle aspects to play a key role
- Sorting of heterogeneous consumers and equilibrium in the housing market

This paper

We develop a dynamic equilibrium model that simulatenously tracks the following mechanisms:

- Choice of residence, work location, housing demand and commuting...
- ...while allowing life cycle aspects to play a key role
- Sorting of heterogeneous consumers and equilibrium in the housing market

This paper

We develop a dynamic equilibrium model that simulateneously tracks the following mechanisms:

- Choice of residence, work location, housing demand and commuting...
- ...while allowing life cycle aspects to play a key role
- Sorting of heterogeneous consumers and equilibrium in the housing market

This paper

We develop a dynamic equilibrium model that simulatenously tracks the following mechanisms:

- Choice of residence, work location, housing demand and commuting...
- ...while allowing life cycle aspects to play a key role
- Sorting of heterogeneous consumers and equilibrium in the housing market

This paper

We develop a dynamic equilibrium model that simulatenously tracks the following mechanisms:

- Choice of residence, work location, housing demand and commuting...
- ...while allowing life cycle aspects to play a key role
- Sorting of heterogeneous consumers and equilibrium in the housing market

This paper

We develop a dynamic equilibrium model that simulatenously tracks the following mechanisms:

- Choice of residence, work location, housing demand and commuting...
- ...while allowing life cycle aspects to play a key role
- Sorting of heterogeneous consumers and equilibrium in the housing market

We structurally estimate this model

- Danish administrative panel data: lots of heterogeneity and rich dynamics
- Model fits key aspects of data well, incl. house price trends

This paper

We develop a dynamic equilibrium model that simulateneously tracks the following mechanisms:

- Choice of residence, work location, housing demand and commuting...
- ...while allowing life cycle aspects to play a key role
- Sorting of heterogeneous consumers and equilibrium in the housing market

We structurally estimate this model

- Danish administrative panel data: lots of heterogeneity and rich dynamics
- Model fits key aspects of data well, incl. house price trends

This paper

We develop a dynamic equilibrium model that simulatenously tracks the following mechanisms:

- Choice of residence, work location, housing demand and commuting...
- ...while allowing life cycle aspects to play a key role
- Sorting of heterogeneous consumers and equilibrium in the housing market

We structurally estimate this model

- Danish administrative panel data: lots of heterogeneity and rich dynamics
- Model fits key aspects of data well, incl. house price trends

We simulate effects on house prices, job mobility, residential sorting and commuting in two counterfactual equilibria:

1. 5% increase in supply of housing stock in central Copenhagen
   - Stylized illustration of actually planned policy of constructing the artificial island Lynetteholm which we have analyzed for the Danish Ministry of Transportation
2. [FOCUS TODAY] Extended use of telecommuting for highly skilled (HS) workers
   - HS workers workers move out of city → prices drop in center → LS move there
   - Non-employment drops and HS better off while LS indifferent

This paper

We develop a dynamic equilibrium model that simulatenously tracks the following mechanisms:

- Choice of residence, work location, housing demand and commuting...
- ...while allowing life cycle aspects to play a key role
- Sorting of heterogeneous consumers and equilibrium in the housing market

We structurally estimate this model

- Danish administrative panel data: lots of heterogeneity and rich dynamics
- Model fits key aspects of data well, incl. house price trends

We simulate effects on house prices, job mobility, residential sorting and commuting in two counterfactual equilibria:

1. 5% increase in supply of housing stock in central Copenhagen
   - Stylized illustration of actually planned policy of constructing the artificial island Lynetteholm which we have analyzed for the Danish Ministry of Transportation
2. [FOCUS TODAY] Extended use of telecommuting for highly skilled (HS) workers
   - HS workers workers move out of city → prices drop in center → LS move there
   - Non-employment drops and HS better off while LS indifferent

This paper

We develop a dynamic equilibrium model that simulatenously tracks the following mechanisms:

- Choice of residence, work location, housing demand and commuting...
- ...while allowing life cycle aspects to play a key role
- Sorting of heterogeneous consumers and equilibrium in the housing market

We structurally estimate this model

- Danish administrative panel data: lots of heterogeneity and rich dynamics
- Model fits key aspects of data well, incl. house price trends

We simulate effects on house prices, job mobility, residential sorting and commuting in two counterfactual equilibria:

1. 5% increase in supply of housing stock in central Copenhagen
    - Stylized illustration of actually planned policy of constructing the artificial island Lynetteholm which we have analyzed for the Danish Ministry of Transportation
2. [FOCUS TODAY] Extended use of telecommuting for highly skilled (HS) workers
    - HS workers workers move out of city → prices drop in center → LS move there
    - Non-employment drops and HS better off while LS indifferent

Overview

Overview

Literature

Model

Estimation

Data

Results

Housing market equilibrium

Counterfactuals

Conclusion

Related literature

- GE models of location choice: Buchinsky et al., 2014, Ahlfeldt et al., 2015
- Life-cycle location choice models w/o commuting: Kennan and Walker, 2011, Oswald, 2019, Dahl, 2002 and Tunali, 2000, Diamond, 2016
- Interactions btw infrastructure, home and work locations: Tsivanidis, 2019, Heblich et al., 2020, Dingel and Tintelnot, 2020, Teulings et al., 2018, Severen, 2021, Chernoff and Craig, 2021, Allen and Arkolakis, 2019 and Monte et al., 2018
- Search models: i.a. Manning and Petrongolo, 2017
- WTP for local non-traded amenities: Sieg et al., 2004, Bayer et al., 2016, Kuminoff et al., 2013

Overview

The Choice Model

Individuals solve the following optimization problem:

$$\max_{\{d_{it}^{rl}, d_{it}^{wl}, h_{it}\}_{t=t_0}^{T}} \sum_{t=t_0}^{T} \rho_t(x_{it})^t \mathbb{E}_t \Big[u_t(x_{it}, wl_{it}, rl_{it}, d_{it}', \mathrm{h}_{it}) \Big]$$

The Choice Model

Individuals solve the following optimization problem:

$$\max_{\{d_{it}^{rl}, d_{it}^{wl}, h_{it}\}_{t=t_0}^{T}} \sum_{t=t_0}^{T} \rho_t(x_{it})^t \mathbb{E}_t \Big[u_t(x_{it}, wl_{it}, rl_{it}, d'_{it}, \mathrm{h}_{it}) \Big]$$

$d'_{it} = (d_{it}^{rl}, d_{it}^{wl})$ *discrete* residential/work location decision

The Choice Model

Individuals solve the following optimization problem:

$$\max_{\{d^{rl}_{it}, d^{wl}_{it}, h_{it}\}_{t=t_0}^{T}} \sum_{t=t_0}^{T} \rho_t(x_{it})^t \mathbb{E}_t \Big[u_t(x_{it}, wl_{it}, rl_{it}, d'_{it}, \mathrm{h}_{it}) \Big]$$

$d'_{it} = (d^{rl}_{it}, d^{wl}_{it})$ *discrete* residential/work location decision

h_{it} *continuous* house size decision (in m^2)

The Choice Model

Individuals solve the following optimization problem:

$$\max_{\{d_{it}^{rl}, d_{it}^{wl}, h_{it}\}_{t=t_0}^{T}} \sum_{t=t_0}^{T} \rho_t(x_{it})^t \mathbb{E}_t \Big[u_t(x_{it}, wl_{it}, rl_{it}, d'_{it}, \mathrm{h}_{it}) \Big]$$

$d'_{it} = (d_{it}^{rl}, d_{it}^{wl})$ *discrete* residential/work location decision

h_{it} *continuous* house size decision (in m^2)

t_0, T household age ($t_0 = 26$, $T = 76$)

The Choice Model

Individuals solve the following optimization problem:

$$\max_{\{d_{it}^{rl}, d_{it}^{wl}, h_{it}\}_{t=t_0}^{T}} \sum_{t=t_0}^{T} \rho_t(x_{it})^t \mathbb{E}_t \left[u_t(x_{it}, wl_{it}, rl_{it}, d'_{it}, \mathrm{h}_{it}) \right]$$

$d'_{it} = (d_{it}^{rl}, d_{it}^{wl})$	*discrete* residential/work location decision
h_{it}	*continuous* house size decision (in m^2)
t_0, T	household age ($t_0 = 26$, $T = 76$)
$\rho_t(x_{it})$	survival probability times discount factor ($\beta = 0.95$)

The Choice Model

Individuals solve the following optimization problem:

$$\max_{\{d_{it}^{rl}, d_{it}^{wl}, h_{it}\}_{t=t_0}^{T}} \sum_{t=t_0}^{T} \rho_t(x_{it})^t \mathbb{E}_t \left[u_t(x_{it}, wl_{it}, rl_{it}, d'_{it}, \mathrm{h}_{it}) \right]$$

$d'_{it} = (d_{it}^{rl}, d_{it}^{wl})$	*discrete* residential/work location decision
h_{it}	*continuous* house size decision (in m^2)
t_0, T	household age ($t_0 = 26$, $T = 76$)
$\rho_t(x_{it})$	survival probability times discount factor ($\beta = 0.95$)
$u_t(\cdot)$	instantaneous utility function

The Choice Model

Individuals solve the following optimization problem:

$$\max_{\{d_{it}^{rl}, d_{it}^{wl}, h_{it}\}_{t=t_0}^{T}} \sum_{t=t_0}^{T} \rho_t(x_{it})^t \mathbb{E}_t \Big[u_t(x_{it}, wl_{it}, rl_{it}, d_{it}', \mathrm{h}_{it}) \Big]$$

$d_{it}' = (d_{it}^{rl}, d_{it}^{wl})$	*discrete* residential/work location decision
h_{it}	*continuous* house size decision (in m^2)
t_0, T	household age ($t_0 = 26$, $T = 76$)
$\rho_t(x_{it})$	survival probability times discount factor ($\beta = 0.95$)
$u_t(\cdot)$	instantaneous utility function
(wl_{it}, rl_{it})	work and residential location in beginning of t, i.e "decision outcome" component of state

The Choice Model

Individuals solve the following optimization problem:

$$\max_{\{d_{it}^{rl}, d_{it}^{wl}, h_{it}\}_{t=t_0}^{T}} \sum_{t=t_0}^{T} \rho_t(x_{it})^t \mathbb{E}_t \left[u_t(x_{it}, wl_{it}, rl_{it}, d_{it}', \mathrm{h}_{it}) \right]$$

$d_{it}' = (d_{it}^{rl}, d_{it}^{wl})$	*discrete* residential/work location decision
h_{it}	*continuous* house size decision (in m^2)
t_0, T	household age ($t_0 = 26$, $T = 76$)
$\rho_t(x_{it})$	survival probability times discount factor ($\beta = 0.95$)
$u_t(\cdot)$	instantaneous utility function
(wl_{it}, rl_{it})	work and residential location in beginning of t, i.e "decision outcome" component of state
x_{it}	(time-varying) individual states, $x_{it} = (ms_{it}, cs_{it}, age_{it}, edu_i)$

The Choice Model

Individuals solve the following optimization problem:

$$\max_{\{d_{it}^{rl}, d_{it}^{wl}, h_{it}\}_{t=t_0}^{T}} \sum_{t=t_0}^{T} \rho_t(x_{it})^t \mathbb{E}_t \left[u_t(x_{it}, wl_{it}, rl_{it}, d_{it}', \mathrm{h}_{it}) \right]$$

$d_{it}' = (d_{it}^{rl}, d_{it}^{wl})$	*discrete* residential/work location decision
h_{it}	*continuous* house size decision (in m^2)
t_0, T	household age ($t_0 = 26, T = 76$)
$\rho_t(x_{it})$	survival probability times discount factor ($\beta = 0.95$)
$u_t(\cdot)$	instantaneous utility function
(wl_{it}, rl_{it})	work and residential location in beginning of t, i.e "decision outcome" component of state
x_{it}	(time-varying) individual states, $x_{it} = (ms_{it}, cs_{it}, age_{it}, edu_i)$
s_{it}	$\{x_{it}, wl_{it}, rl_{it}\}$

Utility specification

$$u_t(x, d, d^{rl}, d^{wl}, h) =$$

$$\kappa(inc, s) \underbrace{\left[inc(s) - \psi_{uc} p^h(d^{rl}) \cdot h\right]}_{\text{consumption}} - swcost_r^p(x) + \underbrace{\phi_{h1}(s)h + \phi_{h2}h^2}_{\text{utility housing}}$$

$$+ \text{ttimecost}(x, d) + \text{amenities}(d^{rl}) + u_r$$

▸ Functional forms

Utility specification

$$u_t(x, d, d^{rl}, d^{wl}, h) =$$

$$\kappa(inc, s) \underbrace{\left[inc(s) - \psi_{uc} p^h(d^{rl}) \cdot h\right]}_{\text{consumption}} - swcost_r^p(x) + \underbrace{\phi_{h1}(s)h + \phi_{h2}h^2}_{\text{utility housing}}$$

$$+ \text{ttimecost}(x, d) + \text{amenities}(d^{rl}) + u_r$$

$\kappa(inc, s)$ Marginal utility of money ($\sim$ consumption). Depends on income, marital status, age, schooling and year.

Functional forms

Utility specification

$$u_t(x, d, d^{rl}, d^{wl}, h) =$$

$$\kappa(inc, s) \underbrace{\left[inc(s) - \psi_{uc} p^h(d^{rl}) \cdot h\right]}_{\text{consumption}} - swcost_r^p(x) + \underbrace{\phi_{h1}(s)h + \phi_{h2}h^2}_{\text{utility housing}}$$

$$+ \text{ttimecost}(x, d) + \text{amenities}(d^{rl}) + u_r$$

$\kappa(inc, s)$ Marginal utility of money ($\sim$ consumption). Depends on income, marital status, age, schooling and year.

$p^h(d^{rl})$ (rental) price per m^2 in chosen region

▸ Functional forms

Utility specification

$$u_t(x, d, d^{rl}, d^{wl}, h) =$$

$$\kappa(inc, s)\underbrace{\left[inc(s) - \psi_{uc} p^h(d^{rl}) \cdot h\right]}_{\text{consumption}} - swcost_r^p(x) + \underbrace{\phi_{h1}(s)h + \phi_{h2}h^2}_{\text{utility housing}}$$

$$+ \text{ttimecost}(x, d) + \text{amenities}(d^{rl}) + u_r$$

$\kappa(inc, s)$ Marginal utility of money ($\sim$ consumption). Depends on income, marital status, age, schooling and year.

$p^h(d^{rl})$ (rental) price per m^2 in chosen region

$swcost_r^p$ psychological switching cost of moving residence depend on marital status, children, age and schooling

▸ Functional forms

Utility specification

$$u_t(x, d, d^{rl}, d^{wl}, h) =$$

$$\kappa(inc, s) \underbrace{\left[inc(s) - \psi_{uc} p^h(d^{rl}) \cdot h\right]}_{\text{consumption}} - swcost_r^p(x) + \underbrace{\phi_{h1}(s) h + \phi_{h2} h^2}_{\text{utility housing}}$$

$$+ \text{ttimecost}(x, d) + \text{amenities}(d^{rl}) + u_r$$

$\kappa(inc, s)$	Marginal utility of money ($\sim$ consumption). Depends on income, marital status, age, schooling and year.
$p^h(d^{rl})$	(rental) price per m^2 in chosen region
$swcost_r^p$	psychological switching cost of moving residence depend on marital status, children, age and schooling
$\phi_{h1}(s), \phi_{h2}$	utility from housing depends on marital status, children and age

Functional forms

Utility specification

$$u_t(x, d, d^{rl}, d^{wl}, h) =$$

$$\kappa(inc, s) \underbrace{\left[inc(s) - \psi_{uc} p^h(d^{rl}) \cdot h\right]}_{\text{consumption}} - swcost_r^p(x) + \underbrace{\phi_{h1}(s)h + \phi_{h2}h^2}_{\text{utility housing}}$$

$$+ \text{ttimecost}(x, d) + \text{amenities}(d^{rl}) + u_r$$

$\kappa(inc, s)$	Marginal utility of money ($\sim$ consumption). Depends on income, marital status, age, schooling and year.
$p^h(d^{rl})$	(rental) price per m^2 in chosen region
$swcost_r^p$	psychological switching cost of moving residence depend on marital status, children, age and schooling
$\phi_{h1}(s), \phi_{h2}$	utility from housing depends on marital status, children and age
ttimecost	depends on travel-time between d^{rl} and d^{wl} and on year

Functional forms

Utility specification

$$u_t(x, d, d^{rl}, d^{wl}, h) =$$

$$\kappa(inc, s)\ \underbrace{\Big[inc(s) - \psi_{uc} p^h(d^{rl}) \cdot h\Big]}_{\text{consumption}} - swcost_r^p(x) + \underbrace{\phi_{h1}(s)h + \phi_{h2}h^2}_{\text{utility housing}}$$

$$+ \text{ttimecost}(x, d) + \text{amenities}(d^{rl}) + u_r$$

$\kappa(inc, s)$ Marginal utility of money ($\sim$ consumption). Depends on income, marital status, age, schooling and year.

$p^h(d^{rl})$ (rental) price per m^2 in chosen region

$swcost_r^p$ psychological switching cost of moving residence depend on marital status, children, age and schooling

$\phi_{h1}(s), \phi_{h2}$ utility from housing depends on marital status, children and age

ttimecost depends on travel-time between d^{rl} and d^{wl} and on year

amenities depend on regional attributes of residential location, d^{rl}

Functional forms

Utility specification

$$u_t(x, d, d^{rl}, d^{wl}, h) =$$

$$\kappa(inc, s) \underbrace{\left[inc(s) - \psi_{uc} p^h(d^{rl}) \cdot h \right]}_{\text{consumption}} - swcost_r^p(x) + \underbrace{\phi_{h1}(s)h + \phi_{h2}h^2}_{\text{utility housing}}$$

$$+ \text{ttimecost}(x, d) + \text{amenities}(d^{rl}) + u_r$$

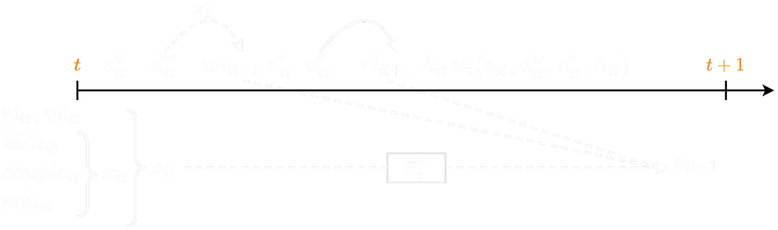
$\kappa(inc, s)$	Marginal utility of money ($\sim$ consumption). Depends on income, marital status, age, schooling and year.
$p^h(d^{rl})$	(rental) price per m^2 in chosen region
$swcost_r^p$	psychological switching cost of moving residence depend on marital status, children, age and schooling
$\phi_{h1}(s), \phi_{h2}$	utility from housing depends on marital status, children and age
ttimecost	depends on travel-time between d^{rl} and d^{wl} and on year
amenities	depend on regional attributes of residential location, d^{rl}
u_r	utility of retiring for those eligible (≥ 60)

Functional forms

Model structure

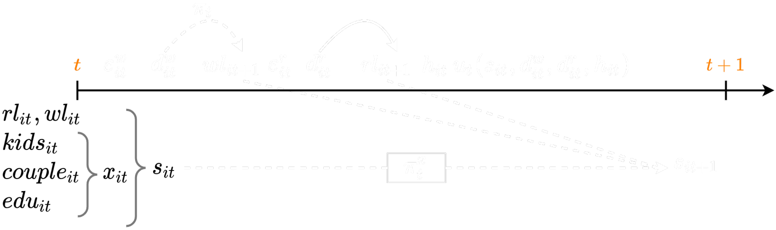
Model is based on ▸ Bellman equations whose content is illustrated in the following:

Figure 1: Timeline of decisions and states



Model is based on ▸ Bellman equations whose content is illustrated in the following:

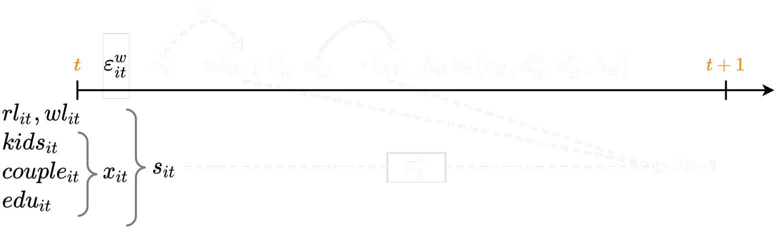
Figure 1: Timeline of decisions and states



Model structure

Model is based on ▸ Bellman equations whose content is illustrated in the following:

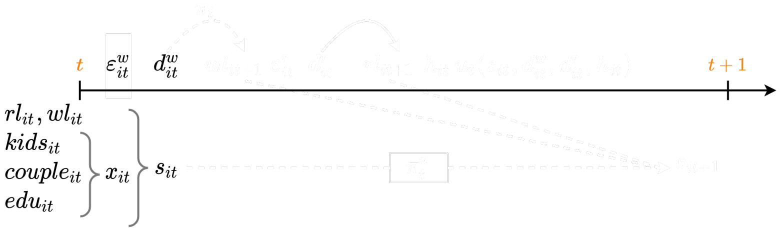
Figure 1: Timeline of decisions and states



Model structure

Model is based on ▸ Bellman equations whose content is illustrated in the following:

Figure 1: Timeline of decisions and states

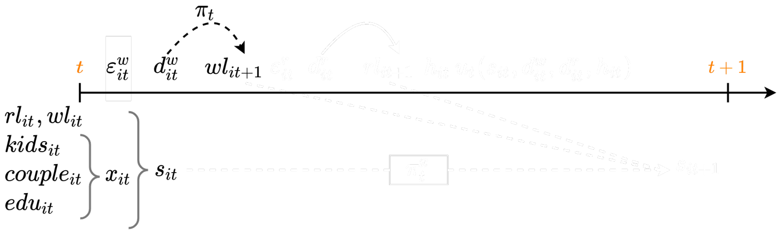


- R regions, $d_{it}^{w} \in R+1$ (voluntary and involuntary unemployment: ø)

Model structure

Model is based on ▸ Bellman equations whose content is illustrated in the following:

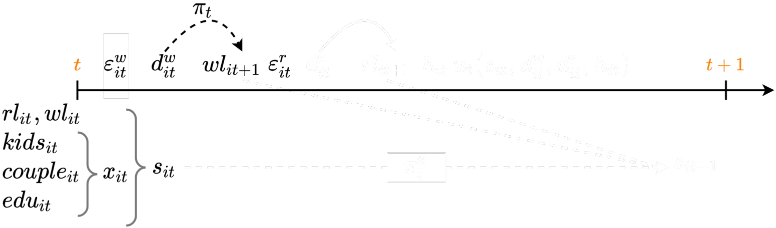
Figure 1: Timeline of decisions and states



- $\pi_t(d_t^w, wl_t, x_t)$ informs about transition from desired work location to work location outcome

Model is based on ▸ Bellman equations whose content is illustrated in the following:

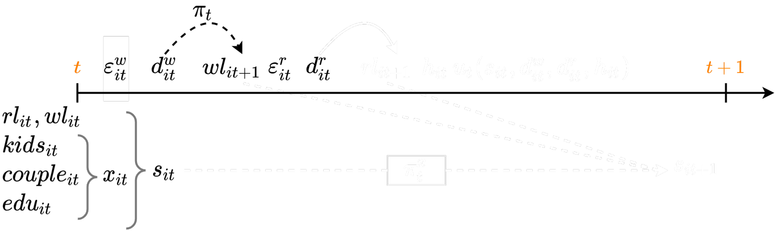
Figure 1: Timeline of decisions and states



Model structure

Model is based on ▸ Bellman equations whose content is illustrated in the following:

Figure 1: Timeline of decisions and states

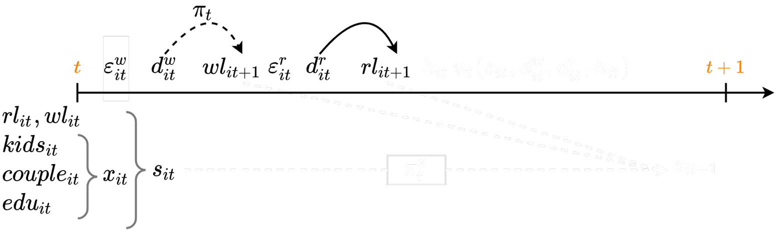


- $d^r_{it} \in R$

Model structure

Model is based on ▸ Bellman equations whose content is illustrated in the following:

Figure 1: Timeline of decisions and states

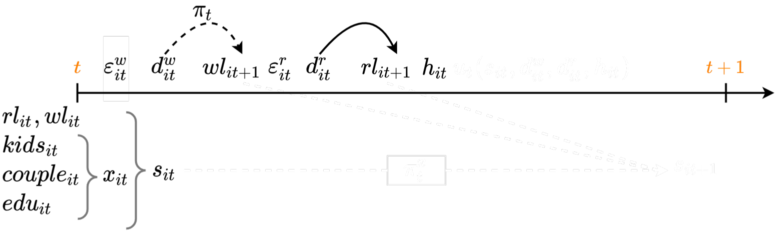


- $d^r_{it} \in R$ and perfect control over residential decision: $d^r_{it} = rl_{it+1}$

Model structure

Model is based on ▸ Bellman equations whose content is illustrated in the following:

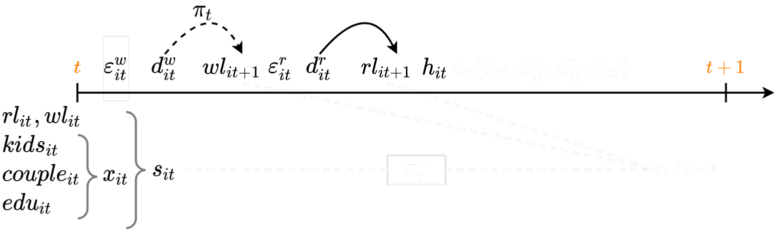
Figure 1: Timeline of decisions and states



- No dynamic implications of housing demand ⇒ static choice

Model is based on ‣ Bellman equations whose content is illustrated in the following:

Figure 1: Timeline of decisions and states

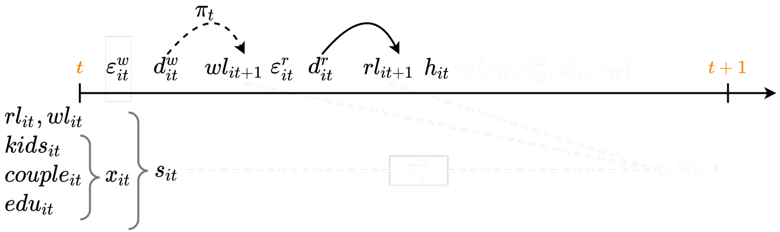


- No dynamic implications of housing demand $\Rightarrow$ static choice
- $\Rightarrow$ Next period value function $V_{t+1}(x, d, \varepsilon)$ independent of h

Model structure

Model is based on ▸ Bellman equations whose content is illustrated in the following:

Figure 1: Timeline of decisions and states



- $\Rightarrow$ Optimal housing demand given by FOC

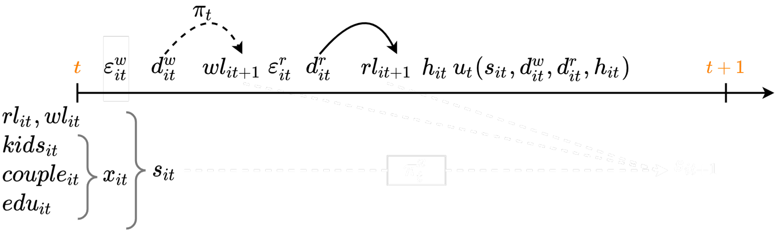
$$\frac{\partial u_t(\cdot)}{\partial h} = \phi_{h1}(s) + 2\phi_{h2}h - \kappa(s)p^h(d^{rl}) = 0 \quad \Rightarrow$$

$$h_t^* = \frac{\phi_{h1}(s) - \kappa(s)p^h(d^{rl})}{-2\phi_{h2}}$$

Model structure

Model is based on ▸ Bellman equations whose content is illustrated in the following:

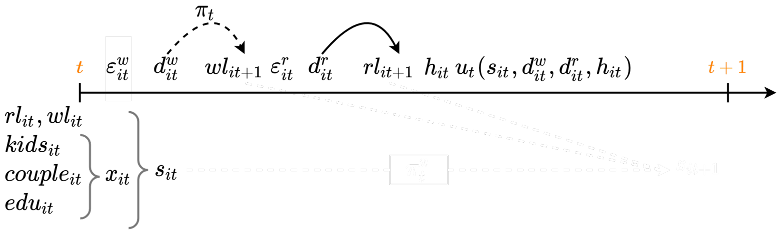
Figure 1: Timeline of decisions and states



Model structure

Model is based on ▸ Bellman equations whose content is illustrated in the following:

Figure 1: Timeline of decisions and states

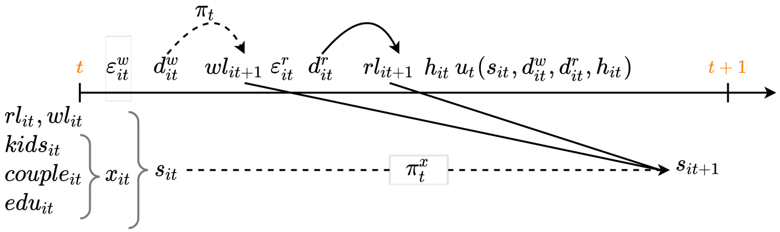


- Substituting expression of optimal housing demand into the utility function defined above, we obtain the *indirect utility function* $u(s_{it}, w_{it+1}, rl_{it+1})$
- Pure discrete choice model conditional on housing demand

Model structure

Model is based on ▸ Bellman equations whose content is illustrated in the following:

Figure 1: Timeline of decisions and states

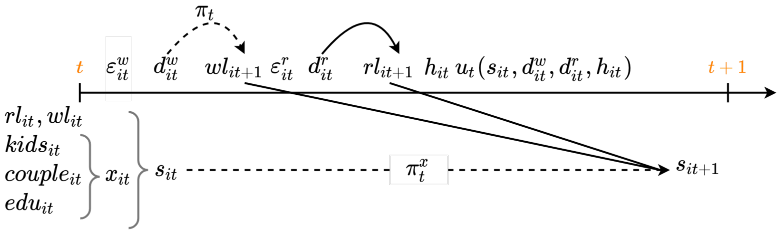


- Location states transition as a deterministic function of decisions

Model structure

Model is based on ▸ Bellman equations whose content is illustrated in the following:

Figure 1: Timeline of decisions and states

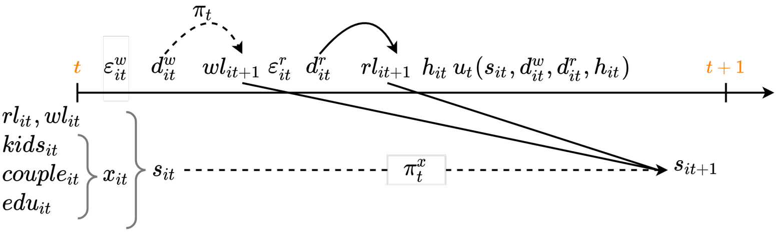


- Location states transition as a deterministic function of decisions
- edu_i is time-constant

Model structure

Model is based on ▸ Bellman equations whose content is illustrated in the following:

Figure 1: Timeline of decisions and states

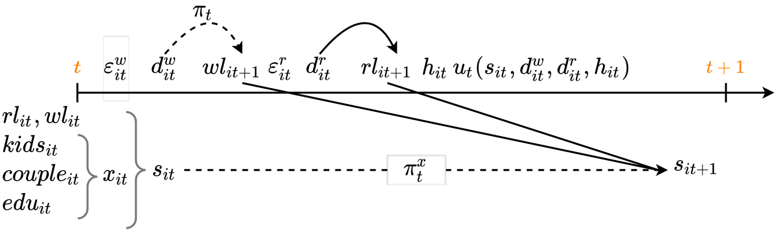


- Location states transition as a deterministic function of decisions
- edu_i is time-constant
- Non-location states:

Model structure

Model is based on ▸ Bellman equations whose content is illustrated in the following:

Figure 1: Timeline of decisions and states



- Location states transition as a deterministic function of decisions
- edu_i is time-constant
- Non-location states:

$$(cs_{it+1}, ms_{it+1}) \sim \mu_{cs,ms}(\cdot | cs_t, ms_t, edu, age_t) \equiv \pi^x(x_t, x_{t+1}) \tag{1}$$

Overview

Estimation strategy

We estimate the model sequentially in three separate steps:

1. Estimate the parameters governing the pre-tax income equations, income tax system and transition probabilities of children and marital status
2. Estimate a reduced form housing demand equation
3. Estimate the remaining structural parameters by maximum likelihood applying the parameters obtained in 1) and 2) *conditional* on house prices

We solve the model via backwards induction for each evaluation of the likelihood function

On top of this comes an equilibrium solver in the counterfactual simulations.

▸ Functional forms ▸ Log-likelihood function

Overview

Literature

Model

Estimation

Data

Results

Housing market equilibrium

Counterfactuals

Conclusion

Data sources

We use full population Danish administrative panel data and focus estimation on the period 2000-2004 and 2009-2013

- Exclude years around housing boom in financial crisis. No attempt to model temporary price hike during this period

Data sources

We use full population Danish administrative panel data and focus estimation on the period 2000-2004 and 2009-2013

- Exclude years around housing boom in financial crisis. No attempt to model temporary price hike during this period
- Focus: explain spatial variation in house prices

Data sources

We use full population Danish administrative panel data and focus estimation on the period 2000-2004 and 2009-2013

- Exclude years around housing boom in financial crisis. No attempt to model temporary price hike during this period
- Focus: explain spatial variation in house prices

We observe and exploit the following data

- Each individual's choice $d_{i,t} \equiv \{rl_{i,t+1}, wl_{i,t+1}, h_{i,t+1}\}$ and state $s_{i,t}$ on an annual basis

Data sources

We use full population Danish administrative panel data and focus estimation on the period 2000-2004 and 2009-2013

- Exclude years around housing boom in financial crisis. No attempt to model temporary price hike during this period
- Focus: explain spatial variation in house prices

We observe and exploit the following data

- Each individual's choice $d_{i,t} \equiv \{rl_{i,t+1}, wl_{i,t+1}, h_{i,t+1}\}$ and state $s_{i,t}$ on an annual basis
- Transaction prices of homes (we compute average by region and year)

Data sources

We use full population Danish administrative panel data and focus estimation on the period 2000-2004 and 2009-2013

- Exclude years around housing boom in financial crisis. No attempt to model temporary price hike during this period
- Focus: explain spatial variation in house prices

We observe and exploit the following data

- Each individual's choice $d_{i,t} \equiv \{rl_{i,t+1}, wl_{i,t+1}, h_{i,t+1}\}$ and state $s_{i,t}$ on an annual basis
- Transaction prices of homes (we compute average by region and year)
- Local amenities (we use # cafes and bars per km^2 and regional fixed effects)

Data sources

We use full population Danish administrative panel data and focus estimation on the period 2000-2004 and 2009-2013

- Exclude years around housing boom in financial crisis. No attempt to model temporary price hike during this period
- Focus: explain spatial variation in house prices

We observe and exploit the following data

- Each individual's choice $d_{i,t} \equiv \{rl_{i,t+1}, wl_{i,t+1}, h_{i,t+1}\}$ and state $s_{i,t}$ on an annual basis
- Transaction prices of homes (we compute average by region and year)
- Local amenities (we use # cafes and bars per km^2 and regional fixed effects)
- Local labor market attributes (we use # employees by *edu* to proxy for labor demand)

Data sources

We use full population Danish administrative panel data and focus estimation on the period 2000-2004 and 2009-2013

- Exclude years around housing boom in financial crisis. No attempt to model temporary price hike during this period
- Focus: explain spatial variation in house prices

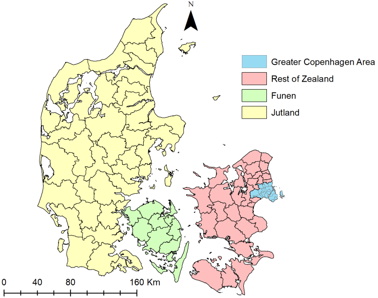
We observe and exploit the following data

- Each individual's choice $d_{i,t} \equiv \{rl_{i,t+1}, wl_{i,t+1}, h_{i,t+1}\}$ and state $s_{i,t}$ on an annual basis
- Transaction prices of homes (we compute average by region and year)
- Local amenities (we use # cafes and bars per km^2 and regional fixed effects)
- Local labor market attributes (we use # employees by *edu* to proxy for labor demand)
- Travel time between all regions

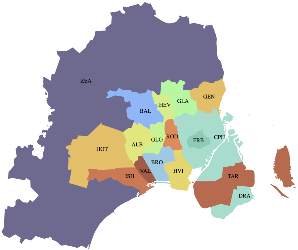
Regional definition

Figure 2: Definition of regions

(a) Entire Denmark



(b) Greater Copenhagen Area

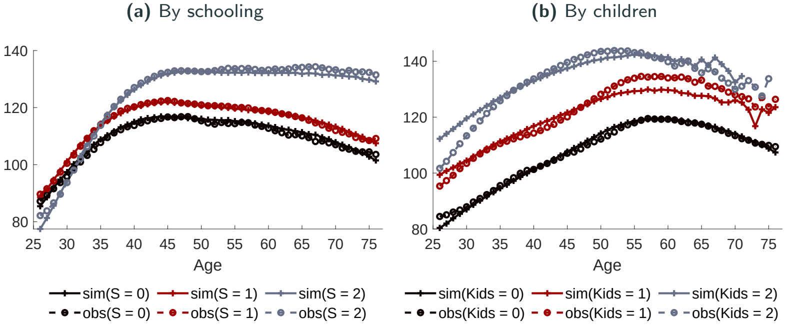


Note: The abbreviations denote the following regions: Copenhagen (CPH), Frederiksberg (FRB), Ballerup (BAL), Broendby (BRO), Dragoer (DRA), Gentofte (GEN), Gladsaxe (GLA), Glostrup (GLO), Herlev (HEV), Albertslund (ALB), Hvidovre (HVI), Hoeje-Taastrup (HOT), Roedovre (ROD), Ishoej (ISH), Taarnby (TAR), Vallensbaek (VAL), rest of Zealand (ZEA). Figure 2b only displays subset of ZEA.

Overview

Model fit: Housing demand

Figure 3: House size in square meters over the life cycle

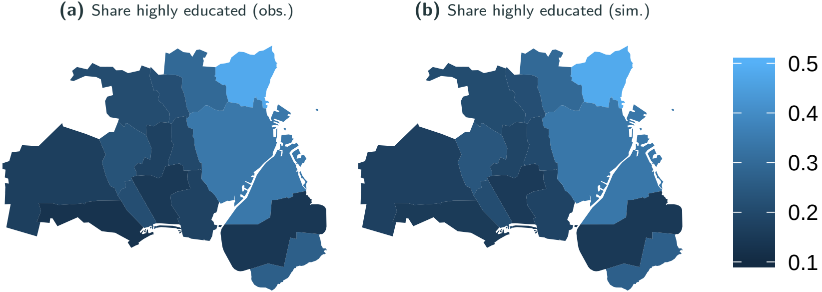


- Clear demographic differences in housing demand over the life cycle
- The reduced form model captures the crucial dependence between household composition and housing demand and difference over the life cycle
- Though some challenges capturing differences in demand at the beginning of the life cycle

▸ Spatial ▸ Marital status ▸ Params housing ▸ Params user costs

Model fit: Spatial educational sorting

Figure 4: Residential sorting by home region



Residential sorting driven mainly by:

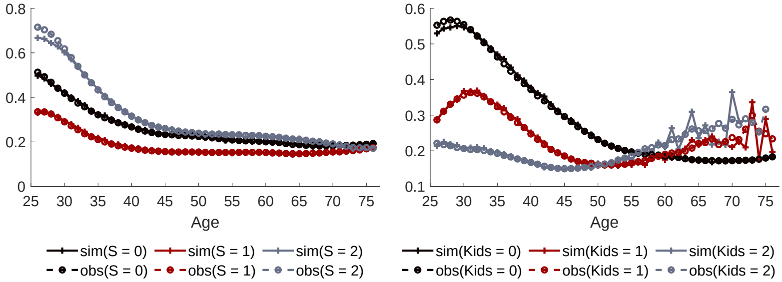
- Regional variation in house prices and regional-specific amenities
- Individual differences in housing demand
- Individual differences in marginal utility of money (main channel of educational sorting)
- Distance to local labor markets

▸ Params cafes ▸ Params amenities FE

Model fit: Sorting over the life cycle

Figure 5: Share living in Copenhagen over the life cycle

(a) By schooling **(b)** By children



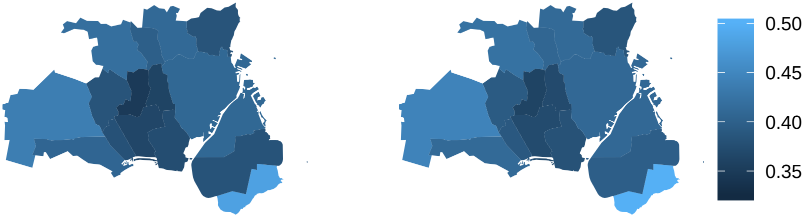
- Only for the youngest cohorts a slight under-prediction
- Reason: not modelling educational choice. Poorer fit only evident for highly-educated

▸ Marital status ▸ Moving ▸ Params moving costs

Model fit: Commuting across space

Figure 6: Commute times (hours) by residential location

(a) (obs.) **(b)** (sim.)

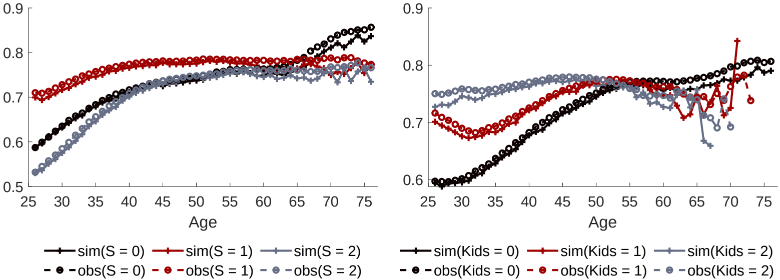


▸ Work in Cph ▸ Job moves ▸ Params commute costs ▸ Params job finding/dismissal

Model fit: Commute time over the life cycle

Figure 7: Commute time (hours)

(a) By schooling **(b)** By children



- Model captures shorter commute by highly educated as they can afford housing close to dense labor markets
- Above age 60 harder to explain the strong selection among working individuals at that age

Marital status

Overview

Demand and supply of housing

- Equilibrium prices, P^h: adjust so total expected demand $D_t(rl, P^h)$ for housing measured in square meters equals the (inelastic) supply $S_t(rl)$ in each residential region

Demand and supply of housing

- Equilibrium prices, P^h: adjust so total expected demand $D_t(rl, P^h)$ for housing measured in square meters equals the (inelastic) supply $S_t(rl)$ in each residential region
- Housing supply $S_t(rl)$: micro aggregated observed square meters of housing h_{it} for people who live in region $rl_{it} = rl$ at the beginning of each period t

$$S_t(rl) = \sum_{i=1}^{N} h_{it} 1(rl_{it} = rl)(1 - ms_{it}/2)$$

Demand and supply of housing

- Equilibrium prices, P^h: adjust so total expected demand $D_t(rl, P^h)$ for housing measured in square meters equals the (inelastic) supply $S_t(rl)$ in each residential region
- Housing supply $S_t(rl)$: micro aggregated observed square meters of housing h_{it} for people who live in region $rl_{it} = rl$ at the beginning of each period t

$$S_t(rl) = \sum_{i=1}^{N} h_{it} 1(rl_{it} = rl)(1 - ms_{it}/2)$$

- *Expected* housing demand $D_t(rl, P^h)$: population average of housing demand weighted by choice probabilities of either staying or moving to region rl at the end of period t.

$$D_t(rl, P^h) = \sum_{i=1}^{N} h(rl, x_{it}; P^h(rl)) \Pi_t(rl | wl_{it+1}, rl_{it}, x_{it}; P^h)(1 - ms_{it}/2),$$

where $\Pi_t(rl | wl_{it+1}, rl_{it}, x_{it}; P^h)$ is the choice probability that individuals in state $s_{it} = (wl_{it+1}, rl_{it}, x_{it})$ choose to live in region rl, given the vector of regional house prices, P^h

Equilibrium house prices

To compute the housing market equilibrium, P^h is set to solve

$$\begin{array}{rcl} S_t(1) & = & D_t(1, P^h) \\ & \vdots & \\ S_t(R) & = & D_t(R, P^h) \end{array}$$

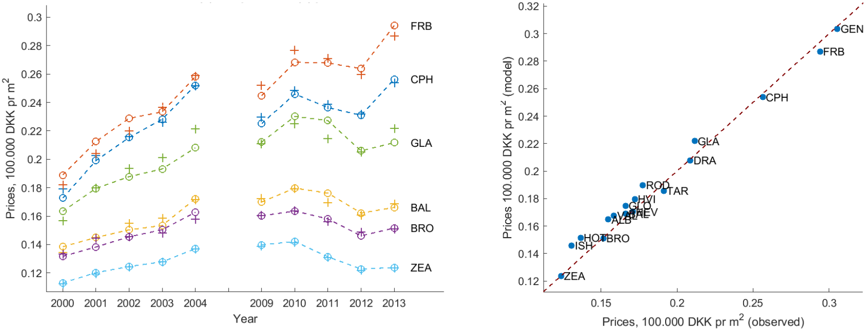
where

- $P^h = (P^h(1), .., P^h(R))$ s the R-dimensional vector of regional square meter prices in each residential region $rl = \{1, \ldots R\}$
- $S_t(rl)$ the inelastic, exogenously fixed supply of total square meters of housing in region rl
- $D_t(rl, P^h)$ is the demand for available square meters of housing in region rl
- We can easily solve the R equilibrium equations with R unknowns using Newton's Method.

Empirical vs. model equilibrium prices

Figure 8: Observed and predicted equilibrium house prices per m^2

(a) Selected regions, 2000-2004 and 2009-2013 **(b)** All regions, 2013



▸ Baseline fit

Counterfactual: increased access to telecommuting - effects on locations

Lowering commute time by 50% for highly-educated

Table 1: Counterfactual II: % change of home and work locations by schooling (2013)

	$rl(s_0)$	$wl(s_0)$	$rl(s_1)$	$wl(s_1)$	$rl(s_2)$	$wl(s_2)$
Center of CPH	3.77	0.44	4.52	0.39	-10.79	3.95
West of CPH	3.46	0.05	4.13	0.01	-13.23	-0.41
North of CPH	7.31	-0.06	7.51	-0.05	-9.62	-0.21
East of CPH	2.08	-0.11	4.81	-0.51	-10.40	1.07
RestOfZealand	-3.95	-0.44	-3.09	-0.76	11.69	7.64
Non-employment	-	-0.27	-	-0.20	-	-8.38

Counterfactual: increased access to telecommuting - effects on locations

Lowering commute time by 50% for highly-educated

Table 1: Counterfactual II: % change of home and work locations by schooling (2013)

	$rl(s_0)$	$wl(s_0)$	$rl(s_1)$	$wl(s_1)$	$rl(s_2)$	$wl(s_2)$
Center of CPH	3.77	0.44	4.52	0.39	-10.79	3.95
West of CPH	3.46	0.05	4.13	0.01	-13.23	-0.41
North of CPH	7.31	-0.06	7.51	-0.05	-9.62	-0.21
East of CPH	2.08	-0.11	4.81	-0.51	-10.40	1.07
RestOfZealand	-3.95	-0.44	-3.09	-0.76	11.69	7.64
Non-employment	-	-0.27	-	-0.20	-	-8.38

- Easier for highly educated to keep high-paying jobs in city centre while living in attractive suburban areas

Counterfactual: increased access to telecommuting - effects on locations

Lowering commute time by 50% for highly-educated

Table 1: Counterfactual II: % change of home and work locations by schooling (2013)

	$rl(s_0)$	$wl(s_0)$	$rl(s_1)$	$wl(s_1)$	$rl(s_2)$	$wl(s_2)$
Center of CPH	3.77	0.44	4.52	0.39	-10.79	3.95
West of CPH	3.46	0.05	4.13	0.01	-13.23	-0.41
North of CPH	7.31	-0.06	7.51	-0.05	-9.62	-0.21
East of CPH	2.08	-0.11	4.81	-0.51	-10.40	1.07
RestOfZealand	-3.95	-0.44	-3.09	-0.76	11.69	7.64
Non-employment	-	-0.27	-	-0.20	-	-8.38

- Easier for highly educated to keep high-paying jobs in city centre while living in attractive suburban areas
- Lower-income households better chance to reside closer to dense labor markets
- All regions more mixed on sociodemographics

Counterfactual: increased access to telecommuting - effects on locations

Lowering commute time by 50% for highly-educated

Table 1: Counterfactual II: % change of home and work locations by schooling (2013)

	$rl(s_0)$	$wl(s_0)$	$rl(s_1)$	$wl(s_1)$	$rl(s_2)$	$wl(s_2)$
Center of CPH	3.77	0.44	4.52	0.39	-10.79	3.95
West of CPH	3.46	0.05	4.13	0.01	-13.23	-0.41
North of CPH	7.31	-0.06	7.51	-0.05	-9.62	-0.21
East of CPH	2.08	-0.11	4.81	-0.51	-10.40	1.07
RestOfZealand	-3.95	-0.44	-3.09	-0.76	11.69	7.64
Non-employment	-	-0.27	-	-0.20	-	-8.38

- Highly-educated more likely to work in Cph and RoZ as they only have to commute half of the week

Counterfactual: increased access to telecommuting - effects on locations

Lowering commute time by 50% for highly-educated

Table 1: Counterfactual II: % change of home and work locations by schooling (2013)

	$rl(s_0)$	$wl(s_0)$	$rl(s_1)$	$wl(s_1)$	$rl(s_2)$	$wl(s_2)$
Center of CPH	3.77	0.44	4.52	0.39	-10.79	3.95
West of CPH	3.46	0.05	4.13	0.01	-13.23	-0.41
North of CPH	7.31	-0.06	7.51	-0.05	-9.62	-0.21
East of CPH	2.08	-0.11	4.81	-0.51	-10.40	1.07
RestOfZealand	-3.95	-0.44	-3.09	-0.76	11.69	7.64
Non-employment	-	-0.27	-	-0.20	-	-8.38

- Highly-educated more likely to work in Cph and RoZ as they only have to commute half of the week
- Main part of extra workers in RoZ come from reduction in non-employment

Counterfactual: increased access to telecommuting - effects on locations

Lowering commute time by 50% for highly-educated

Table 1: Counterfactual II: % change of home and work locations by schooling (2013)

	$rl(s_0)$	$wl(s_0)$	$rl(s_1)$	$wl(s_1)$	$rl(s_2)$	$wl(s_2)$
Center of CPH	3.77	0.44	4.52	0.39	-10.79	3.95
West of CPH	3.46	0.05	4.13	0.01	-13.23	-0.41
North of CPH	7.31	-0.06	7.51	-0.05	-9.62	-0.21
East of CPH	2.08	-0.11	4.81	-0.51	-10.40	1.07
RestOfZealand	-3.95	-0.44	-3.09	-0.76	11.69	7.64
Non-employment	-	-0.27	-	-0.20	-	-8.38

- Highly-educated more likely to work in Cph and RoZ as they only have to commute half of the week
- Main part of extra workers in RoZ come from reduction in non-employment
- Less educated only slight increase in tendency to work in more urbanized areas

Counterfactual: increased access to telecommuting - effects on prices

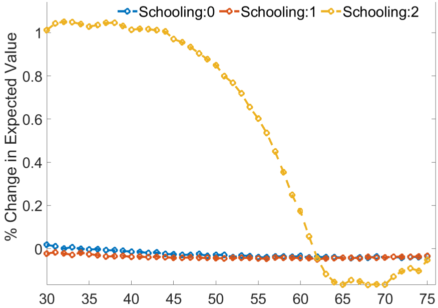
Table 2: Counterfactual II: % change in equilibrium prices 2009-2013

	2009	2010	2011	2012	2013
Center of CPH	-0.53	-1.01	-1.35	-1.64	-1.95
West of CPH	-0.36	-0.64	-0.93	-1.15	-1.60
North of CPH	-0.57	-1.08	-1.46	-1.58	-1.75
East of CPH	-0.06	-0.10	-0.79	-1.04	-1.66
RestOfZealand	0.79	1.55	2.08	2.45	3.03

- Dynamic model → Gradual changes in prices
- Lower demand for living in Cph for highly-educated → prices fall by 0.53% immediately
- Over time prices lower by 1.6-2.0% in the GCA while higher by 3.0% in RoZ
- → affordable for lower-income hhs to live in GCA

Counterfactual: increased access to telecommuting - welfare effects

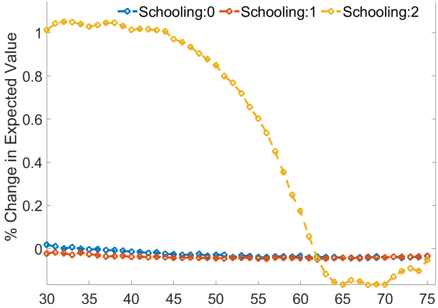
Figure 9: Simulated change in welfare (%)



- Highly-educated better off: access to high-paying jobs in Cph without paying high house prices for this access

Counterfactual: increased access to telecommuting - welfare effects

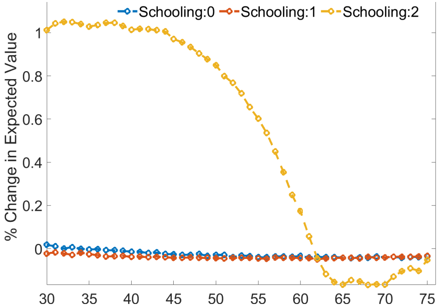
Figure 9: Simulated change in welfare (%)



- Highly-educated better off: access to high-paying jobs in Cph without paying high house prices for this access
- As they age, their marginal utility of money declines → welfare effects drops to 0 around retirement
- At retirement-eligible age: most not working → no benefit of telecommuting but facing increased house prices in their popular regions

Counterfactual: increased access to telecommuting - welfare effects

Figure 9: Simulated change in welfare (%)



- Highly-educated better off: access to high-paying jobs in Cph without paying high house prices for this access
- As they age, their marginal utility of money declines → welfare effects drops to 0 around retirement
- At retirement-eligible age: most not working → no benefit of telecommuting but facing increased house prices in their popular regions
- ≈ 0 welfare effect for less educated: gain from lower commute times, but only by paying higher house prices

Overview

Conclusion

- Dynamic equilibrium model of joint home and work location decisions as well as housing demand for individuals
- Focused on the Greater Copenhagen Area in Denmark and analyzed the counterfactual effects of encouraging more telecommuting for highly educated
- Provides understanding of how location and movement patterns are driven by cost of living and commuting and are very heterogeneous
- Counterfactual: highly educated move out of the city to peripheral regions $\rightarrow$ consume larger homes at a reduced price
- $\rightarrow$ freed up space in the center $\rightarrow$ lower educated people could afford living closer by their jobs in the new equilibrium
- Reducing commute times allows locations to become more specialized in either jobs or residence
- Welfare gains positive in total, but unequally distributed across household types with higher educated being better off and lower educated indifferent

Overview

The residential choice

Let EV^r be the *ex ante* expected value of residence r,
... but conditional on the employment location outcome wl'
... and before learning about the residential location shocks $\epsilon^r(d^r)$.

EV^r is given by the usual log-sum formula

$$EV^r(wl, rl, wl', x) =$$

$$\sigma_r \log\left(\sum_{d^r} \exp\{[u(wl, rl, wl', d^r, x) + \beta EV(wl', d^r, x)]/\sigma_r\}\right).$$

Residence location choice probabilities are given by logit formulas

$$P^r(d^r|wl, rl, wl', x) = \frac{\exp\{[u(wl, rl, wl', d^r, x) + \beta EV(wl', d^r, x)]/\sigma_r\}}{\sum_{d^r} \exp\{[u(wl, rl, wl', d^r, x) + \beta EV(wl', d^r, x)]/\sigma_r\}}.$$

Timeline

The work location choice

- Let $EV^w(wl, rl, x)$ be the *ex ante* expected value of location (wl, rl)
  ...before learning about the work location shocks $\epsilon^w(d^w)$
  ...and the outcome of the job search process

$$EV^w(wl_t, rl_t, x) = \sigma_w \log \left(\sum_{d^w} \exp\left\{ v^w(wl, rl, x, d^w)/\sigma_w \right\} \right).$$

- $v^w(wl, rl, x, d^w)$ is the expected choice-specific value corresponding to the particular choice of job location d^w.

$$v^w(wl, rl, x, d^w) = \sum_{wl} \pi(d^w, wl, x, wl) EV^r(wl, rl, wl, x).$$

- $\pi(d^w, wl, x, wl')$ governs how job search location d^w translates into the realized one wl'

- The work location *choice* probabilities are given by logit formulas

$$P^w(d^w | wl, rl, x) = \frac{\exp\{v^w(wl, rl, x, d^w)/\sigma_w\}}{\sum_{d^w} \exp\{v^w(wl, rl, x, d^w)/\sigma_w\}}.$$

◂ Timeline

Overview

Utility function

Parsimonious utility function:

$$u = u_m + u_r + u_h + \underbrace{amenities - swcost_r^p - ttimecost}_{u_o} \tag{2}$$

- u_m: *monetary utility* (disposable income net of housing expenditures)
- u_r: utility of retirement for the eligible individuals ($t \geq 60$)
- u_h: *housing utility* obtained from the utilization of a chosen home size
- *amenities*: *regional-specific* attractiveness of housing options
- $swcost_r^p$: *psychological costs* of changing the location of residence
- *ttimecost*: *cost of commuting* between the chosen locations of work and residence

◂ Econometrics ◂ Utility

Housing demand

Quadratic utility of housing

$$u_m + u_h = \kappa(inc_t)(inc_t - hcost_t) + \Phi(x_t)h_{t+1} + \frac{1}{2}\phi_{h2}\, h_{t+1}^2,$$

where $\phi_{h2} < 0$ (diminishing returns to house size)

Marginal utility of money depends on income, year and x

$$\kappa(inc_t) = \kappa_0 + \sum_{\tilde{y}=1}^{Y} \kappa_{year,\tilde{y}} \mathbb{1}_{\{year=\tilde{y}\}} + \kappa_y inc_t + \kappa_{ms} ms_t +$$

$$+ \sum_{k=1}^{2} \kappa_{c,k} \mathbb{1}_{\{cs_t=k\}} + \kappa_a age_t + \sum_{j=1}^{2} [(\kappa_{s,j} + \kappa_{as,j} age_t) \mathbb{1}_{\{edu_t=j\}}]$$

Housing demand

$\Phi(x_t)$ allows for heterogeneity in marginal utility of housing

$$\Phi(x_t) = \phi_0 + \sum_{y=1}^{Y} \phi_{year,y} \mathbb{1}_{\{year=y\}} + \phi_a age_t + \phi_{a2} age_t^2 + \phi_{ms} ms_t$$
$$+ \sum_{k=1}^{2} \phi_{c,k} \mathbb{1}_{\{cs_t=k\}} + \sum_{j=1}^{2} \phi_{s,j} \mathbb{1}_{\{edu_t=j\}} + \sum_{rl=1}^{R} \phi_{rl} \mathbb{1}_{\{rl_{t+1}=rl\}}.$$

Housing costs are given by

$$hcost_t(rl_{t+1}, h_{t+1}) = \psi_{uc} p^h(rl_{t+1}) h_{t+1},$$

Implied housing demand (linear regression).

$$h_{t+1} = \frac{\kappa(inc_t) p^h(rl_{t+1}) \psi_{uc} - \Phi(x_t)}{\phi_{h2}}.$$

◂ Econometrics ◂ Utility

Regional amenities

Amenities of regions come as a bundle of attributes that each contributes to the experienced utility of a region.

$$
\begin{aligned}
amenities(rl_{t+1}) = &\left(\alpha_0^{cafe} + \alpha_a^{cafe} age_t + \sum_{k=1}^{2} \alpha_{c,k}^{cafe} \mathbb{1}_{\{cs_t = k\}}\right) cafes_{rl_{t+1}} \\
&+ \sum_{rl=1}^{R} \boldsymbol{\alpha}_{rl} \mathbb{1}_{\{rl_{t+1} = rl\}}
\end{aligned}
$$

- We can include a rich set of amenities almost without any additional computational cost associated with solving the model.
- This may require more parameters as number of amenities increases, but the number of parameters in the chosen specification is independent of the number of regions

◂ Econometrics ◂ Utility

Job arrival and dismissal

Probability of getting a new job

$$\pi_t^n(d^w, wl, x) = \Bigg[1 + exp\Bigg(- \Big(\beta_0^{\pi(n)} + \beta_a^{\pi(n)} age + \beta_\o^{\pi(n)} 1_{wl=\o} + \beta_{jobdens}^{\pi(n)} jobdens(d^w) + \sum_{k=1}^{2} (\beta_s^{\pi(n)}(k) 1_{edu=k})\Big)\Bigg)\Bigg]^{-1}, \tag{3}$$

Probability of keeping current job

$$\pi^k(wl, x) = \Bigg[1 + exp\Bigg(- \Big(\beta_0^{\pi(k)} + \beta_a^{\pi(k)} age + \sum_{k=1}^{2} (\beta_s^{\pi(k)}(k)\{1_{edu=k}))\Big)\Bigg)\Bigg]^{-1}. \tag{4}$$

◂ Econometrics ◂ Utility

Utility cost of moving, $swcost_r^p(x_t)$

The utility cost of moving residence

$$swcost_r^p(x_t) = \mathbb{1}_{\{rl_t \neq rl_{t+1}\}}\Big[\gamma_0 + \gamma_a age_t + \sum_{k=1}^{2} \gamma_{c,k} \mathbb{1}_{\{cs_t = k\}} + \gamma_{ms} ms_t + \sum_{j=1}^{2} \gamma_{s,j} \mathbb{1}_{\{edu_t = j\}}\Big]$$

◂ Econometrics ◂ Utility

Travel time costs

The commuting cost between d^{rl} and d^{wl}

$$ttimecost = (\eta_0 + \sum_{y=1}^{Y} \eta_{year,y} \mathbb{1}_{\{year=y\}}) f^{tt}(rl_{t+1}, wl_{t+1})$$

where

- The $f^{tt}(\cdot)$ function denotes the shortest travel-time by any means of transportation between locations.
- Commute cost are zero when unemployed

Commute cost are assumed to be a function of

- Travel time between the two destinations.
- Year (allowing disutility of commuting to change over time)

◂ Econometrics ◂ Utility

Overview

Log-likelihood function

- The MLE is obtained as

$$\hat{\theta} = \text{argmax}_\theta \; \frac{1}{N} \sum_i \sum_t \{ \log P_t^r(rl_{it+1} | wl_{it}, rl_{it}, wl_{it+1}, x_{it}; \theta) + \\ \log \sum_{d^w} P_t^w(d^w | wl_{it}, rl_{it}, x_{it}; \theta) \pi_t(d^w, wl_{it}, x_{it}, wl_{it+1}; \theta) \}, \tag{5}$$

  where N is the number of individuals.
- We solve the model via backwards induction for each evaluation of the likelihood function.

Back

Overview

Appendix: Bellman equations

Appendix: Functional forms

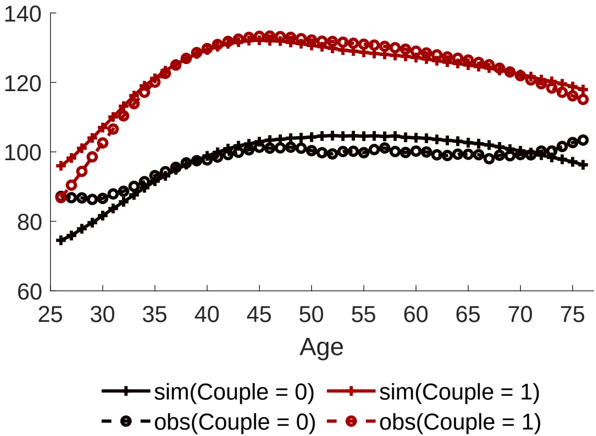
Appendix: Econometrics

Appendix: Model fits

Appendix: Model parameters

Model fit: Housing demand by marital status

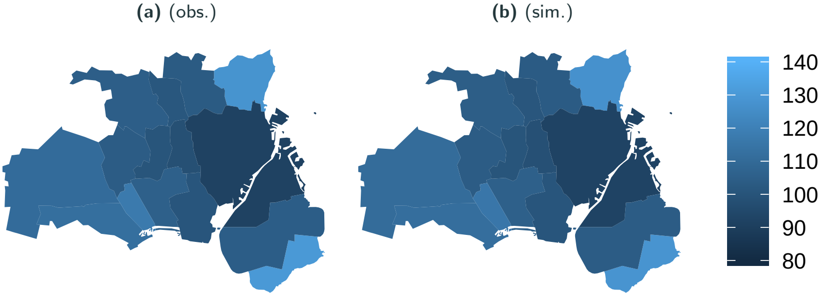
Figure 10: House size in square meters over the life cycle



Back

Model fit: Housing demand by region

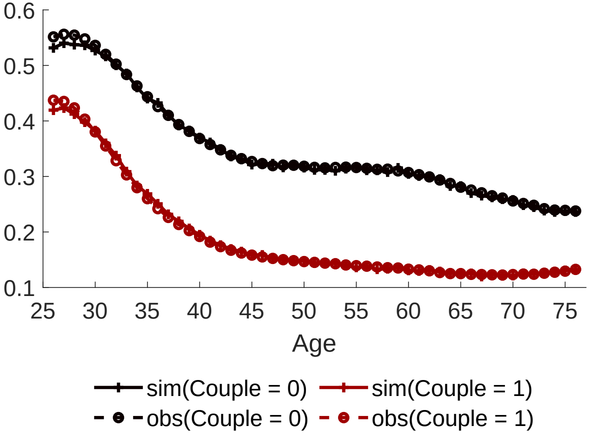
Figure 11: House size by home region



Back

Model fit: Share living in Copenhagen by marital status

Figure 12: Share living in Copenhagen over the life cycle by marital status

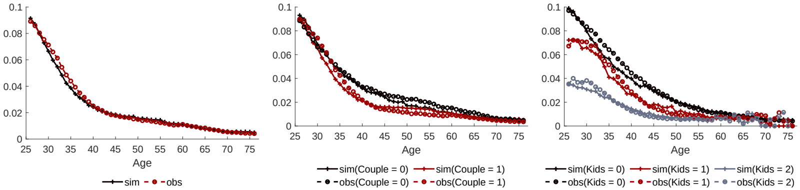


Back

Model fit: Moving propensity

Figure 13: Share moving residential location over the life cycle

(a) Overall **(b)** By marital status **(c)** By children

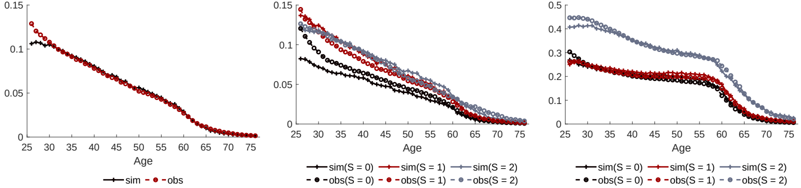


Back

Model fit: Sorting on workplace over the life cycle

Figure 14: Job moves

(a) Move job **(b)** Move job by schooling **(c)** Working in CPH by schooling

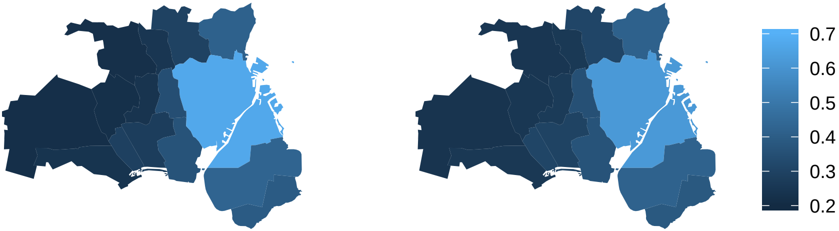


◂ Back

Model fit: Working in Cph by home location

Figure 15: Work in Copenhagen by residential location

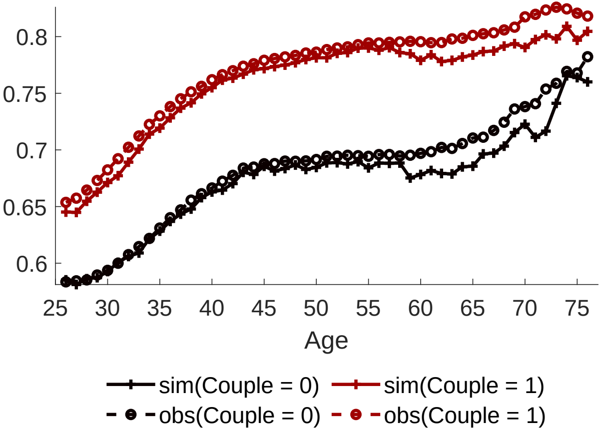
(a) (obs.) **(b)** (sim.)



◂ Back

Model fit: Commuting by marital status

Figure 16: Commute time (hours) by marital status



Back

Overview

Model parameters: Housing demand

Table 3: First Stage Parameter Estimates, Reduced form Housing Demand

	Coeff. Estimates	Standard Error	Z-statistic
Const., $\tilde{\phi}_0$	70.2740	0.16419	428.0
Married, $\tilde{\phi}_{ms}$	27.8578	0.03868	720.2
Children, $\tilde{\phi}_c$ (1)	5.7386	0.05149	111.4
Children, $\tilde{\phi}_c$ (2)	14.6098	0.04911	297.5
Age, $\tilde{\phi}_a$	2.1723	0.00348	624.8
Age^2/1000, $\tilde{\phi}_{a2}$	-19.1718	0.03074	-623.6
Price pr. sqm, $\tilde{\kappa}_0$	-296.2954	0.91043	-325.4
Price pr. sqm $\times$ Income, $\tilde{\kappa}_y$	20.2790	0.07002	289.6
Price pr. sqm $\times$ Age, $\tilde{\kappa}_a$	0.0209	0.00853	2.4
Price pr. sqm $\times$ Age x Schooling, $\tilde{\kappa}_{a,s}$ (1)	1.0073	0.00476	211.5
Price pr. sqm $\times$ Age x Schooling, $\tilde{\kappa}_{a,s}$ (2)	2.9563	0.00529	558.4
Price pr. sqm $\times$ Schooling, $\tilde{\kappa}_s$ (1)	-51.7247	0.24317	-212.7
Price pr. sqm $\times$ Schooling, $\tilde{\kappa}_s$ (2)	-95.3167	0.25080	-380.1
Price pr. sqm $\times$ Children, $\tilde{\kappa}_c$ (1)	0.4389	0.29673	1.5
Price pr. sqm $\times$ Children, $\tilde{\kappa}_c$ (2)	13.4067	0.28757	46.6
Price pr. sqm $\times$ Married, $\tilde{\kappa}_{ms}$	-63.4794	0.22117	-287.0

Dependent variable: House size in square meters

Other controls: Regional dummies, $\tilde{\phi}_{rl}$ and time effects $\tilde{\phi}_{year}$ and $\tilde{\kappa}_{year}$

◂ Back

Model parameters: User costs of housing demand curvature

Table 4: User Cost of Housing and Curvature Parameter of Housing Demand

	Coeff. Estimates	Standard Error	Z-statistic
Coef. on $h^2, \phi_{h2} \times 1000$	-0.0465	0.00036	-127.4
Baseline user cost of housing, ψ_0	0.0239	0.00024	99.9
Time effect, ψ_{2001}	-0.0052	0.00015	-34.9
Time effect, ψ_{2002}	-0.0045	0.00016	-28.8
Time effect, ψ_{2003}	-0.0063	0.00015	-41.0
Time effect, ψ_{2004}	-0.0090	0.00016	-57.3
Time effect, ψ_{2009}	-0.0035	0.00015	-23.6
Time effect, ψ_{2010}	-0.0076	0.00015	-50.2
Time effect, ψ_{2011}	-0.0089	0.00015	-57.7
Time effect, ψ_{2012}	-0.0088	0.00015	-57.1
Time effect, ψ_{2014}	-0.0110	0.00016	-67.4

Back

Model parameters: Amenities (time-varying)

Table 5: Taste Variation in Regional Amenities

	Coeff. Estimates	Standard Error	Z-statistic
Taste for cafes and bars, α^{cafe}			
Constant, α_0^{cafe}	0.0118	0.00005	257.9
Age, α_a^{cafe}	-0.0002	0.00000	-257.5
Children, α_c^{cafe} (1)	-0.0047	0.00005	-97.8
Children, α_c^{cafe} (2)	-0.0074	0.00003	-254.4

Other controls: Regional dummies, $\alpha_{rl_{t+1}}$, shown in online appendix.

Back

Model parameters: Amenities (fixed effects)

Table 6: Time-Constant Regional Amenities

	Coeff. Estimates	Standard Error	Z-statistic
α_{rl} (1)	-0.0976	0.00045	-219.3
α_{rl} (2)	-0.0827	0.00095	-87.0
α_{rl} (3)	-0.1244	0.00107	-115.8
α_{rl} (4)	-0.2733	0.00207	-131.9
α_{rl} (5)	-0.1956	0.00126	-155.7
α_{rl} (6)	-0.0721	0.00081	-89.4
α_{rl} (7)	-0.1329	0.00127	-104.5
α_{rl} (8)	-0.1086	0.00116	-93.6
α_{rl} (9)	-0.1246	0.00121	-102.7
α_{rl} (10)	-0.0713	0.00088	-81.5
α_{rl} (11)	-0.1310	0.00101	-129.9
α_{rl} (12)	-0.0801	0.00094	-85.3
α_{rl} (13)	-0.1888	0.00139	-135.9
α_{rl} (14)	-0.1037	0.00099	-104.6
α_{rl} (15)	-0.2247	0.00163	-137.8
α_{rl} (16)	-0.0274	0.00127	-21.6

Back

Model parameters: Moving costs

Table 7: Utility Cost of Moving Residence

	Coeff. Estimates	Standard Error	Z-statistic
Const., γ_0	1.8750	0.00521	360.0
Age, γ_a	0.0579	0.00012	495.1
Children, γ_c (1)	0.4934	0.00382	129.2
Children, γ_c (2)	1.1926	0.00450	265.0
Married, γ_{ms}	-0.0368	0.00291	-12.6
Schooling, γ_s (1)	0.0163	0.00309	5.3
Schooling, γ_s (2)	-0.1803	0.00317	-56.9

Back

Model parameters: Commuting

Table 8: Commute Cost

	Coeff. Estimates	Standard Error	Z-statistic
Cost of travel time, η_0	0.1789	0.00149	120.3
Time effect, η_{2001} (1)	-0.0014	0.00174	-0.8
Time effect, η_{2002} (2)	-0.0081	0.00178	-4.5
Time effect, η_{2003} (3)	-0.0210	0.00184	-11.4
Time effect, η_{2004} (4)	-0.0451	0.00181	-24.9
Time effect, η_{2009} (5)	0.0406	0.00191	21.2
Time effect, η_{2010} (6)	0.0323	0.00190	17.0
Time effect, η_{2011} (7)	0.0450	0.00194	23.2
Time effect, η_{2012} (8)	0.0484	0.00195	24.8
Time effect, η_{2013} (9)	0.0202	0.00197	10.2

Back

Model parameters: Job arrival and dismissal

Table 9: Job Arrival and Dismissal

	Coeff. Estimates	Standard Error	Z-statistic
Probability of keeping job: $\pi_t^k(wl_t, x_t; \beta^k)$			
Const., $\beta_0^{\pi(keep)}$	0.3066	0.01085	28.3
Age, $\beta_a^{\pi(keep)}$	0.0558	0.00030	186.7
Schooling, $\beta_s^{\pi(keep)}$ (1)	0.9288	0.00536	173.4
Schooling, $\beta_s^{\pi(keep)}$ (2)	1.0818	0.00575	188.0
Probability of new job: $\pi_t^n(d_t^w, wl_t, x_t : \beta^n)$			
Const., $\beta_0^{\pi(new)}$	-1.0998	0.00466	-235.9
Age, $\beta_a^{\pi(new)}$	-0.0431	0.00010	-415.9
Schooling, $\beta_s^{\pi(new)}$ (1)	0.1980	0.00253	78.3
Schooling, $\beta_s^{\pi(new)}$ (2)	0.2264	0.00278	81.5
Job density $\beta_{jobdensity}^{\pi(new)}$	0.2608	0.00045	583.9
Prev. unempl., $\beta_{unemp}^{\pi(new)}$	1.0474	0.00236	443.9

Back

Model parameters: Baseline fit

Table 10: Baseline fit: Change in home and work locations (share by schooling) and equilibrium prices (100,000 DKK)

(data - baseline)	$rl(s_0)$	$rl(s_1)$	$rl(s_2)$	$wl(s_0)$	$wl(s_1)$	$wl(s_2)$	P^{eq}
Center of CPH	0.000	0.006	-0.048	0.003	0.033	-0.009	-0.023
West of CPH	-0.005	-0.003	0.012	0.119	0.120	0.111	0.010
North of CPH	-0.004	0.000	0.006	0.019	0.019	0.007	0.000
East of CPH	0.001	-0.001	0.003	0.024	0.025	0.024	0.004
RestOfZealand	0.008	-0.002	0.027	-0.142	-0.222	-0.170	0.012
Unemployment	-	-	-	-0.024	0.024	0.037	-

Back

Ahlfeldt, G. M., Redding, S. J., Sturm, D. M., & Wolf, N. (2015). The economics of density: Evidence from the berlin wall. *Econometrica*, *83*(6), 2127–2189.

Allen, T., & Arkolakis, C. (2019). *The Welfare Effects of Transportation Infrastructure Improvements* (Working Paper No. 25487). National Bureau of Economic Research.

Bayer, P., McMillan, R., Murphy, A., & Timmins, C. (2016). A Dynamic Model of Demand for Houses and Neighborhoods. *Econometrica*, *84*(3), 893–942.

Buchinsky, M., Gotlibovski, C., & Lifshitz, O. (2014). Residential Location, Work Location, and Labor Market Outcomes of Immigrants in Israel. *Econometrica*, *82*(3), 995–1054.

Chernoff, A., & Craig, A. N. (2021). Distributional and Housing Price Effects from Public Transit Investment: Evidence from Vancouver. *International Economic Review*, *Accepted Manuscript*.

Dahl, G. B. (2002). Mobility and the return to education: Testing a roy model with multiple markets. *Econometrica*, *70*(6), 2367–2420.

References ii

Diamond, R. (2016). The Determinants and Welfare Implications of US Workers' Diverging Location Choices by Skill: 1980â€"2000. *American Economic Review*, *106*(3), 479–524.

Dingel, J. I., & Tintelnot, F. (2020). *Spatial Economics for Granular Settings* (Working Paper No. 27287). National Bureau of Economic Research.

Heblich, S., Redding, S. J., & Sturm, D. M. (2020). The Making of the Modern Metropolis: Evidence from London*. *The Quarterly Journal of Economics*, *135*(4), 2059–2133.

Kennan, J., & Walker, J. R. (2011). The effect of expected income on individual migration decisions. *Econometrica*, *79*(1), 211–251.

Kuminoff, N. V., Smith, K. V., & Timmins, C. (2013). The new economics of equilibrium sorting and policy evaluation using housing markets. *Journal of Economic Literature*, *51*(4), 1007–62.

Manning, A., & Petrongolo, B. (2017). How Local Are Labor Markets? evidence from a Spatial Job Search Model. *American Economic Review*, *107*(10), 2877–2907.

Monte, F., Redding, S. J., & Rossi-Hansberg, E. (2018). Commuting, Migration, and Local Employment Elasticities. *American Economic Review*, *108*(12), 3855–3890.

Oswald, F. (2019). The effect of homeownership on the option value of regional migration. *Quantitative Economics*, *10*(4), 1453–1493.

Severen, C. (2021). Commuting, Labor, and Housing Market Effects of Mass Transportation: Welfare and Identification. *Review of Economics and Statistics*, 99.

Sieg, H., Smith, V. K., Banzhaf, H. S., & Walsh, R. (2004). Estimating the General Equilibrium Benefits of Large Changes in Spatially Delineated Public Goods*. *International Economic Review*, *45*(4), 1047–1077 _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.0020-6598.2004.00297.x.

Teulings, C. N., Ossokina, I. V., & de Groot, H. L. F. (2018). Land use, worker heterogeneity and welfare benefits of public goods. *Journal of Urban Economics*, *103*, 67–82.

Tsivanidis, N. (2019). *Evaluating the Impact of Urban Transit Infrastructure: Evidence from BogotÃ¡â€™s TransMilenio* (Working Paper). University of California, Berkeley.

Tunali, I. (2000). Rationality of migration. *International Economic Review*, *41*(4), 893–920.