Information Aggregation with Delegation of Votes

Amrita Dhillon Grammateia Kotsialou Dilip Ravindran
Dimitrios Xefteris

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Introduction

Liquid democracy (LD) is a voting system which combines aspects of representative democracy (RD) and direct democracy (DD).

- For each issue, every voter is endowed with a vote.
- She can cast it or delegate it to another voter.

LD has gained traction with new technologies (e.g. blockchain) allowing for secure implementations.

- "Google Votes" is based on allowing delegation of votes (see Hardts and Lopes 2015 for details) `https://www.youtube.com/watch?v=F4lkCECSBFw`

Direct Democracy

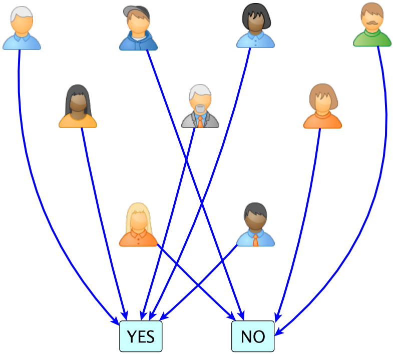

Figure: Direct Democracy

Representative Democracy

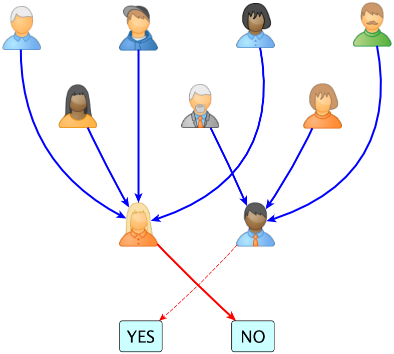

Figure: Representative Democracy

Liquid Democracy

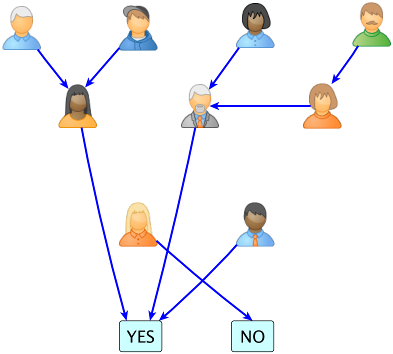

Figure: Liquid Democracy

Information Aggregation

- What are the advantages of delegation?
- Little work studying delegation of votes in strategic settings.

We focus on potential information aggregation advantages: worse informed voters can delegate to better informed voters.

- Setting has 'truth-seeking' (independent) voters and partisans.
- Delegation has tradeoffs:
  - if i delegates to j, she cannot express her own private information through voting.
  - May end up delegating to a partisan.

Preview of results

- Delegation can strictly improve welfare and strictly benefit truth-seeking voters.
  - Delegation trade-off can be ‘worth it’ up to a point.
  - Particular benefits in smaller committees.
- All voting systems considered admit very inefficient equilibria. Delegation may exacerbate this issue.
- Under certain conditions: voters may be able to coordinate on efficient equilibria more easily in LD than DD.

Model of LD

Game timing:

1. State $\omega \in \{A, B\}$ drawn with common prior $\pi = Pr(\omega = A)$
2. Type t_i of voter $i \in \mathcal{N} = \{1, ..., N\}$ commonly observed
3. Each voter i receives private signal $s_i \in \{a, b\}$
4. Voter i chooses to vote/abstain/delegate
5. Outcome is $\mathcal{O} \in \{A, B\}$ receiving majority of votes cast

Voter i has type $t_i = (p_i, q_i)$:

- $p_i \in \{A, B, I\}$ is i's private preference:

$$p_i = A, B \rightarrow u_i(\mathcal{O}, \omega) = \mathbb{1}_{\mathcal{O}=p_i} \text{ ('partisans')}$$
$$p_i = I \quad \rightarrow u_i(\mathcal{O}, \omega) = \mathbb{1}_{\mathcal{O}=\omega} \text{ ('independents')}$$

- $q_i \in [0.5, 1]$ is i's precision: $Pr(s_i = \omega|\omega) = q_i$ for $\omega = A, B$

Delegation, equilibria, comparing mechanisms

A strategy for voter i is: $\sigma_i : \{a, b\} \to \{a, b, x, d_{j \neq i}\}$

- Allow for transitive delegation ($i \xrightarrow{\text{d}} j \xrightarrow{\text{d}} k$)
- Abstain votes in cycle ($i \xrightarrow{\text{d}} j \xrightarrow{\text{d}} i$)
- If $\sigma_i(a) = a$, $\sigma_i(b) = b$, we say i votes *sincerely*

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Equilibrium: Bayes Nash Equilibrium in undominated strategies.

- What is *best* equilibrium in terms of: (1) matching the state, (2) ex-post majoritarian outcome (here: they coincide)
- How does this compare to other mechanisms:
  - DD (no delegation): $\sigma_i : \{a, b\} \to \{a, b, x\}$
  - RD (delegation to fixed set of representatives $\mathcal{J} \subset \mathcal{N}$): for $i \notin \mathcal{J}$ $\sigma_i : \{a, b\} \to \{x, d_{j \in \mathcal{J}}\}$
  - We assume that there there exist at least one of each partisan type in $\mathcal{J}$
- What do worst equilibria look like in LD, DD, RD?

The best equilibrium

Result. The best equilibrium of LD does weakly better than DD/RD.

- Partisan behavior is pinned down in equilibrium.
- In general: better informed independents vote while worse informed delegate/abstain.
- Hard to pin down best equilibrium for general committees $\rightarrow$ consider examples.

An example: single expert committee

Prior $\pi = Pr(\omega = A) = \frac{1}{2}$. Committee $\mathcal{N} = \{1, ..., N-1, e\}$ with:

- All voters are independents (common interest)
- Voters $1, ..., N-1$ have precision $q_i = q \in (0, 1)$
- Voter e has $q_e = r > q$ ('expert')

Intuitively: if $r >> q$, incentive for delegation. But how much?

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Weighted majority voting (Nitzan and Paroush, 1982): $\exists w^* > 1$ s.t. if the expert had w^* votes, sincere voting outcome is first-best. $w^* = \frac{\ln(\frac{r}{1-r})}{\ln(\frac{q}{1-q})}$, number of non expert signals that together have equivalent informational value as the expert's private signal.

An example: single expert committee

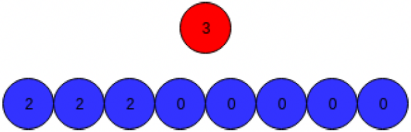
Result. When r is sufficiently large relative to q (i.e. $\lfloor w^* \rfloor > 2$) , in the best equilibrium the expert is delegated $\lfloor w^* \rfloor - 1$ votes. Others vote sincerely. This equilibrium does strictly better than the best equilibrium in DD/RD.

- Worth wasting some information to form optimally weighted' subcommittee
- Result extends to other common-interest settings
- With partisans suppose $n_A - n_B = h > 0$. When $r = 1$, let h of the non experts vote b.

Inefficient equilibria

LD, DD, and RD all admit inefficient equilibria.

- Inefficient overdelegation to few voters
- Example: A single expert committee with $N = 8$, (non experts) with precision $q = 0.6$ and 1 expert with precision $p = 0.7$, $\lfloor w \rfloor^* = 2$. The allocation below (expert in red) is an equilibrium:



Coordination

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Example. $N = 7$. Voters $1, ..., 5$ are independents and $6, 7$ are A partisans. Information: $q_1 = 1$, $q_2, ..., q_5 \in (0.5, 1)$.

- Best equil w/out delegation: 1 votes sincerely, $2, 3$ vote for B always, $4, 5$ abstain
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Result. If committee contains some sufficiently well-informed independent (e.g. some independent with $q_i = 1$) then:

- Game with delegation is (weak) dominance-solvable; game without delegation typically not.
- DS solution is best equilibrium and takes two steps of IEWDS.

Incomplete information about types

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For each voter i, suppose p_i and q_i are drawn from distributions P_i on $\{A, B, I\}$ and Q_i on $[0.5, 1]$; realizations are private information.

Result. There exists an equilibrium in LD. The best equilibrium of LD does weakly better than DD, RD.

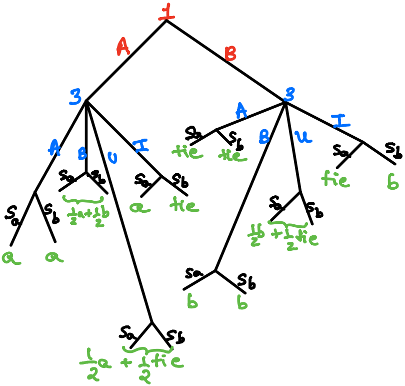
- Partisan behavior again pinned down.
- We look at class of 'threshold' equilibria: independents choose action using thresholds in precision q_i

Incomplete information about types

Example where LD does strictly better than DD/RD.

- $N = 3, \pi = \frac{1}{2}$,
- Player 1 is a partisan with $Prob(p_1 = A) = Prob(p_1 = B) = \frac{1}{2}$.
- Players 2 and 3 are informed independents ($Prob(p_2 = I) = Prob(p_3 = I) = 1/2 - x, q_1 = q_2 = 1$), or uninformed independents ($Prob(p_2 = U) = Prob(p_3 = U) = 1/2 - x, q_1 = q_2 = 1/2$) or partisans $Prob(p_2 = A) = Prob(p_3 = A) = \frac{x}{2}$, $x \in [0, 1)$.
  - Without delegation everybody votes sincerely (and the uninformed are indifferent between voting or abstaining), but the outcome is not always optimal (i.e. when 2 players are independent but only 1 is informed, the correct outcome does not win for sure).
  - With delegation, though, if 2U delegates to 3 and vice versa they can do strictly better: at least one of them is always informed.

Point of view of 2U



The Game from viewpoint of 2U

Literature

Liquid democracy:

- Information aggregation:
  - Christoff and Grossi (2017): delegation affects the rationality postulates satisfied by direct voting.
  - Kahng et al. (2018) use network theory, voters are differentially informed, complete information, non strategic.
  - Armstrong and Larson (2021): Common interest setting where delegation protocols theoretically lead to higher accuracy but experimentally have negative results when the independence assumption is relaxed as in
  - Campbell et al (2021): experimental paper showing that voters delegate too much.
  - Other: Bloembergen et al (2021): accuracy type of voters is known but there is incomplete information on preferences. Delegation is costly: choice between direct voting and delegation. Show existence of Nash equilibrium and average accuracy achieved.
- Preference aggregation: Green-Armytage (2015):

Literature

Information aggregation in elections:

- Condorcet Jury theorem (Condorcet, 1785)- showed with 2 alternatives, two states of the world, common values, if each individual received an informative signal about the state of the world then the probability that a majority would choose the correct alternative is larger than any individual voter choosing the correct alternative, and the probability of a correct choice goes to 1 as $n \to \infty$.
- Austen Smith and Banks (1996) showed that sincere voting was not rational in such a setting.
- Mc Lennan (1998) (for common value elections) and Feddersen and Pesendorfer (1996,1997) - for two candidate elections, a continuum of states, and heterogenous voter preferences - show there is an equilibrium that aggregates information efficiently, asymptotically as the size of the electorate goes to infinity.

Literature

Weighted majority voting:

- Nitzan and Paroush (1982), Shapley and Grofman (1984), Ben-Yashar and Danziger (2015)
  - Binary setting, common interest, different precisions, complete information. What are the *exogenously chosen* optimal weights to ensure the highest probability of reaching the correct outcome.