A Labor Market Sorting Model of Scarring and Hysteresis *

Edoardo Maria Acabbi[1], Andrea Alati[2], and Luca Mazzone[3]

[1] *Universidad Carlos III de Madrid*
[2] *Bank of England*
[3] *International Monetary Fund*

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Abstract

This paper contributes a new framework to account for the interactions between labor misallocation and its interactions with business cycle fluctuations. We propose a tractable search equilibrium model of the labor market with aggregate risk, firm and worker heterogeneity, life-cycle dynamics and endogenous human capital accumulation. We show that sorting of workers to firms is a key factor in increasing the persistence of fluctuations, directly relating labor reallocation to economic hysteresis. We estimate the model on Italian administrative matched employer-employee data. Our estimates highlight the long term supply-side hysteresis effects of business cycles.

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Acabbi: eacabbi@emp.uc3m.es
Alati: andrea.alati@bankofengland.co.uk
Mazzone: lmazzone@imf.org

1 Introduction

We develop a theoretical framework to study the interaction between business cycle fluctuations and the sorting between workers and firms. Research has shown the importance of misallocation in explaining differences in income levels across countries (Restuccia and Rogerson, 2017), but little is known regarding how changes in the sorting between workers and firms affects business cycle dynamics.

We address this issue by proposing a search equilibrium model of the labor market with aggregate risk, firm and worker heterogeneity, life-cycle dynamics and endogenous human capital accumulation. Our model reproduces micro-evidence of labor market dynamics, and generates fluctuations that are consistent with macroeconomic data. In particular, our simulations match the recession patterns of the last three decades. Using the model, we are further able to show that changes in the relative skill premium across education levels accounts for most part of the increased length of recessions compared to the pre-Great Moderation period.

Existing equilibrium search models with two-sided heterogeneity, such as Lise and Robin (2017), Herkenhoff, Phillips and Cohen-Cole (2019), Lise and Postel-Vinay (2020), Jarosch (2021), have separately emphasized the importance of the business cycle and human capital accumulation dynamics for the allocation of workers to firms. We provide a tractable framework allowing a joint analysis of these dynamics, together with the distribution of workers' skills over the life-cycle. Our main contribution is then to characterize the feedback effect between aggregate fluctuations to cross-sectional dynamics in the labor market. The presented framework highlights the role of labor market sorting, defined as the correlation between workers' human capital and firms' productivity across worker-firm matches. It has long been argued that cyclical events can have permanent effects on demand, and therefore on output, generating so-called "hysteresis". In a model that abstracts from demand externalities, we show that sorting is a key factor in increasing the persistence of fluctuations, directly relating labor reallocation to economic hysteresis. In our framework in fact, worker-firm sorting matters for business cycles, both statically and dynamically. The static effect comes from the increase in mismatch of workers to firms in downturns. The dynamic effect is determined by human capital accumulation, which is related to the quality of matches. In our model, therefore, alterations in workers' sorting to firms along the business and life-cycle have persistent effects on the human capital accumulation of affected cohorts, thus distorting long-run aggregate productivity and growth. We further show that search and contracting frictions are crucial in generating the persistence of temporary shocks, as observed empirically.

Our framework generates several empirical predictions that we test in the data. We start by documenting related key empirical facts in our administrative dataset on the universe of Italian private employees between 1996 and 2018. We show that: (1) Macroeconomic conditions at labor market entry can be very consequential for workers' long term earnings (Kahn, 2010, Oreopoulos, Von Wachter and Heisz, 2012, Schwandt and von Wachter, 2019, Huckfeldt, 2022). We do so by analyzing the earnings trajectories of different cohorts of workers depending on the business cycles conditions at entry in the labor market. (2) Which firms young workers start their career in has a fundamental role in determining these initial conditions (Gregory, 2020, Arellano-Bover, 2020), plausibly through different human capital accumulation contributions (Arellano-Bover and Saltiel, 2020, Lise and Postel-Vinay, 2020). We show in fact that quality of past employers affects future earnings for all workers, even the ones hired from unemployment. (3) Long-term consequences of losing a job for workers substantially vary along the business cycle (Schmieder, von Wachter and Heining, 2020, Bertheau et al., 2022 for evidence on Italy in the same data as ours). When taken together these three facts imply that the quality of job matches, especially at entry in the labor market, is particularly important in shaping workers' careers. We further argue that, as suggested in Eeckhout (2018), changes in the sorting between workers and firms over the life- and business-cycles have large aggregate consequences for growth, employment, and inequality. We show that, when the economy enters into a recession, the sorting measure initially goes up, reflecting the separations of less productive workers from their jobs. Sorting then declines as the economy builds its way back to trend, and employment grows. Towards the end of the recovery, as employment stops growing, output and the measure of sorting grow together.

We use our model to analyze the hysteresis dynamics determined by changes in labor market sorting along the business cycle. The model consists of a dynamic, stochastic search model of the labor market with heterogeneous workers and firms. We assume that search is efficient, that is, workers direct search towards their preferred firms (Shi, 2009, Menzio and Shi, 2010). Firms, in turn, post vacancies in submarkets indexed by worker and firm type. Workers are risk averse, and firms are risk neutral. Hence, upon matching, firms will offer workers an incentive compatible contract, conditional on individual characteristics and contingent on all future states of the world (Harris and Holmstrom, 1982, Krueger and Uhlig, 2006, Balke and Lamadon, 2020). In such setting, we assume there is lack of full commitment on both sides: workers cannot commit to stay in their firms, while firms can commit to respect all provisions of the offered contract. Firms, however, won't commit to keep the match in place if the present continuation value of the match is negative. In other words, firms can always borrow to sustain temporary losses, but will exit when the flow of future losses exceeds the flow of future profits, which is akin to saying that entrepreneurs have limited liability. The combination of workers' risk aversion and bilateral lack of commitment has important consequences.

First, because the contract is incentive compatible, it will backload wages in order to maximize worker retention. Second, the optimal contract defines provision of earnings' insurance from firms to risk averse workers. This translates into having endogenous downward wage rigidity in the contract, whereby workers' wages do not decrease after negative shocks, but can at most stay flat when the stream of future profits shrinks to zero. This design of the optimal contract matches empirical properties of wages, and induces endogenous job separations in the model. In some states of the world firms would need to lower wages to reduce costs, but doing so would be incompatible with the commitment of insuring workers against revenue fluctuations. Firms can thus not afford employing workers in unprofitable matches and eventually exit, leaving workers unemployed.

In our model, sorting matters for two reasons: it influences output (static channel), and it determines the level of potential output in the long run (dynamic channel). As production exhibits increasing returns in worker and firm types, positive sorting increases aggregate output. Moreover, workers accumulate human capital on the job and at a rate that depends on the productivity of the firms workers are matched with. Human capital accumulation is then fundamental for climbing the job ladder. Investment in human capital is however limited both on the extensive margin, as matching is frictional, and on the intensive margin, as working life is limited. These limits to investment in human capital matter for business cycles because they increase the persistence of output fluctuations. Surges in unemployment at the onset of recession, for instance, keep workers' productivity below potential for a period longer than the duration of a temporary negative shock, generating economic hysteresis. The joint dynamics of sorting and human capital accumulation along the business cycle are sufficient to reproduce a slowing down of recoveries' dynamics, as observed in the data.

In order to capture the effects of aggregate shocks across different cohorts of workers we populate the economy with overlapping generations (OLG) of heterogeneous agents (see Menzio, Telyukova and Visschers, 2016). Each cohort is exposed to different aggregate conditions at the start of and throughout their the working career, a feature which determines a time- varying cross-sectional distribution of workers. The ensuing OLG structure allows us to identify the different sources of long-run changes in job sorting, wage growth and human capital accumulation affecting each age group.

In presence of aggregate risk, sorting can be altered by inefficient separations and by changes in the search strategy of workers. Inefficient separations occur as a result of the endogenous wage rigidity determined by our contracting protocol. Our model predicts that jobs in firms with lower productivity are more vulnerable to output fluctuations given the compensation rigidity. Because of compositional effects, this is true especially for matches between firms and younger workers, or workers that are closer to retirement. The second channel that impacts sorting acts through alterations in search behaviour, as the targets of workers' search correlate positively with the business cycle. It would seem intuitive that, under some condition, flexibility of entry into post-secondary education could provide an insurance for workers that are displaced or lack investment opportunities during recessions. However, we fail to find evidence of that. In contrast, changes in enrollment seem to be pro-cyclical rather than counter-cyclical.[1]

The search behavior channel has both static and dynamic effects for the economy. Better quality firms provide greater pay and human capital accumulation, but are more coveted by workers and more difficult to be matched with. Therefore, by directing their search to specific submarkets, workers trade off the probability of finding a job against their human capital accumulation at better firms in the long term. In recessions, workers face more unstable employment and less tight labor markets, hence sorting into productive firms becomes more difficult. Especially after separation from their previous employer, they tend to search for worse firms than in good times. In short, not only workers who enter the labor market in bad times, but also those who are displaced during recessions face a worsened job ladder and trade worse employment prospects for a higher likelihood of exiting from unemployment. This lower quality of search targets and realized matches has persistent effects for affected cohorts, generating a "scarring effect" of recessions on their careers.

Finally, our model is used to shed light on the emergence of hysteresis effects in advanced economies. In this sense, our work relates to seminal studies on the roots of the hysteresis phenomenon, as Blanchard and Summers (1986) and Ljungqvist and Sargent (1998), in which the eurosclerosis from the 1970s' to the 1990s' is associated with an analysis of a rigid labor market with slow human capital accumulation by workers. Even in absence of demand externalities, we find recessions to be long lasting and able to generate scarring effects. However, a comparative statics exercise using our model highlights the role of labor market polarization: when the skill premium across education levels is decreased, recessions become shallower and shorter in duration. What drives the amplification and increased persistence of negative shocks is then the lenghtening of the job ladder of educated workers. While their earnings are less cyclical than the earnings of relatively less educated ones, the missed human capital growth opportunities generate longer term effects that affect the aggregate performance of the economy.

The directed search framework allows the model to maintain a high degree of tractability while accommodating two-sided heterogeneity and aggregate risk. Our framework has the advantage of explicitly modelling the dynamics of aggregate shocks and their effects on the entire earnings-skills distribution. Given this tractability, the

[1]We calculate the raw correlation between Hamilton-filtered real GDP per capita and Hamilton-filtered enrollment rates in graduate school, and obtain a positive correlation, of 1.45%. We plot the ratio of enrolled individuals to the population of young adults in Figure F.18: recessions are associated with a slowing down of the trend of increased enrollment.

framework naturally lends itself to the analysis of different policy options affecting job flows and labor market fluidity (employment protection legislation, unemployment benefits, minimum wages, furlough). A natural avenue for future research will be to conduct welfare and policy evaluation analyses on these fundamental topics.

2 Model

In this section we present our model of the labor market. We start by discussing the environment, the timing and the preference structure of the agents populating the economy. We then discuss the features of the frictional labor market with directed search, and finally we characterize the workers problem and the optimal recursive contract.

2.1 Environment

Time is discrete, runs forever and is indexed by $t \in \mathbb{Z}$. The economy is populated by two kinds of agents: a unit measure of finitely-lived risk-averse households (workers) and a continuum of risk neutral entrepreneurs who have the ability to invest in enterprises and thus run an endogenously chosen number of operating firms. All agents in the economy share the same discount factor $\beta \in (0, 1)$.

We populate the economy with $T \geq 2$ overlapping generations of households, which face both aggregate and idiosyncratic risk. Each household lives for T periods, with age $\tau \in \mathcal{T} \equiv \{1, 2, 3, \dots, T\}$. Every period workers participate to the labor market and direct their search towards different submarkets (Shi, 2009, Menzio and Shi, 2010). Workers can only search for work and consume, as we do not model saving decisions.[2]

Workers' objective is to maximize its own lifetime flow-utility from non durable consumption:

$$\mathbb{E}_{t_0}\left(\sum_{\tau=1}^{T} \beta^{\tau} u(c_{\tau,t_0+\tau})\right)$$

where t_0 characterizes the time of entry into the labor market and τ characterizes the age of the agent. We denote future values in recursive expressions by adding a $'$ to them, or index elements by t in non-recursive ones.

Once in the labor force, workers can either be employed or unemployed (e and u) and are fully characterized by heterogeneous human capital levels h, with $h \in \mathcal{H} \equiv [\underline{h}, \overline{h}]$.

We consider two types of workers characterized by an education level

[2] Modeling wealth accumulation in a model with two-sided heterogeneity, life-cycle, human capital accumulation and directed search (also on-the-job) is undoubtedly interesting and important and left to future research.

$\iota \in \mathcal{I} \equiv \{\text{graduate, non-graduate}\}$. Both types begin their life with a baseline level of human capital drawn from type-specific exogenous continuous distributions. Upon entry in the labor market, $\mathbb{E}[h|\text{graduate}] > \mathbb{E}[h|\text{non graduate}]$. As we do not model the schooling choice, graduate workers entry in the labor market is exogenously delayed with respect to non-graduates'.

Workers acquire human capital on-the-job. Workers are matched with firms characterized by different levels of (permanent) firm quality $y \in \mathcal{Y} \equiv [\underline{y}, \overline{y}]$, which in our model is isomorphic to capital levels. Following Lise and Postel-Vinay (2020), we assume human capital accumulation depends on the level of quality of the firm y, worker's own level of ability h and an idiosyncratic human capital shock $\psi \sim \mathcal{N}(0, \sigma_\psi)$. The worker thus accumulates human capital according to a law of motion that is match-specific: $h' = \phi(h, y, \iota, \psi) = g(h, y, \iota) + \psi$, $n : \mathcal{H} \times \mathcal{Y} \times \mathcal{I} \to \mathcal{H}$, where g is the deterministic component of the human capital accumulation dynamics, and ψ constitutes the stochastic component. The function n is concave in h and y arguments.

The deterministic component of human capital accumulation is akin to a "catching-up" of the firm's quality, up to a point when the worker will not be able to learn anymore from the match. Coherently with the concept of "mismatch", workers who lose their job and only manage to re-match with a low quality firm see their ability progressively deteriorating with the same g function. The only difference between graduates and non-graduates in our model is the speed of the "catching-up". Graduate workers will catch up faster. Other than this feature, graduate workers face the same problem as non-graduates.

Human capital accumulation is risky: at any period any employed worker is subject to the idiosyncratic human capital shock ψ, which enters additively with respect to the deterministic component.[3] The shock affects workers' ability and can amplify, shrink or even reverse human capital accumulation. We further allow for the possibility that human capital deteriorates while workers are unemployed, according to an arbitrary process g_u.[4]

Firms are as common in the literature one worker-one job matches, and we thus abstract from firm size.[5] Each job match is also characterized by a promised utility to the worker $V \in \mathcal{V}$, the determination and properties of which are described in the next sections.

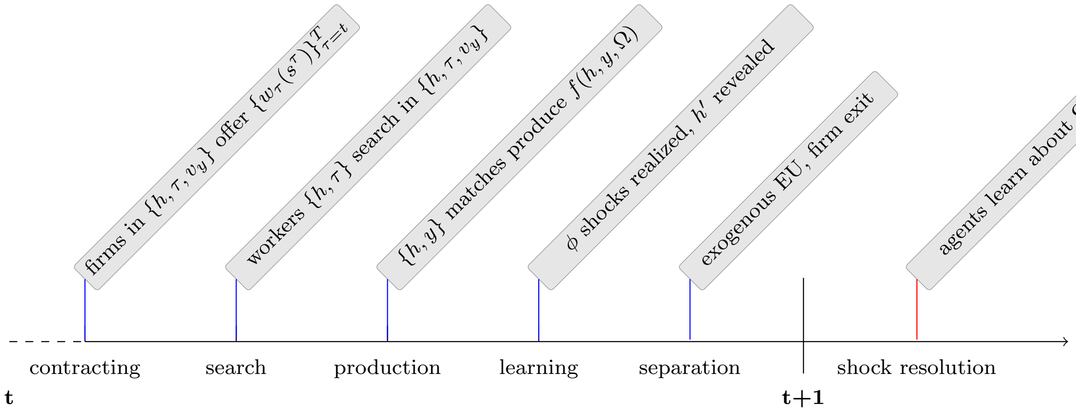
Let us group the worker specific characteristics in a tuple $\chi \in \mathcal{X} \equiv \{\mathcal{H} \times \mathcal{T} \times \mathcal{I} \times \mathcal{V}\}$. The aggregate state of the economy Ω is characterized by the level of aggregate productivity $a \in \mathcal{A} \subseteq \mathbf{R}_0^+$ and by the distribution of agents across states $\mu \in \mathcal{M}$:

[3]The additive nature of the shock keeps the properties of monotonicity and uniqueness of workers' search strategies unaltered, which is essential of tractability.

[4]This process might be without loss of generality deterministic or stochastic, and might or might not depend on current human capital h.

[5]Modeling multi-worker firms in our context is an immensely interesting advancement that we leave for future research.

Figure 1. Timeline of worker-firm match



$\{e, u\} \times \mathcal{Y} \times \mathcal{X} \to [0, 1]$. Let $\Omega = (a, \mu) \in \mathcal{A} \times \mathcal{M}$ represent the aggregate state of the economy and let $\mathcal{M}$ represent the set of distributions μ over the states of the economy. Let $\mu' = \Phi(\Omega, a')$ be the law of motion of the distribution. Aggregate productivity evolves as a stationary monotone increasing Markov process, namely $a' \sim F(a'|a) : \mathcal{A} \to \mathcal{A}$, with the Feller property.

The timing of each period is represented in Figure 1. At the beginning of each period an aggregate productivity shock is drawn; entrepreneurs open vacancies across the submarkets and post their offers; workers search from unemployment or on-the-job, and move to a new job if the search is successful; production of both surviving and newly created firms takes place; workers accumulate human capital depending on their employment status and idiosyncratic shock realization; an exogenous share of matches breaks down, whereas some firms endogenously decide to exit the market and destroy their matches.

2.2 Labor markets

Search is directed. Each labor market is organized as a continuum of submarkets indexed by the expected lifetime utility offered $v_y \in \mathcal{V} \equiv [\underline{v}, \overline{v}]$. Each worker is characterized by the tuple $\chi = (h, \tau, \iota, V)$, where V is the current lifetime utility level, either as unemployed or in its current employment contract. They direct their search, while entrepreneurs decide which kind of firms y to open and, correspondingly, an offered lifetime value v_y.[6] There is free entry for entrepreneurs in submarkets. The process of opening a firm, which amounts to posting a vacancy at a quality-specific cost $\kappa(y)$, will be described in **Section 2.6**. We will also prove that, given any value v_y offered to a worker type χ, there is going to be *only* one kind of firm y offering it. In other words, given χ, v_y is an injective function

[6] As in Menzio and Shi (2010) the equilibrium will be separating. Given a menu of offers from any firm, each worker type χ will visit only a particular submarket. For this reason submarkets can then be indexed directly by a tuple in $\mathcal{X}$.

$f_v : \mathcal{Y} \rightarrow \mathcal{V}$.

The search process is characterized by a constant return to scale twice continuously differentiable matching function $M(u, \nu)$ for each submarket, where the tightness of each submarket in $\mathcal{X}$ is as usual defined as $\theta = \nu/u$, with thus $\theta(\cdot) : \mathcal{X} \rightarrow \mathbf{R}_0^+$. Households job finding rates are defined as $p(\theta(\cdot)) = M(u, \nu)/u$. where $p(\cdot) : \mathbf{R}_0^+ \rightarrow [0, 1]$ is twice continuously differentiable, strictly increasing and strictly concave function with $p(0) = 0$, $\lim_{\theta \rightarrow +\infty} p(\theta) = 1$ and $p'(0) < \infty$. The vacancy-filling is as common defined as $q(\theta(\cdot)) = M(u, \nu)/\nu$, where $q(\cdot) : \mathbf{R}_0^+ \rightarrow [0, 1]$ is twice continuously differentiable, strictly decreasing and strictly convex, with $q(0) = 1$, $\lim_{\theta \rightarrow +\infty} q(\theta) = 0$ and $q'(0) < 0$. Given these properties $q(\theta) = p(\theta)/\theta$, and $p(q^{-1}(\cdot))$ is concave.

Upon match, workers produce period according to the twice-continuous increasing and concave production function $f(a, h, y) : \mathcal{A} \times \mathcal{H} \times \mathcal{Y} \rightarrow \mathbf{R}_0^+$. Workers' compensation is determined by means of dynamic contracts through which firms deliver a promised utility, as described in **Section 2.5**.

Matches are destroyed at an exogenous rate λ_τ each period, with the exogenous separation rate possibly varying by age. Moreover, firms are subject to limited commitment, and matches also separate endogenously either if the worker is poached by another firm or voluntarily decides to quit to unemployment (quit) or if the value of the match for the firm becomes negative (firings).[7] Workers are always allowed to search while unemployed and search while employed with probability λ_e. Lastly, unemployed workers whose job finding probability falls below a threshold, $\underline{\text{p}}$, are assumed to be permanently out of the labor force.

2.3 Informational and contractual structures

The optimal contract in our setting is fully state-contingent, and prescribes an action for each realization of the story of the worker-firm match. We denote this sequence of histories as $\{s^\tau\}_{\tau=1}^T$. The contract defines a transfer of utility from the risk neutral firm to the risk averse worker within the match for all future possible histories of shocks. Given a match formed at a generic hiring time t_0, the state of the match is defined by $s_{t_0} = (h_{t_0}, \tau_{t_0}, \iota, a^{t_0}, \mu^{t_0}) \in \mathcal{S} = \mathcal{H} \times \mathcal{T} \times I \times \Omega$, that is the worker skill, age and the history of aggregate productivity shocks and workers' distributions across the worker's employment history. We define τ_{t_0} as the age at which the worker is hired and T is the retirement age. The history of realizations between t_0, the time of hiring of the worker, and $t_0 + (T - \tau_{t_0})$, the time of maximum duration of the match with the worker before retirement, is thus $s^{t_0 + (T - \tau_{t_0})}$.

[7]Notice that quites and firings happen at different times throughout the period, as shown in Figure 1.

The workers' history and the history of productivity are common knowledge. Realizations of future histories are fully contractible upon. Despite the contract being state-contingent markets are incomplete, given that workers' actions are private knowledge in the search stage. Firms are thus unable to counter outside offers. The contract offered by the firm can thus be defined as:

$$\mathcal{C} := (\mathbf{w}, \zeta) \text{ with } \mathbf{w} := \{w_t(s^{\tau_t - \tau_{t_0} + t_0})\}_{t=t_0}^{t_0+(T-\tau_{t_0})}, \text{ and } \zeta := \{v_t(s^{\tau_t - \tau_{t_0} + t_0})\}_{t=t_0}^{t_0+(T-\tau_{t_0})} \quad (1)$$

According to the contract the firm promises a series of state-contingent wages, to which the worker replies by enacting its own state-contingent search strategy, defined by the series of utility values v_t sought at each node of the history.[8] ζ is the action suggested by the contract, which in our analysis is bound to be incentive compatible for the worker. The contract is otherwise fully flexible in the degree to which the firm can determine wage levels and adjustment paths over the match histories.

2.4 Worker problems

The relationship between workers and firms is characterized by a recursive contract with forward looking constraints. The state space of the worker problem thus needs to include their current lifetime utility, as in Spear and Srivastava (1987). Given a generic current lifetime utility V, any job seeker characterized by human capital h, age τ and education ι has to decide in which submarket to direct the search. Submarkets are indexed by the worker type χ and posted offered utility v_y. As it will be proved in **Section 2.6**, the choice over v will also indirectly determine which kind of firm y the worker matches with, and thus in expectation the human capital accumulation path. For now, assume that this mapping exists, and thus that, given χ, the function $v_y(y)$ is an injective function $f_v : \mathcal{Y} \to \mathcal{V}$. This means that, even if workers only care about offered lifetime utilities v, their choices determine which firm quality y they can match with and the expected human capital accumulation that concurs to deliver the promised utility v itself.

A worker of type (h, τ, ι, V) with the opportunity to search enters the search stage with lifetime utility $V + \max\{0, R(h, \tau, \iota, V; \Omega\}$, where the second component of the expression embeds the option value of the search, and R is the search value function. R is defined as:

$$R(h, \tau, \iota, V; \Omega) = \sup_{\{v_{y,\Omega}\}} \Big[p(\theta(h, \tau, \iota, v_{y,\Omega}; \Omega)) \big[v_{y,\Omega} - V \big] \Big] \quad (2)$$

We denote the solution of the search problem as $v_y^* = v^*(h, \tau, \iota, V; \Omega)$, and

[8] Similarly to Menzio and Shi (2010), Balke and Lamadon (2020), in order to guarantee that the problem is well behaved and the firm profit function is concave, the contract will require a randomization, a two-point lottery, which specifies probabilities over the actions prescribed. We omit it here for conciseness.

$p^*(h,\tau,\iota,v^*_{y,\Omega};\Omega)=p(\theta(h,\tau,\iota,v^*_{y,\Omega};\Omega))$ as the associated optimal job-finding probability. Notice that, given the timing of the choices outlined in **Figure 1**, a job seeker can devise search strategies that are *contingent* on the state in which the search actually takes place.

The lifetime utility of an unemployed worker at the beginning of the production stage can be defined as

$$\begin{aligned} U(h,\tau,\iota;\Omega) = {} & u(b(h,\tau)) + \beta\mathbb{E}_{\Omega,\psi}\Big(U(h',\tau+1,\iota;\Omega') \\ & + \max\{0, R(h',\tau+1,\iota,U(h',\tau+1;\Omega');\Omega')\}\Big) \end{aligned} \tag{3}$$

where $b(h,\tau)$ is a (possibly) skill and age dependent unemployment benefit. Given finite workers' lives, $U(h,\tau,\iota;\Omega)=0\ \forall(h,\tau,\iota;\Omega)\in\mathcal{H}\times\mathcal{T}\times\mathcal{I}\times\mathcal{A}\times\mathcal{M}$ where $\tau>T$.

The corresponding lifetime utility of a worker employed at a firm y with human capital h, age τ, education ι and current promised utility $V_{y,\Omega}$ at the beginning of the production stage can be expressed as:

$$\begin{aligned} V_{y,\Omega}(h,\tau,\iota;\Omega) = {} & u(w_y) + \beta\mathbb{E}_{\Omega,\psi}\Big(\lambda_\tau U(h',\tau+1,\iota;\Omega') + (1-\lambda_\tau)\Big[V_{y,\Omega'}(h',\tau+1,\iota;\Omega') \\ & + \lambda_e \max\{0, R(h',\tau+1,\iota,V_{y,\Omega'};\Omega')\}\Big]\Big) \end{aligned} \tag{4}$$

where w_y is the currently promised wage and $V_{y,\Omega'}$ is next period's state-contingent promised lifetime utility of remaining in the current firm, which becomes the outside option in the search problem.[9] Notice that in the worker's search problem there is nothing specific to firm quality y *per se*. We can however index wages and utilities by y (and the aggregate state Ω, given the state-contingency of promises) as we will prove that upon matching only one kind of firm y will offer a certain level of utility v. These promised utilities are in turn an equilibrium object themselves, as outcomes of the firm dynamic contract optimization.

By means of their search strategy workers indirectly impact their current contract. Firms internalize incentives embedded in workers' strategies in their optimization, and post wages and utility offers to maximize profits *and* retention. Worker quits, in fact, are equivalent to firm exit. Workers future promised utility incorporates both higher wages and higher option values of search, also through the deterministic component of human

[9]IT is here implied that, in case there is an endogenous separation, this future promised value is equivalent to the value of being unemployed.

capital accumulation dynamics $g(h, y)$.

The policy functions are uniquely defined, and allow to identify target y uniquely as long as there exists a injective mapping between the offered utility v and y given $\{h, \tau, \Omega\}$. We assume this is the case for now, and prove it later in **Section 2.6**. Proofs of the uniqueness of policy functions and individuals' optimal policy are on the other hand provided in **Appendix A**.

The solution of employed workers' on-the-job search problem implicitly defines two policy functions, that incorporate workers' incentive compatibility, which firms internalize in their optimization.

Definition 2.1 (Optimal retention probability and utility return)**.** *The solution of the worker's problem defines a retention function* $\widetilde{p} : \mathcal{X} \times \Omega \to [(1-\lambda)(1-\lambda_e), 1-\lambda]$ *and a utility return* $\widetilde{r} : \mathcal{X} \times \Omega \to \mathcal{V}$*:*

$$\widetilde{p}(h, \tau, \iota, V_{y,\Omega}; \Omega) \equiv (1-\lambda_\tau)(1-\lambda_e p^*(h, \tau, v^*_{y,\Omega}; \Omega)) \tag{5}$$

$$\widetilde{r}(h, \tau, \iota, V_{y,\Omega}; \Omega) \equiv \lambda_\tau U(h, \tau, \iota; \Omega) + (1-\lambda_\tau)\Big[V_{y,\Omega} + \lambda_e \max\{0, R(h, \tau, \iota, V_{y,\Omega}; \Omega)\}\Big] \tag{6}$$

2.5 Contract

There is a double sided lack of committed between parties. The firm is subject to limited liability, but commits to the delivery of a utility value to the worker as long as it's profitable. The worker on the other end cannot credibly commit in any circumstance. This means that the worker is able to search at any period he has the possibility to do so. Firms cannot observe poaching offers, and cannot thus counteract them. The sequence of stories s^t is common knowledge, and while the firm cannot observe any of the actions of its workers, it has enough information to incorporate the worker's optimal policy decision.

We define $J(h, \tau, y, \iota, W_{y,\Omega}; \Omega)$ as the profit function of a firm, the difference between revenues from production and the promised wage. Incumbent firms make their exit decisions before the realization of aggregate productivity but after the realization idiosyncratic human capital shock for next period. This implies that at the beginning of a period they already know whether they will exit or not. Given the current state we can define the following indicator function

Definition 2.2 (Exit policy)**.** *The following indicator takes value one if the firm does not decide to exit in the following period:*

$$\eta_{t+1} = \begin{cases} 1 & \textit{if } a \geq \max\{0, a^*\} \\ 0 & \textit{otherwise} \end{cases}$$

with the productivity threshold defined as

$$a^*(h,\tau,y,\iota,W_{y,\Omega}) : \mathbb{E}_\Omega[J_{t+1}(h',\tau+1,y,\iota,W_{y,\Omega'};a',\mu')|h,\tau,y,\iota,W_{y,\Omega},a,\mu,\psi] = 0. \quad (7)$$

Notice that the firm takes its exit decision *before* the realization of a new aggregate shock, but *after* the realization of the worker idiosyncratic human capital shock, which does show up in the expectation operator.[10]

Given $\eta_{t+1} = 1$, the value function of a continuing incumbent y in state $(h,\tau,\iota,W_{y,\Omega};\Omega)$ can be rewritten recursively using the promised utilities to the workers as additional state variables as:

$$\begin{aligned} J_t(h,&\tau,y,\iota,W_{y,\Omega};\Omega) = \\ &\sup_{\pi_i,w_i,\{W_{iy,\Omega'}\}} \sum_{i=1,2} \pi_i \Bigg(f(y,h;\Omega) - w_i \\ &+\mathbb{E}_{\Omega,\psi}\left[\widetilde{p}(h',\tau+1,\iota,W_{iy,\Omega'};\Omega')(J_{t+1}(h',\tau+1,y,\iota,W_{iy,\Omega'};\Omega')\right]\Bigg) \end{aligned} \quad (8)$$

$$s.t.\ W_{y,\Omega} = \mathbb{E}_{\Omega,\psi}\left(u(w_i) + \beta\mathbb{E}_\Omega\widetilde{r}(h',\tau+1,\iota,W_{iy,\Omega'};\Omega')\right), \quad (9)$$

$$\sum_{i=1,2}\pi_i = 1 \quad (10)$$

where **Equation** (9) is the promise keeping constraint ensuring that the current value of the contract is indeed based on the current wage and future utility promises with $\widetilde{r}_t(\cdot)$ implicitly including the incentive constraint of the worker.

In this kind of contracts the firm (principal) optimizes over its possible offers to workers taking into account the utility of the worker (agent) and its incentive compatible best replies. The resulting equilibrium is a subgame perfect Nash equilibrium of the kind identified in leader-follower sequential games, as in Von Stackelberg (1934).

2.6 Vacancy opening and free entry

The economy is populated by a continuum of risk-neutral entrepreneurs. Each entrepreneur can invest to reach the desired level of firm quality y. The start-up costs of the firm are priced in terms of the consumption good and they coincide with vacancy posting costs in the frictional labor market.

The cost of each vacancy is positively related to the quality of the firm being created.

[10]Equivalently, this amounts to having the firm making a state-contingent exit decision in advance of the idiosyncratic shock's realization.

In order to post a vacancy for the creation of a firm with quality y the entrepreneur must thus pay $c(y)$ in terms of the consumption good.[11]

At a generic time t each entrepreneur chooses in which submarket to post the vacancy selecting a lottery over the offered utility W_y, which maps into the set of firms' qualities $y \in \mathcal{Y}$, and worker characteristics $(h, \tau, \iota) \in \mathcal{H} \times \mathcal{T} \times \mathcal{I}$.

As the entrepreneur chooses the submarket in which to open a vacancy, he faces the following problem:

$$\Pi_t(h, \tau, y, \iota, W_{y,\Omega}; \Omega) = \sup_{y,h,\tau,\iota,W_{y,\Omega}} - c(y) + q(\theta(h, \tau, W_y; \Omega))\beta[J_t(h, \tau, y, W_{y,\Omega}; \Omega)] \quad (11)$$

Given perfect competition, free entry and the possibility for all entrepreneurs to choose *any* possible firm kind y the expected profits from opening a vacancy are driven down to 0 in submarkets which actually open.[12] This translates into a free entry condition:

$$\Pi_t(h, \tau, y, \iota, W_{y,\Omega}; \Omega) \leq 0 \text{ for } \forall\{h, \tau, y, \iota, W_{y,\Omega}; \Omega\} \in \{\mathcal{Y} \times \mathcal{V} \times \mathcal{S}\} \quad (12)$$

Assuming that $q(\cdot)$ is invertible, it delivers the equilibrium definition of marker tightness in each submarket:

$$\theta_t(h, \tau, \iota, W_{y,\Omega}; \Omega) = q^{-1}\left(\frac{c(y)}{\beta J(h, \tau, y, \iota, W_{y,\Omega}; \Omega)}\right). \quad (13)$$

2.7 Equilibrium definition

Recursive Equilibrium. Let $\Theta = \mathcal{A} \times \mathcal{M} \times \mathcal{H} \times \mathcal{T} \times \mathcal{I}$. A recursive equilibrium in this economy consists of a market tightness $\theta : \Theta \times \mathcal{V} \to \mathbb{R}_+$, a search value function $R : \Theta \times \mathcal{V} \to \mathbb{R}$, a search policy function $v^* : \Theta \times \mathcal{V} \to \mathcal{V}$, an unemployment value functions $U : \Theta \to \mathbb{R}$, a series of firm value functions, $\{J_t\}_{t=1}^T : \mathcal{S} \times \mathcal{V} \times \mathcal{Y} \to \mathbb{R}$, a series of contract policy functions $\{c_t\}_{t=1}^T : \mathcal{S} \times \mathcal{V} \times \mathcal{Y} \to \mathcal{C}$, an injective mapping between firm qualities and promised utilities at hiring $f_v : \mathcal{Y} \to \mathcal{V}$, an exit threshold for aggregate productivity $a^* : \mathcal{S} \times \mathcal{V} \times \mathcal{Y} \to \mathcal{A}$, a human capital accumulation process $\phi(h, y, \iota, \psi)$, $\mathcal{H} \times \mathcal{Y} \times I \times \Psi \to \mathcal{H}$, a law of motion for the aggregate state of the economy $\Phi_{\Omega,a'} : \mathcal{A} \times \mathcal{M} \to \mathcal{A} \times \mathcal{M}$ such that:

1. given the mapping f_v, market tightness satisfies **Equation** (13)
2. the unemployment value functions solves **Equation** (3)

[11] We assume that entrepreneurs can borrow from risk neutral deep pocketed financiers to finance the opening of a vacancy. Herkenhoff (2019) shows that, through this simplifying assumption, the cost of credit for entrepreneurs coincides with the risk-free rate.

[12] Notice that in this case the expectation does not refer to realizations of the aggregate state Ω or the human capital shock ψ, but to the vacancy-filling probability q.

3. the search value function solves the search problem in **Equation** (2) and v^* is the associated policy function
4. the series of firm value functions and the associated contract policy functions are a solution to **Equation** (8) for each $t \leq T$
5. the exit threshold satisfies **Equation** (7)
6. the law of motion for the aggregate state of the economy respects the search and contract policy functions and the exogenous process of aggregate productivity

Definition 2.3 (Block Recursive Equilibrium)**.** *A block recursive equilibrium is a recursive equilibrium such that the value and policy functions depend on the aggregate state only through aggregate productivity, $a \in \mathcal{A}$ and not through the distribution of agents across states $\mu \in \mathcal{M}$.*

We provide a proof for the existence of a BRE equilibrium in **Appendix F**.

3 Discussion

What does the model imply for the distribution of types of vacancies posted and worker - firm matches along the business cycle? This section will discuss some closed-form results that will pave the way for the following quantitative analysis. We refer the reader to Appendix C for the proofs to all propositions in this Section.

A first, intuitive challenge for a model of dynamic sorting is understanding the basic properties of firm creation and worker search, conditional on a series of other characteristics. The following properties guarantee a high degree of tractability.

Property 3.1 (Unique Mapping)**.** *Firm quality, y, and utility promises in vacancy postings, $v_{y,\Omega}$, are related by an injective mapping conditional on the aggregate state of the economy, Ω, and workers characteristics (h, τ, ι).*

Having established that workers' search being directed towards promised values is equivalent to being directed towards firms, we can focus on the properties of the search strategy to get a complete view of how sorting works in equilibrium.

Property 3.2 (Search Monotonicity)**.** *The optimal search strategy, conditional on age τ and aggregate conditions Ω, is unique and weakly increasing in workers' individual characteristics (h, ι). When workers are employed, it is also increasing in the quality metrics of their existing match, V.*

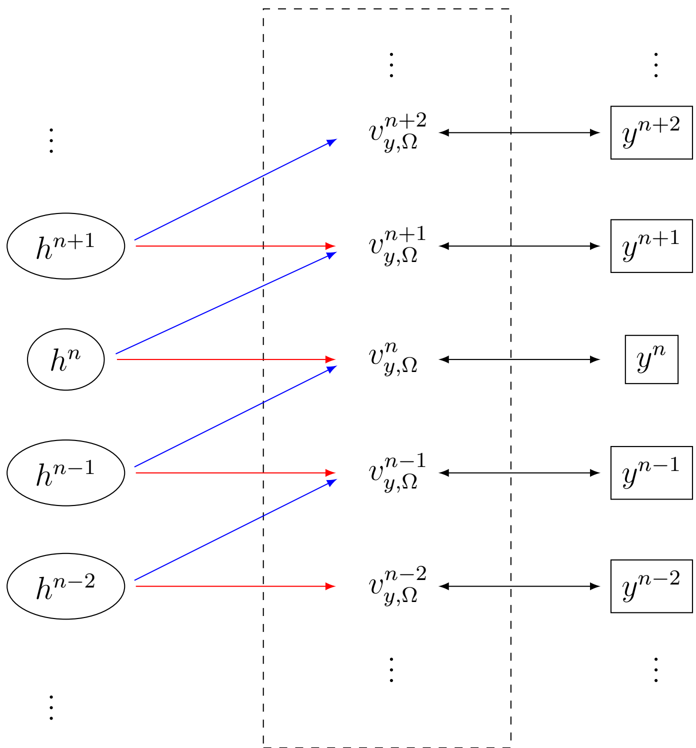
The combination of **Properties 3.1** and 3.2 guarantees that, abstracting from idiosyncratic as well as aggregate shocks, workers sort according to their individual characteristics. Firms are vertically differentiated, and all workers agree to their relative ranking. Differences in optimal strategy arise entirely from the way in which each firm designs the wage protocol, which produces an equilibrium of perfect separation across worker types. We will discuss the properties of such wage protocol below.

Furthermore, because we are interested in the role of aggregate fluctuations in shaping the distribution of matches, we need to look at how search strategies change across aggregate states.

Property 3.3 (Search in Good and Bad Times)**.** *The optimal search strategy is increasing in the aggregate productivity level, a.*

At this point we are able to illustrate one of the main mechanisms of the model, which is represented in **Figure** 2.

Figure 2. Schematic representation of labor market sorting along the business cycle. Unemployed workers, ordered by human capital levels, search in bad times and good times towards (ordered) values, each offered by the (unique) corresponding firm type



The figure highlights one way in which aggregate fluctuations modify sorting in the labor market. The value of vacancies posted by each firm in equilibrium changes with the business cycle, with sub-markets becoming less tight in bad times. Faced with a lower probability of successfully matching with the same firm, risk averse workers will adjust their search downwards. In turn, firms will adjust downwards their utility offers given the lower expected values of matches across the board.

In absence of separations and human capital accumulation, the cross-sectional distribution of matches will directly reflect the history of aggregate shocks. A first element that complicates this relationship is search on the job.

Property 3.4 (Optimal Retention)**.** *Retention probabilities,* $\widetilde{p}(h, \tau, W_{y,\Omega}; a, \mu)$ *are:*

(i) increasing in the value of promised utilities, $W_{y,\Omega}$*.*

(ii) decreasing in aggregate productivity, a

Despite continuation values for staying in the match being pro-cyclical and workers searching more ambitiously in good times firms are more likely to see workers leave in expansions. This is consistent with the data, as employment-to-employment transitions are strongly pro-cyclical. The first part of **Property 3.5** highlights an important aspect of the incentives that shape the optimal contract designed by firms: retention is increasing in continuation values, a classical foundation for wage backloading. To close the model, what we need is in fact a rule for surplus sharing between firms and workers.

Property 3.5 (Wage Protocol)**.** *The optimal contract delivers a wage that satisfies:*

$$\frac{\partial \widetilde{p}(\Theta)}{\partial W_{iy,\Omega'}} \frac{J_{t+1}(\Theta)}{\widetilde{p}(\Theta)} = \frac{1}{u'(w_{i,\Omega'})} - \frac{1}{u'(w_i)} \tag{14}$$

with $\Theta \equiv (\phi(h, y), \tau + 1, W_{iy,\Omega'}; \Omega')$ *being the definition of the relevant state and* $w_{i,\Omega'}$ *is the wage paid in the future state.*

This result extends the wage equation in Balke and Lamadon (2020) to an environment with double-sided heterogeneity. Wage growth is proportional to two elements: the residual continuation value of the match, J_{t+1},[13] and the elasticity of worker's retention probability to the offers she will receive in equilibrium. Limited liability provides both the foundation of wage rigidity, as it ensures that both elements in equation C.1 are weakly positive, and the rationale for inefficient separations. Since match continuation values J are hit by idiosyncratic as well as aggregate shocks, matching the pattern of separations is an important validation test for the model.

Property 3.6 (Countercyclical Separations)**.** *Conditional on the existing contract, and on worker and firm types, there exists an aggregate state* a^* *below which firms will not continue the contract. The threshold* a^* *is cœteris paribus decreasing in the value promised to workers, and increasing in worker and firm types.*

[13]Notice that, in presence of risky human capital accumulation, J will fluctuate together with the human capital levels of the worker even in absence of aggregate fluctuations. However, because the contract provides insurance to the worker, changes in her human capital will have asymmetric effects on wage growth.

A clear implication of **Property 3.6** is that, especially at the onset of recessions, firms will be significantly more likely to lay off workers. In addition, more fragile workers and firms will be more likely to separate in recessions. The counter-cyclicality of separations is a common feature of labor market data, together with the lower job security enjoyed by less productive workers, or provided by less productive firms.

4 Numerical experiment

In this Section we present preliminary results of the model solution, with qualitative indications regarding its properties.[14]

We simulate a population of overlapping generations working for 45 years (180 quarters, from 18 to 63 years old, the average retirement age). Upon entry in the market workers draw a starting skill from a lognormal distribution and then exit the market once they reach their retirement age or when the expected value of employment is too low. [15]

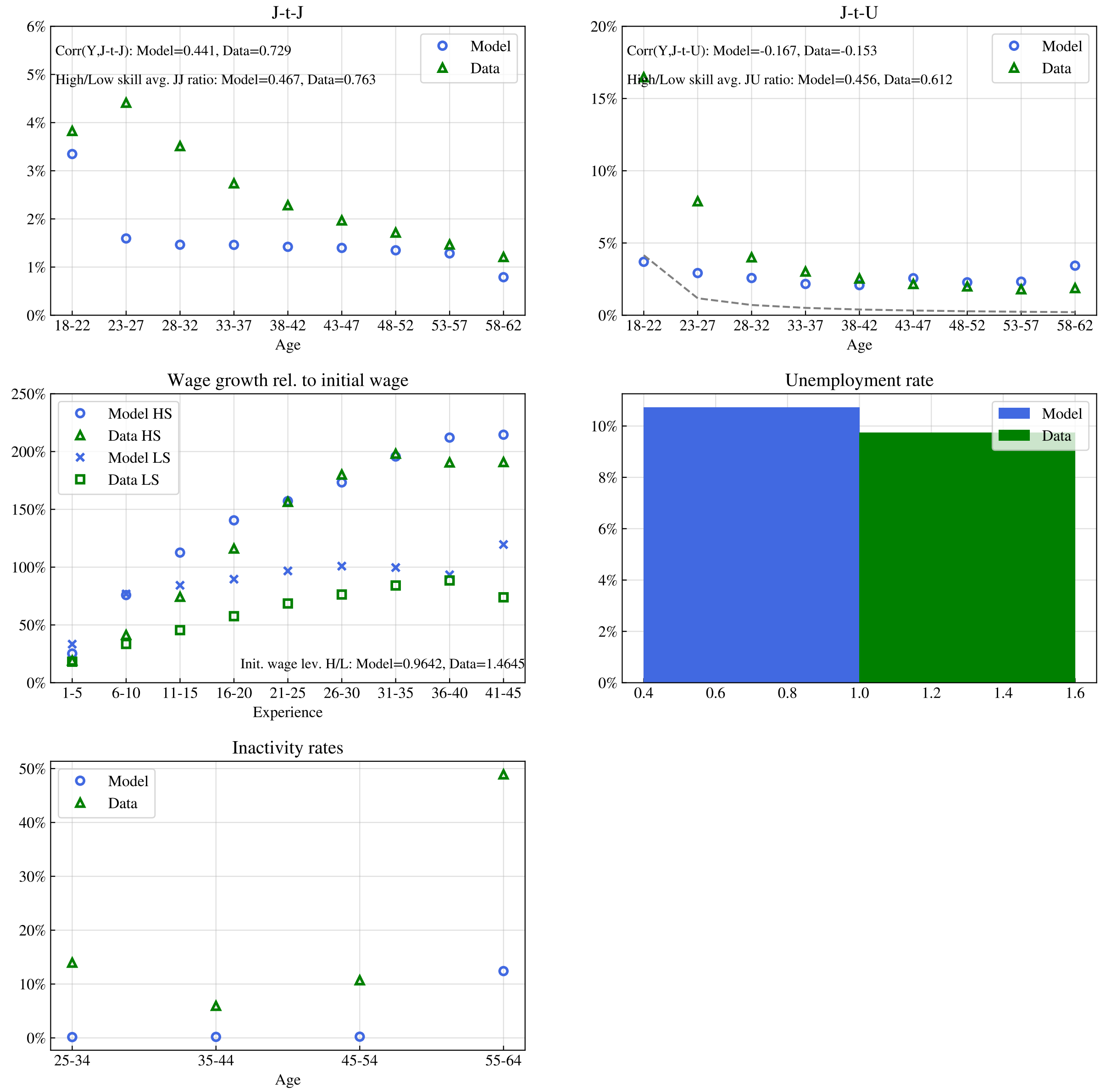
Calibration. The model is characterized by 20 parameters. We set some of them externally: the discount factor (β) and agents' risk aversion (ν), the vacancy cost (κ) as well as the persistence and volatility of the aggregate shock process, (ρ_a, σ_a) and the distribution for initial human capital draws (μ_h, σ_h). We set the rest to fit some standard labor market moments: labor market flows by age, EE and EU, as well as their correlations with aggregate output; the profile of wage growth over workers' careers, the average unemployment rate in the model and the inactivity rates by age. However, for the purpose of this illustrative experiment and given the computational burden of estimating the model, we solve and simulate the model over a sparse grid of the parameter space and we select the combination of parameters that delivers the smallest error on the grid.[16] **Table 3** reports this preliminary calibration and the specific functional forms. The comparison of the model and empirical moments are reported in **Figure 3**.

[14] As we plan to structurally estimate the model on administrative data, we provide for now only results of a generic and untargeted calibration. **Table 3** reports the parameter values and function forms used for the numerical results discussed in this section.

[15] For now workers either deterministically exit the market (die) at the average Italian retirement age in the data or choose to exit the labor force once the job finding probability is below a constant threshold $\underline{p}$.

[16] The advantage of selecting the parameters with this procedure is that the grid is built so that the function domain is optimally covered with the least amount of possible points compared to other forms of approximation (e.g. equi-spaced or random grids). The sparse grid is build using the *Tasmanian* libraries (Stoyanov, 2015, Stoyanov and Webster, 2016). We are currently conducting a structural estimation of the model using SMM.

Figure 3. Empirical and model-based moments



Note: The figure plots the moments used to evaluate the model performance on a sparse-grid of parameters. The dashed-grey line in the EU panel indicates the exogenous separation probability. Referenced on page(s) [17] .

4.1 Model properties

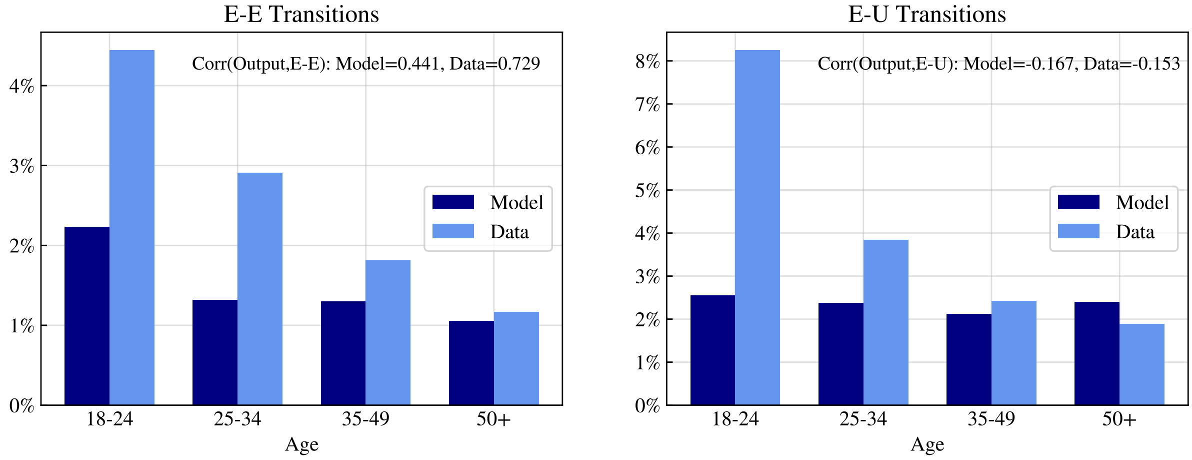
We start by discussing the qualitative behavior of the model along some relevant labor outcomes both in the cross-section and along workers life-cycle.

Labor market flows. In **Figure 4** we compare the main moments for the cross-section of the Italian labor market and the model simulation.

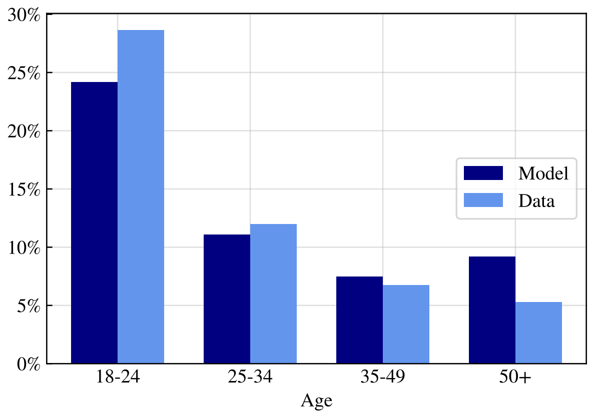
We report transition rates from employment to employement (EE) and from employment to unemployment (EU) in **Figure 4a**.

Figure 4. Main labor market moments

(a) Cross sectional transition rates, model and data



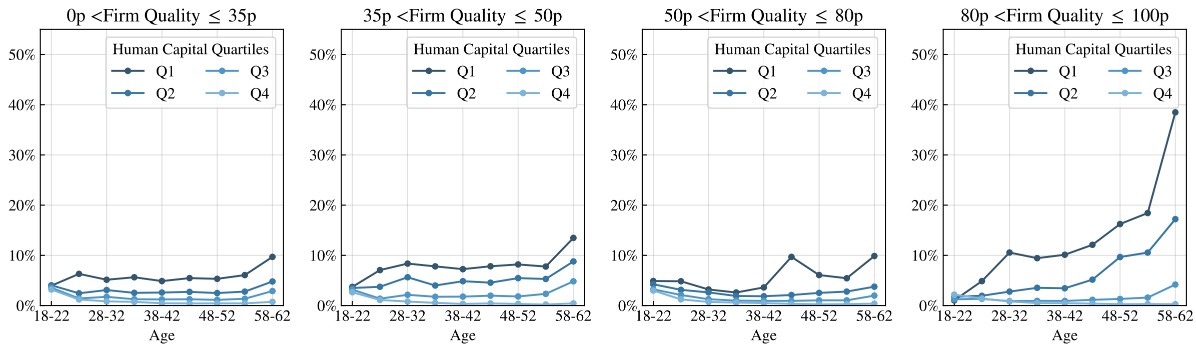
(b) Unemployment rates, model and data



Note: The figure reports the average employment to employment and employment to unemployment transitions as well as their correlation with output plus the unemployment rate by age groups in the model and in the data. *Sources:* Transition rates are computed on administrative data while the unemployment rates are taken from the Italian National Statistical Agency (ISTAT). Referenced on page(s) [18]

Our model, even in absence of a precisely targeted simulation, exhibits qualitatively interesting and coherent traits. EE transition rates peak early in the career and decrease over time, remaining in a tight range for most of the workers lifetime. On the other hand, the model cannot yet fully capture the initial spike in the EU transition rates, as workers generally tend to have a stable rate in the model, although young workers do have stronger separations than their older counterparts. The behaviour of both EE and EU transitions along the business cycle is also in line with the data. The model in fact generates realistic comovements of labor market flows with aggregate output, it generates a mild procyclicality of EE transitions, the correlation between output and EE transition of 0.44, versus 0.73 in the data, and a mild countercyclicality of EU transitions, the correlation between output and EU transition rates is -0.24 in the model and -0.16

Figure 5. Separations rates by age, firm and worker qualities



Note: The figure plots the average separation rates by firm quality, human capital quartiles and age in the model. Referenced on page(s) [20,20]

in the data.

Despite the illustrative purposes of the current calibration the model generates also reasonable unemployment dynamics, as shown in **Figure 4b**. The average unemployment rate in the simulations is approximately 5.6%, slightly more than half of the average unemployment rate in Italy over our sample period. Unemployment rates are higher for younger workers and decline fast to until workers reach approximately 55 years.

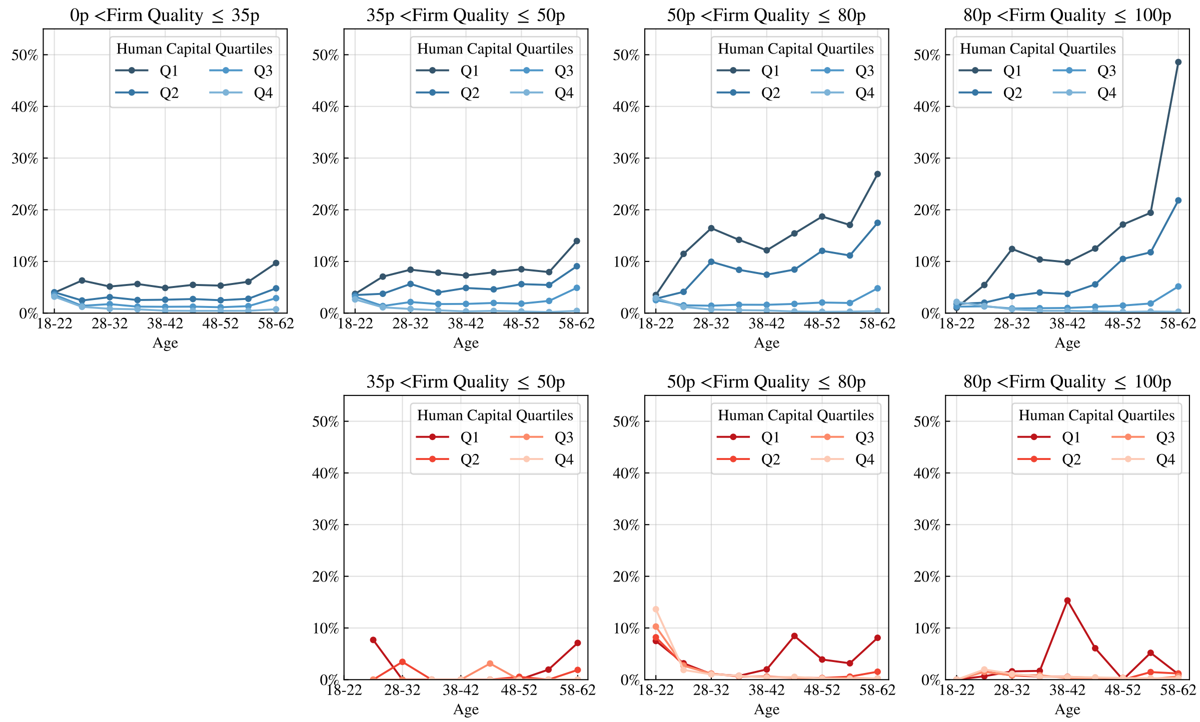
In **Figure 5** we report the separation rates by firm quality and human capital quartiles. Each panel reports the proportion of matches that are dissolved, on average, by age and human capital in each firm class.[17] The patterns that emerge are informative of the leveraging dynamics at work. In particular, the figure shows how separations are prevalent for old, low-skilled workers in relatively good firms. These matches are particularly susceptible to recessions as older workers command higher wages thanks to their longer labor market experience. This increases the operating leverage associated with labor costs making firms more susceptible to aggregate shocks. This mechanisms is stronger for more productive firms as they are also providing a steeper accumulation of human capital and consequently a steeper wage profile, which makes the leveraging component of labor costs more salient for them compared to lower quality firms.

Similar to **Figure 5**, **Figure 6** reports the separation rates by age, firm and worker qualities across the two types of workers that populate the economy, non-graduates (top panels) and graduates.[18] In our simulation, graduate workers are absent from the left tail of the firm distribution and exhibit significantly lower separation rates than their counterparts. The higher ability of graduate workers to accumulate human capital on

[17]Under this tentative calibration the firm distribution is highly positively skewed so that the median is actually the lower bound of available firm qualities.

[18]Recall that for the purpose of our model, graduate workers differ from non-graduates along two margins: a faster on-the-job human capital accumulation, and, on average, a higher initial human capital.

Figure 6. Separations rates by age, firm and worker qualities - Non-graduate vs Graduate



Note: The figure plots the average separation rates by firm quality, human capital quartiles and age in the model for workers *without* (top panels) and *with* (bottom panels) a college education. Referenced on page(s) [20]

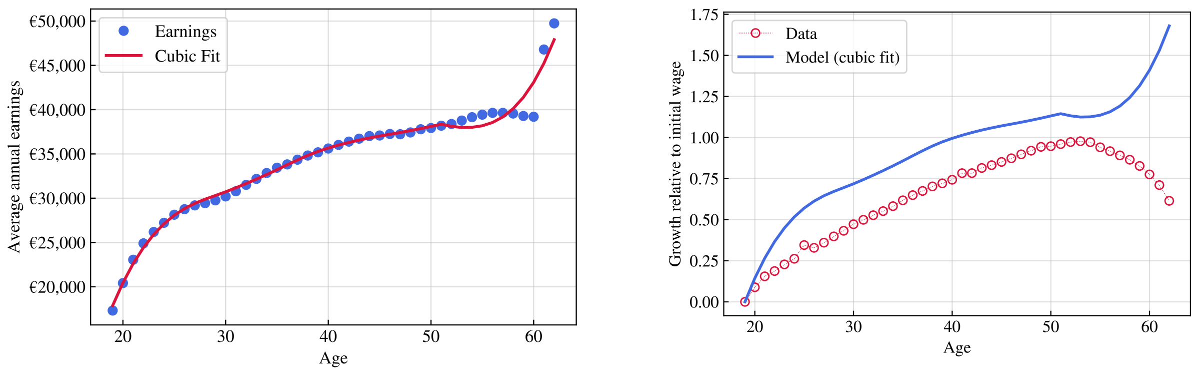
the job, in fact, allows them to search and be matched with relatively better firms than their counterparts, resulting in better matches and lower separation rates.

Wage dynamics. In **Figure 7** we plot respectively the level (**7a**) and the growth rate (**7b**) of labor market earnings in the model. The simulation matches qualitative features of both the time series and the cross sectional dimensions of the wage distribution. The model generate a concave wage profile as in the data, albeit the current calibration generates an implausible strong growth at beginning of workers careers. Nonetheless, the resulting overall wage distributions, reported in **Figure 8**, are very similar between the data and the model.

Exploiting again the rich features of the model, in **Figure 9**, we plot a decomposition of the average growth in wages *within* jobs, i.e. within the same contract, and *between* jobs, i.e. after EE transitions. On one hand, the model simulation implies that the bulk of wage growth is due to EE transitions, replicating a well known feature of labor market flows. On the other hand, the average *within* wage growth is declining in age and firm-quality. This is because high quality firms are matched on average with better workers that are paid more on average. In addition, given that the search strategies of employed

Figure 7. Wage and unemployment dynamics over the life-cycle

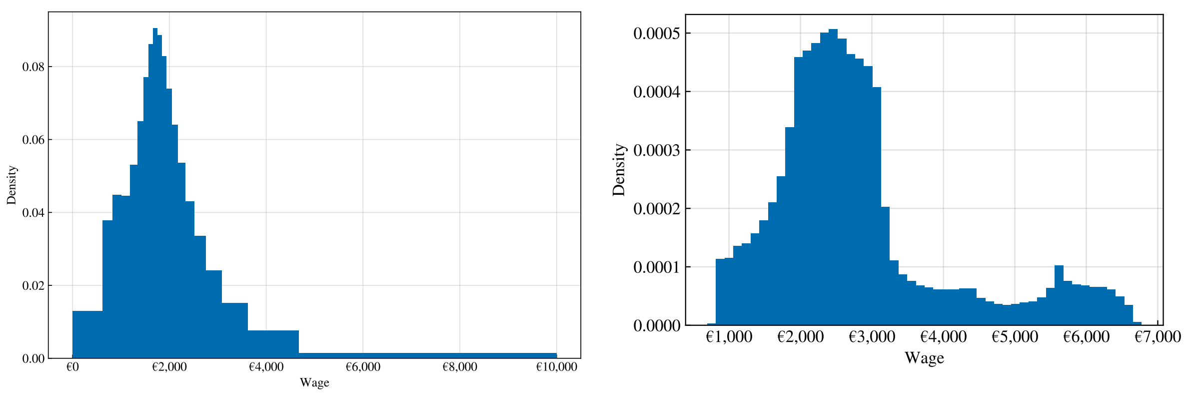
(a) Level **(b)** Growth rates, Model vs Data



Note: The figure plots the life cycle profiles for the average wage in the model and the growth profile in the model and in the data. Referenced on page(s) [21]

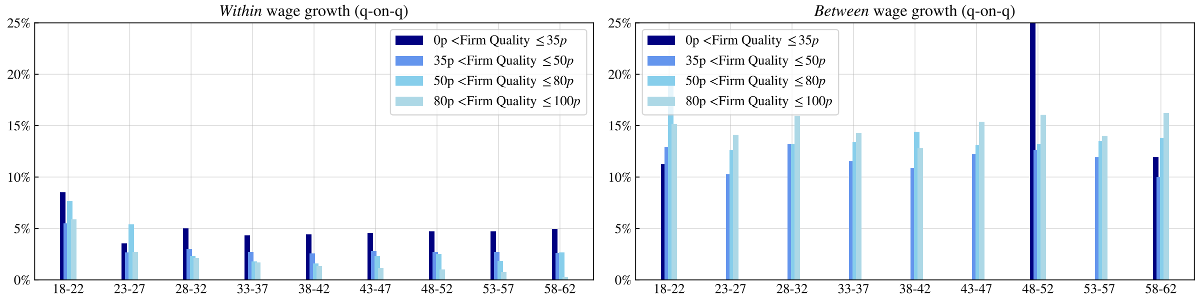
Figure 8. Wage distributions

(a) Data **(b)** Model



Note: Referenced on page(s) [21]

Figure 9. Within vs Between wage growth by age groups and firm quality in the model



Note: The figure plots the average wage growth, by age and firm quality, *within* employment spell and after EE transitions (*between*). For the *betweeen* component, the firm quality quartiles are computed on the distribution of origin firms. Referenced on page(s) [21]

workers is targeted towards better firms, firms in the right tail of the distribution can offer flatter (i.e. less backloaded) wage contracts as they benefit from a higher retention probability.

5 Aggregate consequences of contractions

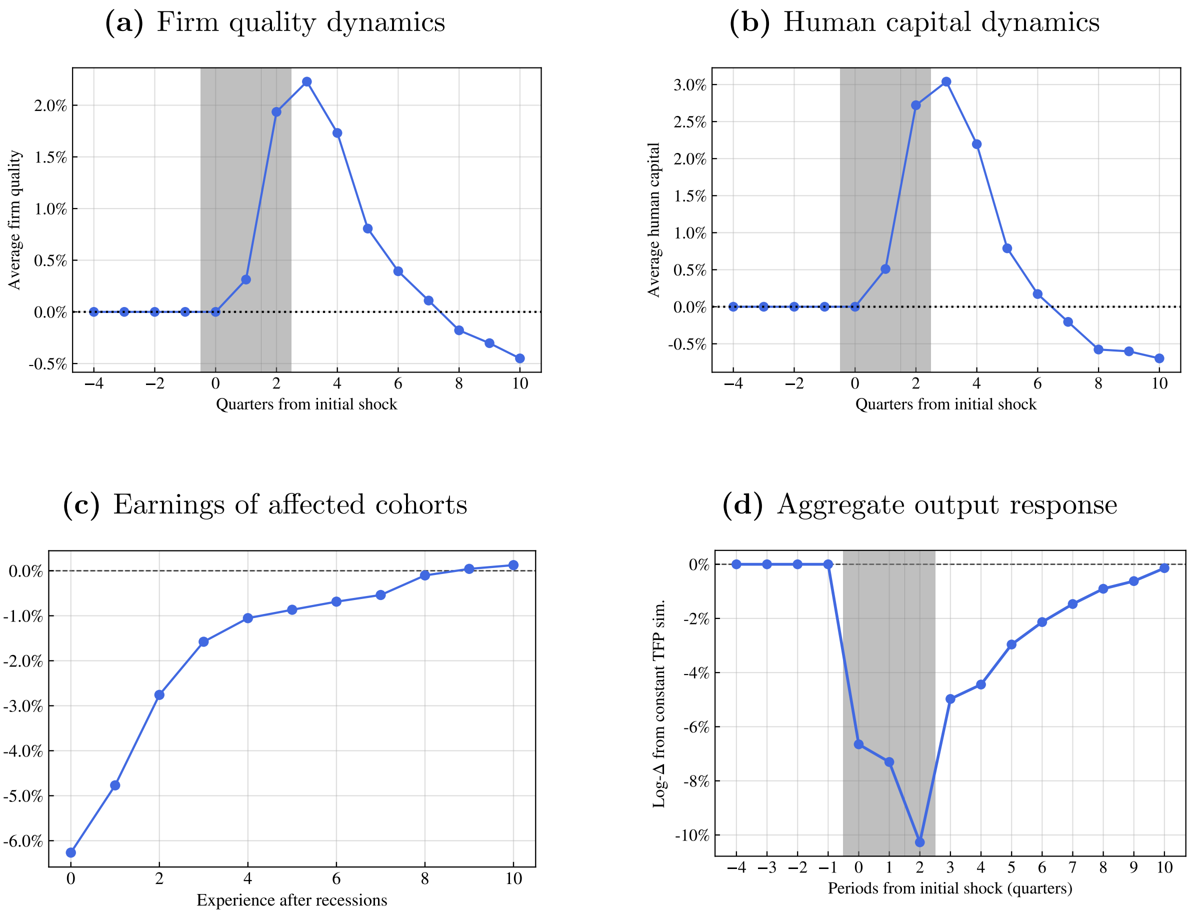
In this section we describe how the mechanism at the core of our model interacts with business cycles. In particular, we show how temporary shocks can trigger a wide reaching set of effects that influence the cross-sectional characteristics of both workers and firms in the economy.

5.1 Shock transmission in the model

In this section, we check how aggregate shocks transmit in our model. Specifically we compare a simulation of the model that does not have aggregate risk and a simulation in which we hit the economy with three consecutive negative realization of the TFP process. We then look at both labor market outcomes of affected cohorts and the response of aggregate GDP to the recession. The main results are reported in **Figure 10**.

The dynamics of firm quality and average human capital, respectively in **Figure 10a** and **Figure 10b**, offer a clearer picture of the transmission of aggregate shocks in the economy. While the onset of the recession is accompanied by a sort of "Schumpeterian" response, as implied by the initial marginal increase in human capital shown in **Figure 10b**, firm quality is persistently crippled by the recession, with the average quality of firms active in the economy remaining approximately 1% below the no-recession economy even two years after the end of the recession. The average human capital in the long run settles to a similar lower level despite the initial increase. This prolonged reduction in the quality of the factors of production increases the persistence

Figure 10. Recession experiment



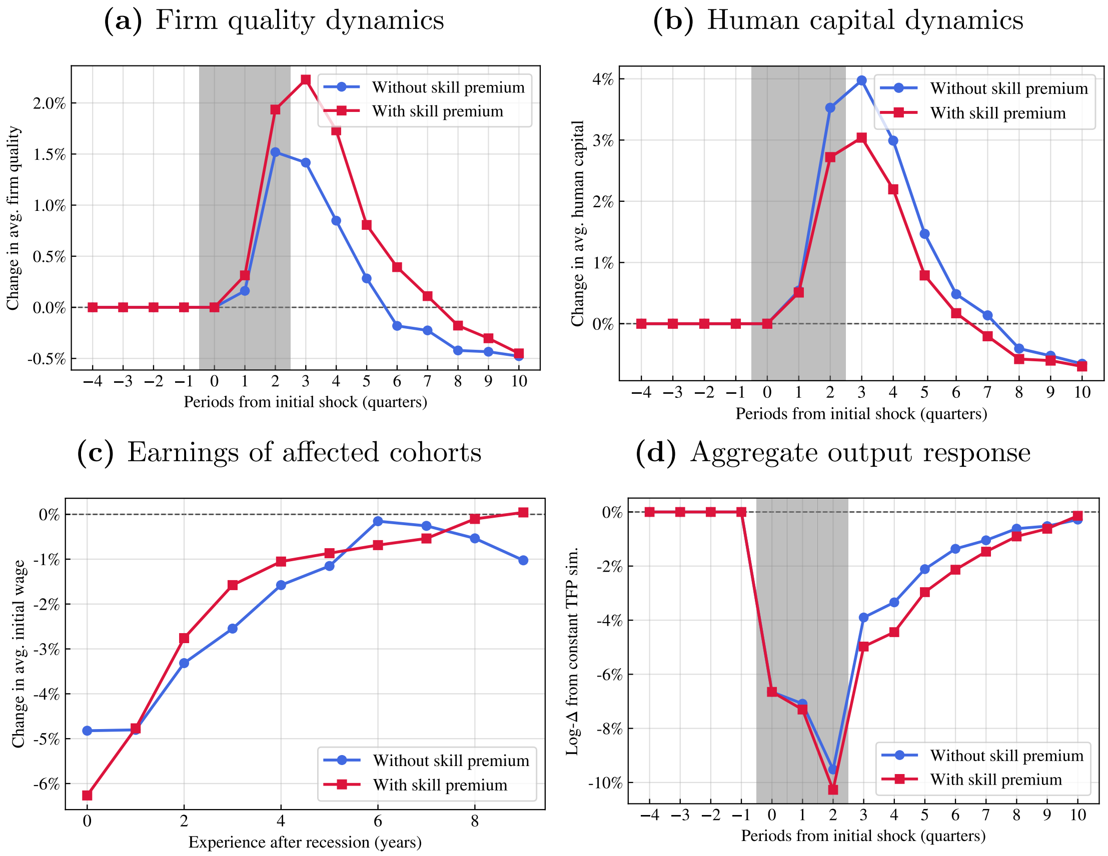
Note: The top panels in the figure plot: (**a**) the behaviour of average firm quality and (**b**) average human capital in an economy with no aggregate shocks, that serves as benchmark, and an economy in which we impose a three-quarter recessions. The bottom panels report: (**c**) the ratio between the two simulations of average wage for the cohorts of workers that enter the labor markets in recessions; (**d**) the aggregate effects of the recession of real GDP. Referenced on page(s) [23,24,24,25]

of the initial shock on output beyond the original duration of the recession. **Figure 10d**, in fact, shows that even after two years aggregate output its still approximately 1.5% below its counterfactual level.

5.2 The importance of the human capital accumulation channel

In this section, we discuss the role of human capital accumulation in shaping the aggregate features of business cycles in our model. In particular, we show that despite a similar transmission mechanism, an economy in which graduate workers can accumulate human capital at a higher rate with respect to non-graduates is characterized by harsher and more prolonged recessions. Remarkably, we find that in our baseline model the average recession is approximately 40% longer than the case where there are no differences in human capital accumulation between graduates and non-graduates. This is roughly in

Figure 11. Recession experiment in an economy *with* and *without* skill premium



Note: For the economy *without* skill premium we set the speed of human capital accumulation equal between graduates and non-graduates. The economy *with* skill premium is our baseline calibration and we therefore report the same data as in **Figure 10**. The top panels in the figure plot: (**a**) the behaviour of average firm quality and (**b**) average human capital in an economy with no aggregate shocks, that serves as benchmark, and an economy in which we impose a three-quarter recessions. The bottom panels report: (**c**) the ratio between the two simulations of average wage for the cohorts of workers that enter the labor markets in recessions; (**d**) the aggregate effects of the recession of real GDP. Referenced on page(s) [25]

line with what we obtain comparing the duration of Italian recessions after 1990 with the duration of recessions before 1990, a period when reasonably the labor market was characterized by less polarization and a lower skill premium.

Transmission mechanism with different skill premia. Let us start by comparing the transmission mechanism of aggregate shocks for two parametrizations of our model. Our baseline one, where graduate workers enjoy a faster accumulation of human capital while employed, and a counterfactual one, where there is no skill premium for graduate workers, i.e. where graduates and non-graduates accumulate human capital at the same rate when matched with the same firm.

As in **Figure 10** we compare the average dynamics for an economy without aggregate shocks and the same economy with a unique three quarter recessions, with and without skill premium between graduates and non-graduates. **Figure 11** reports the effects of the recession on the main aggregate variables in the economy as well as the loss in earnings for workers that enter the labor market during the recession.

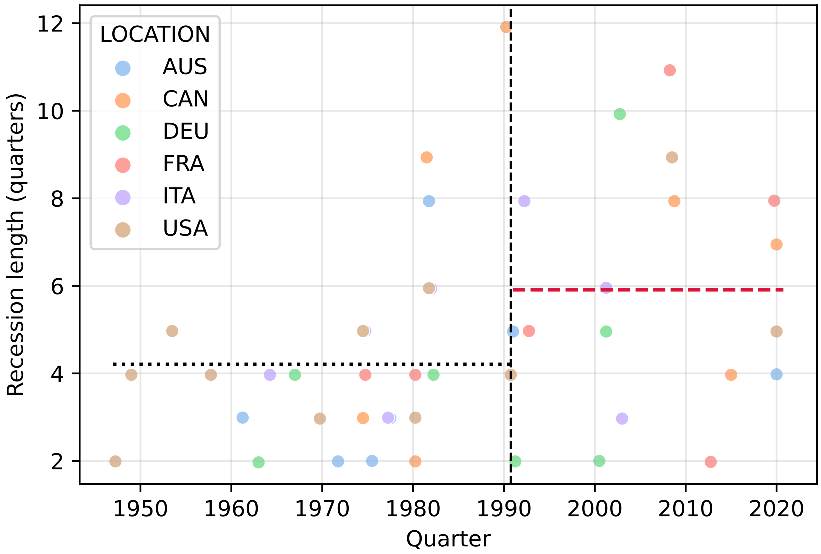
While the economy without skill premium generates similar scarring effects on the cohorts directly affected by the recession, the behaviour of average firm quality, human capital and aggregate output is very different. In particular, the recession has a smaller initial cleansing effect for firms, but not for human capital reflecting how the absence of skill premium influences the viability of a different set of matches.

Amplification and propagation. The time economies take in order to recover from recessions has not always been constant. **Figure 12** shows the average number of quarters aggregate GDP has taken to recover in a subset of advanced economies. Notably, from the mid-80s early 90s there has been a widespread increase in this measure. Among other factors, the increase in job polarization and the rise in the skill premium are phenomena contemporaneous in time with this rise in the time economies need to recover from recessions (Goldin and Katz, 2007, Goos, Manning and Salomons, 2009). Our model provides a useful structure to check whether human capital accumulation and the sorting dynamics in the labor market can explain, at least in part, these aggregate developments.

In order to calculate the duration of recessions and the subsequent recoveries in our baseline economy and a counterfactual one where there are no differences between graduated and non-graduates, we consider two 300 periods simulations of the model, one with and another one without skill premium. The economy without skill premium, on average, fully recovers the loss in aggregate GDP caused from a recession after approximately 12 quarters.[19] With the same shock realization, in our baseline parametrization that implies a strong skill premium for graduate workers, the economy takes approximately 17 quarters to reach its pre-recession GDP levels. Hence, the presence of a high skill premium leads to recoveries that are approximately 40% slower than those occurring without skill premium. In relative terms, this change in the time that the economy takes to fully recover from a recession is close to what observed for the Italian economy when comparing the period before and after 1990, when the average duration of a recovery went from approximately 4 quarters to 6, a 50% increase.

[19] As in the data, we define a recession as occurring after two consecutive quarters of negative GDP growth.

Figure 12. Length of recessions over time



Note: The figure reports the average duration of recoveries for a set of OECD countries. Specifically, for each recession, we compute the number of quarters each economy takes to fully reach its pre-recession GDP level. Horizontal lines are cross country averages before and after 1990Q4.

6 Model validation

The model and the contractual environment developed in this paper imply a strong dependence of workers' careers and consequently aggregate output dynamics on the history of aggregate shocks. In the following sections, we present empirical evidence on these dynamics using Italian administrative data. In all cases, we replicate the empirical analysis on model simulated data to check the ability of our model to replicate the micro-level dynamics of the Italian labor market.

6.1 Empirical evidence

First, in accord with the previous literature on the topic, we adopt a reduced-form approach to show that at a micro-level, aggregate conditions at the start of the career have persistent on workers' careers and earnings dynamics.[20]

Second, we show that even while controlling for outside options the quality of past employers affects future earnings. We do so by analyzing earnings dynamics of workers after reallocating from unemployment, thus controlling for outside options unrelated to differences in human capital.

Last, we adopt a vector autoregressive regression model (VAR) to show how variations in sorting, determined by the dynamics of workers' career at the micro level, end up

[20] See for instance (Oreopoulos, Von Wachter and Heisz, 2012, Kahn, 2010, Schwandt and von Wachter, 2019).

impacting aggregate activity.

6.1.1 Cohort-effects in the Italian labor market

In our model, human capital accumulation is a byproduct of matching between firms and workers, and the only exogenous process that affects matching is that of aggregate productivity. Hence a direct prediction of the model is that aggregate conditions, especially at the beginning of workers' careers, should have persistent effects on their labor market outcomes. The fact that workers accumulate human capital while working gives a disproportionate importance to the quality of matches early in workers' careers. The intuition relies on noticing that the human capital accumulation throughout the early part of the working life tends to be particularly important. This is due both to the fact that workers' human capital is lower while young and, at the same time, the net present value of human capital investments increases with the amount of time in which a worker is able to benefit from them. Younger workers searching for their first job are therefore strongly motivated by human capital accumulation, as opposed to older workers possibly trying to reallocate. As a consequence, any shock that impairs investments in human capital accumulation, especially early in workers careers, will generate persistent losses in workers' earnings.[21]

Even if the magnitude and the persistence of aggregate conditions of cohorts of workers and graduates have been documented by various papers in the labor literature, the structural channel that links aggregate conditions to persistent earning losses has not been fully characterized. In this section, we follow the empirical strategy of these studies and we estimate, by means of a reduced form approach, the magnitude and the persistence of cohort effects for the Italian labor market.[22]

Age-period-cohort models allow to isolate the effect of belonging to a particular cohort from the effects of contemporaneous aggregate conditions and aging, which in the context of labor market outcomes is equivalent to accumulating experience.

Formally a generic age-period-cohort model could be expressed as follows:

$$y_{i,c,t} = \Gamma_t + \Gamma_{t-c} + \Gamma_c + \gamma' \mathbf{X}_i + \varepsilon_{i,c,t}$$

where the dependent variable is our outcome of interest for a worker i, belonging to cohort c in year t and where the right hand side is composed by a collection of functions

[21]For example, Arellano-Bover (2020) shows that, in Spanish data, the size of the initial firm has persistent effects on workers careers. He shows that when the first job of a worker is in a firm one-standard deviation higher than the average, this results in a lifetime income (20 years) one-third higher than the average.

[22]As the empirical part of the project awaits INPS approval, we are not at freedom at this time to disclose any empirical result beyond the ones briefly presented below.

aimed at controlling for year, cohort and age effects respectively and where $\mathbf{X}$ is a set of worker-level controls.

Ideally, instead of assigning parametric structures to the Γ functions we would use fixed effects to allow the data to determine the functional form for each of these effects. However this approach would lead to a well-know identification problem as the fixed effects for age, period and cohort are perfectly collinear between each other. As a consequence, identifying the levels of these three factors requires additional normalizations or exclusion assumptions. The approach we follow to construct the preliminary evidences, as discussed in Heckman and Robb (1985), consists of proxying one of the fixed effects with a variable that is not linearly dependent with the other fixed effects. We use cyclical real GDP realizations at the time of first job as proxies for cohort effects.[23]

We formally evaluate the correlation between annual wage and aggregate conditions at the time of first job by considering the following regression:

$$\log(w_{i,c,y}) = \alpha + \beta\tilde{Y}_c + \phi_e + \phi_y + \gamma'\mathbf{X}_i + \varepsilon_{i,c,y}, \tag{15}$$

in which the dependent variable is the logarithm of annual real wage for worker i, belonging to cohort c in year y. As each cohort is identified by the year of entry in the labor market, the function for age is replaced by a set of experience fixed effects ϕ_e while the function for the period effects is substituted by a set of year fixed effects ϕ_y. The matrix of worker level controls $\mathbf{X}$ includes a series of fixed effects aimed at controlling for worker specific factors, such as sex, type of contract (part-time vs full-time), contract maturity (fixed term vs open ended), sector and qualification. Under the standard exogeneity restrictions the coefficient β measures the average percentage change in annual wage resulting from a one-percent cohort-specific variation in the business cycle measure $\tilde{Y}$. [24]

As our main identifying variation is at the annual level, we estimate a cell-level version of **Equation 15**, in which we aggregate our worker-level data at the cohort, sex and local labor markets and we estimate the persistence of initial aggregate conditions interacting our main proxy variable with experience fixed-effects. In practice, we

[23] We use the Hamilton filtered (HF) series of real log-GDP with 1 period lag and 2 periods horizon. As a robustness with check also the Hodrick-Prescott (HP) filtered series (smoothing parameters, 6.25) and the quadratically detrended real-GDP. The results are qualitatively similar but we choose HF for our baseline specification as it is a filter that does not use any future information, which could be problematic in our regressions, and captures business cycles better than the quadratic detrended measure.

[24] An obvious threat to identification would be the ability of workers to withdraw from the labor market in according to changes in aggregate conditions, while at the same time having access to a technology for investing in human capital. Depending on their individual traits, e.g. their learning ability, there would be heterogeneity in responding to downturns. Due to characteristics of Italian labor demand and its the education system, however, it is reasonable to assume that workers in the country do not have access to such a technology.

estimate the following regression:

$$\log(w_{c,t}) = \alpha + \sum_{e=1}^{\bar{e}} \beta_e \tilde{Y}_c \mathbf{1}\{c_t = e\} + \phi_e + \phi_t + \varepsilon_{c,t}, \tag{16}$$

where $\mathbf{1}\{c_t = e\}$ is an indicator function that takes value one when workers belonging to cohort c reach e years of potential experience. We assign workers to cohorts using the year of their first job and we weight our specification by cell size.

Results. Preliminary analyses on the correlations between aggregate conditions at the moment of first job indicate a large and persistent effect of aggregate conditions at the time of first employment. Results indicate that, when business cycles are two standard deviations below trend, in the first ten years of their careers workers experience a loss in earnings equal to approximately 8% of their wage - cumulated over the same period the loss in earnings generated by facing adverse initial aggregate conditions accounts for approximately 78% of the average monthly wage, a significant amount.[25]

Figure 13 plots the age profiles for workers that experience different aggregate conditions at the beginning of their careers, an expansion (characterized as *positive* two-standard deviations change in the cyclical component of GPD) and a recession (defined as a *negative* two-standard deviations change in the cyclical component of GDP). The figure shows how the losses in earnings associated to starting a career during a recession are persistent and the earnings profiles are only slowly converging.

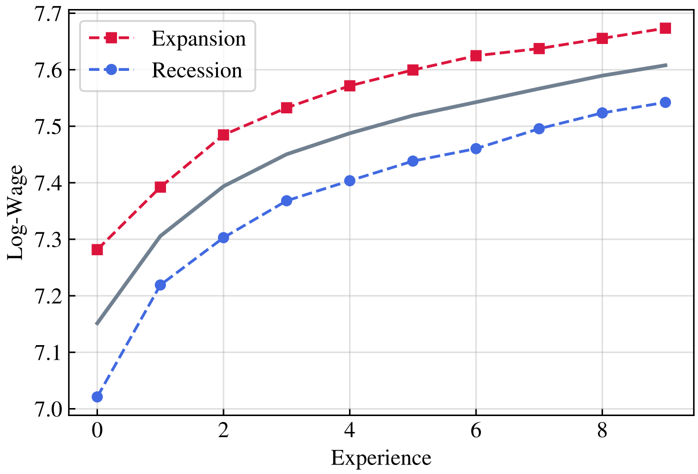
6.1.2 Wage dynamics and past quality of employment

A key feature of our model with human capital accumulation on the job, firms' heterogeneity and sorting is that better quality firms would provide greater human capital accumulation. Arellano-Bover (2020), Arellano-Bover and Saltiel (2020) show that workers experience greater human capital accumulation on the job in some "high quality" firms, and that the earnings prospects of young workers are distinctively better when their career start in bigger (that is, more productive) firms.

The role of human capital in shaping workers' labor market outcomes can become more

[25]This result is in the ballpark of what Schwandt and von Wachter (2019) found for the US, who show results for the period from 1976 to 2015. Our results are however plausibly just a lower bound of the true value, given the implicit endogeneity in selecting first jobs instead of year of graduation for the start of measurement. Analogously, the fact that we use GDP growth instead of unemployment variations in the specifications might help explaining why we do not even indirectly take into account the 0 earnings of unemployed people. For the Spanish labor market, that shares many similarities to the Italian context (e.g. high youth unemployment and high degree of segmentation) Fernández-Kranz and Rodríguez-Planas (2017), Garcia-Cabo (2018) estimate the loss of an average recession in a range between 6-12% of annual earnings over 10 years, stronger and more persistent for less educated workers (7 years persistence for high-school graduates versus 5 years for college graduates).

Figure 13. Effect of initial aggregate condition on workers' wage profiles



Note: The figure plots the age profiles for workers facing different realization of the business cycle at the onset of their careers. In particular, for each experience e we plot $w_0 + \hat{\phi}_e + Z\hat{\beta}_e D_e$ where ˆ variables are estimated using **Equation 16**, D_e are experience dummies and $Z = \{2\sigma, -2\sigma\}$ is a measure of initial conditions with σ denoting the standard deviation of the Hamilton-filtered real GDP and w_0 is the average log-wage for workers with no experience in our sample. **Table 1** reports the corresponding coefficients. Referenced of page(s) [30,38,38] .

apparent examining wage dynamics after Employment-to-Unemployment-to-Employment (E-U-E) transitions. Looking at how wage changes around these particular transitions is useful as workers that land a job after an E-U-E transitions are all coming from the unemployment pool. This allows to indirectly control for possible confounders in the set of the available outside options, which, for our purpose, implies that the observed differences in realized wages are influenced by employers' qualities only through workers characteristics. A positive correlation between firm quality and realized wage then, can be taken as an indication of a positive influence of employers on workers long-term labor market outcomes.

We proxy firm quality by quintiles of firm AKM fixed effects (Abowd, Kramarz and Margolis, 1999) and run the following regression:[26]

$$\log(w_{i,t+2}) = \alpha + \beta \log(w_{i,t}) + \phi_{p_{J(i,t)}} + \Gamma \mathbf{X_{i,t}} + \varepsilon_{i,t}, \tag{17}$$

The worker's salary after an unemployment spell $w_{i,t+2}$ is regressed on its past salary when employed $w_{i,t}$, firm quality quintiles fixed effects $\phi_{p_{J(i,t)}}$, where $J(i,t)$ is the firm when the worker was employed before being unemployed and $p(\cdot)$ is its assigned quality quintile. $\mathbf{X_{i,t}}$ comprehends workers' and contract characteristics: age, sex, fixed effects for years and contract types and controls for experience (years since entering in the job

[26] Herkenhoff et al. (2018) run a similar specification ton analyze the effect of co-workers earnings in past employment on workers earnings in their next employment. Our results are qualitatively equivalent if we proxy firm quality by yearly quintiles of value added per worker.

market).

The coefficients of interest are $\phi_{p_{J(i,t)}}$, the quality of the previous firm fixed effects. The sign and direction of the profile traced by these fixed effects is indicative of the role of previous employers in determining the differences in realized wages after E-U-E transitions. As shown in **Table 2a**, these coefficients are positive and increasing in our sample.[27] This result indicates that, after an unemployment spell, workers that were employed in better firms are indeed able to command a higher wage compared to those employed in lower quality firms. As we estimate the specification in **Equation 17** on workers that experienced an unemployment spell, the beneficial effect of past employer influences workers' wages only indirectly. In the remainder of the paper, we show how human capital accumulation, paired with sorting in the labor market, is a crucial element in replicating this finding.

6.1.3 Sorting in the labor market and the business cycle: aggregate evidence

The long-lasting deterioration of workers' earning in downturns, albeit a crucial point in the link between business cycles and human capital, does not capture the aggregate effects of business cycles on the quality of labor market sorting.

We provide a first-pass empirical analysis of the linkages between business cycles and labor market sorting by estimating a version of Galí (1999) VAR augmented with a measure of labor market sorting. We rely on Italian data annual data from 1998 to 2017.[28] Assuming that TFP shocks are the only structural shocks that are able to influence labor productivity in the long-run we can use them as exogenous shifts to output that are not directly related to the labor market and therefore look at the impulse responses of to this shock to check how exogenous movements in output translate to labor market sorting. Due to the low frequency and short time series, the IRF are only imprecisely estimated, as shown in **Figure 14**. However, the point estimates are still suggestive of the qualitative relationship between business cycles and labor market sorting. A shock to TFP, in fact, triggers an hump-shaped response in our sorting measure. Sorting declines on impact and then overshoots its long-run value, converging back to zero from above. We take this hump-shaped response as a signal of possible long-lasting feedback effects between business cycles and the quality of matches formed in the labor market.

In the remainder of the paper we show how our model is able to provide a framework

[27] As the Italian administrative data report the reason of the separation with run a robustness check excluding separations that were deemed justified (*giusta causa*) under the Italian legislation. Excluding these separations can alleviate possible concerns on the possible selection in workers' quality in the pool of unemployed. Reassuringly, the results are unchanged.

[28] For each year, we measure sorting as the correlation between the fixed effects of an AKM-style regression (Abowd, Kramarz and Margolis, 1999) in which we cluster firms in ten quality bins based on their wage distribution (Bonhomme, Lamadon and Manresa, 2019).

Figure 14. Impulse responses to a 1% technology shock.

Note: The figure plots the IRFs from an annual VAR with one-lag based on *Log-GDP per worker*, *Log-Employment* and *Sorting*. We measure *Sorting* as the correlation between firm quality FE and worker FE from an AKM-style regression (Abowd, Kramarz and Margolis, 1999, Bonhomme, Lamadon and Manresa, 2019). All variables are first-differenced and technology shocks are identified using long-run restrictions as in Galí (1999). Sample period:1998-2017, shaded areas: 68% error bands. Referenced on page(s) [32] .

to rationalize these complex feedback effects between business cycles and labor market sorting both at the micro and at the aggregate level.

6.2 Model vs Data: scarring effects, sorting dynamics and human capital

To check if the model is a good proxy for the data generating processes underlying our empirical analyses we perform the same empirical exercises on artificial data constructed from model simulations, both for the scarring effects of recessions and for the aggregate relationship between aggregate shocks and labor market sorting.[29]

Scarring effects of recessions. Given the dynamics highlighted in the experiment with a controlled recession, we replicate the analysis discusses in Section 2 and we estimate the effects of a two-standard-deviation change in aggregate output on the average wage of workers using a version of **Equation 15**, adapted for our baseline simulation. **Figure 15** shows that, despite the steeper growth profile, our framework is able to generate losses in earning that are qualitatively consistent to the empirical estimates of the scarring effects of recessions at the micro-level.

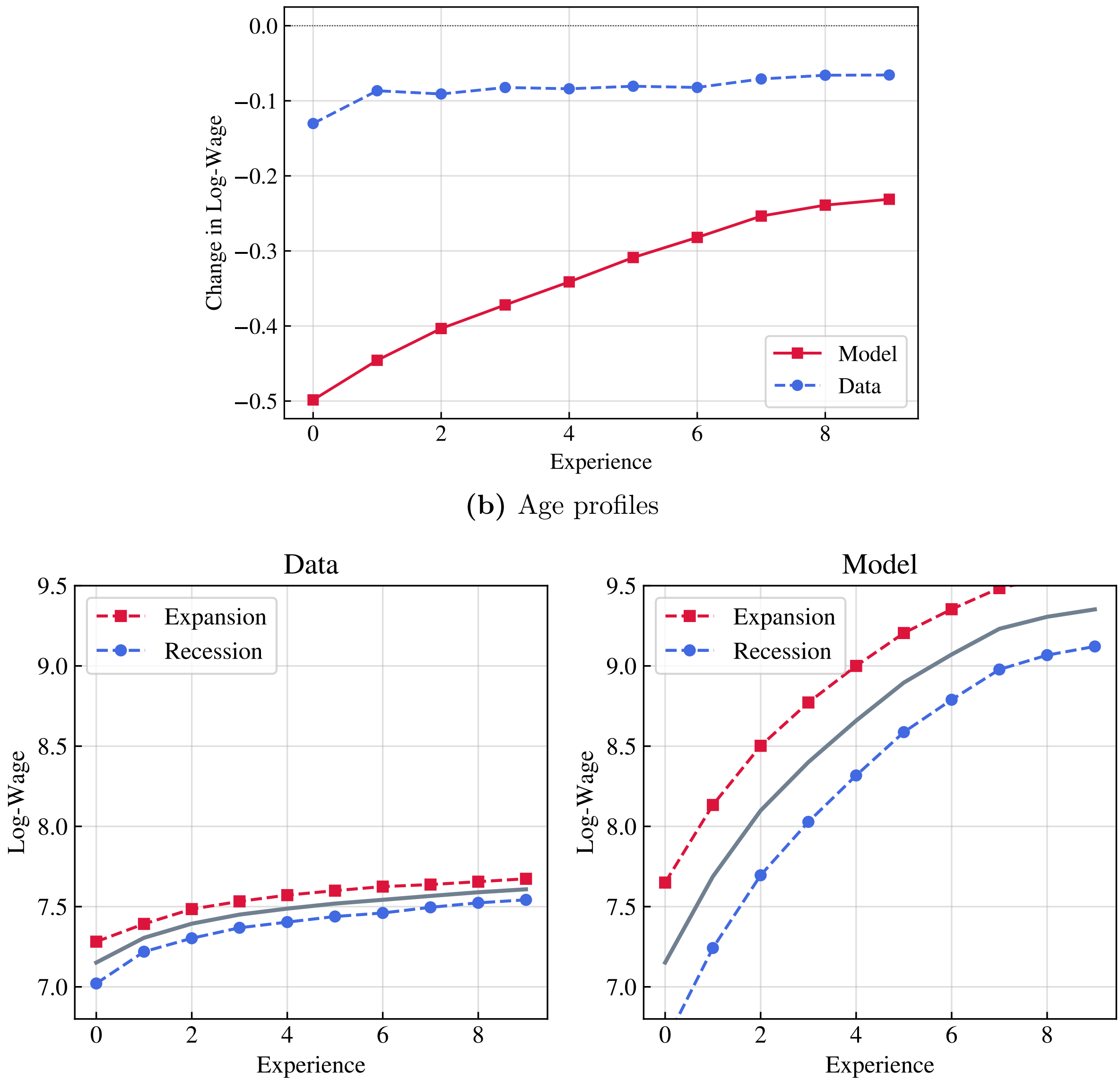
Business cycles and sorting. **Figure 16** plots the IRFs from the same structural VAR discussed in **Section 6.1.3**. On impact, the positive shock to productivity increases output and employment while putting downward pressure on the correlation between

[29]We simulate 480 quarters populating each cohort of with 60 agents.

Figure 15. Effect of initial aggregate condition on workers' wage profiles, Model vs Data

(a) Scarring

(b) Age profiles



Note: The figure plots the estimated effects of a two-standard-deviation recession on workers earnings and the associated profiles for workers facing different realization of the business cycle at the onset of their careers in the model simulation and in the data. For *Model* and *Data* panels, we plot $w_0 + \hat{\phi}_e + Z\hat{\beta}_e D_e$ for each experience e, where ˆ variables are estimated using **Equation 15**, D_e are experience dummies and $Z = \{2\sigma, -2\sigma\}$ is a measure of initial conditions with σ denoting the standard deviation of real GDP and w_0 is the average log-wage for workers with no experience in our sample. Referenced of page(s) [33]
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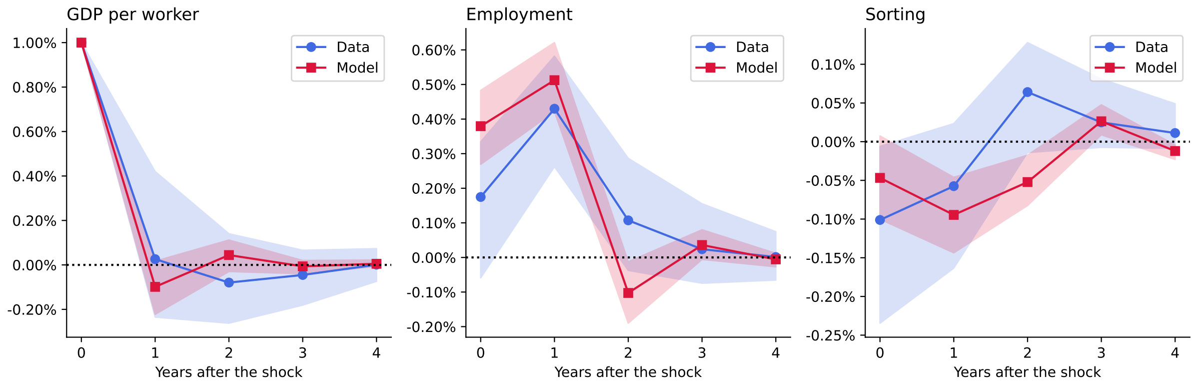
workers and firm qualities. However, as we have discussed, the expansion pushes workers to search in better submarkets laying the foundations for the overshoot in our sorting measure as observed in the data. We consider this result as a further qualitative validation of the mechanisms at the core of our model.

Figure 16. Impulse responses to a 1% technology shock, Model vs Data

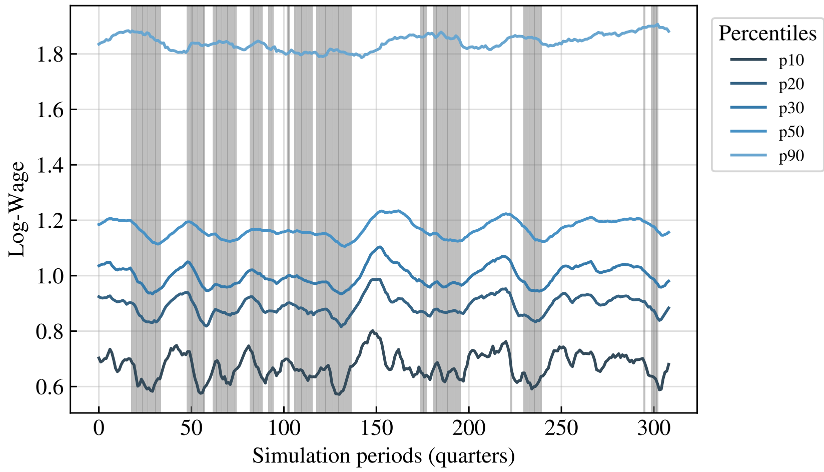


Note: The figure plots the IRFs from an annual VAR with one-lag based on *Log-GDP per worker*, *Log-Employment* and *Sorting*. In the data, we measure *Sorting* as the correlation between firm quality FE and worker FE from an AKM-style regression while in the model, *Sorting* is the correlation between firm and worker qualities. Both in the model and in the data, all variables are first-differenced and technology shocks are identified using long-run restrictions as in Galí (1999). Shaded area are 68% error bands. Referenced on page(s) [33] .

Human capital around E-U-E transitions. **Table 2a** and **2b** report the dynamics of log-wages around E-U-E transitions in the Italian data and in model simulations, respectively. As discussed in **Section 6.1.3** in the data, we observe a significant, positive and increasing relationship between the quality of the last employer and the level of wages workers can obtain after making an E-U-E transition. A similar effect, albeit stronger than in the data, is present also in our simulations, as shown in **Table 2b**. Through the lenses of our model, the influence of employers on workers careers materializes in workers' human capital accumulation. Thus, a positive correlation between realized wages and firm qualities in the model is capturing the stronger human capital accumulation provided by employers of higher quality.

Inequality dynamics. As in the data, the model predicts that negative aggregate conditions at the onset of workers' careers generate a persistent loss in earnings, **Figure 10c**. In the model, this persistent effect is generated through the fact that worse matches, more likely to happen in recessions and especially at the beginning of workers' lives, cause a slower human capital accumulation, putting a persistent drag on the ability of workers to climb the job ladder. Even in our illustrative numerical exercise, the model already generates dynamics in earning losses that are comparable in

Figure 17. Inequality dynamics over the cycle



Note: The figure plots the time series for the different percentiles of the income distribution as in Heathcote, Perri and Violante (2020). Referenced on page(s) [35]

magnitude and duration to those estimated in the data. In addition, we can show that all of this dynamics is concentrated in the left tail of the earnings distribution: **Figure 17** illustrates this point by displaying the pattern of losses across the earnings distribution. Consistently with recent evidence, recessions hit disadvantaged workers the hardest.

7 Conclusions

We have set up a new and tractable model of on-the-job search and human capital accumulation that features heterogeneity both on the worker and on the firm side. Ex-ante heterogeneous workers accumulate on-the-job experience which augments their skills and moves them up in the job ladder. Our contractual framework endogenously accounts for the difference incentives between risk-averse workers and risk-neutral entrepreneurs. We characterize how insurance incentives are of paramount importance in shaping the response of the labor market, the efficiency of workers-firms matches and the overall dynamic of human capital accumulation. Most importantly, we show how even in absence of institutional frictions optimal contract can endogenously generate rigidities in compensation. Employment relationships are subject to a one-sided limited commitment problem, where the firm can commit to a contingent wage path but has limited liability, and are regulated by a dynamic contract that endogenously determines the optimal provision of insurance to workers. Within this framework we show that limited liability on the firm side generates downward wage rigidity as the optimal contract commits the firm to pay the worker a non-decreasing compensation path.

Consistent with the data, wage rigidity amplifies negative shocks to firms, and generates inefficient separations. We establish that workers that look for employment in bad economic times direct their search towards less productive firms. Search frictions and aggregate uncertainty prevent an efficient allocation of workers to firms and expose different cohort of workers to different human capital accumulation paths depending on the aggregate state at the time of entry in the labor force. Limits to workers' ability to accumulate human capital imposes a drag on the overall labor productivity of the economy after recessions that persists as long as these cohorts of workers are active in the labor force.

A numerical solution of the model is provided, with simulated data displaying earnings paths, equilibrium levels of unemployment and patterns of transition within and between jobs that qualitatively match the data even in our initial tentative calibrations. The model is then used to understand the long run effects of business cycle fluctuations on worker's human capital accumulation and earning profiles. We find that, because of the missed investments in human capital due to longer unemployment spells, workers whose initial years are affected by a recession direct their search towards less selective firms. Lower investment and different search behavior compound over time, generating persistent scarring effects that qualitatively match the micro-level empirical evidence. At the same time, workers' search behaviour is a fundamental premise for generating a feedback effect between business cycles and sorting in the labor market also at the aggregate level.

There are two natural directions to expand the research agenda. One is to investigate the effects of welfare policies - like targeted unemployment benefits, targeted hiring subsidies, training programs or fiscal devaluations of labor costs to support unemployed cohorts and employment in recessions, and the minimum wage. The other is to exploit the flexibility of our model to understand how labor markets are shaped by long run trends, e.g. in fertility, interest rates, or firm dynamism.

Tables

Table 1. Effects of initial aggregate conditions along the experience profile and experience growth profile

Dep.Variable: Log-Wage	Experience × Cycle	Experience
Experience Dummy		
0	1.968	
	(0.452)	
1	1.463	0.150
	(0.238)	(0.011)
2	1.473	0.246
	(0.283)	(0.014)
3	1.239	0.304
	(0.343)	(0.014)
4	1.250	0.342
	(0.349)	(0.015)
5	1.239	0.375
	(0.349)	(0.015)
6	1.301	0.399
	(0.287)	(0.015)
7	1.087	0.422
	(0.320)	(0.016)
8	0.985	0.445
	(0.305)	(0.017)
9	0.981	0.463
	(0.334)	(0.017)
Age FE	✓	✓
Year FE	✓	✓
Sex FE	✓	✓
LLM FE	✓	✓
R^2	0.89	0.89
N	254,000,000	254,000,000

Note: The table reports regression coefficients from specifications in Equation 16, Figure 13.

Table 2. Human capital from E-U-E transitions

(a) Data

Dep.Var.: Log-wage after E-U-E transition	(1)	(2)
Quality of origin firm (FQ):		
2^{nd} quint.	0.100	0.128
	(0.002)	(0.004)
3^{rd} quint.	0.143	0.210
	(0.002)	(0.004)
4^{th} quint.	0.153	0.182
	(0.002)	(0.004)
5^{th} quint.	0.230	0.260
	(0.002)	(0.004)
Log-wage at origin	0.669	0.609
	(0.001)	(0.002)
Experience controls	✓	✓
Sex FE	✓	✓
Year FE	✓	✓
Contract type FE	✓	✓
Full- & Part-Time FE	✓	✓
Justified dismissals	✓	
R^2	0.48	0.36
N	955,602	338,975

(b) Model

Dependent Variable:	Log-wage after E-U-E transition
Quality of origin firm (FQ)	
1^{st} quart.	0.988
	(0.004)
2^{nd} quart.	0.983
	(0.006)
3^{rd} quart.	0.990
	(0.005)
4^{th} quart.	0.993
	(0.006)
Log-wage at origin	0.004
	(0.001)
Human capital at origin	0.102
	(0.001)
Experience controls	✓
R^2	0.544
N	15,385

Note: Standard errors in parentheses. The tables report a specification on a dataset of E-U-E transitions in the data and in the model. In Panel **(a)**, column (2) excludes separations that are justified in the Italian labor law (*giusta causa*).Referenced on page(s) [32,35] .

Table 3. Functional forms and parameter values

(a) Functional Forms

Functions	
Production function	$f(y,h) = Ay^{\alpha}h^{1-\alpha} - x(A-1)$
Job finding probability	$p(\theta) = \theta(1+\theta^{\gamma})^{-\frac{1}{\gamma}}$
Vacancy creation cost	$c(y) = \kappa y$
Utility function	$U(c) = \frac{c^{1-\nu}}{1-\nu}$
Human capital accumulation	$g(h,y) = (\xi y)^{\phi}h^{1-\phi} + \psi$
Home production	$b(h,\tau) = b + \xi_b h$
Exogenous exit rate	$\lambda = \lambda_a + \frac{\lambda_b}{\lceil \tau/4 \rceil}$

(b) Parameters

Parameters	**Values**
ν, Risk aversion	2.000
β, Discounting	0.990
α, Production function elasticity to firm quality	0.530
γ, Matching function	0.963
ϕ, Human capital adjustment rate	0.033
ϕ_g, Human capital adjustment rate, Graduate	0.255
b, Unemployment benefit	0.875
λ_a, Exogenous separation prob., limit	0.000
λ_b, Exogenous separation prob., initial	0.091
κ, Vacancy cost	2.000
λ_e, On-the-job-search prob.	0.500
ξ, Scaling factor in human capital accumulation	0.443
ξ_b, UB dependence on human capital	0.010
l, Linear loss of humanc capital while unemployed	0.106
τ_{ee}, Human capital retention after EE	1.007
τ_{eu}, Human capital loss after EU	0.859
x, Cyclical component of cost function	-2.139
$\underline{p}$, Out of labor force threshold	0.050
(μ_h, σ_h), Shape and scale of initial human capital dist.	(1.000,1.100)
(ρ_A, σ_A), Mean and std of TFP process	(0.900, 0.010)
σ_{ψ}, Std of idiosyncratic human capital shock	0.854

Note: As time is discrete we have to pick a matching function bounded between zero and one. This rules out Cobb-Douglas functions and therefore we follow Schaal (2017) and Menzio and Shi (2010) picking a CES function in market tightness. Referenced on page(s) [17] .

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Appendices

For compactness of notation, we omit the dependence on education level, which is a fixed characteristic, and the idiosyncratic human capital shock, which is additive, from the proof in Appendices. The logic of the proofs follows without loss of generality.

A Properties of worker optimal behavior

The following propositions characterize the properties of workers' optimal search strategies that solve the search problem in (2), restated here for convenience:

$$R(h,\tau,V;\Omega) = \sup_{\{v_{y,\Omega}\}} \Big[p(\theta(h,\tau,v_{y,\Omega};\Omega))\big[v_{y,\Omega} - V\big]\Big]. \tag{A.1}$$

Lemma A.1. *The composite function $p(\theta(h,\tau,v;\Omega))$ is strictly decreasing and strictly concave in v.*

Proof. For this proof we follow closely Menzio and Shi (2010), Lemma 4.1 (ii). From the properties of the matching function we know that $p(\theta)$ is increasing and concave in θ, while $q(\theta)$ is decreasing and convex. Consider that the equilibrium definition of $\theta(\cdot)$ is

$$\theta(h,\tau,v;\Omega) = q^{-1}\left(\frac{c(y)}{\beta J(h,\tau,y,v;\Omega)}\right),$$

and that the first order condition for the wage and the envelope condition on V of the optimal contract problem in (8) implies

$$\frac{\partial J(h,\tau,y,v;\Omega)}{\partial v} = -\frac{1}{u'(w)}.$$

so that as $u'(\cdot) > 0$, $J(\cdot)$ is decreasing in v.

From the equilibrium definition of $\theta(\cdot)$ and noting that $q^{-1}(\cdot)$ is also decreasing due to the properties of the matching function we have that

$$\frac{\partial\theta(h,\tau,v;\Omega)}{\partial v} = \frac{\partial q^{-1}(\xi)}{\partial\xi}\bigg|_{\xi=\frac{c(y)}{\beta J(h,\tau,y,v;\Omega)}} \cdot \left(-\frac{\partial J(h,\tau,y,v;\Omega)}{\partial v}\right) \cdot \frac{c(y)}{\beta(J(h,\tau,y,v;\Omega))^2} < 0,$$

which, in turn, implies that

$$\frac{\partial p(\theta(h,\tau,v;\Omega))}{\partial v} = \frac{\partial p(\theta)}{\partial\theta}\bigg|_{\theta=\theta(h,\tau,v;\Omega)} \cdot \frac{\partial\theta(h,\tau,v;\Omega)}{\partial v} < 0.$$

Suppressing dependence on the states (h,τ,y,Ω) for readability, to prove that

$p(\theta(v))$ is concave, consider that $J(v)$ is concave[30] and a generic function $\frac{c}{v}$ is strictly convex in v. This implies that with $\alpha \in [0,1]$ and $v_1, v_2 \in \mathcal{V}$:

$$\frac{c}{J(\alpha v_1 + (1-\alpha)v_2)} \leq \frac{c}{\alpha J(v_1) + (1-\alpha)J(v_2)} < \alpha \frac{c}{J(v_1)} + (1-\alpha)\frac{c}{J(v_2)}.$$

As $p(q^{-1}(\cdot))$ is strictly decreasing the inequality implies that

$$\begin{aligned} p\left(q^{-1}\left(\tfrac{c}{J(\alpha v_1+(1-\alpha)v_2)}\right)\right) &\geq p\left(q^{-1}\left(\tfrac{c}{\alpha J(v_1)+(1-\alpha)J(v_2)}\right)\right) \\ &> \alpha p\left(q^{-1}\left(\tfrac{c}{J(v_1)}\right)\right) + (1-\alpha)p\left(q^{-1}\left(\tfrac{c}{J(v_2)}\right)\right), \end{aligned}$$

and as $\theta(v) = q^{-1}(\frac{c}{J(v)})$:

$$p(\theta(\alpha v_1 + (1-\alpha)v_2)) > \alpha p(\theta(v_1)) + (1-\alpha)p(\theta(v_2))$$

so that $p(\theta())$ is strictly concave in v. □

Proposition A.1. *Given the worker search problem, the following properties hold:*

(i) The returns to search, $p(\theta(h, \tau, v_{y,\Omega}; \Omega))\left[v_{y,\Omega} - V\right]$, *are strictly concave with respect to promised utility,* $v_{y,\Omega}$.

(ii) The optimal search strategy

$$v^*(h, \tau, V; \Omega) \in \arg\max_{v_y} \left\{p(\theta(h, \tau, v_{y,\Omega}; \Omega))\left[v_{y,\Omega} - V\right]\right\}$$

is unique and weakly increasing (and Lipschitz continuous) in V.

(iii) For all promised utilities, the search gain $R(h, \tau, V; \Omega)$ *is positive, weakly decreasing in* V.

(iv) The survival probability of the match, given the optimal choice of the worker, is increasing in the value of promised utilities, so $\widetilde{p}_t(h, \tau, W_{y,\Omega}; \Omega)$ *is increasing (and Lipschitz continuous) in* $W_{y,\Omega}$.

Proof. The proofs follow closely Shi (2009), Lemma 3.1 and Menzio and Shi (2010), Lemma 4.4. More formally, for each triplet (h, τ, Ω) given at each search stage, we can re-define the search objective function as $K(v, V) = p(\theta(v))(v - V)$ and $v^*(V) \in \arg\max_v K(v, V)$ as the function that maximises the search returns (i.e. the optimal search strategy of the worker) and prove the following

[30] $J()$ concave give the two-point lottery in the structure of the contract. See Menzio and Shi (2010) Lemma F.1.

(i) To show that $K(v, V)$ is strictly concave in v consider two values for v, (v_1, v_2) such that $v_2 > v_1$ and define $v_\alpha = \alpha v_1 + (1-\alpha)v_2$ for $\alpha \in [0,1]$.

Then by definition:

$$\begin{aligned}
K(v_\alpha, V) &= p(\theta(v_\alpha))(v_\alpha - V) \\
&\geq [\alpha p(\theta(v_1)) + (1-\alpha)p(\theta(v_2))][\alpha(v_1 - V) + (1-\alpha)(v_2 - V)] \\
&= \alpha K(v_1, V) + (1-\alpha)K(v_2, V) + \alpha(1-\alpha)[(p(\theta(v_1)) - p(\theta(v_2))](v_2 - v_1) \\
&> \alpha K(v_1, V) + (1-\alpha)K(v_2, V)
\end{aligned}$$

where the first inequality follows from the concavity of $p(\theta(\cdot))$ (this is true if $J()$ concave with respect to V) and the second inequality stems from the fact that $p(\theta(\cdot))$ is strictly decreasing hence $\alpha(1-\alpha)[(p(\theta(v_1)) - p(\theta(v_2))](v_2 - v_1) > 0$.

(ii) **Weakly Increasing.** Given that $v \in [\underline{v}, \overline{v}]$, and submarkets are going to open depending on realizations of the aggregate productivity, a, there is only one region in the set of promised utilities where the search gain is positive, conditional on being in a job that pays lifetime utility V. That is $[V, v(a)]$ with $v(a)$ being the highest possible offer that a firm makes in the submarket for the worker (h, τ). As any submarket that promises higher than $v(a)$ is going to have zero tightness, the optimal search strategy for $V \geq v(a)$ is $v^*(V) = V$. For $V \in [V, v(a)]$, instead, as $K(v, V)$ is bounded and continuous,[31] the solution $v^*(V)$ has to be internal and therefore respect the following first order condition

$$V = v^*(V) + \frac{p(\theta(v^*(V)))}{p'(\theta(v^*(V)) \cdot \theta'(v^*(V))}. \tag{A.2}$$

Now consider two arbitrary values V_1 and V_2, $V_1 < V_2 < \overline{v}$ and their associated solutions $W_i = v^*(V_i)$ for $i = 1, 2$. Then, V_1 and V_2 have to generate two different values for the right-hand side of (A.2). Hence, $v^*(V_1) \cap v^*(V_2) = \emptyset$ when $V_1 \neq V_2$.

This also implies that the search gain evaluated at the optimal search strategy is higher than the gain at any other arbitrary strategy so that $K(W_i, V_i) > K(W_j, V_i)$ for $i \neq j$. This implies that

$$\begin{aligned}
0 >&[K(W_2, V_1) - K(W_1, V_1)] + [K(W_1, V_2) - K(W_2, V_2)] \\
&= (p(\theta(W_2)) - p(\theta(W_1)))(V_2 - V_1),
\end{aligned}$$

thus, $p(\theta(W_2)) < p(\theta(W_1)))$. As $p(\theta(\cdot))$ is strictly decreasing (see Corollary A.1), then $v^*(V_1) < v^*(V_2)$.

[31] Recall that $K(v, V)$ is the search objective function, so that $K(v, V) = p(\theta(v))(v - V)$.

Unique. Uniqueness follows directly from strict concavity shown in (i). Lipschitz continuity still to show but coming from assumption of $J()$ being bi-Lipschitz continuous and $\theta(), p()$ being bounded functions.

(iii) The Bellman equation for the search problem is:

$$R(h, \tau, V; \Omega) = \sup_{\{v_{y,\Omega}\}} \Big[p(\theta(h, \tau, v_{y,\Omega}; \Omega)) \big[v_{y,\Omega} - V \big] \Big]$$

hence a simple envelope argument shows that

$$\frac{\partial R(h, \tau, V; \Omega)}{\partial V} = -p(\theta(h, \tau, v_y; \Omega)) \leq 0,$$

as the job finding probability is weakly positive for all utility promises.

As $p(\theta(\cdot)) \geq 0$, $v^*(\cdot) \in [\underline{v}, \overline{v}]$ then $R(\cdot) \geq 0$.

(iv) Given the optimal search strategy, $v^*(h, \tau, V; \Omega)$, we can define the survival probability of the match as in (5):

$$\widetilde{p}(h, \tau, V_{y,\Omega}; \Omega) \equiv (1 - \lambda)(1 - \lambda_e p(\theta(h, \tau, v^*_{y,\Omega}; \Omega))).$$

Then, given (h, τ, Ω)

$$\frac{\partial \widetilde{p}(V)}{\partial V} = -\beta(1 - \lambda)\lambda_e \left. \frac{\partial p(\theta)}{\partial \theta} \right|_{\theta = \theta(v^*)} \left. \frac{\partial \theta(v)}{\partial v} \right|_{v = v^*(V)} \frac{\partial v^*(V)}{\partial V} > 0,$$

because $p(\cdot)$ and $v^*(\cdot)$ are both increasing functions while $\theta(\cdot)$ is a decreasing function in promised utilities.

□

B Properties of the optimal contract

Lemma B.1. *The Pareto frontier $J(h, \tau, y, W_{y,\Omega}; \Omega)$ is concave in $W_{y,\Omega}$.*

Proof. This is a direct consequence of using a two-point lottery for $\{w_i, W_{iy,\Omega'}\}$ as shown by Menzio and Shi (2010), Lemma F.1. □

Lemma B.2. *The Pareto frontier $J(h, \tau, y, W_{y,\Omega}; \Omega)$ is increasing in y.*

Proof. The intuition for this proof follows the fact that a higher y firm, once the match exists, can always deliver a certain promise V and have resources left over. Within a dynamic contract, future retention is already optimized as the match is formed. This means that the promise V can be delivered by the greater capacity on the part of producing with respect to a close y firm. In presence of human capital accumulation, the worker is compensated through greater option values in the future, which again means that, even with lower retention, the firm cashes in more profits while decreasing wages (and respecting the V promise). The reason why one does not have to worry about, for instance, variation in retention is that we are evaluating changes in y given the optimal contract, and given that by definition J is maximized, any indirect derivative of controls over y will get to their respective first order conditions and thus have no direct impact on the comparative static.

One can get to the same conclusion by starting from time T, noticing that the function J is trivially increasing in y in the last period, and the stepping back. At $T-1$, given V, any higher y function can make greater profits with the same delivery of value V, given the contract's optimal promise, which is a fortiori true with human capital accumulation (the option value is greater, so the firm can decrease w as a response). $\square$

Proposition B.1. *The Pareto frontier $J(h, \tau, y, W_{y,\Omega}; a, \mu)$ is strictly increasing in the aggregate productivity shock a, while retention probabilities, $\widetilde{p}(h, \tau, y, W_{y,\Omega}; \Omega)$ decrease in aggregate productivity.*

Proof. For a generic period t, a firm matched to a worker in submarket $\{h, T-1, y, W_{y,\Omega}\}$ will face the following Pareto frontier

$$J_t(h, T-1, y, W_{y,\Omega}; a, \mu) = \sup_{w_i, \{W_{iy,\Omega'}\}} \Big(f(y, h; \Omega) - w \\ + \mathbb{E}_\Omega \left[\widetilde{p}(h', \tau+1, W_{y,\Omega'}; a', \mu')(f(y, h'; a') - w') \right] \Big)$$

The fact that period flows are increasing in a is immediate and follows from the properties of contracts with one-sided lack of commitment, as in Thomas and Worral (1988), Kocherlakota (1996) or Krueger and Uhlig (2006). At the same time, following the logic of **Lemma B.2**, the envelope condition on controls guarantees that one does not have to worry about the variation in optimal retention. This proves that J is increasing in a.

For the second part of the statement, notice that, in equilibrium,

$$\frac{\partial p(\theta)}{\partial a} = \underbrace{\frac{\partial p(\theta)}{\partial \theta}}_{>0} \cdot \underbrace{\frac{\partial \theta}{\partial J(\cdot)}}_{>0} \cdot \frac{\partial J(\cdot)}{\partial a}$$

where the sign of the second derivative on the right hand side comes from the free entry condition and the properties of vacancy filling probability function $q(\cdot)$. Given this, it has to be that $\frac{\partial p(\theta)}{\partial a}$ and $\frac{\partial J(\cdot)}{\partial a}$ have the same sign in equilibrium. This immediately implies that $\frac{\partial \tilde{p}}{\partial a} < 0$ according to the optimal contract. □

Corollary B.1. *There exists a productivity threshold $a^*(h, \tau, y, W_{y,\Omega})$ below which firms will not continue the contract.*

Proof. The proof follows immediately from **Proposition B.1** and the timing of the shock. Given the timing of the shock, exit is fully determined by the current productivity shock and incumbent firms know in advance whether they are willing to produce in the next period.

Therefore, as the Pareto frontier is strictly increasing in a, firms are willing to continue the contract if $\mathbb{E}_\Omega[J_{t+1}(h', \tau+1, y, W_{y,\Omega'}; a', \mu')|h, \tau, y, W_{y,\Omega}, a, \mu] \geq 0$, so that the threshold that determines exit is

$$a^*(h, \tau, y, W_{y,\Omega}) : \mathbb{E}_\Omega[J_{t+1}(h', \tau+1, y, W_{y,\Omega'}; a', \mu')|h, \tau, y, W_{y,\Omega}, a, \mu] = 0.$$

□

Corollary B.2. *The productivity threshold $a^*(h, \tau, y, W_{y,\Omega})$ below which firm y in match with worker (h, τ) and given promised utility $W_{y,\Omega}$ exits the market in the aggregate state Ω is increasing in y.*

Proof. Consider two firms characterized by y_1, y_2 with $y_1 < y_2$. Consider the threshold for firm y_1, $a_1^* = a^*(h, \tau, y_1, W_{y,\Omega})$. Firm y_1 makes 0 profits if state a_1^* materializes next period. Consider firm y_2 trying to mimic the current contract offered by y_1 to (h, τ). We know that J is increasing in y from **Lemma B.2**, which implies that the firm is making a profit at a_1^*. This completes the proof. □

Lemma B.3. *The Pareto frontier $J(h, \tau, y, W_{y,\Omega}; \Omega)$ is strictly concave in y.*

Proof. The proof follows from the fact that the flow component of the profit function is always a concave function in y.

More formally, start from the last period T. The concavity is trivially given by the concavity of f. Now moving backwards to the problem at $\tau = T - 1$, one can still consider the behavior of J given a promise $W_{y,\Omega}$. Again, given the option to search, the flow value is concave in y, retention probability is constant in $W_{y,\Omega}$, and the continuation value is a concave function. By induction, the statement holds for J at all $\tau \in [0, T]$.

□

Corollary B.3. *As $J_t(h, \tau, y, W_{y,\Omega}; \Omega)$ is concave, the tangent line at a generic $y_0 \in \mathcal{Y}$ is above the graph of $J_t(h, \tau, y, W_{y,\Omega}; \Omega)$ so that*

$$J_t(h, \tau, y_0, W_{y,\Omega}; \Omega) + \left.\frac{\partial J_t(h, \tau, y, W_{y,\Omega}; \Omega)}{\partial y}\right|_{y=y_0} (y - y_0) \geq J_t(h, \tau, y, W_{y,\Omega}; \Omega).$$

Proof. Dropping dependence on $(h, \tau, W_{y,\Omega}; \Omega)$, consider two values for firm quality $y_0 < y_1$ both in $\mathcal{Y}$. Then, as $J_t(\cdot)$ is concave in y, taking $\alpha \in [0, 1]$ the following inequalities are true:

$$\begin{aligned}
&J(\alpha y_0 + (1-\alpha)y_1) \geq \alpha J(y_0) + (1-\alpha)J(y_1) \\
\Rightarrow &J(\alpha y_0 + (1-\alpha)y_1) - J(y_0) \geq (1-\alpha)(J(y_1) - J(y_0)) \\
\Rightarrow &\frac{J(\alpha y_0 + (1-\alpha)y_1) - J(y_0)}{\alpha y_0 + (1-\alpha)y_1 - y_0} \geq \frac{J(y_1) - J(y_0)}{y_1 - y_0}.
\end{aligned}$$

where the third inequality comes from noting that $y_1 > y_0$ and $\alpha y_0 + (1-\alpha)y_1 - y_0 = (1-\alpha)(y_1 - y_0)$.

Taking the limit for $\alpha \to 1$, we have that the left hand side tends to $\left.\frac{\partial J_t(y)}{\partial y}\right|_{y=y_0}$ and hence

$$J(y_0) + \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_0} (y_1 - y_0) \geq J(y_1). \tag{B.1}$$

Note that if $y_0 > y_1$ then $\frac{J(\alpha y_0 + (1-\alpha)y_1) - J(y_0)}{\alpha y_0 + (1-\alpha)y_1 - y_0} \leq \frac{J(y_1) - J(y_0)}{y_1 - y_0}$ but multiplying again the left hand side and the right hand side for $(y_1 - y_0) < 0$ still delivers (B.1). □

Proposition B.2. *Define the mapping between promised values and firm installed capital by the function $f_v : \mathcal{Y} \to \mathcal{V}$. Then f_v is an injective function for each couple of worker characteristics (h, τ).*

Proof. Note: throughout the proof we drop the dependence of the functions to the state (h, τ, Ω) to ease readability.

If the function f_v is an injective function then it defines a one-to-one mapping between $\mathcal{Y}$ and $\mathcal{V}$ so that for $(y_1, y_2) \in \mathcal{Y}$, and $f_v(y_1) = W_1$ and $f_v(y_2) = W_2$, $(W_1, W_2) \in \mathcal{V}$, $f_v(y_1) = f_v(y_2) \Rightarrow y_1 = y_2$.[32] We proceed by contradiction. To begin, assume that $f_v(y_1) = f_v(y_2)$ and $y_1 \neq y_2$.

As the optimal contract is a concave function in firm quality, we know that the tangents at each point are above the graph of the function. Thus, we can define the tangents at the two points y_1, y_2 as

$$T_1(y) \equiv J(y_1) + \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_1} (y - y_1) \quad \text{and} \quad T_2(y) \equiv J(y_2) + \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_2} (y - y_2).$$

[32] As the contrapositive of Definition 2.2 in Rudin (1976), that defines a one-to-one mapping for $(x_1, x_2) \in A$ as $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.

Without loss of generality, consider the case in which $y_2 > y_1$. Knowing that $T_i(y) \geq J(y)$ for $i = 1, 2$ due to the concavity of $J(\cdot)$, we can define the following inequalities:

$$T_1(y_2) - J(y_2) \geq 0 \quad \text{and} \quad T_2(y_1) - J(y_1) \geq 0.$$

Using the definitions for the tangents at y_1 and y_2 they imply that

$$\frac{J(y_2) - J(y_1)}{y_2 - y_1} \leq \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_1} \quad \text{and} \quad \frac{J(y_2) - J(y_1)}{y_2 - y_1} \geq \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_2},$$

hence combining the inequalities we get that

$$\left.\frac{\partial J(y)}{\partial y}\right|_{y=y_2} \leq \frac{J(y_2) - J(y_1)}{y_2 - y_1} \leq \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_1}. \tag{B.2}$$

However, the free-entry condition in vacancy posting implies that in the submarket (h, τ, W) both firms must be respecting $c(y_i) = q(\theta)\beta J(y_i)$ for $i = 1, 2$. As $c(y_i)$ is a linear function of firm quality $\frac{\partial c(y_i)}{\partial y_i} = c$ for $i = 1, 2$ and therefore from the free-entry condition:

$$c = q(\theta)\beta \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_i}$$

which is a contradiction of the slopes of the two tangents being decreasing as shown in **Equation** (B.2). Note that if $c(y)$ is convex and twice differentiable, then the derivatives of $c(y)$ are increasing in y while the derivatives of $J(\cdot)$ are decreasing leading again to a contradiction. The proof for the case in which $y_1 > y_2$ follows the same arguments and leads to a similar contradiction on the implied slopes of the optimal contract and those implied by the free entry condition. □

Lemma B.4. *Given a state $(y, h, \tau, W_{y,\Omega})$ the optimal contract implies that*

$$-\frac{\partial J_t(h, \tau, y, W_{y,\Omega}; \Omega)}{\partial W_{y,\Omega}} = \frac{1}{u'(w)}$$

so that promised utilities and wages move in the same direction.

Proof. The proof follows directly from the envelope theorem and the concavity of the utility function $u(\cdot)$, as discussed in the proof of Proposition B.4. □

Corollary B.4. *The Pareto frontier $J(h, \tau, y, W_{y,\Omega}; \Omega)$ is decreasing in promised utilities $W_{y,\Omega}$.*

Proof. The envelope condition in **Lemma B.4** and note that $u'() \geq 0$. □

Proposition B.3. *Assume $q(\theta(h, \tau, W_{y,\Omega}; \Omega))$ is not too convex.*[33] *Then utility*

[33]More precisely, we need q to be such that $q_{WW} < 2\frac{q_W J_W + q J_{WW}}{J}$

promises are unique and increasing in y, $\frac{\partial W}{\partial y} > 0$.

Proof. Uniqueness follows directly from the concavity of the composite function.

The increasing property follows from the maximization of the entrepreneur in the free entry condition **Equation 12**.

Assuming the same (h, τ, y), the entrepreneur has to choose which is the optimal value $W_{y,\Omega}$ to deliver in the contract. We know it is unique by assuming concavity of the composite function (which eventually amounts to assuming that the functional form of $q(\theta(W)))$ is not too convex in W.

For the rest of the proof we consider as given the dependence of the functions on (h, τ) and consider directly the composite function $q(\theta(W))$ as $q(W)$. The optimization involves a trade-off which respects the following first order condition:

$$q_W J(y, W) + q(W) J_W = 0 \tag{B.3}$$

For this to be be solved by a unique sup, the second order condition must be negative:

$$q_{WW} J + 2q_W J_W + q J_{WW} < 0 \tag{B.4}$$

where, as mentioned above, the only element which might lead to a violation is q_{WW} in case it is too convex ($J_{WW} < 0$) by B.1. Notice this hypothesis amounts to assuming that $q(\theta(h, \tau, W_{y,\Omega}; \Omega))J(h, \tau, y, W_{y,\Omega}; \Omega)$ is concave.

By the implicit function theorem, the derivative of **Equation B.3** is:

$$(q_{WW} J + 2q_W J_W + q J_{WW}) W_y + q_W J_y + q J_{Wy} = 0 \tag{B.5}$$

The first term in parenthesis is negative, as second order condition. The second term is positive, given **Lemma B.2** and the fact that q_W is positive. The third term is 0, as the partial derivative of J in y does not contain V (which is the reason why **Lemma B.2** trivially holds). This means that, in order for the equality to be respected, $W_y > 0$.

□

Proposition B.4. *For each state in which the firm is willing to continue the contract, the optimal contract delivers a wage path that follows firms profits according to the* *wage Euler equation:*

$$\frac{\partial \widetilde{p}(\Theta)}{\partial W_{iy,\Omega'}} \frac{J_{t+1}(\Theta)}{\widetilde{p}(\Theta)} = \frac{1}{u'(w_{i,\Omega'})} - \frac{1}{u'(w_i)},$$

with $\Theta \equiv (\phi(h,y), \tau+1, W_{iy,\Omega'}; \Omega')$ *being the definition of the relevant state.*

Proof. Consider the firm problem in **Equation** (8), restated here for convenience

$$\begin{aligned} J_t(h,\tau,y,W_{y,\Omega};\Omega) = & \sup_{\pi_i, w_i, \{W_{i,\Omega'}\}} \sum_{i=1,2} \pi_i \Big(f(y,h;\Omega) - w_i \\ & + \mathbb{E}_\Omega \left[\widetilde{p}(h',\tau+1,W_{iy,\Omega'};\Omega') J_{t+1}(h',\tau+1,y,W_{i,\Omega'};\Omega') \right] \Big) \end{aligned}$$

$$\begin{aligned} s.t.\ [\lambda]\ W_{y,\Omega} = & \sum_{i=1,2} \pi_i \left(u(w_i) + \mathbb{E}_\Omega \widetilde{r}(h',\tau+1,W_{iy,\Omega'};\Omega') \right), \\ & \sum_{i=1,2} \pi_i = 1, \quad h' = \phi(h,y). \end{aligned}$$

For $i = 1,2$, the first order conditions with respect to the wage and the promised utilities are:

$$[w_i]:\ \lambda = \frac{1}{u'(w_i)} \tag{B.6}$$

$$[W_{iy,\Omega'}]:\ \pi_i \frac{\partial \widetilde{p}()}{\partial W_{iy,\Omega'}} J_{t+1}() + \widetilde{p}() \frac{\partial J_{t+1}()}{\partial W_{iy,\Omega'}} + \lambda \frac{\partial \widetilde{r}()}{\partial W_{iy,\Omega'}} = 0. \tag{B.7}$$

Note that by definition,

$$\widetilde{r}(h,\tau,V_{y,\Omega};\Omega) \equiv \lambda U(h,\tau;\Omega) + (1-\lambda)\Big[W_{y,\Omega} + \lambda_e \max\{0, R(h,\tau,V_{y,\Omega};\Omega)\} \Big]$$

therefore we can use the envelope theorem as in Benveniste and Scheinkman (1979), Theorem 1 and the definition in **Equation** (5) to derive an expression for the derivative of the employment value in $t+1$ as the period ahead of the following:

$$\frac{\partial \widetilde{r}(h,\tau,W_{y,\Omega};\Omega)}{\partial W_{y,\Omega}} = \widetilde{p}(h,\tau,W_{y,\Omega};\Omega).$$

Similarly, using the envelope condition on the firm problem and the first order condition for the wage, we can establish that

$$\frac{\partial J_t(h,\tau,y,W_{y,\Omega};\Omega))}{\partial W_{y,\Omega}} = -\lambda \ \therefore\ \frac{\partial J_t(h,\tau,y,W_{y,\Omega};\Omega))}{\partial W_{y,\Omega}} = -\frac{1}{u'(w_i)}. \tag{B.8}$$

Moving these two expressions one period ahead, substituting them in (B.7), taking

$\pi_i > 0$ and rearranging we have that:

$$\frac{\partial \widetilde{p}(\Theta)}{\partial W_{y,\Omega'}} \frac{J_{t+1}(\Theta)}{\widetilde{p}(\Theta)} = \frac{1}{u'(w_{\Omega'})} - \frac{1}{u'(w)},$$

with $\Theta \equiv (\phi(h,y), \tau+1, W_{y,\Omega'}; \Omega')$ and where $w_{\Omega'}$ is the wage next period in state Ω'. □

C Discussion

In this section we briefly discuss the properties of the equilibrium of the model economy developed in the previous sections. All propositions and corresponding proofs are reported in **Appendix A** and **B**.

C.1 Workers optimal behavior

In the following proposition we summarize the main results regarding the behavior of the workers and their objective functions.

Proposition C.1. *Given the worker search problem, the following properties hold:*

(i) The returns to search, $p(\theta(h,\tau,v_{y,\Omega};\Omega))\big[v_{y,\Omega} - V\big]$, are strictly concave with respect to promised utility, $v_{y,\Omega}$.

(ii) The optimal search strategy

$$v^*(h,\tau,V;\Omega) \in \arg\max_{v_y} \big\{p(\theta(h,\tau,v_{y,\Omega};\Omega))\big[v_{y,\Omega} - V\big]\big\}$$

is unique and weakly increasing in V.

(iii) For all promised utilities, the search gain $R(h,\tau,V;\Omega)$ is positive, weakly decreasing in V.

(iv) The survival probability of the match, given the optimal choice of the worker, is increasing in the value of promised utilities, so $\widetilde{p}_t(h,\tau,W_{y,\Omega};\Omega)$ is increasing in $W_{y,\Omega}$.

Proof. See **Proposition A.1** in **Appendix A**. □

The first statement implies that the marginal returns of searching towards better firms are decreasing. The intuition is that as workers search for work at firms granting better values, their job-finding probability decreases as better employment prospects are also subject to higher competition.

As a consequence of the strict concavity established in the first statement, workers' optimal search strategy is unique. The search strategy is also (weakly) increasing in the value of lifetime utilities granted by the current contract, which is the outside option for the worker.

The third statement follows from the fact that marginal returns to search are decreasing and the set of feasible utility promises is compact. The intuition is that employees at firms with higher utility promises have a relatively fewer chances of improving their position. Given a high outside option, the utility gain from moving is relatively lower, whereas the probability of matching with any firm does not depend on the *current* utility promise.

The fourth statement finally follows from considering the implication of the previous ones. Given that the optimal search strategy is increasing in V workers probability of leaving the firm at any time ends up depending negatively on V. This guarantees a longer expected duration of the match at, and generates retention probabilities that are increasing in promised utilities.

As human capital accumulation is tightly linked to the quality of the employer, workers that are able to start their working careers in good times have a greater chance of finding themselves on an higher path of human capital growth. As worker careers are limited and human capital accumulation follows a slow-moving process, business cycle effects on human capital quality fade only slowly and the quality of initial matches bears a long-standing effect on workers' careers.

C.2 Characteristics of the optimal contract

The optimization in the contracting problem balances a trade-off between insurance provision and profit maximization for firms. The contract implicitly takes into account workers' search incentives and their inability to commit to stay. The following proposition characterizes workers' incentives along the business cycle from the firms' standpoint.

Proposition C.2. *The Pareto frontier* $J(h, \tau, y, W_{y,\Omega}; a, \mu)$ *is increasing in the aggregate productivity shock* a*, while retention probabilities,* $\widetilde{p}(h, \tau, W_{y,\Omega}; a, \mu)$ *decrease in aggregate productivity.*

Proof. See **Proposition B.1** in **Appendix B**. □

The intuition behind this proposition relies on the observation that higher productivity realization are associated not only with better outcomes on impact but also to better future prospects, given that the productivity process is an increasing Markov chain.

A key property of the model is that it allows to characterize the workers' optimal behaviour along the business cycle. The following proposition summarizes how the search strategy changes depending on the aggregate productivity realization.

Corollary C.1. *The optimal search strategy of the workers is increasing in aggregate productivity.*

Proof. The claim follows directly from the fact that retention probabilities at the Pareto frontier, $\widetilde{p}$, are decreasing in a as discussed in **Proposition C.2**. □

Proposition B.1 and Corollary C.1 have an important implication regarding firms' vacancy posting and workers' search decisions. The fact that at the posting stage profits J are increasing in aggregate productivity implies that more entry will take place in good times, and ceteris paribus more entrepreneurs will open up vacancies across the whole firms' distribution.[34] The resulting higher tightness impacts workers' optimal search behaviour as the job finding probability increases in all submarkets. As a consequence, workers respond optimally to the productivity increase searching in submarkets that guarantee higher lifetime utility promises.

Firms utility promises depend on the structure of the optimal contract. The contract provides insurance to workers through wage paths that are downward rigid, and at the same time allows firms to profit as wages only partially adjust to productivity realizations.

The following propositions provide a clear picture of the growth path prescribed by the optimal contract for a continuing firm. First, let us define the productivity threshold that determines whether a worker-firm match does not survive.

Corollary C.2. *There exists a productivity threshold $a^*(h, \tau, y, W_{y,\Omega})$ below which firms will not continue the contract.*

The intuition of why this has to be the case is linked to the fact that the Pareto frontier is strictly increasing in a and decreasing in the level of promised utilities to the worker. Hence once the aggregate state realizes a firm is able to perfectly predict whether next period it will exit the market or stay in (given the timing, the decision is based on expected profits, and is thus *not* state-contingent to next period's productivity). The choice is taken *before* new realizations of productivity, so it is possible that a firm makes negative profits for at most one period.

Proposition C.3. *For each state in which the firm is willing to continue the contract, the optimal contract delivers a wage path that follows firms profits according to the*

[34]In our model a better firm is a more productive firm. We do not specifically model the determinant of quality heterogeneity but we take the existence of profound differences in firm quality as a fact.

wage Euler equation:

$$\frac{\partial \widetilde{p}(\Theta)}{\partial W_{iy,\Omega'}} \frac{J_{t+1}(\Theta)}{\widetilde{p}(\Theta)} = \frac{1}{u'(w_{i,\Omega'})} - \frac{1}{u'(w_i)} \tag{C.1}$$

with $\Theta \equiv (\phi(h,y), \tau+1, W_{iy,\Omega'}; \Omega')$ *being the definition of the relevant state and* $w_{i,\Omega'}$ *is the wage paid in the future state.*

Proof. See **Proposition B.4** in **Appendix B**. □

The optimal contract links the wage growth to the realization of firms profits. The right hand side of **Equation C.1** shows that, in providing insurance to the worker, the firm links wage growth to profits and to the incentive to maximize retention, incorporated in $\frac{\partial \log \widetilde{p}}{\partial W_y}$, the semi-elasticity of the retention probability to the utility offer. As the production stage takes place *after* exit choices are taken by the incumbent firms, the wage growth related to the continuation value of the contract is bound to be (weakly) positive, hence workers enjoy an non-decreasing wage profile under the optimal contract.[35]

A feature that the optimal contract derived in our model shares with the literature on long-term contracts with lack of commitment on the worker side is thus the backloading of wages.[36] Workers in our model make search decisions that affect the survival probability of the match. They do not however appropriate the full future value of the current match while making these search decisions (unless the firm makes zero profits). This makes it optimal for the firm to front-load profits and back-load wages. The reason is that the firm provides insurance and income smoothing to the worker, but given its risk neutrality it prefers to front-load its profits while providing an increasing compensation path to maximize retention. The contract thus optimally balances the consumption smoothing motives (i.e. the insurance provision of the contract) with the commitment problem of the worker.

Special case with log-utility. The wage Euler equation discussed in **Proposition B.4** can be simplified to a more intuitive interpretation in the log-utility case. In case of log-utility, in fact, $u'(w_{i,\Omega}) = \frac{1}{w_{i,\Omega}}$. Multiplying and dividing by wage levels and rearranging, we can express the elasticity of retention

[35] As the exit decision takes place by considering *expected* profits next period, a firm might continue operating low but positive expected profits and end up, at most for a period, to have a negative continuation value. This would imply that wage growth *can* be negative before a firm's closure, which is actually a common finding in empirical studies (firstly observed in Ashenfelter (1978)).

[36] See for instance, Thomas and Worral (1988), Tsuyuhara (2016) and Balke and Lamadon (2020).

probability to offered utility as

$$\varepsilon_{\widetilde{p},W_y} = \underbrace{\frac{(w_{i,\Omega'} - w_i)}{w_i}}_{\text{Wage growth}} \underbrace{\frac{w_i}{J_{t+1}(\Theta)}}_{\substack{\text{Ratio of wage}\\ \text{to match value}}} . \tag{C.2}$$

with $\varepsilon_{\widetilde{p},W_y} \equiv \frac{\partial \widetilde{p}(\Theta)}{\partial W_{iy,\Omega'} \widetilde{p}(\Theta)}$.

The interpretation of this result is of interest to analyses that relate labor market dynamism to wage dynamics, like Engbom (2020). This is because $\varepsilon_{\widetilde{p},W_y}$, being a function of the structural parameters of the matching technology, γ, search frictions λ_e, and with measures of labor market tightness θ, provides us with a good proxy of labor market fluidity. The right hand side of (C.2), is composed entirely of observable quantities, as the ratio of wages to match value can be proxied with value added. The quantity can then be used to compare the dynamism of different regional, or national, labor markets.
The next proposition, instead, confirms our initial conjecture that in equilibrium firm qualities and utility promises are related to a one-to-one mapping.

Proposition C.4. *The mapping defined by the function $f_v : \mathcal{Y} \to \mathcal{V}$ is an injective function for each worker characteristic (h, τ).*

Proof. See **Proposition B.2** in **Appendix B**. □

The proof is based on the fact that the Pareto frontier J is concave, the vacancy filling probability q is weakly positive and vacancy costs are both weakly positive and increasing in y. As shown in **Appendix B** these features are enough to guarantee that only one kind of firm y, given workers' characteristics h, τ, can optimally offer a given lifetime utility promise W. **Appendix B** also provides further proofs and very general conditions under which the injective mapping also guarantees monotonicity between y and W. We thus obtain a unique monotonic solution in which higher quality firms offer higher lifetime utility promises to workers.

Finally, we provide the alternative recursive formulation for the contracting problem described in the paper. The saddle-point functional equation that can be alternatively used to define the recursive contract in **Equation** (8) is expressed in the following proposition.

Proposition C.5. *The solution to the contracting problem in **Equation** (8) is the same as the solution to the following saddle-point functional equation:*

$$\begin{aligned}\mathcal{P}_t(h_t, \tau_t, y_t, a_t, \gamma_t) =& \underset{\gamma_t}{inf}\, \underset{w_t}{sup} (f(a_t, y_t, h_t) - w_t) + \mu_t^1 W_{y,t} - \gamma_t^1 (W_{y,t} - u(w_t)) + \\ & \beta \mathbb{E}_t (\lambda U_{t+1} + (1-\lambda)\lambda_e p_{t+1} v_{t+1}^*) + \beta \mathbb{E}_t \widetilde{p}_{t+1} \mathcal{P}_{t+1}(h_{t+1}, \tau_{t+1}, y_{t+1}, a_{t+1}, \gamma_{t+1})\end{aligned}$$

with $\mu_t = \gamma_{t_1}$ *for some starting* γ_0.

Proof. See **Appendix D** for the details of the derivation of the SPFE following Marcet and Marimon (2019). □

D Derivation of recursive contract SPFE

Solving the optimal contract and the overall model given the recursive structure obtained by following the promised utility method of Spear and Srivastava (1987) is computationally infeasible. This is due to the fact that the optimal contract requires to define a valid recursive domain and codomain of promised values that respects all the future forward looking constraints. Known solution methods for these kinds of models (Abreu, Pearce and Stacchetti, 1990), although robust, easily become computationally unmanageable as the number of states of the model increases. We thus follow Marcet and Marimon (2019) in deriving a recursive expression for the optimal contract in which the Lagrange multiplier for the promise keeping constraint **Equation B.8** is added as a co-state of the model, and allows us to circumvent the problem of searching for valid promised values domains altogether.

The reason why the recursive contracts method in Marcet and Marimon (2019) simplifies our problem is simple. As shown in **Equation B.8**, wage growth and levels in any next period and at every node are determined by the state-contingent multiplier on tomorrow's promise keeping constraints. This considerably reduces the complexity of the problem, as by definition Lagrange multipliers are defined over $\mathbb{R}^+$.

We follow Marcet and Marimon (2019) (hereby MM) and their terminology to define how a recursive saddle point functional equation (SPFE) can be obtained from the sequential formulation of the problem. For the present exposition of the constructive method to obtain the SPFE, for simplicity and without loos of generality, we ignore the randomization of the contract over the lotteries and the limited liability constraint. The latter choice, in particular, does not create any problem in terms of thinking about of developing the sequential problem over time: our choice of timing of exit decision is such as that exiting firms know form the start of their period whether the productivity level is below the critical one $a^*_{h,\tau,y,W}$ for the match (h, τ, y, W_y), and thus whether they will exit or not. The lack of uncertainty and optimization over the next periods makes the problem of these firms, at some low states, equivalent to the problem of a firm with a lower maximum length (which is T, the retirement age, in general). At an exiting state t the firm knows *with certainty* that any $J_j = 0$ for $j > t$, match with a worker of age T.

Consider the problem

$$J_t(h_t, \tau_t, y_t, W_{y_t}, a_t) = \sup_{w_t, \{W_{y,s^{t+1}}\}} \Big(f(a_t, y_t, h_t) - w_t$$

$$+ \mathbb{E}_{s^t} \left[\widetilde{p}(h_{t+1}, \tau_{t+1}, W_{y,s^{t+1}}, a_{s^{t+1}})(J_{t+1}(h_{t+1}, \tau_{t+1}, y_t + 1, W_{y,s^{t+1}}, a_{s^{t+1}})\right] \Big) \tag{D.1}$$

$$s.t. \; W_t = u(w_t) + \beta \mathbb{E}_{s^t} \Big(\lambda U_t(h_{t+1}, \tau_{t+1}, a_{t+1}) +$$

$$(1 - \lambda)(\lambda_e p_{t+1}(h_{t+1}, \tau_{t+1}, W_{y,s^{t+1}}, a_{s^{t+1}}) v^*(h_{t+1}, \tau_{t+1}, W_{y,s^{t+1}}, a_{s^{t+1}})$$

$$+ (1 - \lambda_e p_{t+1}(h_{t+1}, \tau_{t+1}, W_{y,s^{t+1}}, a_{s^{t+1}})) W_{y,s^{t+1}}) \Big) \tag{D.2}$$

We define as endogenous states $\mathbf{x}_t = [h_t, \tau_t, y_t, W_{y,t}]$, controls $\mathbf{c}_t = [w_t, W_{y,s^{t+1}}]$ $\forall t, s^{t+1}$, whereas the only exogenous state is a_t. The endogenous states follow the law of motion

$$\mathbf{x}_{t+1} = \begin{bmatrix} h_{t+1} \\ \tau_{t+1} \\ y_{t+1} \\ W_{y,s^{t+1}} \end{bmatrix} = l(\mathbf{x}_t, \mathbf{c}_t, a_{s^{t+1}}) = \begin{bmatrix} \phi(h_t, y_t) \\ \tau_t + 1 \\ y_t \\ W_{y,s^{t+1}}) \end{bmatrix} \tag{D.3}$$

In the subsequent notation, where appropriate, we omit listing all states on which elements in the equation, and subsume their dependence under just listing the time t. J can be rewritten, by developing forward the recursion until time T, at which the match surely dissolves, as

$$J_t(\{h_t, \tau_t, y_t, W_{y,t}, a_t\}_{t=t_0}^{T-t_0}) = \mathbb{E}_{t_0} \sum_{t=t_0}^{T} \beta^{t-t_0} \prod_{i=0}^{t-t_0} \widetilde{p}_{t_o+i} \Big(f(a_t, y_t, h_t) - w_t \Big) \tag{D.4}$$

where $\widetilde{p}_{t_0} = 1$. Notice that the forward-looking constraint in **Equation D.2** is state contingent and an instance of it applies at *every* node of any possible history s^t $\forall t$ given the prevailing W_y promised at that node. The equilibrium is an instance of subgame perfect Nash equilibrium in which an agent chooses its strategies while anticipating the best response of the following agent, as common in dynamic games with a leader-follower component introduced by Von Stackelberg (1934). The structure of the problem and the solution also shares some commonality with Ramsey optimal policy problems in which a policy maker (in this case the firm) optimizes the utility of all agents according to some weights and taking into account their optimal behavior. [37]

[37] In the terminology of MM, we treat constraints coming from **Equation D.2** as a set of one period ahead forward looking constraint, which makes the analysis of our case akin to their case where one have $j = 1$ forward looking constraints, and $N_1 = 0$. The difference with their problems, however, is that our problem features finite time, and thus each one period ahead forward looking constraint

We can redefine the problem:

$$V_{t_0}(\mathbf{x}_t, a_t) = \underset{\{w_{s^t}, W_{y,s^t}\}}{sup} \mathbb{E}_{t_0} \sum_{t=t_0}^{T} \beta^{t-t_0} \prod_{i=0}^{t-t_0} \widetilde{p}_{t_o+i}\Big(f(a_t, y_t, h_t) - w_t\Big) \tag{D.5}$$

$$s.t.\ [j=0]: \sum_{t=t_0}^{T} \beta^{t-t_0} \prod_{i=0}^{t-t_0} \widetilde{p}_{t_o+i}\Big(f(a_t, y_t, h_t) - w_t\Big) - R \geq 0 \tag{D.6}$$

$$[j=1, s^t]:\ W_{y,s^t} - u(w_{s^t}) - \beta \mathbb{E}_{s^t}\Big(\lambda U_{s^{t+1}} + (1-\lambda)(\lambda_e p_{s^{t+1}} v^*_{s^{t+1}}) + \widetilde{p}_{s^{t+1}} W_{y,s^{t+1}})\Big) \geq 0 \tag{D.7}$$

where the constraint D.13 is a slack participation constraint for a sufficiently small R, so that the principal (the firm) is willing to enter the contract in the first place.

In the terminology of MM we can label

$$h_0^0(\mathbf{x}_t, \mathbf{c}_t, a_t) = f(a_t, y_t, h_t) - w_t \tag{D.8}$$

$$h_1^0(\mathbf{x}_t, \mathbf{c}_t, a_t) = f(a_t, y_t, h_t) - w_t - R \tag{D.9}$$

$$h_0^1(\mathbf{x}_t, \mathbf{c}_t, a_t) = W_{y,t} \tag{D.10}$$

$$h_1^1(\mathbf{x}_t, \mathbf{c}_t, a_t) = W_{y,t} - u(w_t) + \beta \mathbb{E}_t(\lambda U_{t+1} + (1-\lambda)\lambda_e p_{t+1} v^*_{y,t+1}) \tag{D.11}$$

and define the Pareto problem ($\mathbf{PP}_\mu$)

$$\mathbf{PP}_\mu : V_{\mu,t_0}(\mathbf{x}_t, a_t) = \underset{\{w_{s^t}, W_{y,s^t}\}}{sup} \mathbb{E}_{t_0} \sum_{t=t_0}^{T} \beta^{t-t_0} \prod_{i=0}^{t-t_0} \widetilde{p}_{t_0+i}\mu^0\Big(f(a_t, y_t, h_t) - w_t\Big) + \mu^1 W_{y,t_0} \tag{D.12}$$

$$s.t.\ [j=0;\ \gamma^0]: \sum_{t=t_0}^{T} \beta^{t-t_0} \prod_{i=0}^{t-t_0} \widetilde{p}_{t_o+i}\Big(f(a_t, y_t, h_t) - w_t\Big) - R \geq 0 \tag{D.13}$$

$$[j=1, s^t;\ \gamma^1_{s^t}]:\ W_{y,s^t} - u(w_{s^t}) - \beta \mathbb{E}_{s^t}\Big(\lambda U_{s^{t+1}} + (1-\lambda)(\lambda_e p_{s^{t+1}} v^*_{s^{t+1}}) + \widetilde{p}_{s^{t+1}} W_{y,s^{t+1}})\Big) \geq 0 \tag{D.14}$$

Still following the notation from Marcet and Marimon (2019), we can define the Saddle Point Problem ($\mathbf{SPP}_\mu$) as:

technically applies to a *different* function j_t (indexed by t).

$$\mathbf{SPP}_\mu : SV_{\mu,t_0}(\mathbf{x}_{t_0}, a_{t_0}) = \underset{\{\gamma\in\mathbb{R}^l_+\}}{inf} \underset{\{w_st, W_{y,s}t_0\}}{sup} \mu^0\Big(f(a_{t_0}, y_{t_0}, h_{t_0}) - w_{t_0}\Big) + \mu^1 W_{y,t_0} +$$
$$+ \beta\mathbb{E}_t\left(\phi(\mu,\gamma)\sum_{i=0}^{T-t_0}\left[\beta^{t_0+i}\prod_{i=0}^{T-t_0-1}\widetilde{p}_{t_0+1+i}\left(f(a_{t_0+i}, y_{t_0+i}, h_{t_0+i}) - w_{t_0+i}\right) + W_{y,t_0+i}\right]\right) +$$
$$+ \gamma^1\left(u(w_{t_0} + \beta\mathbb{E}_{t_0}\left(\lambda U_{t_0+1} + (1-\lambda)\lambda_e p_{t_0+1} v^*_{y,t_0+1}\right)\right) +$$
$$+ \gamma^0\left(f(a_{t_0}, y_{t_0}, h_{t_0}) - w_{t_0} - R\right) \tag{D.15}$$

The problem can be restated as a saddle-point problem over a Lagrangian equation

$$\underset{\gamma_t}{inf} \underset{\{w_st, W_{y,s}t\}}{sup} \mu^0\Big(f(a_{t_0}, y_{t_0}, h_{t_0}) - w_{t_0}\Big) + \mu^1 W_{y,t_0} +$$
$$\gamma^0\Big(\big(f(a_{t_0}, y_{t_0}, h_{t_0}) - w_{t_0}\Big) - R) +$$
$$\gamma^1_{t_0}\Big(-W_{y,t_0} + u(w_{t_0}) + \beta\mathbb{E}_{t_0}(\lambda U_{t_0+1} + (1-\lambda)(\lambda_e p_{t_0+1} v^*_{t_0+1} + \widetilde{p}_{t_0+1} W_{y,t_0+1})\Big) +$$
$$\beta\mathbb{E}_{t_0}\left[(\mu_0 + \gamma_0)\sum_{t=t_0+1}^{T}\beta^{t-t_0-1}\prod_{i=0}^{T-t_0-1}\widetilde{p}_{t_0+1+i}\Big(f(a_t, y_t, h_t) - w_t\Big) + \right.$$
$$\sum_{t=t_0+1}^{T}\mathbb{E}_t\beta^{t-t_0-1}\prod_{i=0}^{t-t_0-1}\widetilde{p}_{t_0+1+i}\gamma^1_t\Big(-W_{y,t} + u(w_t) +$$
$$\left.\beta(\lambda U_{t+1} + (1-\lambda)(\lambda_e p_{t+1} v^*_{t+1} + \widetilde{p}_{t+1} W_{y,t+1})\Big)\right] \tag{D.16}$$

which, thanks to some algebra and the law of iterated expectations becomes

$$\underset{\gamma_t}{inf} \underset{\{w_st, W_{y,s}t\}}{sup} -\gamma^0 R + \mathbb{E}_{t_0}\sum_{t=t_0}^{T}\beta^{t-t_0}\prod_{i=0}^{t-t_0}\widetilde{p}_{t_0+i}\left[\Big(\mu^0_t + \gamma^0_t\Big)\Big(f(a_t, y_t, h_t) - w_t\Big) + \mu^1_t W_{y,t} - \right.$$
$$\left.\gamma^1_t\Big(W_{y,t} - u(w_t) - \beta(\lambda U_{t+1} - (1-\lambda)\lambda_e p_{t+1} v^*_{t+1}\Big)\right] \tag{D.17}$$

where $\mu^0_t = \mu^0 = 1$, $\gamma^0_t = \gamma_0 = 0$, $\mu^1_t = \gamma^1_{t-1}$ for some starting $\gamma^1_{t_0-1}$.

The problem can now be written in recursive form. Define

$$\mathcal{P}_t(h_t, \tau_t, y_t, a_t, \gamma_t) = \underset{W_{y,t}}{sup}\, J_t(h_t, \tau_t, y_t, W_{y,t}, a_t) + \mu^1_t W_{y,t} \tag{D.18}$$

Given **Equation D.17** the SPFE of the problem can be written as

$$\mathcal{P}_t(h_t,\tau_t,y_t,a_t,\gamma_t) = \underset{\gamma_t}{inf}\, \underset{w_t}{sup}(f(a_t,y_t,h_t)-w_t)+\mu_t^1 W_{y,t}-\gamma_t(W_{y,t}-u(w_t))+ \\ \beta\mathbb{E}_t(\lambda U_{t+1}+(1-\lambda)\lambda_e p_{t+1}v^*_{t+1})+\beta\mathbb{E}_t\widetilde{p}_{t+1}\mathcal{P}_{t+1}(h_{t+1},\tau_{t+1},y_{t+1},a_{t+1},\gamma_{t+1}) \tag{D.19}$$

One can easily verify that the solution of this equation is the same we found in the maximization of **Equation 8** in the main text. Take the first order conditions and compute the envelope condition:

$$[FOC\ w_t]:\ -1+\gamma_t u'(w_t)=0 \tag{D.20}$$

$$[ENV\ W_{y,t}]:\ \frac{\partial \mathcal{P}_t}{\partial W_{y,t}}=\mu_t^1-\gamma_t \tag{D.21}$$

$$[FOC\ W_{y,t+1}]:\ -\widetilde{p}_{t+1}W_{y,t+1}\gamma_t+\frac{\partial \widetilde{p}_{t+1}}{\partial W_{y,t+1}}\mathcal{P}_{t+1}+\widetilde{p}_{t+1}\frac{\partial \mathcal{P}_{t+1}}{\partial W_{y,t+1}}=0 \tag{D.22}$$

where **EquationD.22** is obtained by adding and subtracting from **Equation D.19** $\beta\gamma_t\widetilde{p}_{t+1}W_{y,t+1}$. The reader should also keep in mind that the condition in **Equation D.22** is actually state contingent and applied to *all* future states next period, with a different set of co-states $\gamma_{s^{t+1}}$ for each realization of a_{t+1}.

Some rearranging of the **Equation D.22** leads to the following result

$$\frac{\partial \log \widetilde{p}_{t+1}}{\partial W_{y,t+1}}\Big(\mathcal{P}_{t+1}-\gamma_t W_{y,t+1}\Big)=\gamma_{t+1}-\mu_{t+1}^1 \tag{D.23}$$

which, given the law of motion of the co-states and the definition in **Equation D.18** can be re-written as:

$$\frac{\partial \log \widetilde{p}_{t+1}}{\partial W_{y,t+1}}J_{t+1}=\frac{1}{u'(w_{t+1})}-\frac{1}{u'(w_t)} \tag{D.24}$$

which is exactly **Equation C.1**, namely the Euler equation that governs the behavior of wage setting and disciplines the provision of insurance within the contract.

E Existence of a Block Recursive Equilibrium

In order to show that a Block Recursive Equilibrium (BRE) exists in our model we need to show that the equilibrium contracts, the workers' and the entrepreneurs value and policy functions do not depend on the distribution of employed and unemployed workers.

Most of the results are tightly linked to our search protocol, directed versus random search, and our contracting structure whereby workers have finite lives and therefore contracts end in finite time. The intuition for why directed search is paramount for the existence of a BRE is linked to the fact that with directed search, workers that are matched with a particular job accept that job with certainty as they are actively looking for it in the labor market. This certainty of acceptance makes the probability of filling a vacancy, and consequently the return of opening it in a particular submarket, independent from the type of worker a firm meets. This implies that the only element of the aggregate state that matters for a firm when making an hiring decision is the state of aggregate productivity but not the distribution of worker types (e.g. employed vs unemployed). Without loss of generality in the proof we omit the idiosyncratic shock to human capital for the workers, as it does not have any bearing on the proposition to be proved.

Proposition E.1. *A block recursive equilibrium as defined in Definition 2.3 exists.*

Proof. We follow the approach in Menzio, Telyukova and Visschers (2016), Herkenhoff, Phillips and Cohen-Cole (2019) and prove the existence of a BRE using backward induction.

Consider the lifetime values of an unemployed and an employed worker before the production stage in the last period of households lives with $\tau = T$:

$$U(h, T; \Omega) = u(b(h, T)) \tag{E.1}$$

$$V(h, T, W; \Omega) = u(w(a)), \tag{E.2}$$

their values trivially do not depend on the distribution of types as both valuations are 0 from $T+1$ onward. Hence, $U(h, T; \Omega) = U(h, T; a)$ and $V(h, T, W; \Omega) = V(h, T, W; a)$.

The optimal contract for agents aged $\tau = T$, instead, solves the following problem

$$J_t(h, T, y, W; \Omega) = \sup_w [f(y, h; a) - w] \quad s.t.\ W = u(w),$$

that clearly does not depend on the distribution of worker types due to the directed search protocol and where the aggregate state only affects the promised utility and the optimal wage through realization of the aggregate productivity processes. Therefore, $J_t(h, T, y, W; \Omega) = J_t(h, T, y, W; a)$.

This also implies that the equilibrium market tightness

$$\theta(h,T,W;\Omega) = q^{-1}\left(\frac{c(y)}{J_t(h,T,y,W;a)}\right)$$

is independent from the distribution of worker types and it is only affected by realization of aggregate productivity, so $\theta(h,T,W;a)$.

This in turn implies that the search problem workers face at the beginning of the last period of their lives depends on the aggregate state only through aggregate productivity a:

$$R(h,T,V;a) = \sup_{\{v_{y,\Omega}\}} \Big[p(\theta(h,T,v_{y,\Omega};a)) \big[v_{y,\Omega} - V \big] \Big],$$

does not depend on the distribution of worker types.

Stepping back at $\tau = T-1$, the value functions for the unemployed and the employed agents are solutions to the following dynamic programs

$$\sup_{\{v_{y,\Omega'}\}} u(b(h,T-1)) + \beta\mathbb{E}_{\Omega,\psi}\Big(U_{t+1}(h',T;a') + p(\theta(h,T,v_{y,\Omega'};a'))\big[v_{y,\Omega'} - U_{t+1}(h^prime,T;a')\big]\Big)$$

$$u(w) + \beta\mathbb{E}_{\Omega,\psi}\left(\begin{array}{c} \lambda U_{t+1}(h',T;a') + \beta(1-\lambda)W_{\Omega'} + \\ +\beta(1-\lambda)\lambda_e \max(0,R(h',T,W_{\Omega'});a')]\big] \end{array}\right),$$

where both do not depend on the distribution of worker types.

The optimal contract at this step is a solution to

$$\begin{aligned} J_t(h,T-1,y,V;a) = & \sup_{w_i,\{W_{i,\Omega'}\}} \sum_{i=1,2} \pi_i \Big(f(y,h;a) - w_i \\ & + \mathbb{E}_{\Omega,\psi}\left[\widetilde{p}_{t+1}(h',T,W_{i,\Omega'};a')(J_{t+1}(h',T,y,W_{i,\Omega'};a')\right] \Big) \end{aligned}$$

$$\begin{aligned} s.t.\ V = & \sum_{i=1,2} \pi_i \left(u(w_i) + \mathbb{E}_{\Omega,\psi}\widetilde{r}_{t+1}(h',T,W_{i,\Omega'};a')\right),\ h' = g(h,y)\psi \\ \mathbb{E}_{\Omega,\psi} & \sum_{i=1,2} \pi_i \left(\mathbb{E}_{\Omega,\psi} J_{t+1}(h^prime,T,y,W_{i,\Omega'};a')\right) \geq 0 \text{ and } t \leq T \end{aligned}$$

which does not depend on types distribution.

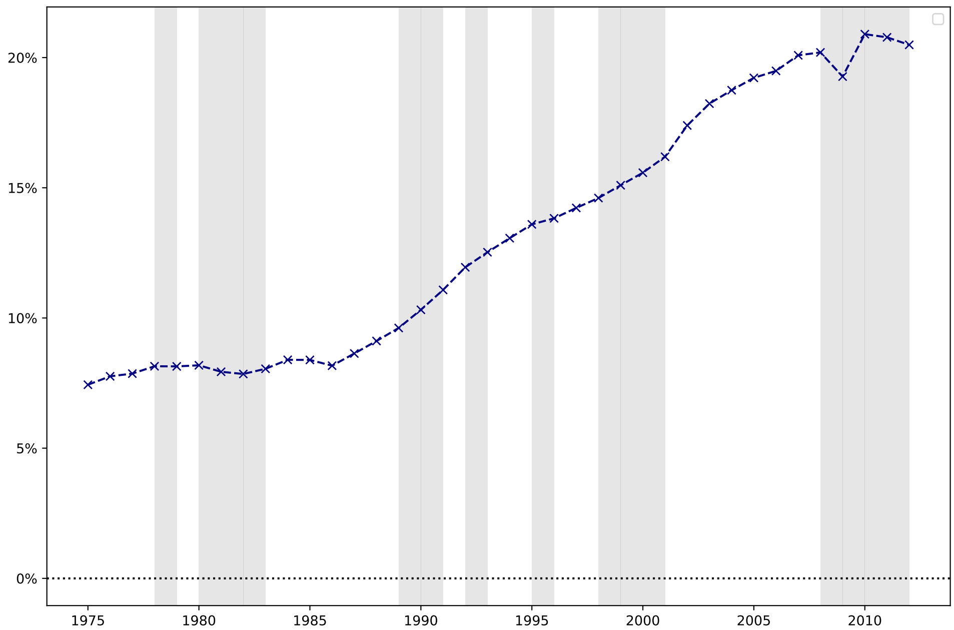
Therefore, also the equilibrium tightness and the search gain at $T-1$ are independent from types' distributions, as

$$\theta(h, T-1, W; a) = q^{-1}\left(\frac{c(y)}{J_t(h, T-1, y, W; a)}\right)$$
$$R(h, T-1, V; a) = \sup_{\{W_{y,\Omega}\}} \Big[p(\theta(h, T-1, v_{y,\Omega}; a))\big[v_{y,\Omega} - V\big]\Big].$$

Stepping back from $\tau = T-1, ..., 1$ and repeating the arguments above completes the proof. □

F Additional Figures and Tables

Figure F.18. Tertiary School Enrollment and the Business Cycle



Note: The figure plots the ratio of students enrolled in tertiary education to population aged 16-29 in every year. Source: Italian National Institute of Statistics (ISTAT). Shaded areas indicate the OECD based Recession Indicators for Italy.