Striking While the Iron Is Cold: Fragility after a Surge of Lumpy Investments

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Investment surges precede recessions

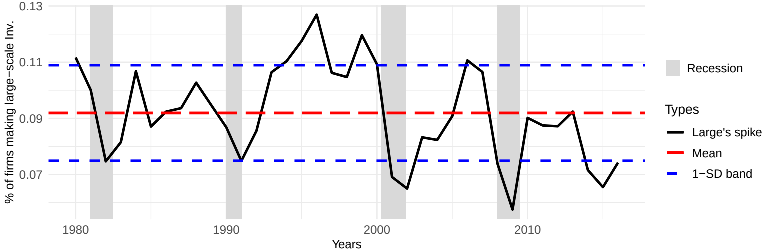

Figure: Surges of large firms' lumpy investments preceded three recessions

- Since 1980, there have been four periods of surges in the number of large firms making large-scale investments.
- Three events were followed by recessions within two years.
- Conversely, three out of the four recessions were preceded by the surges of lumpy investments.
- The exception (1990) was the mildest recession.

This paper

Research question

How do large firms' investments affect the business cycle?

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What this paper does

1. Points out that the existing models' limitations: the elasticity ranking is counterfactually flipped!
2. Develops a model that can correctly capture the ranking.
3. Studies the role of large firms' investments on the business cycle.

Why large firms?

- Large firms are insensitive to fluctuations in macroeconomic conditions including the interest rate.
  - Crouzet and Mehrotra (2020)
  - Zwick and Mahon (2017)
- Large firms are the most observable group of firms as most of them are listed and subject to financial disclosure.
  - Any forward-looking information contained in the large firms' investment dynamics can be conducive to designing a policy.
- Large firms account for a substantial portion of the aggregate investments.
  - The investments of the top 5% of firms cover more than 60% of entire investments.

Bird's-eye view of the main findings

Model side

Size-dependent fixed cost helps correctly capture the cross-section of the elasticities.
TFP-induced recessions are especially severe, after a surge of large firms' lumpy investments.

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40% of $I_t \downarrow$ during Dot-com bubble crash is accounted for by fragility fluctuations.

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Policy implication

The elasticity of aggregate investment drops after a surge of lumpy investment of large firms.

Related papers

Two contrasting views on firm-level lumpy investments

- Firm-level lumpiness in investment affects aggregate investment
  - Cooper et al. (1999), Abel and Eberly (2002), Gourio and Kashyap (2007), Bachmann, Caballero and Engel (2013), Winberry (2020), Koby and Wolf (2020)
  - This paper: Micro-level lumpiness generates state-dependency in macro-level shock sensitivity
- General equilibrium effect washes out micro-level lumpiness for aggregate investments
  - Khan and Thomas (2003), Khan and Thomas (2008), House (2014)
  - This paper: If interest-inelastic large firms are included in the model, the lumpiness survives aggregation.

State-dependent macro-level sensitivity

- Uncertainty shocks lead to nonlinear aggregate fluctuations
  - Fernandez-Villaverde et al. (2011), Bloom et al. (2018)
- Financial frictions generate endogenous risk
  - Adrian et al. (2019), Fernandez-Villaverde et al. (2020)
- This paper introduces a novel mechanism where nonlinear aggregate fluctuations arise from firm-level heterogeneity.

Interest-elasticities in the data

Large firms are less elastic than small firms

$$f(k_{it}, I_{it+1}) = \beta MP_t + \alpha_i + \alpha_{sy} + Controls_{it} + \epsilon_{it}$$

	Dependent variables:			
	$log(I_{it})$		$\mathbb{I}\{\frac{I_{it}}{k_{it}} > 0.2\}$	
	L	S	L	S
$MP_{Tight,t}$	-2.201 (0.606)	-7.025 (2.41)	-0.870 (0.366)	-2.072 (0.676)
Obs.	29,400	7,903	29,400	7,903
R^2	0.929	0.791	0.603	0.558
Firm FE	Yes	Yes	Yes	Yes
Sect.-year FE	Yes	Yes	Yes	Yes
Firm-level ctrl.	Yes	Yes	Yes	Yes
Two-way cl.	Yes	Yes	Yes	Yes

Table: Investment sensitivities to the monetary policy shocks

- Zwick and Mahon (2017): S/L elasticity ratio = 1.95.

Interest-elasticities in the existing models

Roadmap

Interest-elasticities in the existing models

- Can they capture less elastic large firms?

1. A two-period canonical model with convex adjustment cost
2. A two-period canonical model with fixed adjustment cost
3. Quantitative analysis of full model

$$\frac{\partial}{\partial k}\left(\frac{\partial}{\partial q}I\right)$$

Two-period canonical model with convex adjustment cost

Consider a two-period firm-level investment problem:

$$\max_{I} \quad -I - \frac{\mu}{2}\left(\frac{I}{k}\right)^2 k + q\mathbb{E}_z z'((1-\delta)k + I)^\alpha$$

$$\text{FOC: } 1 + \mu\left(\frac{I^*}{k}\right) = q\mathbb{E}z'\alpha((1-\delta)k + I^*)^{\alpha-1}$$

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Using an approximation of $log(1+x) \approxeq x$ for small x,

$$\mu\left(\frac{I^*}{k}\right) \approxeq log(q) + log(\mathbb{E}z'\alpha) + (\alpha-1)log(k) + (\alpha-1)\left(\frac{I^*}{k} - \delta\right)$$

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$$\mu\left(\frac{I^*}{k}\right) \approxeq log(q) + log(\mathbb{E}z'\alpha) + (\alpha-1)log(k) + (\alpha-1)\left(\frac{I^*}{k} - \delta\right)$$

Then, I re-arrange the terms to obtain the following equation:

$$\frac{I^*}{k} \approxeq A(\mu)log(q) + B(\mu,k) \implies I^* \approxeq A(\mu)log(q)k + B(\mu,k)$$

where $A(\mu) = \frac{1}{\mu+(1-\alpha)}$ and $B(\mu,k) = A(\mu)(log(\mathbb{E}z'\alpha) + (\alpha-1)log(k) - (\alpha-1)\delta)$.

Three predictions

$$I^* \approxeq A(\mu) log(q) k + B(\mu, k)$$
$$\text{where } A(\mu) = \frac{1}{\mu + (1-\alpha)}.$$

1. As q increases, I^* increases.
2. As q increases, I^* increases more when k is greater: $\frac{\partial}{\partial k}\left(\frac{\partial}{\partial q} I\right) > 0$.
3. As q increases, I^* increases less when μ is greater. $(A(\mu) \downarrow)$

Note: Formal proof without an approximation is in the paper.

Large firms are more elastic in the existing models

	Fixed	Convex only	Convex + Fixed	Data
Investment				
All	382.73	18.18	5.01	7.2
Small	313.76	14.8	4.32	
Large	481.93	21.79	6.99	
S/L ratio	0.65	0.68	0.62	1.95
Spike ratio				
All	25.61	1.97	1.04	
Small	37.97	0.74	1.24	
Large	16.39	1.35	1.14	
S/L ratio	2.32	0.55	1.09	

Table: Semi-elasticity comparison across models

- Spike ratio is the fraction of firms making large-scale investments.
- Models are calibrated to match $mean(i/k)$ and $mean(spikeRatio)$.

Model

Roadmap

Model

1. Overview of the model
2. Production technology
3. Firm-level investment
4. Recursive formulation
5. Household
6. Recursive competitive equilibrium

Overview of the model economy

Firms

Heterogeneous firms holding capital stocks operate using labor and capital

Convex adj. cost + **Size-dependent fixed cost**

Household

A representative household consumes, works, and saves (claim for all firms).

Competitive market

Fin. Story

Firm-level investment

Total adjustment costs = Size-dependent fixed cost + convex adjustment cost

- Size-dependent fixed cost: $wF(k) = w\xi k^{\zeta}$
  - Incurs only if $I \notin \Omega(k) := [-\nu k, \nu k]$ $(\nu < \delta)$
  - $\xi \sim_{iid} Unif([0, \bar{\xi}])$
  - ζ captures the cross-sectional dispersion of elasticities.
  - The micro foundation of k^{ζ}: Inter-dependence across establishments. ▸ Microfoundation
  - As in Khan and Thomas (2008), there exists a threshold rule for the extensive-margin adjustment: Adjust if $\xi^*(k, z; S) > \xi$.
  - The cost is regarded as an overhead labor cost.
- Convex adjustment cost: $c(k, I) = \frac{\mu}{2}\left(\frac{I}{k}\right)^2 k$
  - Essential component to match the empirical elasticity of the aggregate investment.

Recursive formulation

$$J(k,z;S) = \quad \pi(k,z;S) + (1-\delta)k + \int_0^{\overline{\xi}} \max\left\{R^*(k,z,\xi;S), R^c(k,z;S)\right\} dG_\xi(\xi)$$

$$R^*(k,z,\xi;S) = \max_{k'} \quad -k' - c(k,k') - w(S)F(k,\xi) + \mathbb{E}m(S,S')J(k',z';S')$$

$$R^c(k,z;S) = \max_{k^c-(1-\delta)k\in\Omega(k)} \quad -k^c - c(k,k^c) + \mathbb{E}m(S,S')J(k^c,z';S')$$

(Operating profit)	$\pi(z,k;S) := \max_{n_d} zAk^\alpha n_d^\gamma - w(S)n_d$ (n_d: labor demand)
(Convex adjustment cost)	$c(k,k') := \left(\mu^I/2\right)\left((k'-(1-\delta)k)/k\right)^2 k$
(Size-dependent fixed cost)	$F(k,\xi) := \xi k^\zeta$
(Constrained investment)	$I^c \in \Omega(k) := [-k\nu, k\nu] \quad (\nu < \delta)$
(Idiosyncratic productivity)	$z' = G_z(z)$ (AR(1) process)
(Stochastic discount factor)	$m(S,S') = \beta\left(C(S)/C(S')\right)$
(Aggregate states)	$S = \{A, \Phi\}$
(Aggregate law of motion)	$\Phi' := H(S),\ A' = G_A(A)$ (AR(1) process)

Quantitative Analysis

Roadmap

Quantitative Analysis

1. Calibration
2. Synchronization
3. Fragility after a surge of lumpy investments
4. Policy implication: State-dependent interest elasticity of aggregate investment
5. Discussion

Calibration - Fitted moments ▸ Fixed

Moments	Data	Model	Reference
Targeted moments			
Semi-elasticity of investment (%)	7.20	6.63	Koby and Wolf (2020)
Cross-sectional semi-elasticity ratio (%)	1.95	2.13	Zwick and Mahon (2017)
Cross-sectional average of i_t/k_t ratio	0.10	0.10	Zwick and Mahon (2017)
Cross-sectional dispersion of i_t/k_t $(s.d.)$	0.16	0.16	Zwick and Mahon (2017)
Cross-sectional average spike ratio	0.14	0.14	Zwick and Mahon (2017)
Positive investment rate	0.86	0.86	Winberry (2021)
$sd(log(Y_t))$	0.06	0.07	NIPA data (Annual)
Untargeted moments			
Average inaction periods (years)	6.38	7.72	Compustat data
Dispersion of inaction periods (years)	4.87	5.50	Compustat data
Average of lag difference of inaction periods	0.27	0.67	Compustat data
Dispersion of lag difference of inaction periods	6.47	8.36	Compustat data

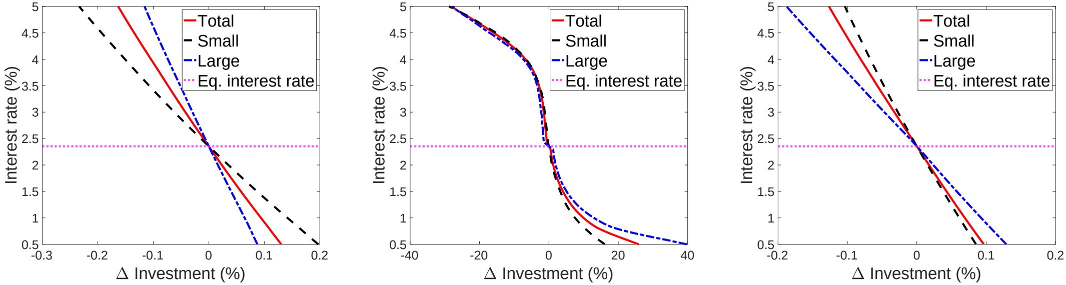
Table: Fitted Moments

Calibration - Parameters ▸ Fixed

Parameters	Description	Value
Internally calibrated parameters		
ζ	Fixed cost curvature	3.500
$\overline{\xi}$	Fixed cost upperbound	0.440
μ^I	Capital adjustment cost	0.780
ν	Small investment range	0.041
σ	Standard deviation of idiosyncratic TFP	0.130
σ_A	Standard deviation of aggregate TFP shock	0.025
Externally estimated parameters		
ρ	Persistence of idiosyncratic TFP	0.750

Table: Calibrated Parameters

Elasticities of investment across models



(a) Baseline (b) Fixed (c) Convex + Fixed

Figure: Semi-elasticities of investments across different models

Elasticities of investment across models (Cont'd)

	Baseline	Fixed	Convex	Fixed + Convex	Linear-Fixed	Quadratic-Fixed
Investment						
All	6.63	382.97	18.18	5.01	5.49	5.57
Small	9.85	343.62	14.8	4.32	5.41	6.35
Large	4.62	413.84	21.79	6.99	6.38	4.7
S/L ratio	2.13	0.83	0.68	0.62	0.85	1.35
Spike ratio						
All	1.3	25.61	1.97	1.04	1.27	1.33
Small	2.36	37.97	0.74	1.24	1.67	2.42
Large	0.98	16.39	1.35	1.14	0.91	1.06
S/L ratio	2.4	2.32	0.55	1.09	1.84	2.29

Table: Semi-elasticity of investment

Macroeconomic implications

Synchronization

- Upon a negative aggregate TFP shock, firm-level large-scale investment timings are synchronized. (e.g., Covid-19)

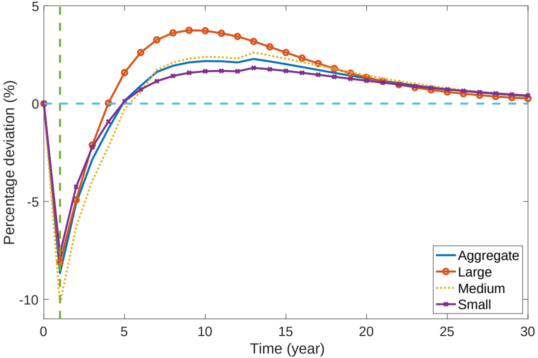
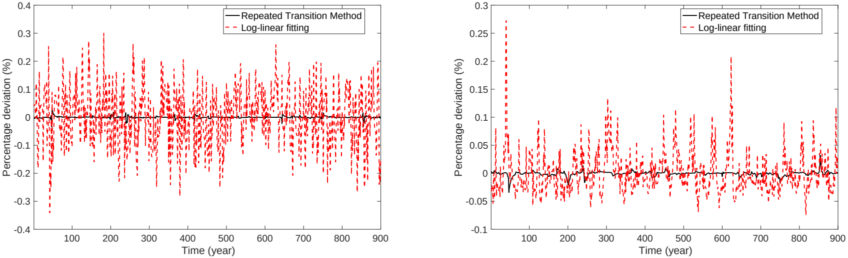


Figure: Impulse response of spike ratio

Dynamic stochastic general equilibrium simulation ▸ Price ▸ Capital

- 5,000 firms are simulated for 1,000 periods (years) using the DSGE allocations.
- Due to high nonlinearity, Krusell and Smith (1998) (Khan and Thomas, 2008) methodology is not helpful (R^2 is less than 0.999).
- I concurrently developed "**Repeated transition method**" that can globally solve the nonlinear DSGE problem with heterogeneous firms (agents).



(a) Marginal utility, $p_t = \frac{1}{c_t}$ (b) Aggregate capital stock K_t

Figure: Prediction errors in the marginal utility and the aggregate capital stock

Fragility index

Firm-level lumpy investments take 6-7 years on average, and the periodicity is significantly regular.

If a negative TFP shock hits the economy,

- Large firms that have not recently made lumpy investments tend to invest, regardless.
- Large firms that have recently finished lumpy investments do not.

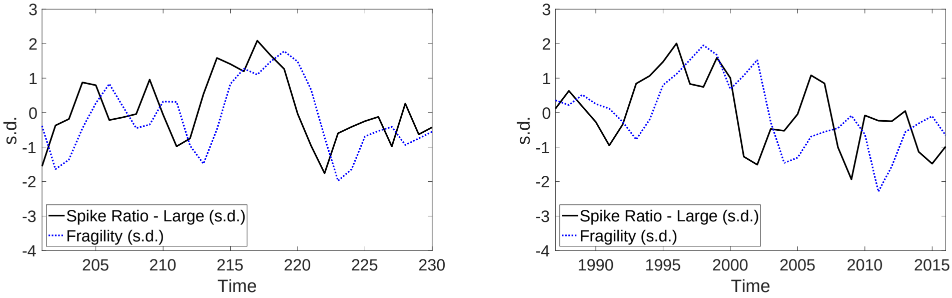
I define a **fragility index** based on observables at t:

$$Fragility_t := \frac{\sum_{Large} \mathbb{I}\{tFromInv < \bar{s}\}}{\#(Large)}$$

where *tFromInv* is time (years) from the last lumpy investment. I use $\bar{s} = 3$.

Time series of fragility index

Figure: Time-series of fragility indices in simulation and data



(a) Simulation (b) Data

Fragility over the business cycle

$$[\text{Data}]: \quad \Delta log(I_t) = \underset{(0.477)}{3.231} * OutputShock_t - \underset{(0.047)}{0.140} * log(Fragility_t) + \epsilon_t, \quad R^2 = 0.628$$

$$[\text{Model}]: \quad \Delta log(I_t) = \underset{(0.025)}{2.868} * OutputShock_t - \underset{(0.005)}{0.175} * log(Fragility_t) + \epsilon_t, \quad R^2 = 0.936$$

	Investment growth rate (%): $\Delta log(I_t)$		
	Raw data (NIPA)	Fragility-adjusted	Adjusted portion (%)
Recession-1991	-2.140	-1.889	11.729
Recession-2001	-7.627	-4.340	43.097
Recession-2009	-16.359	-16.551	-1.174

- Endogenous component: $s.d.(\Delta log(I_t^{Fragility}))/s.d.(\Delta log(I_t)) \approx 0.36$

State-dependent impulse responses

Let's denote the response of the aggregate investment as $g(I_t; S_t, \Delta A_t)$, where

- $S_t = \{A_t, \Phi_t\}$ is the aggregate states.
- ΔA_t is the magnitude of the impulse.

Suppose we observe a drop of investment ΔI_t^{Obs}, and we want to explain this.

$$\Delta I_t^{Obs} = g(I_t; S_t, \Delta A_t)$$

Traditionally,

$$\Delta I_t^{Obs} = g(I_t; S_{ss}, \Delta A_t)$$

In this paper,

$$\Delta I_t^{Obs} = g(I_t; S_t, \Delta A)$$

Depending on the aggregate state, the post shock responses of the investment vary for the same exogenous shock: The focus is on the role of S_t.

State-dependent instantaneous responses

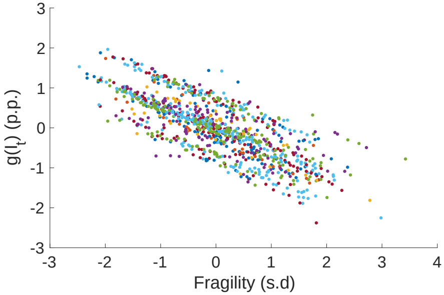


Figure: State-dependent instantaneous responses to a negative aggregate TFP shock

$$g(I_t)\ (p.p.) = \underset{(0.0151)}{-\,0.5605} * Fragility_t\ (s.d.) + \epsilon_t, \quad R^2 = 0.580$$

State-dependent interest elasticity of investments ▸ Decomp

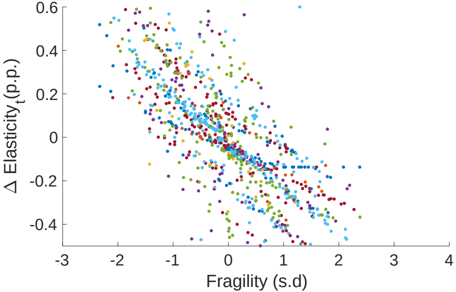


Figure: State-dependent semi-elasticities of aggregate investment

$$\Delta Elasticity_t\ (p.p) = -\underset{(0.0086)}{0.2689} * Fragility_t\ (s.d) + \epsilon_t, \quad R^2 = 0.497$$

Discussion

- Large firms' recent investment histories are starkly visible to policymakers: financial statements are subject to SEC regulation.
- The fragility index constructed from large firms has significant explanatory power on the one-period-ahead investment growth.
- Monetary policy may not be effective after a surge of large firms' lumpy investments:
  - There are not many large firms that can flexibly participate in and out of the large-scale investment after the surge.
  - Given the recent recessions were combined with the fragility effects, the monetary policy might not have been effective during the recession (Tenreyro and Thwaites, 2016).

Conclusion

- This paper sheds light on a precondition of an economy that makes the economy more fragile to a negative aggregate shock.
  - When few large firms are ready to make large-scale investments (after the surge of lumpy investments), the economy falls into a deeper recession after a negative aggregate shock.
  - 0.56 percentage point further drop in the investment growth per s.d. of fragility.
- Low interest-elasticity of large firms' lumpy investments generates the nonlinearity in the business cycle.
- The interest-elasticity of aggregate investment decreases after a surge of lumpy investment of large firms.
  - 0.27 percentage point drop in the interest-elasticity per s.d. of fragility.

Non-large firms' spike ratio ▸ Back

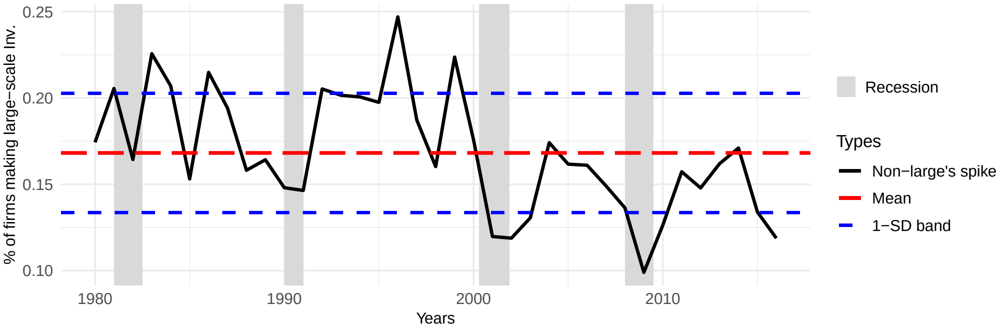


Figure: Small firms' spike ratio

Compustat data coverage

Back

- Since 1980, almost 90% of total listed firms are covered by Compustat data.
- In my cleaned version of Compustat data,
  - Total investment is 7 - 8% of annual US GDP.
  - Total investment is around 50% of US private domestic investment.
  - Total employment is around 60% of US private employment.
  - Total sales is around 80% of annual US GDP.
  - Total Value-Add is around 30% of annual US GDP.

Investment spike

Firm-level Investment I_{it}

A gross capital adjustment at the firm level where capital stock is obtained from applying perpetual inventory method to PPENT (Plant, Property and Equipment (NET))

Investment spike$_{it}$

A binary variable indicating a firm-specific incidence: $\mathbb{I}\left[\left(\frac{I_{it}}{k_{it}}\right) > 0.2\right]$

Spike ratio$_{j,t}$

A time-series aggregating investment spikes: $\frac{\sum_{i \in j} \text{Investment spike}_{it}}{\# \text{ of } j\text{-type firms at } t}, \quad j \in \{Non\text{-}large, Large\}$

Facts in the literature

	Zwick and Mahon (2017)	Koby and Wolf (2020)
	Δ Tax Policy (%)	ΔR (%) (Upper bound)
Aggregate	3.69	7.2
Small	6.29	12.27
Large	3.22	6.28
S/L ratio	1.95	1.95

Table: Semi-elasticities of investment

- Aggregate investment displays low interest elasticity (≤ 7.2).
- Small firms (B30) are almost twice more elastic than large firms (T30).
- On top of this, I show small firms are more elastic in the extensive margin. Evidence

Firm-level TFP Estimation (Ackerberg et al. (2015)) Back

$$log(ValueAdd_{it}) = \overline{\alpha} + \alpha log(Capital_{it-1}) + \gamma log(Emp_{it}) + TFP_{it} + \epsilon_{it}$$
$$MaterialExpense_{it} = f(Capital_{it-1}, Emp_{it}, TFP_{it})$$

Following Ackerberg et al. (2015), I assume

- Production, material expenditure and idiosyncratic TFP shocks are all realized simultaneously.
- Before the realization of the idiosyncratic TFP, a firm receives an idiosyncratic TFP signal ($sTFP_{it}$): a firm determines labor demand based on the signal. The idiosyncratic TFP follows a Markov process conditional on the signal of idiosyncratic TFP ($P(TFP_{it}|sTFP_{it})$).
- The idiosyncratic TFP signal follows a Markov process conditional on the past realization of the idiosyncratic TFP ($P(sTFP_{it}|TFP_{it-1})$).
- The function f is invertible with respect to TFP_{it}.

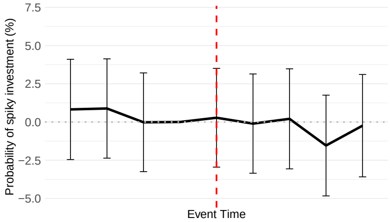
Then, the original model becomes

$$\begin{aligned} log(ValueAdd_{it}) &= \overline{\alpha} + \alpha log(Capital_{it-1}) + \gamma log(Emp_{it}) + f^{-1}(Capital_{it-1}, Emp_{it}, MaterialExpense_{it}) + \epsilon_{it} \\ &= g(Capital_{it-1}, Emp_{it}, MaterialExpense_{it}) + \epsilon_{it} \end{aligned}$$

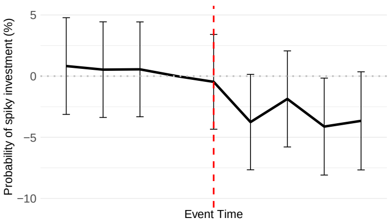
Then I estimate α and γ from

$$\mathbb{E}(\xi(\alpha,\gamma)|Capital_{it-1}, Emp_{it-1}) = 0, \text{ where } \xi(\alpha,\gamma) = TFP_{it} - \mathbb{E}(TFP_{it}|TFP_{it-1})$$

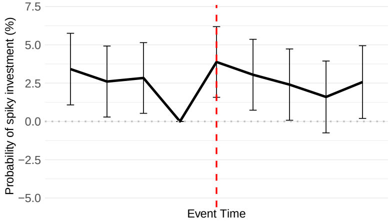
Financial reason? Back



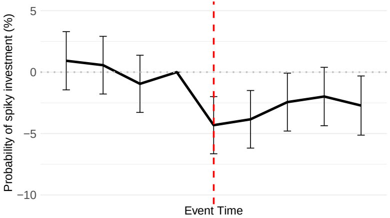
(a) Large firms around positive event



(b) Large firms around negative event



(c) Small firms around positive event



(d) Small firms around negative event

Figure: Extensive-margin investment sensitivity to an idiosyncratic TFP shock

Microfoundation of size-dependent fixed cost ▸ Back

Imagine a firm has three establishments, A, B, and C, and the firm is considering building another establishment, D.

- On the introduction of D, the adjustment happens in all of A, B, and C
  - Reorganization of workforces, product lines, and etc.
- If a fixed cost, ξ arises per an adjustment, and adjustment happens pairwise due to inter-dependence across the establishments.
  - e.g., (A,B), (A,C), and (B,C): the total fixed cost becomes $\binom{3}{2} \times \xi$
- For firms with n establishments, the total fixed adjustment cost becomes $\binom{n}{2} \times \xi = \frac{n(n-1)}{2}\xi$ which increases in n at the quadratic speed (pairwise adjustment case).
- If the average interdependence across establishments is ζ, the total fixed adjustment cost becomes $\binom{n}{\zeta} \times \xi$. The cost increases to the power of ζ.

Household ▸ Back

A representative household consumes, supplies labor, and saves.

$$
\begin{aligned}
V(a;S) = \max_{c,a',l_H} \quad & log(c) - \eta l_H + \beta \mathbb{E}^{A'} V(a';S') \\
\text{s.t. } & c + \int a' q(S,S') dS' = w(S) l_H + a \\
& G(S) = \Phi' \\
& G_A(A) = A' \quad \text{(AR(1) process)}
\end{aligned}
$$

- a: current wealth level, Φ: distribution of firms
  A: aggregate productivity, c: consumption
  a': future wealth level, l_H: labor supply (indivisible)
  q: state-contingent bond price, w: wage
- Household is holding the equity of firms as their wealth.

Fixed Parameters ▸ Back

Parameters	Description	Value
Firm-side Fundamentals		
α	Capital share	0.2800
γ	Labor share	0.6400
δ	Depreciation rate	0.0900
Household		
β	Discount factor	0.9770
η	Labor disutility parameter	2.4000
Aggregate TFP Process		
ρ_A	Persistence of aggregate TFP	0.8145

Table: Fixed Parameters

The nonlinear law of motions: p_t ▸ Back

	Dependent variables: $log(p_t)$								
	R^2			$max(\|error\|)(\%)$			$mean(\|error\|)(\%)$		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
A_1	0.9965	0.9995	0.9999	0.1960	0.0722	0.0393	0.0619	0.0225	0.0098
A_2	0.9951	0.9994	0.9999	0.2613	0.0936	0.0423	0.0756	0.0235	0.0117
A_3	0.9958	0.9993	0.9999	0.2793	0.1394	0.0676	0.0662	0.0263	0.0128
A_4	0.9945	0.9994	0.9999	0.3261	0.0900	0.0468	0.0657	0.0248	0.0115
A_5	0.9966	0.9992	0.9999	0.1954	0.1146	0.0669	0.0532	0.0266	0.0084

The nonlinear law of motions: K_t ▸ Back

	Dependent variables: $log(K_{t+1})$								
	R^2			$max(\|error\|)(\%)$			$mean(\|error\|)(\%)$		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
A_1	1.0000	1.0000	1.0000	0.0793	0.0785	0.0233	0.0150	0.0141	0.0057
A_2	0.9999	0.9999	1.0000	0.1253	0.1295	0.0402	0.0230	0.0237	0.0082
A_3	0.9999	0.9999	1.0000	0.2286	0.2248	0.0481	0.0210	0.0207	0.0090
A_4	0.9999	0.9999	1.0000	0.2503	0.2508	0.0784	0.0254	0.0244	0.0095
A_5	0.9998	0.9998	1.0000	0.1994	0.1886	0.0409	0.0259	0.0227	0.0076

Regularity in investment cycle Back

- Based on the stationary equilibrium, 5,000 firms are simulated for 1,000 periods (years).
- The dependent variable is inaction duration, and the independent variable is the lagged inaction duration from the simulated data.

	Dependent variable: $log(t2Inv_{i,j})$					
	Compustat			Stationary equilibrium		
	All	Large	Non-large	All	Large	Non-large
$log(t2Inv_{i,j-1})$	0.900	0.908	0.877	0.846	0.864	0.852
$(s.e.)$	(0.012)	(0.014)	(0.023)	(0.001)	(0.002)	(0.001)
Observations	2,070	1,501	569	587,041	59,110	508,841

Table: Regression of inaction durations on the lagged terms: Simulated data

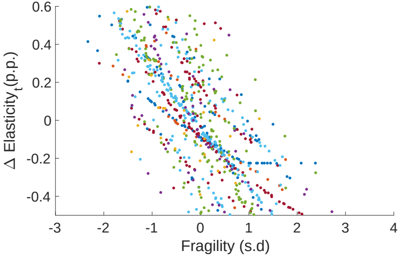
Business cycle statistics ▸ Back

- 5,000 firms are simulated for 1,000 periods (years).
- The data counterpart is from National Income and Product Accounts (NIPA) data.

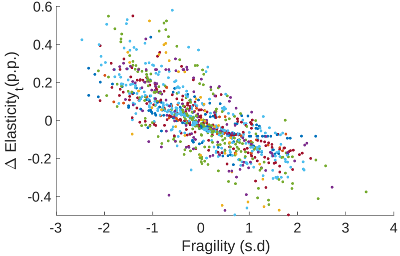
	Data	Model
$corr(Y_t, Y_{t-1})$	0.941	0.843
$corr(I_t, I_{t-1})$	0.742	0.742
$corr(C_t, C_{t-1})$	0.954	0.903
$corr(I_t, Y_t)$	0.795	0.796
$corr(L_t, Y_t)$	0.898	0.771
$corr(C_t, Y_t)$	0.978	0.980
$sd(Y_t)$	0.060	0.065
$sd(I_t)/sd(Y_t)$	1.976	1.809
$sd(C_t)/sd(Y_t)$	0.945	0.823

Table: Business cycle statistics

State-dependent interest elasticity of investments ▸ Back



(a) Large firms



(b) Small firms

$$\Delta Elasticity_t^{Large}\ (p.p) = -\underset{(0.0130)}{0.3992} * Fragility_t\ (s.d) + \epsilon_t, \quad R^2 = 0.484$$

$$\Delta Elasticity_t^{Small}\ (p.p) = -\underset{(0.0039)}{0.1403} * Fragility_t\ (s.d) + \epsilon_t, \quad R^2 = 0.569$$