Short-squeeze bubbles*

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Abstract

This paper argues that short selling might give rise to bubbles that would otherwise not exist in equilibrium. An asset with zero fundamental value might be traded at a positive price because short selling creates some present supply but raises the stock's future demand. The bubble is sustained by short-sellers covering their positions. Agents trade according to their beliefs on how long the bubble will persist. Several features of our model resemble the short-squeeze episodes of 2021.

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Jel Classification: G12, G14, D84

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1. Introduction

Short selling is typically seen as a remedy. A sizable body of research argues that allowing for short selling prevents speculative bubbles and drives asset prices closer to fundamentals. This paper argues that in some situations, the opposite might happen as well. Short selling can actually give rise to bubbles that would otherwise not be possible in equilibrium.

We present the argument through a simple model with an asset that will never yield any dividend. It is common knowledge that the asset's fundamental value is zero. All agents are price takers and risk-neutral. In the absence of short-selling, the asset must be worth zero. Once short-selling is allowed, there might be multiple equilibria. In some of them, *shareholders* lend the stock to *arbitrageurs* at a positive fee, who sell it short to *investors* at a positive price.

Short selling creates some present supply of the asset but also generates a future demand for the stock. Since equilibrium loan fees are positive, short-sellers will eventually need to close their trades. The bubble is sustained by arbitrageurs covering their positions.

The mass of investors, who buy the bubble, is assumed to increase at a rate lower than the market return, implying that a bubble cannot be sustained by their purchases. The realized gross return on the bubble is given by:

$$\text{Return} = \text{investors' purchasing rate} \times \text{short sellers' repurchasing rate}.$$

Many models generate bubbles through the first term, as investors keep buying the asset at high rates. This paper points to a different channel that may enable the emergence of a bubble: short positions create a need for repurchasing the asset in the future.

Short positions and the bubble are thus born together. The positive price creates a reason for short-selling, and short-selling creates a future demand that justifies the positive price early on. In equilibrium, buyers and sellers are effectively betting against each other.

Two ingredients of the model play an important role: (i) agents are allowed to disagree about the odds of bursting of the bubble; and (ii) shareholders, the initial owners of the stock, make portfolio decisions infrequently.

The bubble depends on expectations about future prices and might burst owing to non-fundamental (sunspot) variables that lead agents to coordinate on the zero-price equilibrium. Disagreement about the probability the bubble will burst generates a reason for asset trading. Those who think the bubble is more likely to burst will be willing to pay a positive fee to sell the asset to others. With no disagreement and no reason for trade, positive fees could not be an equilibrium.

Owing to positive fees, shareholders might choose to lend the asset instead of selling it. Were fees always equal to zero, shareholders would sell the asset at any positive price, and a bubble could not exist.

It is also important that shareholders don't make portfolio decisions at every period. They reoptimize their choices at random times. Their real-world counterparts are long-horizon traders like passive funds and pension funds. Eventually, the mass of investors fades out, and so do revenues from loan fees. Hence, at some point, it will be optimal for shareholders to sell the asset if the price is positive. At this point, the bubble bursts for sure. If shareholders were allowed to choose at every period, backward induction starting from this point would eliminate the bubble equilibrium.

In our baseline model, short-selling can only drive the asset price away from its zero fundamental value. We extend the model to include behavioral agents with irrational positive valuations of the asset and assume shareholders believe their number will grow. This gives rise to a speculative bubble: shareholders hold the stock despite believing its fundamental value is zero hoping to sell it to others with high valuation later on.

In this extension, the possibility of shorting stocks is a remedy for the speculative bubble but can also give rise to a short-squeeze bubble. Short-selling can backfire when more buyers are rational agents coordinating in a bubble equilibrium rather than behavioral agents with a high demand for the stock.

Several features of our stylized model resemble the short-squeeze episodes of 2021 and, in particular, the case of GameStop. In mid-January, the stock price was clearly above its fundamental value. Buyers hoped prices would keep rising for reasons unrelated to the company's valuation. Short-sellers hoped the bubble would burst. Then, in the last week of January, sellers capitulated. The ensuing short squeeze was fueled by short-sellers rushing to cover their positions. The stock price skyrocketed. The bubble did not burst in the following months. Throughout 2021, GameStop was worth about two orders of magnitude more than a year earlier. At least in part, the bubble was sustained either by short-sellers covering their positions or by the expectation that more short covering would eventually happen.

The remainder of this introduction discusses the related literature. Section 2 lays down the model, shows when bubbles can arise, and presents examples. Section 3 shows the extention with speculative bubbles. Section 4 highlights the connection between our model and some bubble episodes. Section 5 concludes.

1.1. Relation to the literature

The literature on speculative bubbles studies asset prices in models where agents have heterogeneous priors about the fundamental value of an asset – they agree to disagree. Short-selling is usually related to curbing rather than fueling bubbles in this literature. If shorting is not allowed, the possibility of selling the asset to someone with a larger (and possibly incorrect) valuation leads to deviations between market prices and fundamental values of firms (see e.g. Harrison & Kreps 1978, Morris 1996, Scheinkman & Xiong 2003).[1]

As in this literature, agents buy overpriced shares in our model, hoping to sell them later at a higher price to other agents. However, here, all agents know the asset will never yield any dividend; there is no disagreement about fundamentals. Moreover, short-selling is a necessary condition for a bubble, not an obstacle. Different beliefs about the persistency of the bubble equilibrium induce short-sellers and investors to take opposite positions while shareholders optimally choose to lend.

In line with the literature on speculative bubbles, much of the empirical research has found that short selling improves market efficiency. To cite a few examples,

[1]See Xiong (2013) for a survey. Heterogeneous priors is an important element of these models. In a world with common priors where agents trade for liquidity reasons, short-selling constraints will not bias prices because agents know which kind of information could be missing. But it typically harms price discovery (Diamond & Verrecchia 1987).

Karpoff & Lou (2010) argue that short sellers detect financial misconduct. Saffi & Sigurdsson (2011) show that more supply of equity lending has a positive impact on measures of market efficiency. Chague et al. (2014) show that short sellers are on average well-informed traders. Massa et al. (2015) argue that short selling disciplines earnings management. Taking advantage of a natural experiment, Chu et al. (2020) show that market anomalies became weaker for stocks with lower short-selling restrictions.

Notwithstanding the importance of these points, according to a prominent narrative of the recent episodes involving meme stocks, short-selling stimulated the bubble instead of taming it, and buyers were not betting on the intrinsic value of GameStop. Instead, they were buying overpriced stocks hoping that short-sellers would capitulate and cover their positions, thus raising asset prices further away from fundamental values – which indeed happened.

This logic is broadly related to the idea of predatory trading in Brunnermeier & Pedersen (2005). They show how private information about vulnerable market participants affects liquidity and leads to short-run mispricing. Here instead, we study (potentially) long-lasting bubbles due to short-selling with atomistic investors and no private information. Extending their reasoning, Brunnermeier & Oehmke (2014) argue that predatory shorting could destroy financial institutions that would otherwise survive, while here, the point is that short selling might lead to excessively large prices.[2]

In Duffie et al. (2002), allowing for shorting stocks might lead to higher prices. The reason is that an agent might pay more for an asset because she can later lend it for a loan fee to someone with a different valuation. Here, short selling can boost asset prices for entirely different reasons. All agents know the asset is worthless, but some buy a bubble fueled by short-sellers covering their positions.

Gârleanu et al. (2021) also explore the feedback effects in the equity lending and stock markets in a model with heterogeneous agents. They show that multiple equilibria might arise. The possibility of short selling may lead to instability (as in this paper) but not to overpricing and bubbles (differently from here). Their

[2]Short-selling may also lead to excessively low prices in Goldstein & Guembel (2008), as it open doors to manipulation owing to a feedback effect from the financial market to the real value of a firm.

argument is related to Vayanos & Weill (2008), who show that assets with the same cash flow might trade at different prices owing to search externalities in the equity lending market.

In Abreu & Brunnermeier (2003) and Doblas-Madrid (2012), an asset can be overvalued for a long time because information travels slowly, and arbitrageurs are not sure about what others know. Here, timing frictions and private information play no role. In Bordalo et al. (2021), overpricing arises owing to excessively optimistic beliefs.

Our paper is also broadly related to the literature of rational bubbles started with Tirole (1985). A bubble can only exist if it yields at least the market rate. In the rational bubbles literature, this means that it might emerge if market returns are lower than the growth rate of the economy.[3] In our framework, we rule out the possibility of a classical rational bubble by assuming that the aggregate endowment of the economy grows at a lower rate than the interest rate.[4]

Our model considers infinitely-lived short sellers and overlapping generations of buyers. In an OLG environment, bubbles can be an equilibrium without any market imperfection (Tirole 1985). This is because no transversality condition must be imposed. In an infinite-lived framework, imperfect credit markets are typically needed for bubbles to arise (Bewley 1979, Woodford 1990, Kocherlakota 1992, Santos & Woodford 1997). Here, we don't have imperfect credit markets as in the literature, but we allow for positive fees in the equity lending market.

Kocherlakota (1992) showed that short-sale constraints are necessary for the existence of bubbles in infinite-lived models as they eliminate arbitrage opportunities. In our framework, bubbles emerge precisely when short-selling contracts are in place. This result seems to contradict the earlier literature.

The key difference between our model and this literature is that here short selling requires the payment of fees. Therefore, short-sellers eventually need to cover their positions. When short positions are closed, the aggregate supply of

[3]See Miao (2014) and Martin & Ventura (2018) for a review of the literature on rational bubbles in infinite-lived and overlapping generations environments.

[4]Miao & Wang (2018) propose a framework with infinite-live agents in which stock bubbles may exist even if they grow at rate lower than the interest rate due to a liquidity premium. They obtain this result in a model with uninsured idiosyncratic shocks and endogenous debt constraints. Hellwig & Lorenzoni (2009) and Martins-da Rocha et al. (2019) also analyze the link between self-enforcing debt limits and bubbles.

stocks in the market falls. This may turn possible a path of increasing stock prices despite demand falling over time as well.[5] In contrast, in typical models of bubbles, short-selling is akin to taking a permanent negative position on the asset.

2. The model

2.1. Setup

We consider a discrete-time infinite-horizon economy with two investment opportunities: a risk-free asset in infinite supply with gross return $R > 1$; and the shares of a company with zero fundamental value. The shares are traded in a centralized market, their price is p_t and their quantity is normalized to 1.

There are three types of agents: investors, arbitrageurs and shareholders. They are all risk neutral, rational and know that the stock will never yield any dividend.

Investors are overlapping generations.[6] Each one is born with one unit of resources. They live for two periods and maximize old-age consumption investing in risk-free assets or stocks. In every period t, a new generation of size δ^t enters the economy. Throughout the paper, we assume that the growth rate of investors δ is smaller then R.[7]

A measure-one continuum of infinitely-lived arbitrageurs maximizes utility from consumption c_t

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t c_t \right), \tag{1}$$

with $\beta = \frac{1}{R}$, investing in risk-free assets and stocks, or taking a short position on stocks. We assume that arbitrageurs receive a constant endowment that is large enough to cover possible losses from short-selling strategies.[8] We also exclude the

[5] In a broadly related contribution, Kocherlakota (2008) shows that rational bubbles can be injected in an economy with infinite lived agents by perturbing agents' wealth constraints upwards. This generates a demand for savings in the future that sustains a bubble.

[6] This assumption is not important for our results. Investors are assumed to be overlapping generations in order to capture the idea that they are occasional players who enter and exit the market. Equilibria with our short-squeeze bubbles are still possible if investors were infinitely-lived.

[7] For simplicity, we assume they cannot lend their stock, but relaxing this assumption does not affect our qualitative results. Since investors do not get a loan fee, short positions are not a subsidy to their long positions. The literature has shown this subsidy can lead to overpricing and multiple equilibria (see e.g. Duffie et al. 2002, Gârleanu et al. 2021), but this channel does not affect investors' decisions in our model.

[8] The conditions for existence of bubbles are the same if we assume overlapping generations of arbitrageurs.

possibility of default in the short-selling market by assuming they must pay an arbitrarily high cost in case a contract is not respected.

Shareholders are the initial owners of all stocks. They have the same preferences as arbitrageurs but take decisions in a staggered manner. At $t = 0$, they decide whether to sell or lend their stock. In every following period, they may revise this strategy with probability $1 - \sigma$, with $0 < \sigma \leq 1$.

The real-world counterparts to the model's shareholders are long-horizon traders, such as passive funds and pension funds. Passive investors, such as index funds, are particularly likely to lend stocks (D'avolio 2002, Evans et al. 2017) – which makes intuitive sense since they effectively do not choose what to hold.[9] Passive funds account for more than 40% of assets under management in the US (Anadu et al. 2020), and active funds typically follow benchmarks rather closely (Raddatz et al. 2017).[10]

We model the equity market in a simple way. The loan fee borrowers pay to shareholders is denoted by ϕ. If the supply of equity loans exceeds the demand, some randomly chosen shareholders do not lend their stock – analogously, if demand exceeded supply, some randomly chosen borrowers would not be able to short. This assumption captures in a simple way the frictions in the equity lending market that give rise to positive fees.[11] In the periods in which they are allowed to decide, shareholders choose between selling or keeping the asset, knowing the fee and the odds they will be able to lend the stock in the future. If they choose to keep, the stock is available for lending until the next period in which they can revise their strategy.

Setting a finite endowment generates an additional constraint for the existence of bubbles, but it never eliminates them. We allow the endowment to be arbitrarily large (but finite) to keep our results as simple as possible.

[9]Evans et al. (2017) also argue that restrictions set by fund families prevent some active funds from selling assets.

[10]Lending stocks might also be a good strategy for less informed shareholders. A branch of the literature has established that short-sellers possess superior information and skill as compared to the market (Desai et al. 2002, Boehmer et al. 2018, Chague et al. 2019).

[11]Explicit modeling of the frictions in the lending market that generates a bilateral monopoly problem between borrower and lender would give rise to positive fees. Since this is not a central part of the argument, we chose the simplest set of assumptions for this part of the model.

2.2. Equilibria

An equilibrium consists of a loan fee ϕ and a sequence of prices $\{p_t\}$ that are consistent with profit maximization by all agents taking prices as given.

A bubble in our environment is defined as follows:

Definition 1 *A bubble is an asset with zero fundamental value traded at a price* $p_t > 0$.

Regardless of the assumptions on short-selling, there is always an equilibrium where the asset price is zero. In equilibria with bubbles, we allow agents to disagree about the probability the bubble will burst in the following periods. This provides them with a reason to trade.

Proposition 1 states a well-known result in the literature of rational bubbles with overlapping generations. If short-selling is never available, stocks can be traded at a positive price if and only if the growth rate of investors' endowments is larger than the interest rate R. By assuming $\delta < R$, we exclude the possibility of having a traditional bubble scheme.

Proposition 1 *If short-selling is never allowed and* $\delta < R$*, there are no equilibria with bubbles.*

Proof. *See Appendix A.1.* ■

In the equilibrium without short-selling, all agents must believe that the probability of a positive price in the following periods is zero. No other belief is rationalizable. There is no room for disagreement about the odds that a bubble will persist.

As we show next, when short-selling is allowed, the asset may turn into a bubble.

2.3. Equilibria with bubbles

We will now construct equilibria with bubbles. Shareholders lend all their stocks at $t = 0$ to arbitrageurs, who sell them to investors. At some point, the bubble bursts, and the asset value is worth zero from then on.

We will show equilibria where the bubble bursts when shareholders get a chance to reoptimize again. There are equilibria with bubbles where this does not happen, shareholders lend stocks more than once, but we are focusing on this case which is simpler to explain. Moreover, a bubble can burst for other reasons, since at any point agents might coordinate on the equilibrium with prices equal to zero. We will assume that agents disagree on the probability this will happen: conditional on the bubble being on at t, arbitrageurs believe the bubble will burst with probability π_H at $t+1$; shareholders believe the bubble will burst with probability π_S; and investors believe the bubble will burst with probability π_L.

In the equilibria we focus, investors use all their resources to buy the bubble, as long as it is on.[12] Let q_t be the amount short at t and p_t be the price of the bubble as long as it is on. Then,

$$p_0 = 1 \ , \ p_1 = \frac{\delta}{q_1} \ , \ p_2 = \frac{\delta^2}{q_2} \text{ and so on } \Rightarrow \ p_{t+1}q_{t+1} = \delta p_t q_t. \tag{2}$$

At $t = 0$, investors use all their resources (normalized to 1) to buy the supply of stocks. At $t = 1$, there is a measure δ of investors, and the amount short is q_1, an endogenous object. At every $t > 0$, the resources investors have to buy the bubble, and hence $p_t q_t$ grows (or decreases) by a factor δ. Notice that a growth in prices, p_{t+1}/p_t, can be sustained not only by the purchasing of the investors, δ, but also by the reduction in quantities, q_t/q_{t+1}.

Since arbitrageurs are risk-neutral and have large pockets, they must be indifferent between shorting or not in equilibrium, which happens when

$$p_t - \phi p_t - \sigma(1-\pi_H)\frac{p_{t+1}}{R} = 0 \ \rightarrow \ \frac{p_{t+1}}{p_t} = \frac{R(1-\phi)}{\sigma(1-\pi_H)}. \tag{3}$$

Using (2),

$$\frac{q_{t+1}}{q_t} = \frac{\delta p_t}{p_{t+1}} = \frac{\delta\sigma(1-\pi_H)}{R(1-\phi)}. \tag{4}$$

Investors use all their resources to buy the bubble as long as:[13]

$$-p_t + \sigma(1-\pi_L)\frac{p_{t+1}}{R} \geq 0.$$

[12] There exist equilibria in which investors use only a fraction of their resources to buy the bubble – they are indifferent between buying it or not.

[13] For simplicity, we do not allow investors to lend what they bought and use the resources to buy stocks again in the same period. Allowing them to lend some of their assets does not change our qualitative results.

Using (3), this implies

$$\phi \leq \frac{\pi_H - \pi_L}{1 - \pi_L}. \tag{5}$$

This condition yields an upper bound for ϕ because higher loan fees make arbitrageurs less willing to short, which reduces the return p_{t+1}/p_t.

Using (4), this leads to

$$\frac{q_{t+1}}{q_t} \leq \frac{\delta\sigma(1 - \pi_L)}{R} < 1.$$

In equilibrium, the amount short q_t falls with time, as short-sellers find it optimal to cover some of their positions, given a path of expected prices. If they didn't cover as much, prices would be lower, and any individual short seller would strictly prefer to buy the bubble. On the equilibrium path, they cover the amount that makes them indifferent.

Last, at $t = 0$, shareholders choose between selling the asset at price p_0 and lending it, in which case they will get the loan fee until the bubble bursts. Lending a unit of the asset yields ϕp_t. Since the short interest is q_t (which is the proportion of assets they will be able to lend), and they expect the bubble to burst at every t with probability $\sigma(1-\pi_S)$ (while it is on), the discounted expected revenues from lending the asset are

$$\sum_{i=0}^{\infty} \left[\frac{(1 - \pi_S)\sigma}{R}\right]^i \phi p_i q_i.$$

This is larger than p_0, meaning that lending the asset is the optimal choice, if

$$\phi \geq 1 - (1 - \pi_S)\frac{\sigma\delta}{R} \tag{6}$$

The condition for shareholders in (6) yields a lower bound for ϕ, since higher loan fees make lending more attractive. Combining it with the condition for investors in (5), we obtain the result for existence of bubbles in Proposition 2.

Proposition 2 *Consider a bubble that will burst when shareholders can reoptimize (probability $1-\sigma$) or according to a sunspot variable. The probability attached to this event is π_H for arbitrageurs, π_L for investors and π_S for shareholders. This bubble is an equilibrium as long as*

$$1 - \pi_H \leq (1 - \pi_L)(1 - \pi_S)\frac{\sigma\delta}{R}. \tag{7}$$

The loan fee must satisfy (5) and (6). The return on the bubble, conditional on not bursting, is given by (3). While the bubble is on, arbitrageurs cover their positions at every t so that the short interest q_t decreases according to (4).

Proof. *See Appendix A.2.* ■

The bubble is sustained by short-covering from arbitrageurs. This reduces the supply of stocks and raises the price of the asset. Their (correct) expectations about others' behavior make it optimal for them to cover their trades, but their purchases fuel the short-squeeze bubble.

The bubble requires $\pi_H > \pi_S$ and $\pi_H > \pi_L$. Arbitrageurs short the asset because they think it is more likely that the bubble will burst. Notice that agents agree about the zero fundamental value of the asset. They disagree on the probability of coordinating on a bubble equilibrium that can only exist in the presence of short-selling.

The condition for bubbles is more likely to hold if (i) σ is large (shareholders re-optimize with a small probability), (ii) δ is large (investors grow more over time), and (iii) R is small (interest rate is not very large).

If shareholders have longer horizons (larger σ), the bubble will last longer. This is true in the equilibrium we focus on by construction but is a more general property of the model. As time goes by, the short interest falls, and shareholders are less likely to get loan fees, so they are more willing to sell the asset. However, when they do so, the bubble must burst. With a higher σ, bubbles are expected to last for long, and shareholders are more willing to lend the asset at $t = 0$.

Investors must find it profitable to buy the asset. Hence the return on the bubble, conditional on not bursting, must satisfy

$$\frac{p_{t+1}}{p_t} \geq \frac{R}{\sigma(1 - \pi_L)} > R$$

However, the value of resources used to buy the bubble grows by a factor $\delta < R$ while the bubble is on. A bubble is only possible because short-sellers cover their positions. When δ is larger, the amount of short covering can be smaller, so shareholders get loan fees for longer and are thus more willing to lend the asset.

Last, a higher R makes selling more attractive than lending for reducing the present value of future loans and thus makes it harder for a bubble to arise.

2.3.1. Discussion of assumptions

In order to get trading in equilibrium with positive loan fees, we assume agents disagree about the probability the bubble will burst. In what follows, we discuss the role of these assumptions and possible alternatives.

If we assumed $\pi_H = \pi_L = \pi_S$, with no further change to the model, arbitrageurs and investors would only take opposite positions if both were indifferent. This could only happen if loan fees were zero. Were they positive, combined returns for short-sellers and buyers would be negative, so asset trading would never occur.

Were fees equal to zero, shareholders would only lend if we excluded the possibility of selling the asset. For example, suppose shareholders were passive funds and had to hold an exogenous quantity of stocks until the company left the stock index they follow, an event with probability $1 - \sigma$ at every time t. In this case, shareholders would lend at a zero fee in a Walrasian market.

With a zero loan fee, disagreement between arbitrageurs and investors would not be needed: a bubble equilibrium would exist as long as arbitrageurs were forced to close their positions eventually. With positive fees, different beliefs about the persistence of the bubble are enough to generate trade.

Alternatively, with a positive loan fee, we could have trade between investors and arbitrageurs in case $\pi_H = \pi_L$ if investors were risk lovers and not allowed to short the bubble. There is plenty of evidence that retail investors like to gamble in the stock market and buy lottery-like stocks with low expected return.[14] Investors with a convex utility function could buy a risky asset with an expected return lower than R.

The model assumed constant probabilities of bubble-bursting for each group of agents, but an asset might be overpriced at $t = 0$ even if all agents initially agree on those probabilities but expect some disagreement later. A stock that might be traded at a positive price in the future is worth more than zero now. More generally, the model could be extended to study how asset prices could be affected by shocks to beliefs on the probabilities of bubble-bursting.

[14] See, for example, Ang et al. (2006), Barber et al. (2009), Grinblatt & Keloharju (2009), Kumar (2009), Han & Kumar (2013) and Dorn et al. (2015).

2.3.2. Example

We now provide a numerical example to illustrate the workings of the model. The parameters are $\delta = 0.98$, $\sigma = 0.96$, $R = 1$, $\pi_H = 0.1$, $\pi_L = 0$ and $\pi_S = 0$. Figure 1 shows the paths of asset price and short interest assuming that the bubble bursts in the 12^{th} period. We assume that shareholders break even, so the equilibrium fee ϕ is 0.059.

Figure 1: Example of the bubble described in Proposition 2. Parameters: $\delta = 0.98$, $\sigma = 0.96$, $R = 1$, $\pi_H = 0.1$, $\pi_L = 0$ and $\pi_S = 0$. Equilibrium fee: $\phi = 0.059$.

While the bubble is on, asset prices rise quickly, while the short interest falls as arbitrageurs cover their positions. The value of the short interest also goes down with time. This must happen, at least in the long run, because the value of short positions must be equal to the investors' long positions, and their endowment decreases by a factor δ.

The value of the short interest is thus the demand for the asset. Since this demand is decreasing in time and approaches zero in the long run, a bubble cannot be sustained without short-selling. Short-selling allows for the existence of a bubble because short-covering reduces the supply of the asset allowing for a path of ever-increasing prices.

If short-sellers decided, off equilibrium path, not to change their positions in a given period, the asset price would fall. However, given the expected (bubbly) price path, this would not be optimal because returns on the bubble would be very high in that period, and all arbitrageurs would have an incentive to buy the stock. In equilibrium, they do so up to the point they are indifferent.

3. Short-squeeze bubbles and speculative bubbles

The model presented in the previous section portrays an environment where bubbles can only exist when short-selling is in place. We now modify our setting to take into account the role of short-selling in lowering prices.

3.1. Adding speculative bubbles

Financial bubbles are often described as a result of the irrational exuberance of behavioral investors. To incorporate this view, we add two assumptions to the model described in Section 2.1.

First, there is now a class of 2-period-OLG behavioral agents who hold arbitrarily high valuations of the asset. The size of these behavioral agents at time $t = 0$ is M_0^B. Second, all rational agents agree that behavioral agents will grow at a rate δ from time $t = 1$ on, but we allow them to disagree about their growth from $t = 0$ to $t = 1$.[15] Shareholders, the initial owners of stocks at $t = 0$, expect behavioral agents to grow at a rate $\delta_S > \delta$ from $t = 0$ to $t = 1$.

We first analyze the model assuming that short-selling cannot take place. Shareholders can sell the stock to behavioral agents. In this case, the asset price would be $p_0 = M_0^B$. Alternatively, they can hold some of the asset to sell to

[15] Admitting some disagreement over other periods does not change our qualitative results.

behavioral agents in the future – since they expect their numbers to grow.

If shareholders do not sell all their stocks, their indifference condition is:

$$p_0 = \frac{(1-\sigma)\delta_S}{R} M_0^B \sum_{t=0}^{\infty} \left(\frac{\sigma\delta}{R}\right)^t$$

They expect to sell the assets to behavioral agents as soon as they can. Therefore, the price of the asset in future periods is always equal to the expected size of the behavioral agents. If the RHS is larger than M_0^B, we have a speculative bubble in this model. Proposition 3 summarizes the result.

Proposition 3 *Suppose short selling is not allowed. If* $\delta_S > \frac{R-\sigma\delta}{1-\sigma} > R$*, shareholders do not sell all their stocks and*

$$p_0 = \frac{(1-\sigma)\delta_S}{R-\sigma\delta} M_0^B > M_0^B \tag{8}$$

Proof. *See Appendix A.3.* ■

If the conditions in Proposition 3 are satisfied, the asset price is larger than M_0^B and shareholders keep some of the asset for themselves. This is a speculative bubble. Since shareholders expect behavioral agents to grow more than R from $t = 0$ to $t = 1$, they hold some stocks to sell at a higher price to behavioral agents in the future.

3.2. Short-selling: cure or harm?

We now allow shareholders to lend stocks and arbitrageurs to borrow, as in the previous section. Since arbitrageurs expect behavioral agents to grow at a rate $\delta < R$ and prices to drop, they want to borrow stocks to sell them to behavioral investors. We will consider equilibria where arbitrageurs short all stocks.

Investors and arbitrageurs hold the same beliefs about the growth rate of behavioral agents, so investors do not want to buy for speculative reasons. However, arbitrageurs and investors may still coordinate on a short-squeeze bubble, which generates an overpricing that would not exist if short-selling was not permitted. We assume that the initial size of investors is M_0 and that they grow at a rate δ as before. We still assume that arbitrageurs and investors believe that a short-squeeze bubble component bursts respectively with probabilities π_H and π_L, with

$\pi_H > \pi_L$. For simplicity, we set that shareholders never expect a short-squeeze bubble to persist, or $\pi_S = 1$.[16]

Arbitrageurs are indifferent between shorting or not if

$$(1-\phi)p_t = (1-\sigma)\frac{M^B_{t+1}}{R} + \sigma\pi_H\frac{M^B_{t+1}}{R} + \sigma(1-\pi_H)\frac{p_{t+1}}{R}. \tag{9}$$

With probability $(1-\sigma)+\sigma\pi_H$, arbitrageurs expect that only behavioral agents will buy the asset. When the shareholders can revise their strategy (with probability $(1-\sigma)$), they sell all stocks. Similarly, when a short-squeeze bubble bursts (with subjective probability $\sigma\pi_H$), arbitrageurs borrow all stocks to sell to the behavioral agents. In both cases the price must be equal to the size of behavioral agents M^B_{t+1}. With probability $\sigma(1-\pi_H)$, investors and arbitrageurs may coordinate on a short-squeeze bubble equilibrium: the price can be $p_{t+1} \geq M^B_{t+1}$.

A similar reasoning implies that investors want to purchase assets as long as:

$$p_t \leq (1-\sigma)\frac{M^B_{t+1}}{R} + \sigma\pi_L\frac{M^B_{t+1}}{R} + \sigma(1-\pi_L)\frac{p_{t+1}}{R}. \tag{10}$$

Shareholders can now obtain fees from lending stocks. They prefer to hold and lend as long as

$$p_0 \leq \frac{(1-\sigma)\delta_S}{R-\sigma\delta}M^B_0 + \phi\left[p_0 + \frac{\sigma\delta_S}{R-\sigma\delta}M^B_0\right] \tag{11}$$

The first term on the right hand side refers to the expected return from selling to behavioral agents and it is identical to the one in (8). The second term refers to the lending revenues. Given $\pi_S = 1$, shareholders always expect the price at time t to be equal to the size of behavioral agents $\delta^t M^B_0$.

Proposition 4 puts together these conditions and shows what can happen in this environment.

Proposition 4 *Consider the case where short-selling is allowed.*

1. *There is an equilibrium where behavioral agents are the only buyers with* $p_0 = M^B_0$.

[16] Since $\pi_S = 1$, only the expected demand from behavioral buyers prevents a shareholder from selling the asset. With a smaller π_S, shareholders would also consider they could profit from short-squeeze bubbles, and the set of parameters consistent with a short-squeeze bubble would be larger.

2. *There is an equilibrium where investors also buy the bubble and* $p_0 = M_0 + M_0^B$ *if*

$$\frac{b}{(R - \sigma\delta) + b\sigma\delta_S} \geq \frac{1 - \pi_H}{1 - \pi_L} + \left(\frac{\pi_H - \pi_L}{1 - \pi_L}\right)\frac{b\delta}{R}, \tag{12}$$

where

$$b = \frac{M_0^B}{M_0 + M_0^B}.$$

3. *There is an equilibrium where the asset price is larger than in the absence of short-selling (Equation 8) if (12) holds and*

$$b < \frac{R - \sigma\delta}{(1 - \sigma)\delta_S}. \tag{13}$$

Proof. *See Appendix A.4* ■

If arbitrageurs and investors do not coordinate on a short-squeeze bubble, the behavioral agents are the only buyers: the price at $t = 0$ is $p_0 = M_0^B$ and grows at rate δ in the following periods. In this scenario, short selling reduces prices by eliminating the speculative bubble term.

However, a short-squeeze bubble might arise as long as (12) holds. This condition ensures that arbitrageurs are willing to pay a fee and short the asset, investors are happy to buy the asset, and shareholders want to hold and lend the asset. In this case, the initial price of the bubble is $p_0 = M_0^B + M_0$.

If the condition in (13) is respected, the emergence of a short-squeeze bubble would actually lead to a price larger than in the absence of short-selling (Proposition 3). This is the scenario in which short-selling can harm rather than cure. The condition in (13) is more likely to hold if b, the proportion of behavioral agents among asset buyers, is small. Intuitively, when most investors are rational, the role of short-selling in correcting market valuations is limited, while some disagreement on the persistence of a bubble can support the rise of a short-squeeze bubble. When b is large, mispricing is caused by incorrect beliefs, and short-selling is a cure; when b is small, many buyers understand the asset is overpriced, and short-selling might harm.

In sum, short-selling can reduce the scale of this speculative behavior and can now lower prices. However, as in the previous section, short-selling also allows investors and arbitrageurs to coordinate on short-squeeze bubbles.

3.3. Discussion

Theoretical and empirical papers have shown time and again that short-selling is a remedy for overpricing and market inefficiency. Notwithstanding its importance, we argue that the remedy can backfire. It is thus important to understand what could be done to reduce the likelihood of bubbles.[17]

First, our model implies that regulators should be concerned about preventing agents from coordinating on the bubble equilibria. Although short-squeeze bubbles require a degree of coordination among agents that seems difficult to achieve, social news platforms have arguably enabled agents to orchestrate short squeezes (Allen, Nowak, Pirovano & Tengulov 2021). This is a challenging task for regulators since it is easier for the Securities and Exchange Commission to discipline Wall Street financial institutions than to oversee the actions of individual investors connected through social media.[18]

Second, the bubble is sustained by short-sellers covering their positions. For most companies, the short interest is not large enough to generate a significant buying pressure, but we should be concerned when a large share of the float is short.

Last, the condition for short-squeeze bubbles in Proposition 2 is easier to be satisfied when R is low. This is typically true in models of rational bubbles (as in Tirole 1985) but not in models of speculative bubbles (as in Harrison & Kreps 1978).

4. Short-squeeze bubbles in action

We believe our stylized model captures important features of some bubble episodes.

[17] In our model, bubbles have no impact on capital accumulation and output because its production side is extremely simple. However, the literature has shown that bubbles can generate non-trivial welfare effects. In Tirole (1985), bubbles crowd-out capital but increase total consumption and welfare. In endogenous growth models, this crowding-out of capital reduces long-run welfare (Saint-Paul 1992, Grossman & Yanagawa 1993, King & Ferguson 1993). Recent research has shown how conclusions might differ in models with financial frictions: in Martin & Ventura (2012) and Hirano & Yanagawa (2016), bubbles can crowd investment in and increase output, whereas in Miao & Wang (2014), Basco (2016) and Pannella (2020), bubbles channel resources to less productive sectors or firms.

[18] Although the episodes of early 2021 spread from online forums, Short squeezes and short corners have been typically related to manipulation by large players (Allen et al. 2006, Allen, Haas, Nowak & Tengulov 2021).

A usual narrative of the GameStop episode portrays it as a buying frenzy fueled by retail investors through social media.[19] One possibility is that this buying activity has pushed prices up, and limits to arbitrage such as short-selling costs have allowed a disconnection between asset prices and fundamentals. This view is well explained by a model of agents with heterogeneous priors about fundamentals and short-selling restrictions. Retail investors would be agents with high (and possibly unreasonable) expectations about future dividends. In this case, if short selling were banned, overpricing would be even more pronounced.

The model suggests an alternative interpretation. Investors bought GameStop hoping that short-sellers would eventually give in and cover their positions, which would pump prices up. The key implication is that in the absence of short-selling, overpricing would have been smaller. Indeed, there is little evidence that retail investors believed in the future profitability of GameStop stores. In contrast, their confidence about a short-squeeze event has been well documented.[20]

One key feature of the model is the negative relation between asset prices and short interest. Short covering by arbitrageurs pumps up the stock price. This brings to mind some of the short squeezes episodes of 2021 and, in particular, the case of GameStop. Its price hike in the last week of January (from about USD 65 to USD 325) coincided with a strong asset demand by short-sellers covering their positions. In about a week, the short interest plummeted from a whopping 140% to less than 50% of the available shares. Since then, the short interest has fallen and is around 20% of the float in May 2022, implying that roughly 30 percent of the company has been bought by short sellers covering their positions. [21] Their demand for the stock has helped to keep its price up during this period.

[19] This has stimulated research exploring the role played by naive agents in financial markets and short squeezes. For example, Pedersen (2021) explores the interaction of rational and naive agents in a social network. The static model of Van Wesep & Waters (2021) assumes that some agents use all their wealth to buy GameStop shares. Hasso et al. (2021) study retail-investor activity using transaction-level data from brokerage accounts and conclude that the portrayal of a fight between retail investors and Wall Street is too simplistic.

[20] Online discussions among retail investors about a Gamestop short-squeeze started months before January 2021 ("How WallStreetBets Pushed GameStop Shares to the Moon", Bloomberg, January 25th, available at: https://www.bloomberg.com/news/articles/2021-01-25/how-wallstreetbets-pushed-gamestop-shares-to-the-moon?sref=M8H6LjUF).

[21] The fraction of GameStop short in 2022 looks modest as compared to its past values but is still large in absolute terms. Using a dataset with almost 5000 firms and about 10 years, Beneish et al. (2015) find that the average short interest in the top decile is around 10%.

In models of speculative bubbles, more short-selling activity brings prices closer to fundamentals. One important difference is that here, short sales sow the seeds of future asset purchases and thus stimulate the bubble. Hence, one key prediction of our model is that agents would buy shares of firms with high short interest, hoping that short-sellers would eventually give in. This is in line with popular narratives of the recent short-squeeze episodes.[22]

In models of speculative bubbles, an exceedingly expensive stock must be held by agents with exceedingly high fundamental valuation; in the case of short-squeeze bubbles, buyers understand the stock is overpriced. Studying the Volkswagen short squeeze of 2008, Allen, Haas, Nowak & Tengulov (2021) show evidence that sophisticated agents bought stocks with the understanding that short-sellers would find it difficult to short the stock or cover their positions, and prices would consequently rise.

Like all models of rational bubbles, ours features multiple equilibria and has no say on whether and when agents will coordinate on the bubble equilibrium. By all accounts, retail investors have played an important role in the onset of the Gamestop bubble, and social networks have helped them to achieve coordination (Allen, Nowak, Pirovano & Tengulov 2021).

5. Concluding remarks

January 2021 witnessed a battle between buyers and sellers in the stock market. Those long in GameStop were buying overpriced shares, hoping that short-sellers would capitulate and cover their positions, driving prices up. The hope indeed materialized, and the ensuing short squeeze led to skyrocketing stock prices for a long time.

Here we argue that short selling opens the door for this kind of asset overpricing, portrayed in the model as a pure bubble. At some point, short-sellers must cover their positions. Their short-covering provides fuel for price hikes, making possible a path of increasing prices.

[22]Long et al. (2021) perform textual analysis of 10 million comments on Reddit and relate agents' sentiments to 1-minute GameStop returns. A similar strategy could uncover whether the expectation of short-covering pushing up prices was in agents' minds.

Coordination on short-squeeze bubbles is in principle a difficult task, but has become easier as agents started to interact in social networks. Trading by retail investors has been booming. Short selling, usually associated with curbing excesses, might turn into a source of instability more often.

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A. Proofs

A.1. Proof of Proposition 1

The reasoning that leads to Proposition 1 is well known and standard in the literature. Let us suppose there exists a bubble on stocks, or $p_t > 0$. Agents in our economy may be willing to purchase the asset with the expectation of selling to other agents in the future. All agents are rational and risk neutral, therefore the expected price growth must be $\geq R$. Since the economy has no financial frictions, the OLG investors must be marginal buyers of the bubble for $t \to \infty$. However, such a pyramid scheme is sustainable if and only if their aggregate endowment grew at a rate $\geq R = \frac{1}{\beta}$, which we explicitly excluded.

A.2. Proof of Proposition 2

Most of the argument for the proof is in the main text, here we just complete the proof.

In a bubble equilibrium, arbitrageurs must be indifferent according to (3), and the market clearing condition (2) must be respected. In order to have investors purchasing and shareholders lending, both (5) and (6) must hold, so

$$1 - \pi_H \leq (1 - \pi_L)(1 - \pi_S)\frac{\sigma\delta}{R}. \tag{14}$$

is a necessary condition.

Notice that, since the equilibrium implies positive fees, the arbitrageurs eventually must close their short position, which means that short selling is not equivalent to issuing assets. In fact, an arbitrageur that keeps his short position open forever would have to pay a cost $\phi \sum_{i=0}^{\infty} \left(\frac{1}{R}\right)^i p_i \geq \phi \sum_{i=0}^{\infty} \left(\frac{1}{\sigma(1-\pi_L)}\right)^i = \infty$.

A.3. Proof of Proposition 3

Arbitrageurs and investors would never buy the asset, as they always expect the price to grow at a rate that is lower than R. Given that only behavioral agents would buy the asset, from market clearing, the quantity of purchased stocks is

$$q_0^B = \frac{M_0^B}{p_0} = \frac{R - \sigma\delta}{(1-\sigma)\delta_S},$$

which is smaller than 1 under the conditions in Proposition 3. The condition in (8) follows from shareholders' indifference condition.

A.4. Proof of Proposition 4

First Statement If prices are $p_t = \delta^t M_0^B$, investors do not want to purchase assets. The condition for arbitrageurs in 9 implies that arbitrageurs want to short the asset as long as

$$\phi \leq 1 - \frac{\delta}{R}$$

What happens then depends on whether shareholders want to hold or sell the asset. According to (11), if $\delta_S > \frac{R-\sigma\delta}{1-\sigma}$, they want to hold and lend. So the asset price must be M_0^B.

Second Statement Combining (10) with (9), we obtain:

$$\phi \leq \frac{\pi_H - \pi_L}{1-\pi_L}\left(1 - \frac{\delta}{R}\frac{M_t^B}{p_t}\right). \tag{15}$$

In case of a short-squeeze bubble, the price at $t = 0$ is $p_0 = M_0 + M_0^B$. While the bubble is on, it must be that $p_{t+1}q_{t+1} = \delta p_t q_t$. This implies that the relative price $\frac{M_t^B}{p_t}$ decreases with time. Therefore, condition (15) holds for any t if it holds at $t = 0$:

$$\phi \leq \frac{\pi_H - \pi_L}{1-\pi_L}\left(1 - \frac{\delta}{R}\frac{M_0^B}{M_0 + M_0^B}\right). \tag{16}$$

Combining with (11), we obtain a necessary condition for the emergence of a short-squeeze bubble:

$$\frac{1 - \frac{(1-\sigma)\delta_S}{R-\sigma\delta}\frac{M_0^B}{M_0+M_0^B}}{1 + \frac{\sigma\delta_S}{R-\sigma\delta}\frac{M_0^B}{M_0+M_0^B}} \leq \left(1 - \frac{1-\pi_H}{1-\pi_L}\right)\left(1 - \frac{\delta}{R}\frac{M_0^B}{M_0 + M_0^B}\right).$$

which can be written as the condition (12).

Third Statement The asset price is $M_0 + M_0^B$ in case of a short-squeeze bubble and given by (8) in case no short-selling is allowed. The former is larger if (13) holds.