Gambling in Risk-Taking Contests

Experimental Evidence

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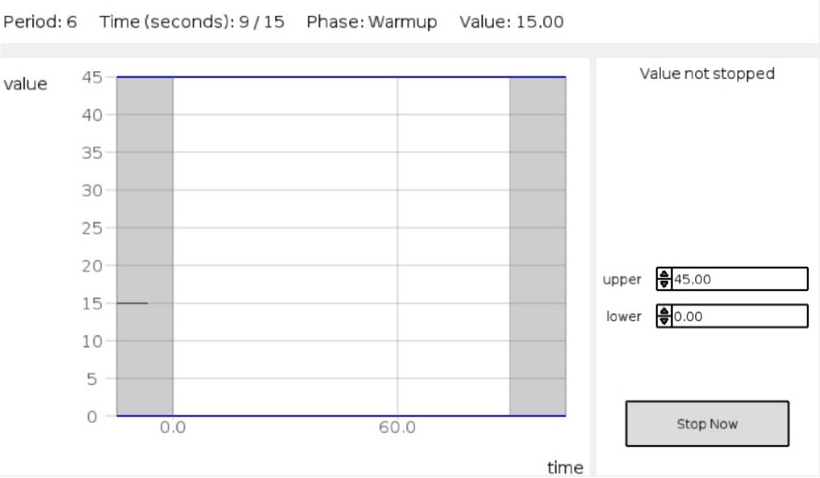
- *Rank-based rewards*: Relative performance schemes (contests and tournaments)
  - Induce effort (Lazear & Rosen, 1981)
  - Economic examples: fund manager rankings, managerial promotions, R&D patent races, electoral campaigns, sales contests, status contests.
- *Risk choice*: Risk a crucial decision variable, not just effort
- *Risk-taking contests*: A more recent theoretical literature
  - Seel and Strack (2013)
  - Many qualitative features extend, including excessive risk-taking to more general stochastic processes (Feng and Hobson, 2014), asymmetric bankruptcy constraints (Seel, 2015), incomplete information on endowment (Feng and Hobson, 2016), flow costs of research (Seel and Strack, 2016), multiple prizes partial observability plus B-S-M model (Fang, Noe and Strack, 2020).

- Implement the stylized model of Seel & Strack (2013):
  - Agents privately observe their own stochastic process; Brownian motion with (negative) drift
  - For example: fund manager competition, competition in a declining industry
- Implementation of the risk-taking contest environment
  - Relatively simple stopping problem that implements choice-over-stopped-value-distribution setting (Fang, Noe and Strack, 2020, Fang and Noe, 2022)
  - Includes dynamic feedback

- Predictions from the theory
  - Excessive risk in equilibrium
  - A moderately bad situation produces greatest inefficiency
  - Win-small loose-big
- Research questions
  1. Does the interaction between dynamic feedback and contest payoffs result in inefficient gambling?
  2. Are the inefficiencies non-monotonic?
  3. What sort of strategies are used in this context?

To Fix Some Ideas
The Main Experimental Task



To Fix Some Ideas
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The Main Experimental Task

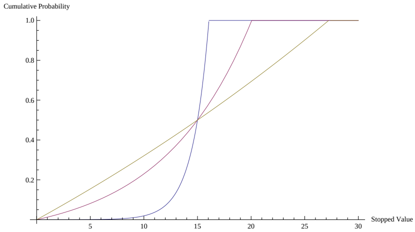
- Seel & Strack (2013) stylized model of gambling in contests
    - Two agents control stochastic process, $X_t^i = x_0 + \mu t + \sigma^2 B_t^i$
    - Independent realisations; $x_0 > 0, \mu < 0, \sigma^2 > 0$ common
    - Bankruptcy if X_t^i hits zero
    - Continuous time; only action is to stop process
    - No information update about other player's realisation
    - Agent with higher stopped value wins prize
    - $\mu < 0$ so processes decrease in expectation
- Player's decision is dynamic, but game is static
  $\Rightarrow$ focus on Nash equilibrium

Proposition (Summary of Results in Seel and Strack 2013)

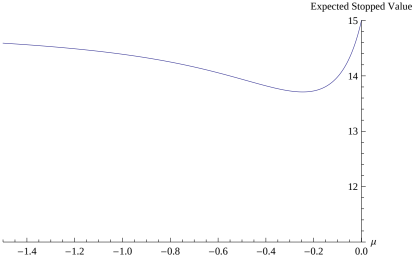
In any Nash equilibrium, both players choose strategies which induce the distribution

$$F^1(x) = F^2(x) = F(x) = \min\left\{\frac{1}{2}\frac{\exp(\frac{-2\mu x}{\sigma^2}) - 1}{\exp(\frac{-2\mu x_0}{\sigma^2}) - 1}, 1\right\}$$

- Key predictions
  - Always stopping immediately not an equilibrium strategy
  - Inefficient gambling in equilibrium
  - Inefficiencies non-monotonic in drift parameter
- Predictions independent of
  - size of prize
  - risk attitude of agents



(a) Equilibrium Distributions



(b) Expected Stopped Value

- Using
  - $x_0 = 15, \sigma^2 = 2$
  - blue: $\mu = -1.3$
    red: $\mu = -0.24$
    yellow: $\mu = -0.03$

- Basic decision task: discretised version of Seel & Strack (2013)
  - Start at $x_0 = 15$; 15 seconds warm-up
  - Every $\frac{1}{4}$ second: with p, $X_t^i \uparrow 1$; with $(1-p)$, $X_t^i \downarrow 1$
  - Time limit is 120 seconds; prize 150 ECU
  - Stop now button, max threshold and min threshold

- Structure of a session
  1. Instructions including control questions and example realisations
  2. 5 periods of individual task; not payoff relevant
  3. 10 periods of contest; random re-matching; payoff relevant
  4. 10 periods of lottery task (optimal to stop immediately); individual task; payoff relevant
- Treatment design: 3×2
  - Min Neg (49.625%), Mod Neg (47%), Ext Neg (33.75%)
  - Contest-Lottery, Lottery-Contest

Treatment	Abbreviation	Process Fundamental Drift (μ)	Process Fundamental Pr(up)	Expected Stopped Value
Minimal Negative	Min-	-0.03	0.49625	14.54
Moderate Negative (baseline)	Mod-	-0.24	0.47	13.71
Extreme Negative	Ext-	-1.3	0.3375	14.53

Notes: The drift parameter corresponds to the brownian motion (continuous-time process); for all treatments the variance parameter is $\sigma = 2$ and the starting value is $X_0 = 15$. The Pr(up) value gives the probability of an increase in value for the associated random walk (discrete-time process); for all treatments the starting value is $X_0 = 15$, the time interval is $\Delta t = 0.25$ seconds and the jump size is $\Delta X = 1$.

1. GAMBLING IN THE CONTEST
   - A In the contest, players do not always stop immediately
   - B The average time until the process is stopped increases in the drift.
   - C The average time until the process is stopped equals 0.72 seconds for $\mu = -1.3$, 10.75 seconds for $\mu = -0.24$, and 30.66 seconds for $\mu = -0.03$.
2. STOPPED VALUE IN THE CONTEST
   - A The average stopped value is non-monotone in the drift, first falling and then rising (U-Shape).
   - B The average stopped value equals 14.53 for $\mu = -1.3$, 13.71 for $\mu = -0.24$, and 14.54 for $\mu = -0.03$.
3. GAMBLING IN THE LOTTERY

   The process is stopped immediately in the individual choice setting.

- Contest

Treatment	Percent Stopped Time > 0	Stopped Time (sec) Average (95% C.I.)	Stopped Time (sec) Predicted	Stopped Value (ECU) Average (95% C.I.)	Stopped Value (ECU) Predicted
Min -ve	95.5 ***	23.74 (18.12, 29.36)	30.66	14.69 (13.60, 15.79)	14.54
Mod -ve	82.0 ***	8.78 (6.78, 10.79)	10.75	12.86 (12.35, 13.37)	13.71
Ext -ve	26.2 ***	0.25 (0.16, 0.34)	0.72	14.70 (14.55, 14.85)	14.53

Notes: *** 1%, ** 5%, * 10% significance.
Statistical tests and confidence intervals use standard errors clustered at the matching-group level.
Data from matches 6-10.

1. In the contest, in line with the predictions
   - Subjects do not always stop the process immediately
   - Time before stopping reduces as the drift becomes more negative
2. In the contest, in line with the predictions
   - Stopped value is non-monotonic in the drift parameter
   - Stopped value is lowest in Mod Neg.
3. In the lottery, <u>contrary</u> to the predictions
   - Subjects also do not always stop the process immediately
   - Stopped time also reduces as the drift becomes more negative
   - Stopped value is also non-monotonic in the drift parameter

1. Experience and payoff structure play a differential role across treatments
   - In Min-ve: Noisy feedback on cost of gambling
   - In Ext-ve: Contest payoffs crowd out intrinsic gambling
   - In Mod-ve: Increase in gambling in response to contest payoff
2. Heterogenous response to payoff structure change
3. Determinants of gambling
   - Strong individual component
   - Response to other's outcome strongest in Mod-ve

- Regret if stopped too soon and there was—in hindsight—a winning strategy
    - Implementation suggests it
    - Needs to be anticipated (e.g. Loomes and Sugden, 1982)
- Follow dynamic model of Strack and Viefers (2019)
    - Regret over stopping below peak stopping time
    - But predicts stopping immediately in the lottery payoff

- Derive utility from gambling itself
  - Decompose into two terms (Diecidue, Schmidt and Wakker, 2004): Expected utility of gamble + intrinsic cost/benefit of gambling
  - In lottery choice case, decision maker aims to maximise

$$\frac{X_\tau}{30} + C(X_\tau) = \frac{X_\tau}{30} + \int_0^\tau c(t, X_t)\,dt$$

  - Consider two cases
    1. Constant joy of gambling: $C(X_\tau) = c\tau \Rightarrow$ "Bang-bang" decision
    2. Decreasing joy of gambling: $C(X_\tau) = \int_0^\tau c(t)\,dt$, with $c(t)$ decreasing in t $\Rightarrow$ threshold decision
  - With decreasing joy of gambling
    - gambling in lottery
    - decreases as drift become more negative
    - optimal stopping time can be based on whether it is profitable to continue for one more period

- Consider reasoning over other's choice to complete explanation
  - In Ext-ve treatment
    - Observe most opponents stop immediately
    - Do not work out complete best-response to this
      (Stopping at 16 or 0)
    - Compare stopping immediately to gambling for one tick
    - In contest, cost is 1/2 reduction in probability (given above belief)
      In lottery, cost is a small reduction in probability
    - Need larger weight on joy of gambling for this to be optimal in contest compared to lottery → contest incentive crowds out intrinsic motivations to gamble
  - In Min-ve treatment
    - Observe more opponents gambling, so less extreme beliefs likely
    - Optimal BR calculation also less knife edge

- Equilibrium strategy in this environment implies a Win-Small-Lose-Big strategy
  - Left-skew stopped value distribution.
- Above Joy of gambling + limited reasoning over others suggests a finite gambling time not dependent on realisation of process
  - Right-skew (in finite sample)/zero-skew stopped value distribution.
- Some evidence, for experienced subjects, for example
  - In Min-ve with experienced subjects:
    Skew is negative (-0.43) and significantly different from normal (i.e.zero; p=0.013) in contest; Skew is less negative (-0.18) but not significantly different from normal (p=0.29) in lottery

Conclusion

- Implement Seel & Strack (2013) model in lab
- Provide a framework to implement probability distribution different from the "distribution builder" (Sharpe et al., 2000)
- Main results
  - Do engage in inefficient gambling
  - Non-monotonicity is at least as problematic as predicted
  - However, also in the non-strategic setting
- Additional results
  - Crowding of out of intrinsic motivations in Ext-ve
  - Opposite in Mod-ve (more response to other's outcome)
- Alternative explanations
  - Joy of gambling: intrinsic utility gain from gambling
  - Joy of gambling + reasoning over others