Identifying Monetary Policy Shocks Through External Variable Constraints*

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Abstract

This paper proposes a new strategy for the identification of monetary policy shocks in structural vector autoregressions (SVARs). It combines traditional sign restrictions with external variable constraints on high-frequency monetary surprises and central bank's macroeconomic projections. I use it to characterize the transmission of US monetary policy over the period 1965-2007. First, I find that contractionary monetary policy shocks induce a drop in output, sharpening the ambiguous implications of standard sign-restricted SVARs. Second, I show that the identified monetary policy shocks and monetary policy equations are consistent, respectively, with an historical reading of the times and Taylor-type rules. Finally, I propose an algorithm for robust Bayesian inference in SVARs identified with external variable constraints, providing further evidence in support of this approach.

Keywords: SVARs, Monetary policy shocks, Set-identification.

JEL Codes: E52; C51

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1 Introduction and Related Literature

Starting with the seminal paper of Sims (1980), a large number of studies has employed structural vector autoregressions (SVARs) to evaluate how monetary policy affects the real economy. Coherently with theoretical predictions, early SVAR literature based on short-run restrictions (e.g. Christiano et al., 1996) found that monetary tightenings have contractionary effects on output. The soundness of contemporaneous zero restrictions, however, has later been questioned by Uhlig (2005), who suggests to identify monetary policy shocks through sign restrictions on the impulse responses (henceforth *standard* or *traditional* sign restrictions). This methodology, that only achieves set-identification of the structural model, has the advantage of hinging on rather uncontroversial identifying assumptions. By using it, Uhlig (2005) finds that the effects of monetary contractions on output are not necessarily negative and may even be expansionary.

This paper proposes a refinement to standard sign restrictions, whose ability to properly identify monetary policy shocks has been recently called into question. In particular, Wolf (2020) provides an insightful interpretation of the ambiguous findings obtained by Uhlig (2005). Using Smets and Wouters's (2007) model as data-generating process, he argues that sign restrictions are likely to mistake positive demand and supply shocks for 'masquerading' contractionary monetary policy shocks, that are thus misleadingly found to raise output. Although theoretically grounded, sign restrictions may therefore not be enough to accurately identify monetary policy shocks. To address this issue, I combine them with external variable constraints. They reduce the number of admissible solutions and are thus a useful tool to sharpen identification. Specifically, I only retain the structural models which, in addition to satisfying standard sign restrictions, deliver monetary policy shocks that exhibit a certain relationship with Greenbook forecasts and high-frequency monetary surprises. The latter measure the changes in the three-month-ahead federal funds rate futures over a 30-minute window around each Federal Open

Market Committee (FOMC) announcement. They capture therefore the unpredictable component of monetary policy and plausible measures of monetary policy shocks should thus display a substantial positive correlation with them. Using the terminology of Wolf (2020), shocks with this feature are likely to actually be monetary policy shocks rather than mere demand or supply shocks 'masquerading' as such. The Greenbook forecasts represent instead a proxy of the information set of the Federal Reserve (Fed) about the current and future state of the economy. Hence, only candidate monetary policy shocks that are uncorrelated with them should be retained as solutions to the identification problem. If not, the effects of changes in monetary policy might be confused with those induced by the release of central bank private information and with the realization of the expected conditions to which the Fed is reacting. Once traditional sign restrictions are combined with external variable constraints, contractionary monetary policy shocks are incontrovertibly found to decrease output. This result sheds light on the ambiguous findings achieved by Uhlig (2005) and contributes to restore the conventional wisdom about the transmission of US monetary policy in set-identified SVARs.

This paper is closely connected to two recent contributions in the literature on set-identification of monetary policy shocks. Antolín-Díaz and Rubio-Ramírez (2018) combine traditional sign restrictions with narrative sign restrictions around key historical episodes, while Arias et al. (2019), motivated by Taylor-type rules, constrain the sign of the coefficients of the monetary policy equation. Importantly, I find that the structural models recovered through my identification strategy meet their identifying assumptions and thus exhibit two crucial features. First, monetary policy shocks are consistent with an historical reading of the times and, second, structural monetary policy equations are reconcilable with Taylor-type monetary policy rules. Conversely, although their results are coherent with mine, the monetary policy shocks identified by these two alternative approaches are found to be correlated with the Greenbook forecasts and to display a weak correlation with monetary surprises. Moreover, Antolín-Díaz and Rubio-Ramírez (2018) and Arias et al. (2019) impose restrictions by assuming a uniform Haar prior on the space of orthonormal matrices. As pointed out by Baumeister and Hamilton (2015), such a prior is however informative about objects of interest as impulse response functions. It is essential to stress that the findings obtained under my identification scheme, unlike those achieved through theirs, are still valid when inference is performed using a prior-robust algorithm based on numerical optimization methods.

The idea of using central bank's forecasts or high-frequency series to better identify monetary policy shocks is not new: monetary surprises are typically employed as an external instrument in proxy-SVARs (e.g. Gertler and Karadi, 2015), while Greenbook projections have been for instance included as endogenous variables in the VAR (Barth and Ramey, 2002). Recently, Miranda-Agrippino and Ricco (2021) have combined them to derive an informationally robust instrument for the identification of monetary policy shocks. Since I rely on the same external information, this paper is inevitably related to theirs. However, their approach is considerably different from the one I implement. First of all, my identification strategy does not require any of the external variables to be a valid instrument. Second, the use of external variable constraints does not achieve point-identification but is only aimed at sharpening set-identification.

My work also relates to Braun and Brüggerman (2022), who combine restrictions on the monetary policy equation with a constraint on the relationship between monetary policy shocks and Romer and Romer's (2004) narrative series. This paper differs from their work in several respects. First, they do not directly control for the central bank's information set and impose the external variable constraint on a narrative series rather than high-frequency surprises. Second they do not impose restrictions on the impulse responses but on the monetary policy equation, as Arias et al. (2019). Third, they only perform inference through the uniform, but informative, Haar prior.

The structure of this paper is as follows. Section 2 sets the econometric framework. Section 3 presents my identification strategy and the main findings. Section 4 relates my approach to narrative sign restrictions and restrictions on the monetary policy equation. Section 5 evaluates if my identification scheme effectively controls for the central bank information channel. Section 6 introduces the prior-robust inference algorithm and the resulting impulse response functions. Section 7 draws conclusions. Finally, Appendix A and Appendix B provide robustness checks and further technical details, respectively.

2 The Econometric Framework

This section sets the econometric framework and introduces the use of sign restrictions (Uhlig, 2005) for the identification of monetary policy shocks.

2.1 The Identification Problem

A reduced-form VAR(p) model takes the form:

$$y_t = \sum_{j=1}^{p} A_j L^j y_t + e_t \tag{1}$$

where L is the lag operator and p is the lag order; y_t is a $k \times 1$ vector of endogenous variables; e_t is a $k \times 1$ vector of reduced-form residuals and A_j, for $j = 1, \dots, p$, are matrices of estimated coefficients. Let $E(e_t e_t') = \Sigma_e$ be the variance-covariance matrix of e_t and $A = [A_1, \dots, A_p]$. If the reduced-form parameters $\omega = (\Sigma_e, A)$ are such that the VAR(p) is stationary, the following infinite-order vector moving average (VMA) representation does exist:

$$y_t = \sum_{h=0}^{\infty} C_h e_{t-h} \tag{2}$$

where C_h is the *h-th* coefficient matrix of $(I_k - \sum_{j=1}^{p} A_j L^j)^{-1}$. For $h = 0, \dots, H$, the (i, l)-element of the $k \times k$ matrix C_h is the reduced-form impulse response at time $t + h$

of the i-th variable in y_t to a unit innovation to the l-th entry of e_t.

Importantly, Σ_e is non-diagonal and the elements of e_t are thus contemporaneously correlated. The identification problem consists therefore in retrieving a linear transformation of e_t of the form

$$e_t = P\varepsilon_t \tag{3}$$

such that the variance-covariance matrix Σ_ε of the resulting structural shocks ε_t is diagonal. Once the structural impact matrix P is known, the structural impulse response functions (IRFs) at horizon h can then be computed as

$$\Theta_h = C_h P \tag{4}$$

where the (i, l)-element of the $k \times k$ matrix Θ_h is the impulse response at time $t + h$ of the i-th variable in y_t to a unit structural shock to the l-th element of ε_t.

2.2 Set-Identification of SVAR Models

The crucial result behind set-identification is that there are infinitely many matrices P such that Σ_ε is diagonal. Let us consider the following linear transformation of e_t,

$$e_t = S\eta_t \tag{5}$$

where S is the unique lower-triangular Cholesky factor of Σ_e. From (5), it follows that the shocks η_t are by construction mutually uncorrelated and have unit variance:

$$\Sigma_\eta = S^{-1}\Sigma_e(S^{-1})' = I \tag{6}$$

The above, however, is not the only solution to the identification problem. In order to see this, consider the following orthonormal transformation of η_t:

$$\hat{\varepsilon}_t = Q'\eta_t \tag{7}$$

where Q' is a square orthonormal matrix such that $Q'Q = QQ' = I$. By exploiting the orthonormality of Q and equation (7), it follows that

$$e_t = SQQ'\eta_t = SQ\hat{\varepsilon}_t \tag{8}$$

As proved in equation (9), such an orthonormal transformation succeeds in delivering a diagonal structural variance-covariance matrix Σ_ε.

$$\Sigma_\varepsilon = Q^{-1}S^{-1}SS'(S^{-1})'(Q^{-1})' = I \tag{9}$$

Hence, there are infinitely many solutions to the identification problem, one for each orthonormal transformation of S. In this setting, an identification strategy may therefore be thought of as a set of identifying restrictions that restrain the admissible support for the orthonormal matrices Q.

2.3 Identification by Sign Restrictions

For a given orthonormal matrix Q and $h = 0, \ldots, H$, the $k \times k$ matrix of candidate structural impulse responses $\hat{\Theta}_h$ can be expressed as:

$$\hat{\Theta}_h = C_h SQ \tag{10}$$

where the (i, l)-element of $\hat{\Theta}_h$ is the structural impulse response at time $t + h$ of the i-th variable in y to a unit structural shock to the l-th element of $\hat{\varepsilon}_t$. Sign restrictions solve the identification problem by constraining the sign of some elements of $\hat{\Theta}_h$. This approach was pioneered by Uhlig (2005), who implements it on the following vector y_t of US monthly variables over the period 1965:M1-2003:M12,

$$y_t' = \begin{bmatrix} gdp_t & pi_t & ff_t & ci_t & tr_t & nr_t \end{bmatrix} \tag{11}$$

where gdp_t and pi_t are the log of real GDP and of the GDP deflator, constructed using interpolation of the quarterly series as in Bernanke and Mihov (1998); ff_t is the federal funds rate; ci_t is the log of the commodity price index from Global Financial Data; tr_t

and nr_t are the log of total and nonborrowed reserves.

The identification of monetary policy shocks is achieved by retaining a large number of structural impact matrices SQ such that the resulting shock $\hat{\varepsilon}_t^m(SQ)$ satisfies Restriction SR.

Restriction SR. A monetary policy shock ε_t^m leads to a negative response of pi_t, ci_t and nr_t, and to a positive response of ff_t at horizons $h = 0, \ldots, 5$.

3 Enhancing Sign Restrictions With External Variable Constraints

The soundness of the shocks recovered through Restriction SR has been recently called into question. Specifically, Wolf (2020) shows that sign restrictions are likely to mistake positive demand and supply shocks for 'masquerading' contractionary monetary policy shocks. Structural models may thus be misidentified if only Restriction SR is imposed.

To address this issue, I enhance sign restrictions with external variable constraints on high-frequency monetary surprises and Greenbook forecasts. The latter proxies the Fed's information set about the current and future state of the economy and monetary policy shocks should thus be not correlated with them. If not, the effects of changes in monetary policy might be confused with those induced by the disclosure of central bank private information and by the realization of the expected conditions to which the Fed is reacting. Monetary surprises measure instead the movements in the three-month-ahead federal funds rate futures over a 30-minute window around each FOMC announcement and capture therefore the unpredictable component of monetary policy. Hence, I retain only the candidate shocks that show a strong positive correlation with them, since they are likely to be 'true' monetary policy shocks and not just demand or supply shocks 'masquerading' as such.

My sample starts in January 1965 and ends in November 2007. This allows to extend the time frame originally considered by Uhlig (2005) while excluding the unconventional monetary policy undertaken after the global financial crisis.[1] The reduced-form VAR specification includes 12 lags of the variables in (11) and, consistently with Uhlig (2005), does not include any deterministic term. The estimation is performed by using Bayesian methods with Jeffrey (non-informative or flat) priors for Σ_e and A.

3.1 The Identification Strategy

In the first step, I impose Restriction SR by implementing the algorithm proposed by Rubio-Ramìrez et al. (2010), described in Appendix B.

Restriction SR. A monetary policy shock ε_t^m leads to a negative response of pi_t, ci_t and nr_t, and to a positive response of $f\!f_t$ at horizons $h = 0, \ldots, 5$.

I generate 100000 draws of SQ satisfying the above restriction and I store them into the set $\mathcal{P}$. For $i = 1, \ldots, 100000$, let $\hat{\varepsilon}_t^{m,i}(SQ)$ be the *i-th* monetary policy shock associated with the *i-th* structural impact matrix $SQ \in \mathcal{P}$. The set $\mathcal{P}$ is then sharpened by only retaining the matrices $SQ \in \mathcal{P}$ such that $\hat{\varepsilon}_t^{m,i}(SQ)$ meet Restriction ER.

Restriction ER. Over the period 1990:M1-2007:M11, a monetary policy shock ε_t^m satisfies the following external variable constraints:

$$corr(\varepsilon_t^m, FF4_t) > g \tag{ER1}$$

$$corr(\varepsilon_t^m, FI_t) = 0 \tag{ER2}$$

where $FF4_t$ is the change in the three-month-ahead federal funds rate futures in the 30-minute window around the FOMC announcement and FI_t is the information set of the Fed, in month t, about current and future economic conditions. The latter is proxied by the Greenbook forecasts for output growth and inflation rate for the previous quarter and up to three quarters ahead and by the Greenbook nowcast for the quarterly unemp-

[1]The findings discussed in the next few sections are still valid over Uhlig's (2005) original sample.

ment rate.[2] The parameter g in (ER1) determines how strong the correlation between monetary surprises $FF4_t$ and $\hat{\varepsilon}_t^m(SQ)$ must be for the latter to be accepted as a solution. Restriction ER ensures therefore the identification of monetary policy shocks that are substantially correlated with monetary surprises and exogenous to the information set of the policymaker.[3] Specifically, I enforce constraint (ER2) by running the following regression at the monthly frequency and requiring the coefficients ϕ_p^{gdp}, ψ_p^{π} and ϑ_0^u to be jointly not significant at the 5% level:

$$\hat{\varepsilon}_t^{m,i} = \alpha_m^i + \sum_{p=-1}^{3} \phi_p^i G_{t,p}^{gdp} + \sum_{p=-1}^{3} \psi_p^i G_{t,p}^{\pi} + \vartheta_0^u G_{t,0}^u + u_{m,t}^i \tag{12}$$

where $\hat{\varepsilon}_t^{m,i}$, with $i = 1, \ldots, 100000$, is the i-th candidate shock satisfying Restriction SR; $G_{t,p}^i$, with $p = -1, \ldots, 3$ and $i = \{gdp, \pi\}$, denotes the p-quarters ahead Greenbook projection for output growth and inflation rate in month t and $G_{t,0}^u$ is the Greenbook nowcast for the quarterly unemployment rate. I consider three alternative calibrations for g in (ER1), fixed at the 75^{th}, 90^{th} or 99^{th} percentile of the set of correlation coefficients between the 100000 shocks $\hat{\varepsilon}_t^m(SQ)$ formed from $SQ \in \mathcal{P}$ and $FF4_t$. The structural impact matrices SQ that generate shocks $\hat{\varepsilon}_t^m(SQ)$ satisfying Restriction SR and ER are then stored into the sets of solutions $\mathcal{P}_{75th}^*$, $\mathcal{P}_{90th}^*$ and $\mathcal{P}_{99th}^*$. It is worth noting that the frequency of the dependent variable in (12) is originally different from the one of the regressors: shocks $\hat{\varepsilon}_t^{m,i}$ are monthly series while Greenbook forecasts are available eight times a year. The latter are in fact prepared by the Federal Reserve Board staff prior to each FOMC meeting, typically in the first and third month of each quarter.

[2] In this I follow Romer and Romer (2004). The inclusion of backcast, nowcast and all the forecasts of the unemployment rate would not bring any additional information, while creating collinearity issues with the output growth series in regression (12).

[3] Restriction ER can only be imposed over the period 1990:M1-2007:M11, since the $FF4_t$ series is available from January 1990. This limitation is common to the entire literature on high-frequency identification of monetary policy shocks, as Gertler and Karadi (2015) and Miranda-Agrippino and Ricco (2021). The latter, for instance, estimate the reduced-form over the period 1979:M1-2014:M12 but only run the 2SLS regression that delivers the structural parameters from January 1990 onward.

As in Barth and Ramey (2002), I convert Greenbook series to monthly frequency by using the initial forecasts of the quarter to proxy the Fed's information set in the first two months, while the projections produced for the second meeting are used to update the series in the third month. This amounts to saying that the information set of the Fed does not change in months without FOMC meetings and probably assumes slightly less information than the Fed actually has.[4]

Finally, note that identification by external variable constraints significantly differs from that achieved in proxy-SVARs. The latter point-identify the structural model by assuming exogeneity and relevance of the external instrument. Conversely, the method I propose only delivers set-identification and, most importantly, does not assume any of the external variables to be a valid instrument. This is an advantage since it is hard to build convincingly exogenous instruments for monetary policy shocks. Several popular instruments in the empirical literature on monetary policy have been in fact found to be correlated with the Fed's information set (as Gertler and Karadi's monetary surprises) or predictable by past information (as Romer and Romer's narrative series).

3.2 Impulse Response Functions

This subsection compares the IRFs derived by imposing standard sign restrictions with those obtained using the sets of solutions recovered through my identification strategy. In Appendix A, I show instead the IRFs obtained using only Restriction ER and in the case in which constraints (ER1) and (ER2) are alternatively released.

Figure 1 displays the IRFs under Restriction SR and those formed from $SQ \in \mathcal{P}^*_{75th}$. This set contains 5171 structural impact matrices SQ (out of the 100000 matrices stored in $\mathcal{P}$) delivering monetary policy shocks uncorrelated with the Greenbook and that show

[4] In Appendix A, I drop this assumption by imposing (ER2) at the FOMC meeting frequency. This alternative approach delivers very similar results and has the advantage of not requiring any frequency conversion. However, it largely reduces the number of observations used to estimate regression (12).

a correlation with monetary surprises higher than 0.09 (the 75^{th} percentile of the set of correlation coefficients between $FF4_t$ and the 100000 shocks formed from $SQ \in \mathcal{P}$). Importantly, once I discard the candidate shocks correlated with the Fed's information

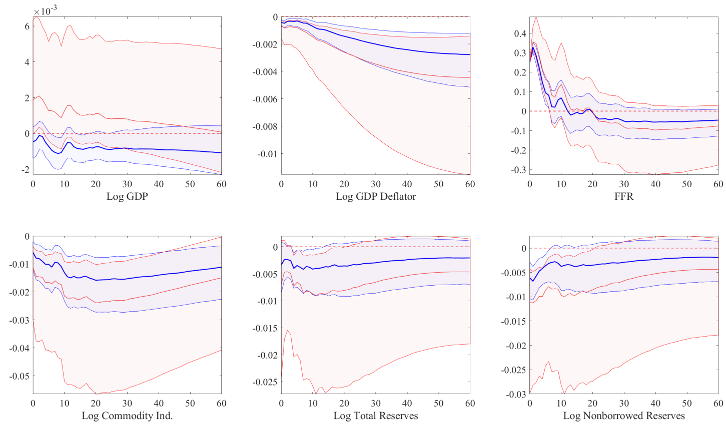

Figure 1: Responses to contractionary monetary policy shocks formed from $SQ \in \mathcal{P}^*_{75th}$ (in blue) and under Restriction SR (in red).

Notes: Monetary policy shocks normalized to induce a 0.25% rise in ff_t. The solid line is the point-wise posterior median response and the shaded bands are the 68% equal-tailed point-wise posterior probability bands.

set or that weakly comove with $FF4_t$, expansionary effects of monetary tightenings are entirely ruled out. Figure 2 illustrates instead the IRFs formed from $SQ \in \mathcal{P}^*_{90th}$. The correlation between monetary policy shocks and $FF4_t$ is in such case constrained to be

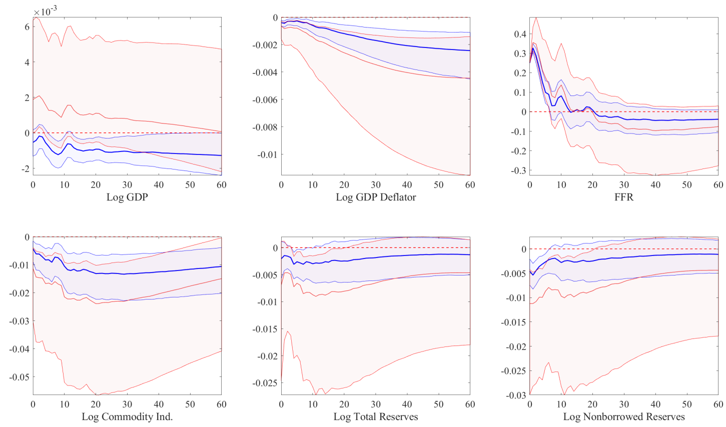

Figure 2: Responses to contractionary monetary policy shocks formed from $SQ \in \mathcal{P}^*_{90th}$ (in blue) and under Restriction SR (in red).

Notes: Monetary policy shocks normalized to induce a 0.25% rise in ff_t. The solid line is the point-wise posterior median response and the shaded bands are the 68% equal-tailed point-wise posterior probability bands.

greater than 0.13 and the number of solutions drops to 2166. In contrast with the results induced by traditional sign restrictions, monetary contractions are found to trigger significantly negative effects on output in the short and medium-term.

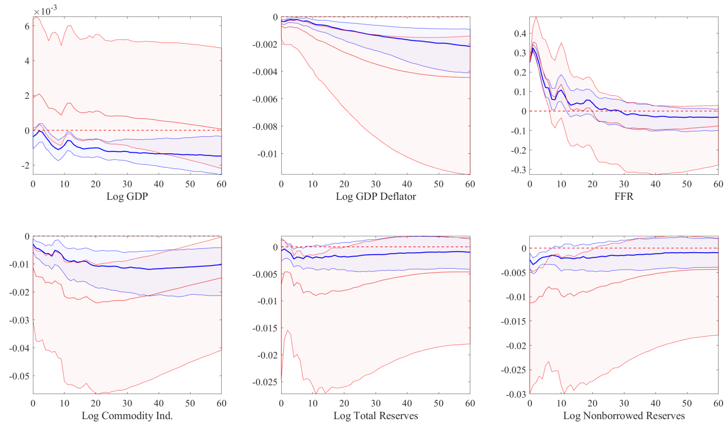

Figure 3: Responses to contractionary monetary policy shocks formed from $SQ \in \mathcal{P}^*_{99th}$ (in blue) and under Restriction SR (in red).

Notes: Monetary policy shocks normalized to induce a 0.25% rise in ff_t. The solid line is the point-wise posterior median response and the shaded bands are the 68% equal-tailed point-wise posterior probability bands.

Finally, Figure 3 shows the IRFs formed from $SQ \in \mathcal{P}^*_{99th}$. The set P^*_{99th}, because of the stricter restrictions, consists of only 227 elements. The associated monetary policy shocks are uncorrelated with Greenbook projections and exhibit a correlation with $FF4_t$ larger than 0.21. Compared to the previous cases, monetary tightenings lower output in a shorter time and with a larger magnitude. External variable constraints appear thus to greatly mitigate the ambiguity surrounding Uhlig's (2005) findings: when Restriction SR is combined with Restriction ER, monetary contractions are unequivocally found to reduce output.[5] Coherently with the point raised by Wolf (2020), the results obtained by Uhlig (2005) seem instead to be driven by a misidentification of the monetary policy shocks. As detailed in Section 4, 73% of the shocks recovered through Restriction SR are in fact found to be correlated with the information set of the Fed, while about 28% of them negatively comove with $FF4_t$.

[5] As shown in Appendix A.2, this result is still valid under only Restriction ER.

4 Relationship With Alternative Set-Identification Strategies

This section relates my approach to narrative sign restrictions (Antolín-Díaz and Rubio-Ramírez, 2018) and to restrictions on the monetary policy equation (Arias et al., 2019). Similarly to my identification strategy, they are thought as refinements to standard sign restrictions and are thus implemented on the model specification detailed in Section 3.

First, I show that the structural models recovered by Restriction SR and ER satisfy their identifying assumptions. This is a crucial result, since it implies that my approach ensures: (i) narrative consistency of monetary policy shocks; (ii) Taylor-rule consistency of monetary policy equations. Second, I test whether the shocks identified through these alternative methods meet Restriction ER. Uncorrelation with the Fed's information set and large correlation with monetary surprises should in fact characterize any plausible measure of monetary policy shock, regardless of the approach used to recover it.[6]

4.1 Identification by Narrative Sign Restrictions

Narrative sign restrictions were introduced by Antolín-Díaz and Rubio-Ramírez (2018), who combine them with Restriction SR. In particular, on the occasion of key historical episodes, they constrain the sign of the monetary policy shocks ε_t^m and the magnitude of their contribution to the historical decomposition of the federal funds rate $f\!f_t$.

More formally, at any t, I can approximate the infinite-order structural VMA representation as follows,

$$y_t = \sum_{h=0}^{t-1} \Theta_h \varepsilon_{t-h} \tag{13}$$

where $\Theta_h = C_h P$ and $e_t = P\varepsilon_t$. Let the federal funds rate $f\!f_t$ and the monetary policy shock ε_t^m be, respectively, the third and first entry of y_t and ε_t. The contribution of ε_t^m

[6] A comparison between the IRFs obtained using my identification strategy and those derived using these two alternative approaches is contained in Appendix A.4.

to the observed unexpected change in ff_t at time t can then be expressed as

$$H^{\mathit{ff}}_{m,t} = \Theta_{0,11}\varepsilon^m_t \tag{14}$$

The anti-inflationary reform adopted by the Fed's chairman Paul Volcker starting in October 1979 is the main historical event exploited by Antolín-Díaz and Rubio-Ramírez (2018). On this episode, they impose the following restrictions.

Restriction NR1. The monetary policy shock ε^m_t for the observation corresponding to October 1979 must be of positive value.

Restriction NR2. In October 1979, the absolute value of $H^{\mathit{ff}}_{m,t}$ is larger than the sum of the absolute value of the contributions of all other structural shocks.

Alternatively, they impose Restriction NR3 and NR4 on a wider set of events for which there is a reasonable agreement that an important monetary policy shock occurred.

Restriction NR3. The monetary policy shock ε^m_t must be positive for the observations corresponding to April 1974, October 1979, December 1988, and February 1994, and negative for December 1990, October 1998, April 2001, and November 2002.

Restriction NR4. For the periods specified in Restriction NR3, the absolute value of $H^{\mathit{ff}}_{m,t}$ is larger than the absolute value of the contribution of any other structural shock.

4.2 Narrative Consistency of Monetary Policy Shocks

Below, I compute the percentages of monetary policy shocks formed from $SQ \in \mathcal{P}^*_{99th}$ that satisfy Restriction NR1, NR2, NR3 and NR4. By doing so, I can evaluate whether they are consistent or not with the historical reading of the times performed by Antolín-Díaz and Rubio-Ramírez (2018).

As shown in Table 1, all the monetary policy shocks recovered through my approach are positive on the Volcker episode. On the same date, as required by Restriction NR2, 90% of them is also the overwhelming driver of unexpected changes in the federal funds rate. This is a key result, since Antolín-Díaz and Rubio-Ramírez (2018) consider the Volcker disinflation as the clearest example of an exogenous shock in the postwar period. Overall, the monetary policy shocks formed from $SQ \in \mathcal{P}^*_{99th}$ are also consistent with the broader set of events in Restriction NR3 and NR4. However, the December 1988 and February 1994 episodes are two exceptions that is worth exploring in greater detail.

Restriction	**1974:4**	**1979:10**	**1988:12**	**1990:12**	**1994:2**	**1998:10**	**2001:4**	**2002:11**
NR1	-	100.0%	-	-	-	-	-	-
NR2	-	90.3%	-	-	-	-	-	-
NR3	99.6%	100.0%	40.0%	94.3%	68.8%	100.0%	100.0%	100.0%
NR4	96.5%	97.4%	32.6%	71.8%	45.0%	99.1%	95.1%	88.6%

Table 1: % of ε^m_t formed from $SQ \in \mathcal{P}^*_{99th}$ satisfying Restriction NR1, NR2, NR3 and NR4

A close scrutiny of Greenbook forecasts and FOMC meetings minutes suggests that, rather than being exogenous shocks, the federal funds rate hikes occurred in December 1988 and February 1994 may indeed constitute the endogenous response of the Fed to good news about future economic conditions.[7] The rise in the policy rate in December 1988 is in fact paired with upward revisions in the nowcast and one-quarter ahead forecast for output growth that cast more than a doubt on the exogeneity of the monetary tightening. In a similar way, the minutes of the FOMC meeting held in February 1994 state that the policy tightening was undertaken based on the confidential access to 'optimistic' employment data that were not available when the Greenbook was prepared.

4.3 Identification by Restrictions on the Monetary Policy Equation

The use of restrictions on the coefficients of the monetary policy equation was proposed by Arias et al. (2019). Denoting the (i, j)-element of $A = (SQ)^{-1}$ by a_{ij}, the structural

[7] This argument is also outlined in Antolín-Díaz and Rubio-Ramírez's (2018) Appendix C. However, given the magnitude of the federal funds rate increase, they nevertheless consider these two episodes as monetary policy shocks.

monetary policy equation can be expressed as follows:

$$ff_t = \phi_{gdp} gdp_t + \phi_{pi} pi_t + \phi_{ci} ci_t + \phi_{tr} tr_t + \phi_{nr} nr_t + \sigma \varepsilon_t^m \tag{15}$$

where $\phi_{gdp} = -\frac{a_{11}}{a_{13}}$, $\phi_{pi} = -\frac{a_{12}}{a_{13}}$, $\phi_{ci} = -\frac{a_{14}}{a_{13}}$, $\phi_{tr} = -\frac{a_{15}}{a_{13}}$, $\phi_{nr} = -\frac{a_{16}}{a_{13}}$ and $\sigma = a_{13}$.

Supported by a large literature on Taylor-type rules, they achieve identification by imposing the following zero and sign restrictions on the coefficients in (15).

Restriction TR1. The federal funds rate is the monetary policy instrument and only reacts contemporaneously to output, prices and commodity prices. Thus, $\phi_{tr}, \phi_{nr} = 0$.

Restriction TR2. The contemporaneous reaction of the federal funds rate to output and prices is positive, that is ϕ_{gdp}, $\phi_{pi} > 0$.

4.4 Taylor-Rule Consistency of Monetary Policy Equations

In this subsection, I compute the monetary policy equations associated with $SQ \in \mathcal{P}^*_{99th}$ and check if they satisfy Restriction TR1 and TR2. This allows to assess whether they are reconcilable or not with Taylor-type monetary policy rules.

As a benchmark, I first compute the posterior median estimates and the 68% probability interval for the coefficients in equation (15) when only Restriction SR is enforced. As shown in Table 2, the results are in this case rather puzzling. At odds with Restriction TR2, the median estimate for ϕ_{gdp} is negative. Moreover, the posterior estimates

Coefficient	ϕ_{gdp}	ϕ_{pi}	ϕ_{ci}	ϕ_{tr}	ϕ_{nr}
Median	-0.38	1.90	0.11	0.09	0.04
68% Prob. Interval	[-2.42;0.82]	[-0.03;6.00]	[0.00;0.35]	[-0.44;0.64]	[-0.40;0.65]

Table 2: Coefficients in the monetary policy equations formed from $SQ \in \mathcal{P}$.

Notes: The entries are the posterior median estimates of the coefficients in the monetary policy equations (15) formed from $SQ \in \mathcal{P}$. The 68% equal-tailed posterior probability interval is reported in brackets.

for ϕ_{tr} and ϕ_{nr} do not exclude large values. Even though the median is close to zero, the 68% interval is in fact quite large and ranges up to about 0.65. The median for ϕ_{pi} is instead positive and thus consistent with Restriction TR2: however, the estimates are quite imprecise and negative values cannot be completely ruled out.

Table 3 displays the coefficients of the monetary policy equation that I derive when Restriction SR is combined with Restriction ER. In this case, they are fully reconcilable with Taylor-type rules. As required by Restriction TR2, the median estimates for ϕ_{gdp}

Coefficient	ϕ_{gdp}	ϕ_{pi}	ϕ_{ci}	ϕ_{tr}	ϕ_{nr}
Median	0.30	1.04	0.03	0.03	-0.03
68% Prob. Interval	[0.11;0.55]	[0.61;1.57]	[0.00;0.06]	[-0.07;0.11]	[-0.10;0.07]

Table 3: Coefficients in the monetary policy equations formed from $SQ \in \mathcal{P}^*_{99th}$.

Notes: The entries are the posterior median estimates of the coefficients in the monetary policy equations (15) formed from $SQ \in \mathcal{P}$. The 68% equal-tailed posterior probability interval is reported in brackets.

and ϕ_{pi} are positive and the 68% probability interval entirely excludes negative values. This result is coherent with the conduct of an inflation-targeting central bank that rises the interest rate to prevent an overheating economy or dampen inflationary pressures. Finally, ϕ_{tr} and ϕ_{nr} are narrowly concentrated around zero and thus consistent with Restriction TR1.

4.5 Alternative Set-Identification Strategies and the Fed's Information Set

Any truly exogenous measure of monetary policy shock should be uncorrelated with the Fed's information set about current and future economic conditions. Below, I identify 1000 shocks ε_t^m through narrative sign restrictions (Restriction SR, NR1 and NR2) and restrictions on the monetary policy equation (Restriction TR1 and TR2) and I project them on the Greenbook by running regression (12) over the period 1990:M1-2007:M11. Then, I test the null of joint nonsignificance of the estimated coefficients the 5% level.[8]

[8] I could perform this test on a larger sample since Greenbook forecasts for CPI inflation are available from 1980. I run it from 1990 to 2007 to ensure consistency with the period on which constraint (ER2)

As a benchmark, I perform the same analysis on a set of 1000 shocks identified through standard sign restrictions (Restriction SR).

Table 4 shows the percentages of acceptance and rejection. 73% of the monetary policy shocks identified by Restriction SR are correlated with the Greenbook forecasts.

F-test result	**SR**	**SR+NR1+NR2**	**TR1+TR2**
Rejection	73.1%	64.0%	51.4%
Acceptance	26.9%	36.0%	48.6%

Table 4: % of acceptances and rejections of the null of joint nonsignificance of the coefficients of equation (12), 1990:M1-2007:M11.

Hence, they embody the Fed's response to the expected future state of the economy and their exogeneity is therefore called into question. The additional use of Restriction NR1 and NR2 partially helps in mitigating this issue and, as a consequence, the rejection rate falls to 64%. The imposition of Restriction TR1 and TR2 is found to overperform these two methods but, also in this case, the results are far from being satisfactory. About half of the shocks are in fact correlated with the Fed's information set.

4.6 Alternative Set-Identification Strategies and Monetary Surprises

In this subsection, I verify whether monetary policy shocks identified by narrative sign restrictions and restrictions on the monetary policy equation display a strong positive comovement with monetary surprises. To this end, I derive the correlation coefficients ρ_m between the 1000 shocks recovered by each identification scheme and $FF4_t$. Then, I check if they are higher than 0 and 0.2, that is the minimum correlation required for SQ to be accepted in the set of solutions $\mathcal{P}^*_{99th}$. By setting it as a threshold, I can thus assess whether these methods deliver monetary policy shocks whose correlation with $FF4_t$ is comparable to that ensured by the approach I implement.

is imposed. Increasing the sample size does not change the results displayed in Table 4.

As displayed in Table 5, 24% of the shocks recovered by Restriction SR negatively comoves with $FF4_t$. Even when positive, the correlation is weak and larger than 0.2 in only 1% of the cases. The additional imposition of Restriction NR1 and NR2 seems to be quite effective in mitigating this issue and all the identified shocks positively comove

ρ_m	**SR**	**SR+NR1+NR2**	**TR1+TR2**
> 0	76.2%	100.0%	99.9%
> 0.2	1.1%	8.9%	10.3%

Table 5: % of monetary policy shocks such that ρ_m is larger than 0 and 0.2, 1990:M1-2007M11

with $FF4_t$. However, the correlation is overall low and only 8.9% of the shocks would meet the threshold implied by constraint (ER1). Similar findings hold for the monetary policy shocks retrieved through Restriction TR1 and TR2.

5 Am I Controlling for the Central Bank Information Channel?

In this section, I evaluate whether the use of Restriction SR and ER succeeds in controlling for the central bank information channel. The logic behind the tests I perform is that 'true' contractionary monetary policy shocks should be associated with a drop in the stock market (e.g. Jarociński and Karadi, 2020). The comovement should instead be positive if the increase in the federal funds rate is related to the disclosure of good news from the Fed about future economic conditions.

ρ_s	**SR**	**SR+NR1+NR2**	**TR1+TR2**	**SR+ER**
< 0	45.7%	81.2%	85.6%	100.0%
> 0	54.3%	18.8%	14.4%	0%

Table 6: Percentages of shocks whose correlation with S&P 500 surprises is < 0 and > 0, 1990:M1-2007:M11.

First, I derive the correlation coefficients ρ_s between the monetary policy shocks ε_t^m formed from $SQ \in \mathcal{P}^*_{99th}$ and stock market surprises SPI_t^{hf}, measured as the changes in the S&P 500 over a 30-minute window around each FOMC announcement. The results in Table 6 appear to support the use of external variable constraints. All the shocks identified through Restriction SR and ER are in fact found to negatively comove with SPI_t^{hf}. On the other hand, non-negligible shares of shocks recovered through alternative approaches exhibit a positive comovement with stock market surprises.

Second, I use local projections to derive the IRFs of US stock prices to contractionary monetary policy shocks. Denoting by $\hat{\varepsilon}_t^{m,i}$ the i-th shock formed from $SQ \in \mathcal{P}^*_{99th}$, I estimate the following regression at the monthly frequency:

$$SPI_{t+h} = \gamma_i^{(h)} + \sum_{l=1}^{2} \alpha_{l,i}^{(h)} SPI_{t-l} + \sum_{j=0}^{5} \beta_{j,i}^{(h)} \hat{\varepsilon}_{t-j}^{m,i} + u_{t+h,i} \tag{16}$$

where $h = 0, \ldots, 48$ and $i = 1, \ldots, 227$. SPI_t is the log of the US share price index produced by the OECD and computed as the average of daily closing data. The estimated parameter $\hat{\beta}_{0,i}^{(h)}$ is the impulse response of SPI_t at time $t+h$ to the i-th identified shock. For each horizon h, I then compute the median response and the 68% equal-tailed credibility interval by calculating the appropriate percentiles of the set of impulse responses $\{\hat{\beta}_{0,1}^{(h)}, \ldots, \hat{\beta}_{0,227}^{(h)}\}$. As shown in Figure 4, US stock prices are found to significantly drop

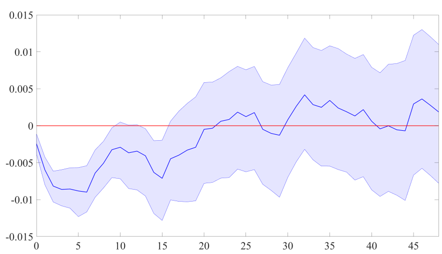

Figure 4: Response of SPI_t to contractionary monetary policy shocks formed from $\mathcal{P}^*_{99th}$.

Notes: The solid line is the median response and the shaded bands are the 68% equal-tailed probability bands. For each horizon h, they are computed point-wise by using the set of impulse responses estimated from (16).

after a monetary contraction. The response is negative on impact and reaches its minimum after a few months. These findings seem to be consistent with the propagation of 'true' contractionary monetary policy shocks, rather than with the disclosure of Fed's private information about future economic conditions.

6 Robust Bayesian Inference

So far, in line with Antolín-Díaz and Rubio-Ramírez (2018) and Arias et al. (2019), I have imposed Restriction SR and ER through Rubio-Ramìrez et al.'s (2010) algorithm. As detailed in Appendix B, it is based on the QR decomposition and assumes a uniform distribution of Q (the so-called Haar prior) on the space of orthonormal matrices $\mathcal{O}(k)$. However, since the likelihood is not dependent on Q, this does not imply that structural parameters are uniformly distributed over the identified set (Baumeister and Hamilton, 2015). In other words, the prior for Q is not updated by the data and, even if uniform, it is informative for objects of interest as IRFs.

In this section, I address this issue by combining numerical methods for constrained optimization with standard sampling from the posterior to compute the infimum and supremum of the IRFs over all admissible rotation matrices Q. Specifically, I build on the algorithm proposed by Volpicella (2022), by adapting it to the case in which identification is achieved by: (i) traditional sign restrictions and external variable constraints (Restriction SR and ER); (ii) traditional sign restrictions and narrative restrictions on monetary policy shocks and historical decomposition (Restriction SR, NR1 and NR2); (iii) restrictions on the monetary policy equation (Restriction TR1 and TR2).

When inference is performed through the Haar prior, these three methodologies lead to similar conclusions about the transmission of contractionary monetary policy shocks. Examining the results under the prior-robust inference algorithm, I can therefore assess whether these findings are still valid when the IRFs do not depend on a specific prior for Q. For the rest of this section, let the monetary policy shock ε_t^m be the first entry of the $k \times 1$ vector ε_t and let $d'_{ih}(\phi)$ denote the i-th row of the $k \times k$ matrix $D_h = C_h S$, where C_h denotes the reduced-form impulse responses at horizon h and S is the unique lower-triangular Cholesky factor.

6.1 Algorithm 1: Sign Restrictions and External Variable Constraints

Algorithm 1 describes the procedure to obtain the prior-robust set of impulse responses of variable i to contractionary monetary policy shocks ε_t^m when Restriction SR is combined with Restriction ER.[9]

Algorithm 1

1. Draw $\omega = (\Sigma_e, A)$ from the posterior distribution of the reduced-form VAR and a $k \times (k-1)$ matrix W, whose columns are derived from a standard multivariate normal distribution $\mathcal{N}(0_{k\times 1}, I_{k-1})$.

2. Check if the following optimization problems have solutions q_1^* at any horizon h:

$$\min_{q_1} \text{ and } \max_{q_1} \; d'_{ih}(\phi) q_1 \qquad \text{subject to:}$$

$$(i)\ S_1(\phi) q_1 \geq 0$$
$$(ii)\ corr(\hat{\varepsilon}_t^m(SQ), FF4_t) > k$$
$$(iii)\ corr(\hat{\varepsilon}_t^m(SQ), FI_t) = 0$$
$$(iv)\ ||q_1|| = 1$$

   where, by abuse of notation, $S_1(\phi)q_1$ denotes the sign restrictions in Restriction SR and Q is an orthonormal matrix. At each horizon h, the latter is derived by applying the QR decomposition to the $k \times k$ matrix $\bar{W}$, which is the concatenation of the candidate solutions q_1 and the matrix W.

[9] Computational details are provided in Appendix B.

3. If Step 2 is satisfied, store the impulse response functions derived using the solutions q_1^* in the sets $\hat{\Theta}_{i,h}^{min}$ and $\hat{\Theta}_{i,h}^{max}$. Otherwise, go back to Step 1.

4. Repeat Steps 1-3 M times.

In Step 2, I compute the QR decomposition of $\bar{W}$, obtained as the concatenation of the candidate solution q_1 and matrix W. This delivers an orthonormal matrix Q that preserves q_1 as first column and that instead draws, as argued by Baumeister and Hamilton (2015), the elements of q_j, for $j = 2, \ldots, k$, from a nonuniform distribution. This prior, importantly, is however not informative about the response of variable i to ε_t^m, that is my unique object of interest. The latter, in fact, only depends on the elements of q_1, that Algorithm 1 selects as solutions to the optimization problems.

Below, I implement Algorithm 1 by drawing from the posterior of the reduced-form VAR described in Section 3. To ensure comparability, the parameter g is calibrated at the 99^{th} percentile of the set of correlation coefficients between $FF4_t$ and the monetary policy shocks formed from $SQ \in \mathcal{P}$. Since external variable constraints considerably truncate the admissible support of Q, this algorithm is computationally quite demanding. Hence, I narrow my focus on the output response and set $M = 1000$. In Figure 5, I compare the 68% equal-tailed credibility region obtained under a uniform prior for Q

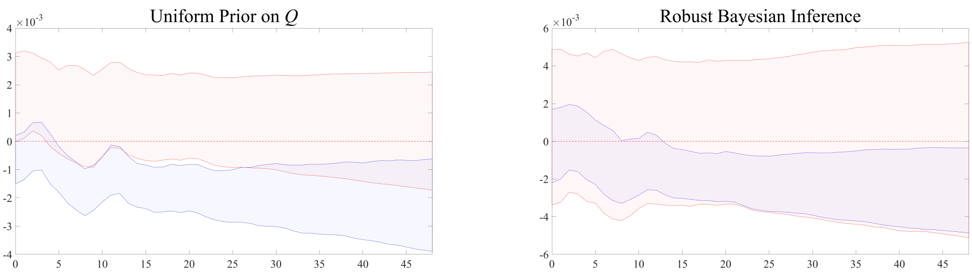

Figure 5: 68% equal-tailed credibility interval for output response using Restriction SR and ER (in blue) and using Restriction SR (in red).

(on the left) and robust Bayesian inference (on the right). Unlike in Section 3, monetary policy shocks are not normalized to induce a 0.25% increase in $f\!f_t$. The resulting intervals may in fact be unbounded when the structural parameter $\hat{\Theta}_{0,31} = d'_{30} q_1^*$ is not bounded away from zero for all ω and q_1^*. Importantly, the findings achieved by running standard inference are equally valid when inference is performed through Algorithm 1: although the bands in the right panel are wider than those in the left one, contractionary monetary policy shocks are still found to induce a significant decrease in output. Under robust Bayesian inference, standard sign restrictions deliver instead impulse responses with even more contradictory economic implications and are thus totally uninformative about the transmission of US monetary policy.

6.2 Algorithm 2: Sign Restrictions and Narrative Sign Restrictions

Algorithm 2 describes the procedure to obtain the prior-robust set of impulse responses of variable i to contractionary monetary policy shocks ε_t^m when Restriction SR is combined with Restriction NR1 and NR2.

Algorithm 2

In Algorithm 1, replace Step 2 with the following.

2. Check if the following optimization problems have solutions q_1^* at any horizon h:

$$\min_{q_1} \text{ and } \max_{q_1} \; d'_{ih}(\phi) q_1 \qquad \text{subject to:}$$

$$(i)\; S_1(\phi) q_1 \geq 0$$

$$(ii)\; \varepsilon_t^m(SQ) > 0 \quad \text{for } t\text{=1979:10}$$

$$(iii)\; H^{f\!f}_{1,t}(SQ) > \textstyle\sum_{j=2}^{k} H^{f\!f}_{j,t}(SQ) \quad \text{for } t\text{=1979:10}$$

$$(iv)\; ||q_1|| = 1$$

   where $S_1(\phi) q_1$ denotes the sign restrictions described in Restriction SR; Q is an orthonormal matrix derived by computing the QR decomposition of an auxiliary matrix $\bar{W}$ with the candidate solution q_1 as first column and the remaining $k-1$ columns drawn from a $\mathcal{N}(0_{k\times 1}, I_{k-1})$; $H^{ff}_{i,t}$, for $i = 1, \ldots, k$, denotes the contribution of shock i in explaining the historical decomposition of ff_t for observation t.

Below, I use Algorithm 2 by drawing from the posterior distribution of the reduced-form VAR described in Section 3. In particular, Figure 6 compares the 68% prior-robust equal-tailed credibility region for output response with the counterpart derived under a uniform prior for Q. In the latter case, the effects of contractionary monetary policy shocks on output are significantly negative. If compared to those obtained by using my

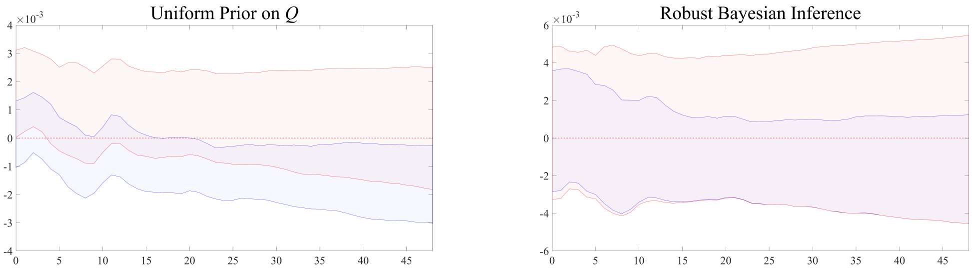

Figure 6: 68% equal-tailed credibility interval for output response using Restriction SR, NR1 and NR2 (in blue) and using Restriction SR (in red).

identification scheme, these effects are smaller and statistically significant with a greater delay. Furthermore, they vanish when the uniform prior on Q is replaced by Algorithm 2: the robust credibility interval includes in fact zero at all horizons.

6.3 Algorithm 3: Restrictions on the Monetary Policy Equations

Algorithm 3 describes the procedure to obtain the prior-robust set of impulse responses of variable i to contractionary monetary policy shocks when identification is achieved through Restriction TR1 and TR2.

Algorithm 3

In Algorithm 1, replace Step 2 with the following.

2. Check if the following optimization problems have solutions q_1^* at any horizon h:

$$\min_{q_1} \text{ and } \max_{q_1} \; d'_{ih}(\phi)q_1 \qquad \text{subject to:}$$

$$(i)\; d'_{31}q_1 \geq 0$$

$$(ii)\; \phi_{gdp}(SQ) > 0,\; \phi_{\pi}(SQ) < 0,\; \phi_{tr}(SQ) = \phi_{nr}(SQ) = 0$$

$$(iii)\; ||q_1|| = 1$$

where Q is an orthonormal matrix derived by computing the QR decomposition to an auxiliary matrix $\bar{W}$ having the candidate solution q_1 as first column and the remaining $k-1$ columns drawn from a $\mathcal{N}(0_{k\times 1}, I_{k-1})$.

In order to ensure that those estimated are indeed responses to contractionary monetary policy shocks, the constraint in (i) requires the solutions q_1^* to induce a contemporaneous increase in the federal funds rate, that is the third variable in the system.

After sampling from the posterior distribution of the reduced-form VAR introduced in Section 3, I derive the 68% equal-tailed credibility region for output response through Algorithm 3 and I compare it with the one obtained under a uniform prior for Q. Under

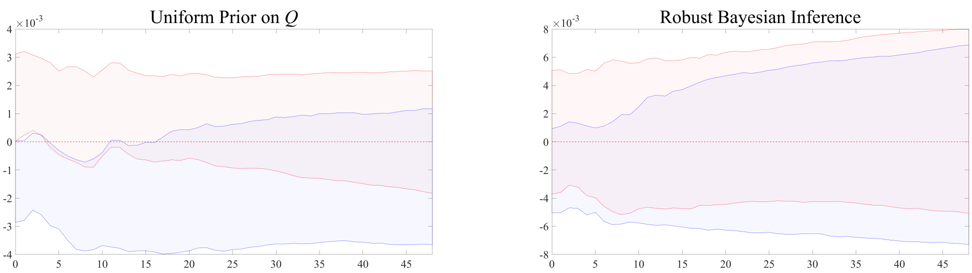

Figure 7: 68% equal-tailed credibility interval for output response using Restriction TR1 and TR2 (in blue) and using Restriction SR (in red).

standard inference, contractionary monetary policy shocks are found to have significant negative effects on output. Differently from the findings discussed in the previous two subsections, these effects are quite short-lived, with the peak that occurs about eight months after the shock. However, when inference is performed by using Algorithm 3, the impact of monetary tightenings is found to be far more ambiguous and not fully consistent with negative effects on output.

7 Conclusion

This paper identifies monetary policy shocks through a combination of sign restrictions on the impulse responses (Uhlig, 2005) and external variable constraints on Greenbook projections and high-frequency monetary surprises.

I employ this approach to evaluate the transmission of US monetary policy over the period 1965:M1-2007:M11. The imposition of external variable constraints considerably mitigates the ambiguity surrounding Uhlig's (2005) findings and monetary contractions are unequivocally found to reduce output. Importantly, these results are still valid when the uniform prior on the rotation matrix Q is replaced by the robust Bayesian inference procedure detailed in Section 6. These effects are larger and more persistent than those derived by imposing narrative sign restrictions (Antolín-Díaz and Rubio-Ramírez, 2018) and restrictions on the monetary policy equation (Arias et al., 2019). Furthermore, the shocks recovered through these alternative methods turn out to be correlated with the Fed's information set about current and future economic conditions and poorly correlated with monetary surprises. On the other hand, the use of external variable constraints delivers monetary policy shocks and monetary policy equations that are reconcilable, respectively, with an historical reading of the times and Taylor-type rules.

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A Robustness Checks

A.1 Imposing the Greenbook Constraint at the FOMC Meeting Frequency

As discussed in Section 3, enforcing constraint (ER2) at the monthly frequency requires inevitable assumptions about the timing with which the Fed updates its information set. In this section, I show the results obtained when constraint (ER2) is instead imposed at the FOMC meeting frequency. This approach does not involve any frequency conversion but considerably reduces the number of observations used to estimate regression (12).

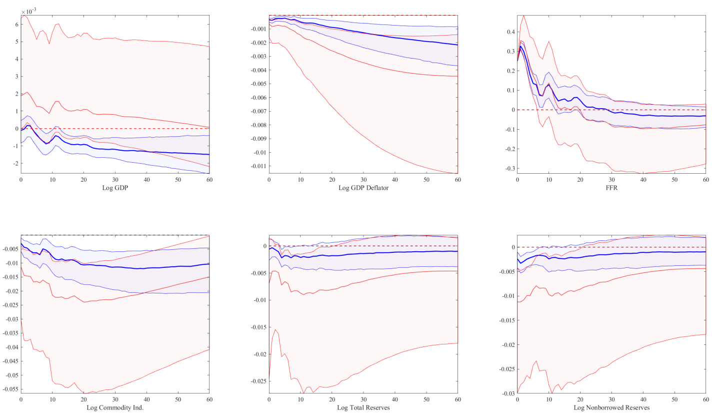

Figure A.1: Response to contractionary monetary policy shocks formed from $SQ \in \mathcal{P}^{*,mf}_{99th}$ (in blue) and under Restriction SR (in red)

Notes: Monetary policy shocks normalized to induce a 0.25% rise in $f\!f_t$. The solid line is the point-wise posterior median response and the shaded bands are the 68% equal-tailed point-wise posterior probability bands.

Figure A.1 compares the IRFs derived under Restriction SR and those formed from the set of solutions $P^{*,mf}_{99th}$. The latter collects the 429 matrices SQ meeting Restriction SR and ER when constraint (ER2) is imposed at the FOMC meeting frequency. Importantly, the output response is basically unchanged compared to the one plot in Figure 3. Table A.1 shows instead the percentages of shocks formed from $SQ \in P^{*,mf}_{99th}$ that satisfy the restrictions imposed by Antolín-Díaz and Rubio-Ramírez (2018). Again, the results are very similar to those in Section 3, thus showing that the narrative consistency of the monetary policy shocks is robust to the time frequency with which constraint (ER2) is enforced. A similar conclusion holds true for the Taylor-rule consistency of the structu-

Restriction	**1974:4**	**1979:10**	**1988:12**	**1990:12**	**1994:2**	**1998:10**	**2001:4**	**2002:11**
NR1	-	100.0%	-	-	-	-	-	-
NR2	-	90.0%	-	-	-	-	-	-
NR3	99.5%	100.0%	47.5%	96.5%	57.3%	100.0%	99.8%	99.3%
NR4	94.6%	96.7%	37.7%	78.5%	44.8%	94.6%	89.7%	87.7%

Table A.1: % of ε_t^m formed from $SQ \in \mathcal{P}_{99th}^{*,mf}$ satisfying Restriction NR1, NR2, NR3 and NR4

ral monetary policy equations. As displayed in Table A.2, the coefficients formed from $SQ \in \mathcal{P}_{99th}^{*,mf}$ are in fact overall consistent with Restriction TR1 and TR2.

Coefficient	$\boldsymbol{\phi_{gdp}}$	$\boldsymbol{\phi_{pi}}$	$\boldsymbol{\phi_{ci}}$	$\boldsymbol{\phi_{tr}}$	$\boldsymbol{\phi_{nr}}$
Median	0.21	1.06	0.03	0.03	-0.02
68% Prob. Interval	[-0.02;0.46]	[0.65;1.59]	[0.01;0.06]	[-0.07;0.12]	[-0.10;0.06]

Table A.2: Coefficients in the monetary policy equations formed from $SQ \in \mathcal{P}_{99th}^{*,mf}$.

Notes: The entries in the table are the posterior median estimates of the coefficients in the monetary equations (15) formed from $SQ \in \mathcal{P}_{99th}^{*,mf}$. The 68% equal-tailed posterior probability interval is reported in brackets.

A.2 IRFs Using Only Restriction ER

This section presents the IRFs obtained when Restriction SR is dropped and identification is achieved through only Restriction ER.

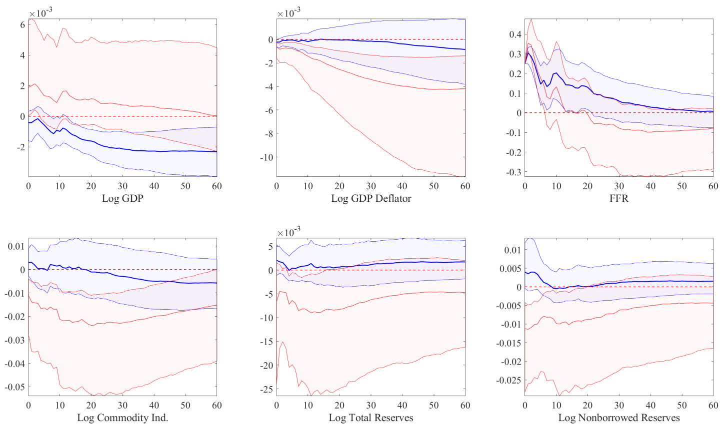

Figure A.2: Response to contractionary monetary policy shocks formed from $\bar{\mathcal{P}}^*_{99th}$ (in blue) and under Restriction SR (in red).

Notes: Monetary policy shocks normalized to induce a 0.25% rise in $f\!f_t$. The solid line is the point-wise posterior median response and the shaded bands are the 68% equal-tailed point-wise posterior probability bands.

As in Section 3, I first generate 100000 candidate structural impact matrices SQ that are then stored into the set of solutions $\bar{\mathcal{P}}$. In this case, they are not required to satisfy Restriction SR but only to ensure that the resulting monetary policy shock $\hat{\varepsilon}^m_t(SQ)$ has a positive impact effect on $f\!f_t$. In the second step, I only retain the matrices $SQ \in \bar{\mathcal{P}}$ such that $\hat{\varepsilon}^m_t(SQ)$ meet Restriction ER.

Restriction ER. Over the period 1990:M1-2007:M11, a monetary policy shock ε^m_t satisfies the following external variable constraints:

$$corr(\varepsilon^m_t, FF4_t) > g \tag{ER1}$$

$$corr(\varepsilon^m_t, FI_t) = 0 \tag{ER2}$$

For the sake of comparability, I set the parameter g equal to the 99^{th} percentile value of the set of correlation coefficients between $FF4_t$ and the shocks $\hat{\varepsilon}^m_t(SQ)$ formed from

$SQ \in \bar{\mathcal{P}}$ (this implies $k = 0.23$). The matrices SQ that deliver monetary policy shocks $\hat{\varepsilon}_t^m(SQ)$ satisfying Restriction ER are finally stored into the set of solutions $\bar{\mathcal{P}}^*_{99th}$. The resulting IRFs are displayed in Figure A.2, where I compare them with those obtained under Restriction SR. Consistently with the results in Section 3, output is found to negatively react in response to contractionary monetary policy shocks.

A.3 IRFs Using Minimal External Variable Constraints

In this section, I display the IRFs obtained by keeping Restriction SR binding and by alternatively imposing constraints (ER1) and (ER2).

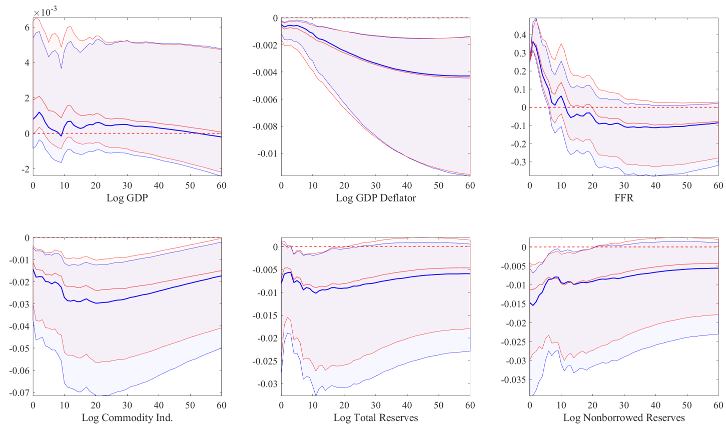

Figure A.3: Responses to contractionary monetary policy shocks formed from $SQ \in \mathcal{P}_f^*$ (in bliue) and under Restriction SR (in red).

Notes: Monetary policy shocks normalized to induce a 0.25% rise in $f\!f_t$. The solid line is the point-wise posterior median response and the shaded bands are the 68% equal-tailed point-wise posterior probability bands.

Figure A.3 plots the IRFs derived if only constraint (ER2) is added to Restriction SR. The resulting set of solutions is denoted by $\mathcal{P}_f^*$ and counts 27508 elements. Despite it

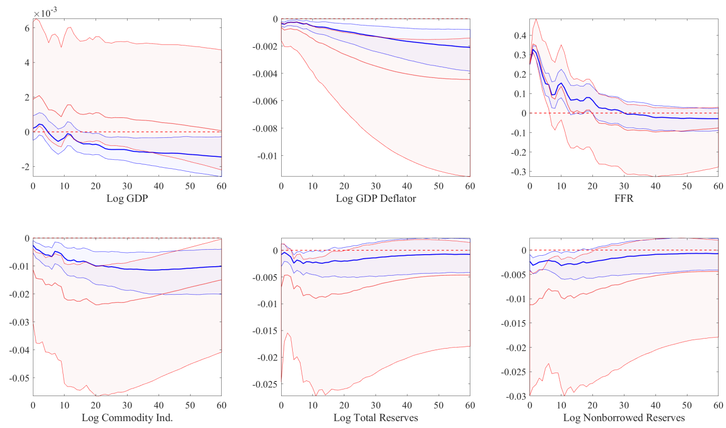

Figure A.4: Responses to contractionary monetary policy shocks formed from $SQ \in \mathcal{P}_m^*$ (in blue) and under Restriction SR (in red).

Notes: Monetary policy shocks normalized to induce a 0.25% rise in $f\!f_t$. The solid line is the point-wise posterior median response and the shaded bands are the 68% equal-tailed point-wise posterior probability bands.

shifts towards negative values, the set of output responses is similar to that obtained under only Restriction SR. Figure A.4 plots instead the IRFs if only constraint (ER1) (with g set at the 99^{th} percentile) is added to Restriction SR. The contractionary shocks formed from the resulting set of solutions $\mathcal{P}_m^*$ have quite ambiguous effects on output in the very short-run: although not statistically significant, the median response is positive on impact. This finding may lend itself to the following interpretation. When constraint (ER2) is not binding, the identified set also includes shocks that are correlated with the Fed's information set. The output increase is thus consistent with a scenario in which the Fed discloses good news about future economic conditions and, given its reaction function, tightens monetary policy to partly offset the expansionary effects of the news and prevent an overheating economy.

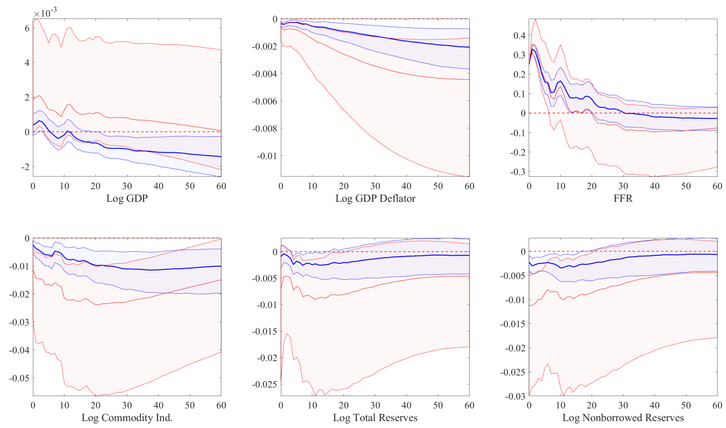

Figure A.5: Responses to contractionary monetary policy shocks formed from $\mathcal{P}_m^{*,info}$ (in blue) and under Restriction SR (in red).

Notes: Monetary policy shocks normalized to induce a 0.25% rise in ff_t. The solid line is the point-wise posterior median response and the shaded bands are the 68% equal-tailed point-wise posterior probability bands.

This argument is even clearer by looking at the IRFs, plotted in Figure A.5, formed from the set of solutions $\mathcal{P}_m^{*,info}$. The latter contains the 773 matrices SQ generating monetary policy shocks that meet constraint (ER1) (with g set at the 99^{th} percentile) but correlated with the Greenbook. The output response is in fact positive and, even if only weakly, statistically significant in the first few months after the shock. Summing up, constraints (ER1) and (ER2) are both necessary to obtain conventional effects of contractionary monetary policy shocks. Specifically, the exclusion of shocks correlated with the Fed's information set is crucial to rule out structural models whose short-run implications are compatible with the information channel of monetary policy.

A.4 Imposing Restriction SR and ER on a Different Model Specification

As already mentioned, I rely on the same external information as Miranda-Agrippino and Ricco (2021), who combine Greenbook projections and monetary surprises to derive an informationally robust instrument. When it is employed in a proxy-SVAR, monetary contractions are found to have unequivocally contractionary effects.

In this section, I check whether Restriction SR and ER induces analogous results when imposed on their model. The latter spans the period 1979:1-2014:12 and includes a constant as well as 12 lags of the following vector of US monthly series:

$$y_t' = \begin{bmatrix} ip_t & pi_t & ff_t & ci_t & u_t & ebp_t \end{bmatrix} \tag{A.1}$$

where ip_t is the log of industrial production, pi_t is the log of the consumer price index, ff_t is the federal funds rate, ci_t is the log of a commodity price index, u_t is the unemployment rate and ebp_t is Gilchrist and Zakrajšek's (2012) excess bond premium. I impose the following restrictions and apply the same procedure described in Section 3.

Restriction SR. A monetary policy shock ε_t^m leads to a negative response of pi_t and ci_t and to a positive response of ff_t at horizons $h = 0, \ldots, 5$.

Restriction ER. Over the period 1990:M1-2007:M11, a monetary policy shock ε_t^m satisfies the following external variable constraints:

$$corr(\varepsilon_t^m, FF4_t) > g \tag{ER1}$$

$$corr(\varepsilon_t^m, FI_t) = 0 \tag{ER2}$$

In particular, I focus on the case in which g is set equal to the 99^{th} percentile value of the set of correlation coefficients between $FF4_t$ and the shocks $\hat{\varepsilon}_t^m(SQ)$ formed from $SQ \in \mathcal{P}$. Out of the 100000 matrices stored in $\mathcal{P}$, the set of solutions $\mathcal{P}^*_{99th}$ retains 566 matrices SQ which deliver monetary policy shocks that are uncorrelated with the

Greenbook and that display a correlation with monetary surprises larger than 0.20 (the value of the 99^{th} percentile). As shown in Figure A.4, coherently with the results found by Miranda-Agrippino and Ricco (2021), contractionary monetary policy shocks turn out to reduce industrial production and increase the unemployment rate.

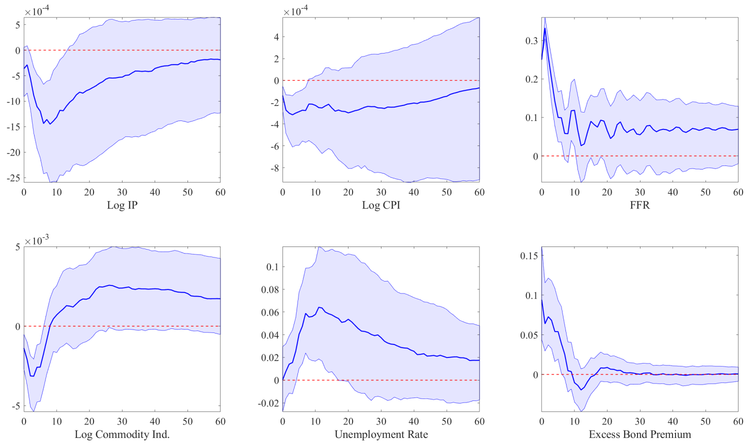

Figure A.6: Response to contractionary monetary policy shocks formed from $SQ \in \mathcal{P}^*_{99th}$ using Miranda-Agrippino and Ricco's (2021) model

Notes: Monetary policy shocks normalized to induce a 0.25% rise in $f\!f_t$. The solid line is the point-wise posterior median response and the shaded bands are the 68% equal-tailed point-wise posterior probability bands.

A.5 Comparison With IRFs From Alternative Set-Identification Strategies

In this section, I compare the IRFs formed from $SQ \in \mathcal{P}^*_{99th}$ with those obtained under narrative sign restrictions and restrictions on the monetary policy equation.

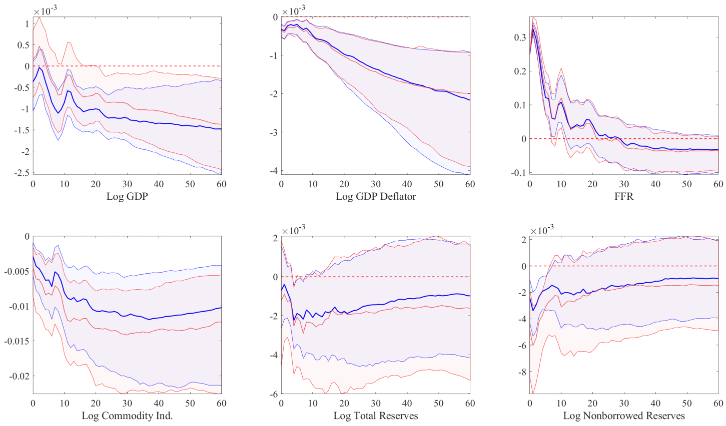

Figure A.7: Response to contractionary monetary policy shocks formed from $SQ \in \mathcal{P}^*_{99th}$ (in blue) and under Restriction SR, NR1 and NR2 (in red)

Notes: Monetary policy shocks normalized to induce a 0.25% rise in ff_t. The solid line is the point-wise posterior median response and the shaded bands are the 68% equal-tailed point-wise posterior probability bands.

As shown in Figure A.7, the output response derived under Restriction SR, NR1 and NR2 is quite similar to that obtained under Restriction SR and ER. In both cases,

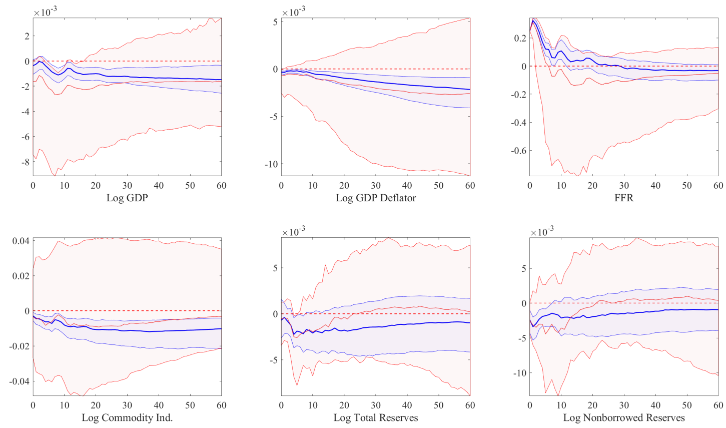

Figure A.8: Response to contractionary monetary policy shocks formed from $SQ \in \mathcal{P}^*_{99th}$ (in blue) and under Restriction TR1 and TR2 (in red)

Notes: Monetary policy shocks normalized to induce a 0.25% rise in ff_t. The solid line is the point-wise posterior median response and the shaded bands are the 68% equal-tailed point-wise posterior probability bands.

consistently with theoretical predictions, output significantly declines in response to a contractionary shock. Under Restriction SR and ER, these effects turn out to be larger and statistically significant with a shorter delay. Figure A.8 compares instead the IRFs formed from $SQ \in \mathcal{P}^*_{99th}$ with those derived through Restriction TR1 and TR2. First, note that in the latter case the 68% confidence intervals are found to be much wider, due to a larger identification uncertainty that makes inference less precise. Second, the negative effects of monetary policy shocks are only significant in the very short-run, with the peak occurring about 8 months after the shock.

A.6 Computing IRFs With Local Projections

A VAR model is a linear global approximation to the data-generating process that is optimally designed for one-period ahead forecasting. However, impulse responses are a function of forecasts at progressively distant horizons and misspecification errors are therefore compounded. Jordà (2005) tackles this issue by computing local projections to each forecast horizon, that are then combined to derive the IRFs.

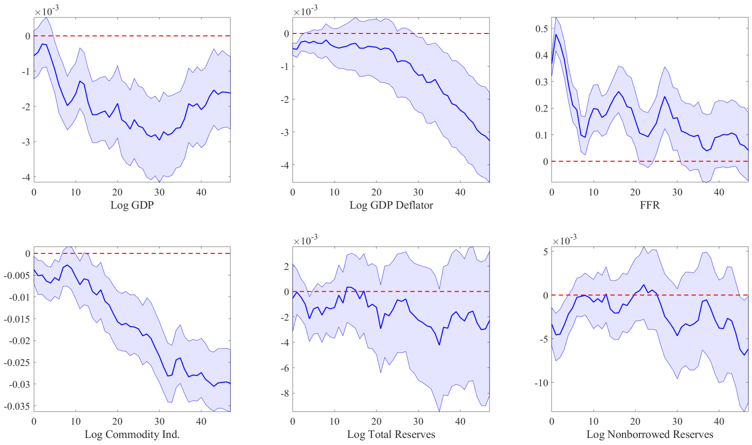

Figure A.9: Responses to contractionary monetary policy shocks formed from $SQ \in \mathcal{P}^*_{99th}$ in a local projections framework

Notes: The solid line is the median response and the shaded bands are the 68% equal-tailed probability bands. For each horizon h, they are computed point-wise by using the set of impulse responses estimated from (A.2).

In this section, I use local projections to evaluate the effects of the monetary policy shocks obtained from $\mathcal{P}^*_{99th}$ on each of the variables in (12). Specifically, denoting by $\hat{\varepsilon}^{m,i}_t$ the i-th shock formed from $SQ \in \mathcal{P}^*_{99th}$, I estimate the following regression at the monthly frequency:

$$y_{t+h} = \gamma^{(h)}_i + \sum_{l=1}^{2} \alpha^{(h)}_{l,i} y_{t-i} + \sum_{j=0}^{5} \beta^{(h)}_{j,i} \hat{\varepsilon}^{m,i}_{t-j} + u_{t+h,i} \tag{A.2}$$

where $h = 0, \ldots, 48$, $i = 1, \ldots, 227$ and $y'_t = [gdp_t \;\; pi_t \;\; ff_t \;\; ci_t \;\; tr_t \;\; nr_t]$. The estimated coefficient $\hat{\beta}^{(h)}_0$ is the impulse response of the variable of interest at time $t+h$ to the i-th identified shock. For each horizon h, following the same procedure described in Section 4.6, I then compute the median response and the 68% equal-tailed credibility interval by calculating the appropriate percentiles of the set of impulse responses $\{\hat{\beta}^{(h)}_{0,1}, \ldots, \hat{\beta}^{(h)}_{0,227}\}$.

As displayed in Figure A.9, the IRFs estimated with local projections, particularly in the short-run, are very similar to those computed through the VMA representation and discussed in Section 3. An alternative approach could consist in selecting a single element from the set of identified monetary policy shocks (as for instance the median) and estimating the regression in (A.2) only once. When I implement it (by computing the bands with Newey-West robust standard errors), I find analogous results.

B Technical Appendix

In this section, I describe Rubio-Ramìrez et al.'s (2010) algorithm and I provide computational details on the robust Bayesian inference algorithm introduced in Section 6.

B.1 Rubio-Ramìrez et al.'s (2010) Algorithm

In Section 3, I impose sign restrictions through Rubio-Ramìrez et al.'s (2010) algorithm, based on the QR decomposition. For a certain draw of $\omega = (\Sigma_e, A)$ from the posterior distribution of the reduced-form VAR, I iterate the following procedure.

1. Draw from a $\mathcal{N}(0_{k\times 1}, I_{k-1})$ and perform a QR decomposition of the matrix, that delivers a $k \times k$ matrix R with positive diagonal elements and a $k \times k$ orthonormal matrix Q.

2. Let S denote the lower-triangular Cholesky factor of Σ_e. I compute the candidate impulse responses $\hat{\Theta}_h = C_h SQ$, where C_h are the reduced-form impulse responses, for $h = 0, \ldots, h$. If $\hat{\Theta}_h$ satisfy the sign restrictions, I store them. If not, I discard them and go back to the first step.

3. I repeat step 1 and 2 until $M = 100000$ responses are obtained.

Once I obtain 100000 draws, I compute the point-wise posterior median and 68% equal-tailed posterior probability bands at each horizon h.

B.2 Computational Details

The minimization and maximization problems in Section 6 are solved by using the Sequential Quadratic Programming (SQP) algorithm in MATLAB's Optimization Toolbox. Table B.1 provides details about the optimization options used in the implementation of the `fmincon` solver. They are calibrated so as to strike a balance between accuracy and speed of the numerical solver. Note that I obtain very similar results under the 'interior-point' algorithm.

Option	Description	Calibration
`OptimalityTolerance`	Termination tolerance on the first-order optimality measure	1e-6
`ConstraintTolerance`	Tolerance on the constraint violation	1e-6
`MaxFunctionEvaluations`	Maximum number of function evaluations allowed	3000
`MaxIterations`	Maximum number of iterations allowed	1000

Table B.1: Computational details