

Firm Sorting and Spatial Wage Inequality

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EEA, 23 Aug 2022

Outline

1. Introduction

2. Theory

3. Quantification

4. Conclusion

- ▶ Spatial inequality is large and pervasive
 - e.g. Germany West-East 28%, Urban-rural 18%
 - Literature mostly focuses on heterogeneous **workers' spatial sorting** (Diamond '16)

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Firms' spatial sorting \Rightarrow Spatial wage inequality?

What drives spatial sorting of firms?

Firms face trade-off when choosing locations:

- ▶ Local TFP: infrastructure, human capital, productivity spillovers ...
- ▶ Local labor market competition between firms
 - 2/3 of hires are local → segmented local labor market
 - 1/2 of hires are from employment → poaching workers from other firms is frequent
(Burdett, Moretenseon '98, Postel-Vinay, Robin '02)
 - firms' **local** productivity **rank** matters!

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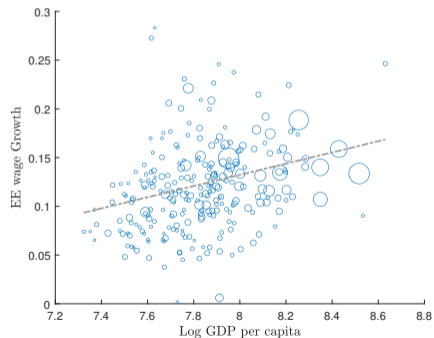
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Trade-off & **heterogeneous firm** ⇒ Spatial **sorting** ⇒ Spatial **wage inequality**

Why are labor market frictions important in understanding spatial wage inequality?

- ▶ Wage growth through EE movements (Job-to-Job) significantly differs
 - This heterogeneity is one of the major driving forces of spatial wage inequality



source: German administrative data LIAB, 2010-2017

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- ▶ Time: continuous
- ▶ Location $\ell \sim R$: TFP $A(\ell)$ ₊
- ▶ Workers: risk-neutral, homogeneous, measure 1 in each ℓ , immobile
- ▶ Firms: type $p \sim Q$ (ex-ante: *before* location choice)
 - choose ℓ paying land price $k(\ell)$
 - productivity $y \sim \Gamma(y|p)$ (ex-post: *after* location choice)₋
- ▶ Production: $z(A(\ell), y)$ ₊ ₊

► Search and matching

- unemployed: meet firms at rate λ^U
- employed: meet firms at rate λ^E (on the job search, EE)
- firms: meet workers at rate λ^F , post a vacancy each period, no capacity constraint
- matching separates exogenously at rate δ

- ▶ Search and matching
 - unemployed: meet firms at rate λ^U
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 - matching separates exogenously at rate δ
- ▶ Wage setting: wage posting with commitment (Burdett, Mortensen 1998)
 - Employed earn $w(y, \ell)$, unemployed earn $b(\ell)$

Focus on steady state. As worker's value is increasing in wage,

- ▶ Unemployed: accept if wage is higher than reservation wage $w^R(\ell)$
- ▶ Employed: accept if wage is higher than the current wage
 - Workers' wages increase as they climb [the local job ladder](#)

Equilibrium: Firm's wage posting

Begin with a firm of prod. y who chose ℓ . Firm posts wages that maximizes profit

- ▶ **Wage posting**: profit per worker vs. firm size (Burdett, Mortensen '98)

$$\tilde{J}(y, \ell) = \max_{w \geq w^R(\ell)} \underbrace{\frac{\lambda^F}{\delta \left(1 + \frac{\lambda^E}{\delta} (1 - F_\ell(w))\right)^2}}_{\text{firm size (+)}} \cdot \underbrace{(z(A(\ell), y) - w)}_{\text{profit per worker (-)}}$$

where F_ℓ is a local rank in the job ladder (wage posting dis.)

- Wage $w(y, \ell)$ is increasing in $y \rightarrow F_\ell(w(y, \ell)) = \text{local productivity rank } \Gamma_\ell(y)$
- ▶ Location ℓ affects profits: local TFA $A(\ell)$ vs. local rank $\Gamma_\ell(y)$ from spatial sorting

Consider spatial sorting of a firm with (ex-ante) type p

- ▶ Firm p chooses ℓ that maximizes the expected value

$$\begin{aligned}\bar{J}(p, \ell) &= \int \tilde{J}(y, \ell) d\Gamma(y|p) - k(\ell) \\ &= \delta \lambda^F \int \int^y \frac{\frac{\partial z(A(\ell), t)}{\partial y}}{[\delta + \lambda^E (1 - \Gamma_\ell(t))]^2} dt d\Gamma(y|p) - k(\ell)\end{aligned}$$

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- ▶ Focus on pure assignment. Then, we have matching function $p = \mu(\ell)$
 - local productivity rank $\Gamma_\ell(y) = \Gamma(y|\mu(\ell))$

Equilibrium: Firm's location choice

▶ Positive assortative matching (PAM) if $\mu'(\ell) > 0$

▶ PAM is an equilibrium if ▶ Proposition

- $\tilde{J}(y, \ell)$ is spm in (y, ℓ)

$$\frac{\partial^2 \tilde{J}(y, \ell)}{\partial y \partial \ell} \stackrel{s}{=} \underbrace{\frac{\partial}{\partial \ell} \log \left(\frac{\partial z(A(\ell), y)}{\partial y} \right)}_{\text{productivity/profit per worker (+)}} + \underbrace{\frac{\partial}{\partial \ell} \log I(y, \ell)}_{\text{Competition/firm size (-)}} > 0$$

- Higher p implies FOSD, i.e. $\frac{\partial \Gamma(y|p)}{\partial p} < 0$

- ▶ Average wage across ℓ :

$$\mathbb{E}[w(y, \ell)|\ell] = \underbrace{w(\underline{y}, \ell)}_{\text{intercept}} + \int_{\underline{y}}^{\bar{y}} \underbrace{\frac{\partial w(y, \ell)}{\partial y}}_{\text{job ladder steepness}} \underbrace{(1 - G_\ell(y))}_{\text{employment composition}} dy$$

where G_ℓ is the steady-state employment distribution

- ▶ Spatial wage inequality:

$$\frac{\partial \mathbb{E}[w(y, \ell)|\ell]}{\partial \ell} \stackrel{s}{=} \underbrace{\frac{\partial w(\underline{y}, \ell)}{\partial \ell}}_{\text{local TFP}} + \int_{\underline{y}}^{\bar{y}} \left(\underbrace{\frac{\partial^2 w(y, \ell)}{\partial y \partial \ell}}_{\text{complementarity, OJS}} (1 - G_\ell(y)) + \frac{\partial w}{\partial y} \underbrace{\left(-\frac{\partial G_\ell(y)}{\partial \ell} \right)}_{\text{spatial sorting}} \right) dy$$

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Goal: how much the spatial sorting of firms explain the West-East wage gap?

▶ Quantitative model

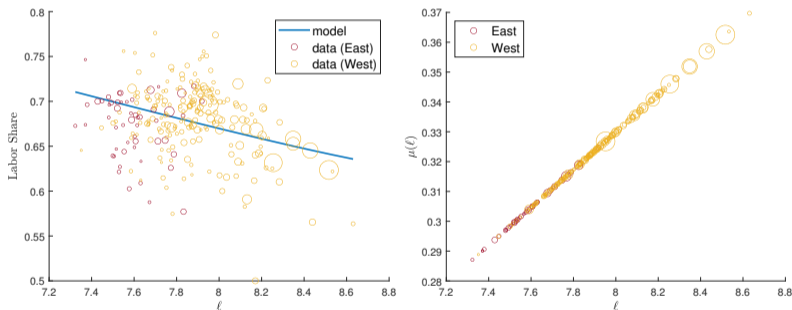
- Worker: demand goods and housing, Local amenity differences
- Free mobility of the unemployed endogenously determine $\lambda^U(\ell)$, $\lambda^E(\ell)$, $\lambda^F(\ell)$ ▶ [Details](#)
- (Exogenous) Heterogeneous $\delta(\ell)$
- Parameterization: (ex-post) productivity $\Gamma(y|p) \sim \text{Pareto}(1, p)$, production $z(A, y) = Ay$

▶ Data

- Regional data from German Federal Statistical Office (2010-2017)
- Linked-Employer-Employee-Data (LIAB) from the Research Data Centre (FDZ)

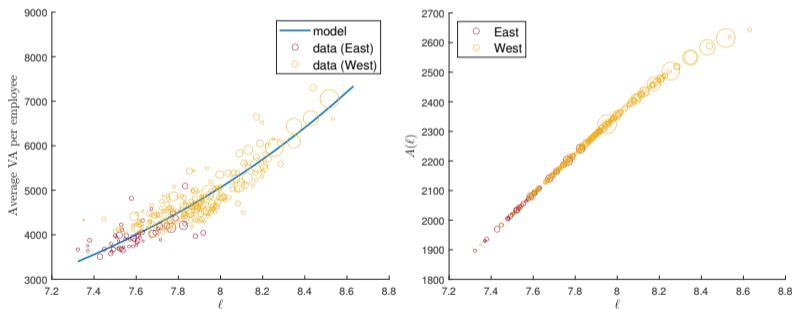
- ▶ Locations ℓ : log GDP per capita
- ▶ Firm (ex-ante) type $\mu(\ell)$: Labor share(ℓ) = $1 - \mu(\ell)$
- ▶ Local TFP $A(\ell)$: average VA (ℓ) = $A(\ell) \cdot$ (contribution from $\mu(\ell)$)

Firm (ex-ante) type $\mu(\ell)$: Labor share(ℓ) = $1 - \mu(\ell)$

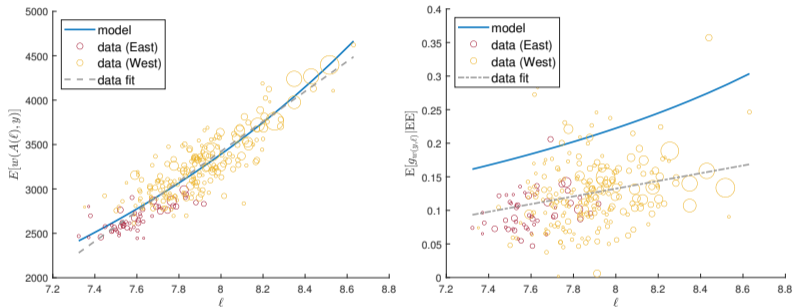


- ▶ cf) labor share pattern: robust to controls/data set ▶ Robustness

Local TFP $A(\ell)$: average $VA(\ell) = A(\ell) \cdot (\text{contribution from } \mu(\ell))$



Estimation: Non-targeted moments



► More results

▶ Wage gap West/East:

	Data	Model	No sorting
West/East	1.278	1.223	1.186

▶ No sorting: random firm allocation across space

- Differences in steepness of job ladders and employment composition [▶ Details](#)
- Firm [sorting explains 16.6%](#) of the West-East wage gap

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- ▶ Firms **positively sort in space**
 - if productivity gain outweighs the costs from local competition particularly for better firms
 - **fuels spatial wage inequality**: job ladder steepness and employment composition
- ▶ Positive firm sorting is **quantitatively important** for spatial wage inequality

Thank You!

- ▶ Value of posting a vacancy: (E_ℓ = a local wage distribution of employed workers)

$$\begin{aligned} \rho \tilde{J}(y, \ell) = & \lambda^F \left(\frac{\lambda^U u(\ell)}{\lambda^U u(\ell) + \lambda^E (1 - u(\ell))} + \frac{\lambda^E (1 - u(\ell))}{\lambda^U u(\ell) + \lambda^E (1 - u(\ell))} E_\ell(w) \right) \\ & \times [z(A(\ell), y) - w(y, \ell) - \delta \tilde{J}(y, \ell) - \lambda^E (1 - F_\ell(w)) \tilde{J}(y, \ell)] \end{aligned}$$

where below equations hold under steady-state

$$\begin{aligned} \delta(1 - u(\ell)) &= u(\ell) \lambda^U \\ u(\ell) \lambda^U F_\ell(w) &= (1 - u(\ell)) (\delta + \lambda^E (1 - F_\ell(w))) E_\ell(w) \end{aligned}$$

Proposition

If $z(A, y)$ is strictly supermodular, and either the productivity gains from sorting into higher ℓ is sufficiently large, or the competition forces are sufficiently small, then there is positive sorting of firms p to location ℓ with $p = \mu(\ell) = Q^{-1}(R(\ell))$

Formally, we require

$$\min_{\ell, y} \frac{\frac{\partial^2 z(A(\ell), y)}{\partial y \partial A(\ell)} \frac{\partial A(\ell)}{\partial \ell}}{\frac{\partial z(A(\ell), y)}{\partial y}} > 2 \frac{\lambda^E}{\delta} \max_{\ell, y} \left(\frac{\partial \Gamma(y | Q^{-1}(R(\ell)))}{\partial p} \right) \frac{r(\ell)}{q(Q^{-1}R(\ell))}$$

where the left hand side is the minimum of the elasticity of firms' marginal product with respect to location and $\frac{\lambda^E}{\delta}$ on the right hand side is the size of competition force.

- ▶ Quantitative model:
 - Cobb-Douglas utility over (tradable) goods and housing with rents $\tilde{r}(\ell)$, (Exogenous)
Local amenity $\tilde{B}(\ell)$, Unemployment subsidy $w^U(\ell)$ from the local government
- ▶ Free mobility & housing market: pin down $L(\ell)$ (and arrival rates)

$$\rho V^U = \tilde{r}(\ell)^{-\omega} \tilde{B}(\ell) \left[b(\ell) + \lambda^U(\ell) \int_{\underline{w}}^{\bar{w}} \max\{V^E(t, \ell), V^U(\ell)\} dF_\ell(t) - V^U(\ell) \right]$$
$$\tilde{r}(\ell) H(\ell) = \omega \left(w^U(\ell) u(\ell) + \mathbb{E}[w(y, \ell)] (1 - u(\ell)) \right) L(\ell)$$

where $H(\ell)$ is housing supply

Estimation: Labor share robustness

Table: Labor Shares

	(1)	(2)	(3)	(4)	(5)
log(GDP per capita)	-0.0625*** (0.0087)	-0.1010*** (0.0211)	-0.0992*** (0.0130)	-0.0794*** (0.0086)	-0.1147*** (0.0236)
Share of employment in industry	N	Y	N	N	Y
Share of branch sizes	N	N	Y	N	Y
log(Population Density)	N	N	N	Y	Y
N	257	257	257	257	257

Notes: Data Source: Statistische Ämter der Länder. All columns are weighted by number of branches (*Niederlassungen*).

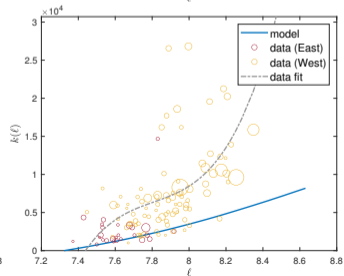
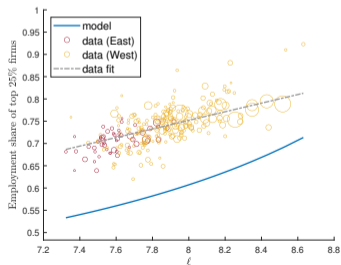
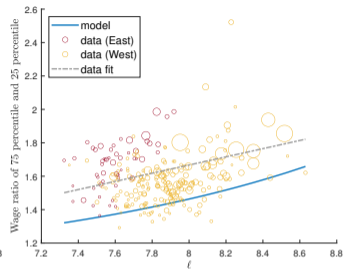
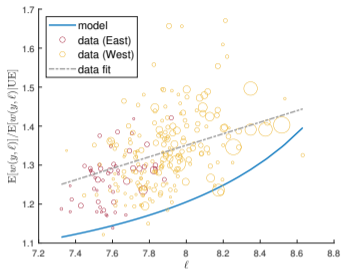
Estimation: Labor share robustness

Table: Labor Shares (FDZ)

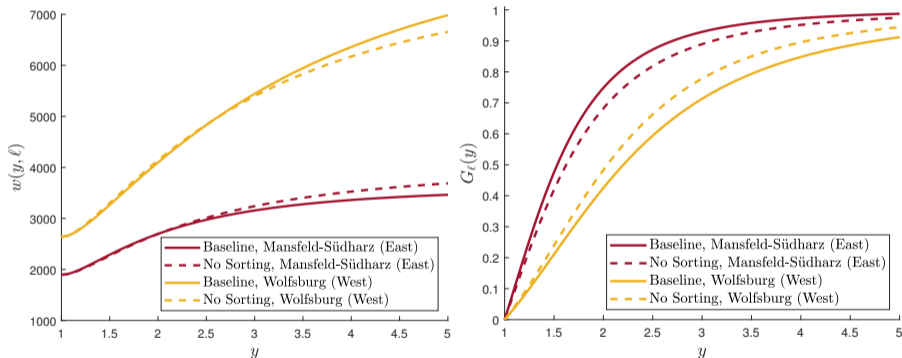
	(1)	(2)	(3)	(4)	(5)
log(VA per FTE)	-0.1291*** (0.0221)	-0.2329*** (0.0348)	-0.1632*** (0.0255)	-0.1475*** (0.0383)	-0.2315*** (0.0354)
Share of employment in industry	N	Y	N	N	Y
Share of branch sizes	N	N	Y	N	Y
log(Population Density)	N	N	N	Y	Y
N	257	257	257	257	257

Notes: Data Source: EP for labor share and value added, BHP for number of firms and employment, Statistische Ämter der Länder for population density. All columns are weighted by number of firms in each commuting zone. Standard errors are in parentheses.

Estimation: Non-targeted moments



- ▶ Compare Wolfsburg (West Top) vs. Mansfeld-Südharz (East Bottom)



job ladder steepness (left) and employment distribution (right)

- ▶ Wage gap Urban-Rural:

	Data	Model	No sorting
Urban/Rural	1.176	1.159	1.132

- ▶ No sorting: random firm allocation across space
- ▶ Firm [sorting explains 17%](#) of the Urban-Rural wage gap