Sufficient Statistics for Nonlinear Tax Systems with General Across-Income Heterogeneity

Antoine Ferey (LMU Munich & CESifo)
Benjamin B. Lockwood (Penn Wharton & NBER)
Dmitry Taubinsky (UC Berkeley & NBER)

Taxation of savings, wealth, bequests, capital

At the forefront of policy discussions. Useful for redistribution?

Atkinson & Stiglitz (1976)

- Should redistribute through an income tax only
- Taxing savings is more distortionary than taxing earnings

Beyond Atkinson & Stiglitz (1976)

- Well-known that A&S does not apply when earnings ability covaries with attributes affecting savings
    - Heterogeneous preferences; but also rates of return, inheritances, shifting between tax bases
- Long literature of special assumptions (e.g., two-type models), modeling aspects of heterogeneity in isolation, qualitative insights (e.g., Saez, 2002), formulas using unobservable primitives
- **Want, need, but don't have:** General sufficient statistics formulas, like Saez (2001) for income tax

This paper: General sufficient statistics formulas

Setting: Standard 2-good model bridging capital and commodity taxation.

Results

1. Optimal unrestricted smooth tax systems
   (i) Can implement optimal direct-revelation mechanism
   (ii) General sufficient statistics characterization of optimal nonlinear tax system...
   (iii) ... including empirically-measurable statistic for across-income heterogeneity
2. "Simpler tax systems" (study three types)
   - Can be characterized using same sufficient statistics and similar techniques
3. Extensions
   - Multidimensional heterogeneity; many dimensions of consumption; corrective motives to encourage more saving; additional efficiency considerations with heterogeneous returns
4. Application to saving and capital taxation in the US economy
   - Estimate progressive optimal tax on savings

Model

Baseline model

Agents

- Heterogeneous ability, preferences, indexed by unidimensional type $\theta \in \mathbb{R}$.
- Preferences: $U(c, s, z; \theta)$
- Numeraire consumption c. Labor earnings z.
- Commodity s, with marginal rate of transformation p.
    - Examples: electricity, education, housing ...
    - Today: savings, where $p = \frac{1}{1+r}$

Policymaker

- Maximizes weighted sum of utilities subject to resource constraint,

$$\max \int_{\Theta} \left\{ \alpha(\theta)\, U\Big(c(\theta), s(\theta), z(\theta); \theta \Big) \right\} dF(\theta)$$

$$\text{s.t.} \int_{\Theta} \Big\{ z(\theta) - c(\theta) - ps(\theta) \Big\} dF(\theta) \geq R$$

Optimal mechanisms and smooth tax systems

Optimal allocation: $\mathcal{A} = \{c(\theta), s(\theta), z(\theta)\}_{\theta \in \Theta}$ subject to individual IC constraints:

$$U(c(\theta), s(\theta), z(\theta); \theta) \geq U(c(\theta'), s(\theta'), z(\theta'); \theta) \quad \forall \theta, \theta'$$

Theorem 1: *Under regularity assumptions and an extended Spence-Mirrlees condition, an optimal allocation can be* ***implemented by a smooth tax function*** $\mathcal{T}(s, z)$**.**

- Why is this new? with smooth $\mathcal{T}(s, z)$, θ can choose bundles not chosen by θ'.
  - Such "double deviations" make implementation theorems much harder!

Sufficient statistics for optimal smooth tax systems

Sufficient statistics for optimal $\mathcal{T}(s, z)$

Individuals' maximization problem is

$$\max_{z} \left\{ \max_{c,s} U(c, s, z; \theta) \text{ s.t. } c + ps \leq z - \mathcal{T}(s, z) \right\},$$

Familiar statistics:

- $\zeta_z^c(z)$: compensated earnings elasticity
- $\zeta_{s|z}^c(z)$: compensated savings elasticity (fixing z)
- $\hat{g}(z)$: social marginal welfare weights augmented with income effects
- $h_z(z)$: income density

Plus a sufficient statistic for local *across-income heterogeneity.*

Sufficient statistic for across-income heterogeneity

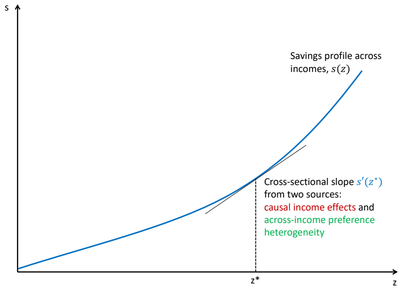
- Let $\vartheta(z)$ be the type choosing earnings z
- Let $s(z;\vartheta(z))$ its s choice with earnings z

- Cross-sectional var = income effect + across-z heterogeneity

$$\underbrace{\left.\frac{ds\left(\tilde{z};\vartheta(\tilde{z})\right)}{d\tilde{z}}\right|_{\tilde{z}=z}}_{s'(z)} = \underbrace{\left.\frac{\partial s\left(\tilde{z};\vartheta(z)\right)}{\partial\tilde{z}}\right|_{\tilde{z}=z}}_{s'_{inc}(z)} + \underbrace{\left.\frac{\partial s\left(z;\vartheta(\tilde{z})\right)}{\partial\tilde{z}}\right|_{\tilde{z}=z}}_{s'_{het}(z)}$$

- $s'_{het}(z)$ is the sufficient statistic for across-z heterogeneity
  - Intuition: When $s'(z)$ driven by $s'_{het}(z)$, s tags ability

- Atkinson-Stiglitz assumptions: $s'_{inc}(z) = s'(z) \implies s'_{het}(z) = 0.$



$s'_{het}(z)$ captures all relevant across-income heterogeneity

$s'(z) - s'_{inc}(z)$ captures all type-specific across-income heterogeneity

- Heterogeneous “prices” (e.g., rates of return)
- Heterogeneous income-shifting abilities/opportunities (e.g., from labor to capital income)
- Heterogeneous endowments (e.g., inheritances)

In every case:

- *Scale*-dependence related to s or z captured by $s'_{inc}(z)$
- *Type*-dependence associated with earnings-ability captured by $s'_{het}(z)$

Note:

- Also captures failures of weak separability (e.g. Corlett, Hague, 1953)

Optimal savings tax rates

Theorem 2:

In an optimal smooth tax system, at each bundle (s, z), marginal savings tax rates satisfy:

$$\frac{\mathcal{T}_s'(s,z)}{1+\mathcal{T}_s'(s,z)} = s_{het}'(z)\frac{1}{s\,\zeta_{s|z}^c(z)}\frac{1}{h_z(z)}\int_{x=z}^{\bar{z}}\Big(1-\hat{g}(x)\Big)\,h_z(x)\,dx$$

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And Pareto efficiency implies

$$\frac{\mathcal{T}_s'(s,z)}{1+\mathcal{T}_s'(s,z)} = s_{het}'(z) \frac{z\,\zeta_z^c(z)}{s\,\zeta_{s|z}^c(z)} \frac{\mathcal{T}_z'(s,z)+s_{inc}'(z)\mathcal{T}_s'(s,z)}{1-\mathcal{T}_z'(s,z)}$$

- Savings tax rate is proportional to local preference heterogeneity $s_{het}'(z)$.
- Atkinson-Stiglitz as a corollary: $s_{het}'(z) = 0 \implies \mathcal{T}_s'(s,z) = 0$.

Optimal earnings tax rates

Theorem 2, continued:

In an optimal smooth tax system, at each bundle (s, z), marginal earnings tax rates satisfy:

$$\frac{\mathcal{T}_z'(s,z)}{1-\mathcal{T}_z'(s,z)} = \frac{1}{z\,\zeta_z^c(z)}\frac{1}{h_z(z)}\int_{x=z}^{\bar{z}}\left(1-\hat{g}(x)\right)h_z(x)\,dx - s_{inc}'(z)\frac{\mathcal{T}_s'(s,z)}{1-\mathcal{T}_z'(s,z)}$$

- Equity-efficiency trade-off, extended with savings responses through $s_{inc}'(z)$.
- Under Atkinson-Stiglitz, $\mathcal{T}_s'(s,z) = 0 \implies$ last term drops out, recover Saez (2001) formula

Simple tax systems

A taxonomy of simple tax systems

Focus on three common functional restrictions on general $\mathcal{T}(s, z)$

Type of tax system	$\mathcal{T}(s, z)$
SL: Separable Linear	$\tau_s s + T_z(z)$
SN: Separable Nonlinear	$T_s(s) + T_z(z)$
LED: Linear Earnings-Dependent	$\tau_s(z) s + T_z(z)$

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Selected examples (more in paper)

Country	Wealth	Capital Gains	Property	Pensions	Inheritance
France	–	Other	Other	SL, SN	SN
Italy	SL, SN	SL	SL	SL	SL, SN
New Zealand	–	Other	SN	SL, LED	–
Norway	SN	SL	SL	SN	–
United States	–	LED	SL	SN	SN

Appendix Props: Conditions where optimal $\mathcal{T}(s,z)$ can be implemented by SN or LED tax system.

Optimal *simple* tax systems

Proposition 2: Optimal simple tax systems satisfy

$$SL: \frac{\tau_s}{1+\tau_s} = \frac{1}{\overline{s\zeta^{c}_{s|z}}} \int_z s'_{het}(z) \left[\int_{x\geq z} (1-\hat{g}(x)) dH_z(x)\right] dz$$

$$SN: \frac{T'_s(s)}{1+T'_s(s)} = \frac{1}{s\,\zeta^{c}_{s|z}(z)} \frac{1}{h_z(z)} s'_{het}(z) \int_{x\geq z} (1-\hat{g}(x))\, dH_z(x)$$

$$LED: \frac{\tau_s(z)}{1+\tau_s(z)} = \frac{1}{s\,\zeta^{c}_{s|z}(z)} \frac{1}{h_z(z)} s'_{het}(z) \int_{x\geq z} (1-\hat{g}(x))\, dH_z(x)$$

Optimal *simple* tax systems

Proposition 2, continued: Pareto-efficient simple tax systems satisfy

$$SL: \frac{\tau_s}{1+\tau_s} = \frac{1}{\bar{s}\overline{\zeta^c_{s|z}}} \int_z s'_{het}(z)\, z\zeta^c_z(z)\, \frac{T'_z(z) + s'_{inc}(z)\tau_s}{1-T'_z(z)}\, dH_z(z)$$

$$SN: \frac{T'_s(s)}{1+T'_s(s)} = s'_{het}(z)\, \frac{z\zeta^c_z(z)}{s\zeta^c_{s|z}(z)}\, \frac{T'_z(z) + s'_{inc}(z)\, T'_s(s)}{1-T'_z(z)}$$

$$LED: \frac{\tau_s(z)}{1+\tau_s(z)} = s'_{het}(z)\, \frac{z\zeta^c_z(z)}{s\zeta^c_{s|z}(z)}\, \frac{T'_z(z) + \tau'_s(z)\, s + s'_{inc}(z)\tau_s(z)}{1-T'_z(z) - \tau'_s(z)\, s}$$

Empirical application

Calibrating a model of savings taxes in the U.S.

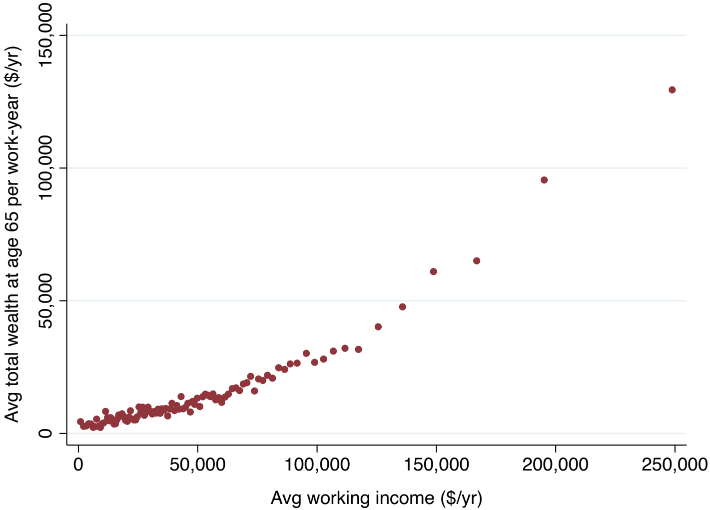
Model and calibration sources

- 2 representative periods: work-life (ages 20-64), and retirement (ages 65+)
- z : annualized labor income during work-life (Piketty, Saez, Zucman, 2018)
- s : annualized retirement savings (Piketty, Saez, Zucman, 2018)
  - housing, business, and financial assets, net of liabilities + pension and life insurance
  - Net-of-tax: avg. tax rates computed using Bricker et al. (2019) asset composition
- $p = \frac{1}{(1+r)^N}$: price of retirement savings, returns compounded N years
  - $r = 3.8\%$ (Fagereng et al. 2020)
- τ_s, $T_s(s)$, $\tau_s(z)$: remap model to report these in 2nd-period dollars. [Details]

Elasticities

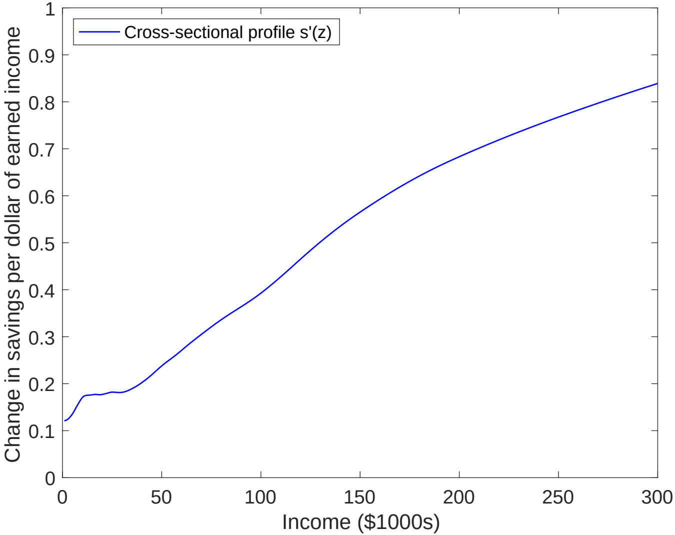
- Earnings elasticity $\zeta_z^c = 0.33$ (Chetty, 2012)
- Savings elasticity $\zeta_{s|z}^c$ between 0.7 and 3, baseline $\zeta_{s|z}^c = 1$ (similar to Golosov et al. 2013)

Input: cross-sectional savings profile $s(z)$



Source: DINA micro-files for the US (Piketty, Saez, Zucman, 2018)

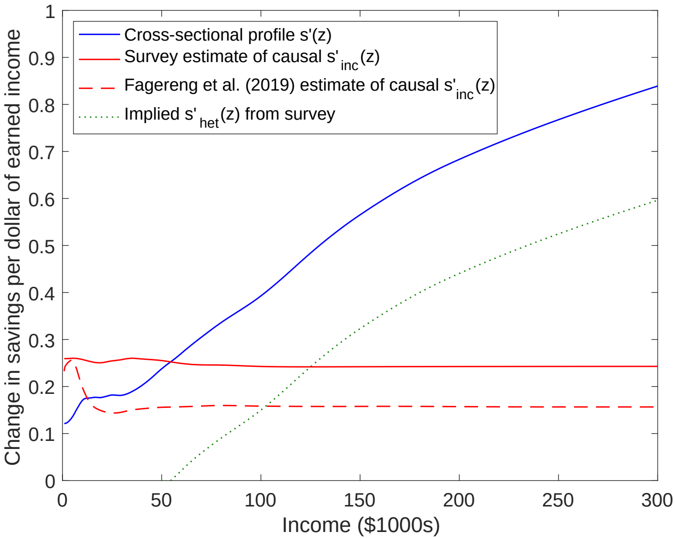
Slope of cross-sectional savings profile $s'(z)$



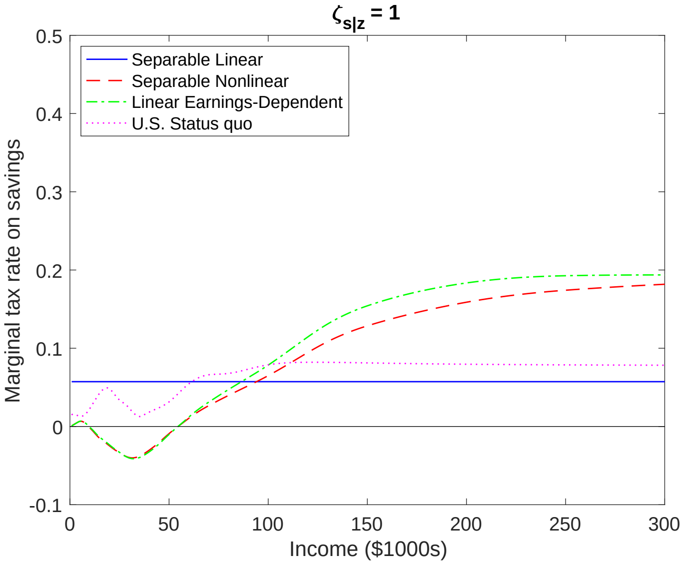
Estimating the causal income effect $s'_{inc}(z)$

1. Fagereng et al. (2020) uses lottery prizes linked with admin data in Norway
   - Estimates 1-year causal MPC of net-of-tax windfall income is 0.52.
   - Estimates a 5-year causal MPC of 0.9, constant across incomes.
   - Imposing that $1 - MPC$ is saved $\implies$ $s'_{inc}(z) = (1 + r)0.1(1 - T'(z))$
2. New survey of US adults about MPS from $1000 increase in earned income
   - Fielded to 1,703 adults through nationally representative AmeriSpeak panel
   - Asks directly about *savings* response to *earned* income.
     (Caveats: hypothetical, short-run.)
   - Average short-run MPS = 0.6, constant across incomes.

$s'_{het}(z) = s'(z) - s'_{inc}(z)$



Implied optimal savings taxes (from Pareto-efficiency)



Conclusion

Conclusion

General and **empirically-grounded** formulas for nonlinear tax systems.

Generality:

- Synthesis of prior work studying aspects of across-income heterogeneity (heterogeneous preferences, prices, endowments, ...) without particularly restrictive assumptions

Empirical grounding and quantitative prescriptions:

- Empirically-oriented guide for optimal tax design accounting for broad forms of heterogeneity, using a relatively small set of sufficient statistics measured in current empirical work

Thank you!

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Appendix

Regularity assumptions

Regularity assumptions on utility

- $U(.)$ is twice continuously differentiable
- Increasing and weakly concave in c and s
- Decreasing and strictly concave in z
- U'_c and U'_s are bounded.

Regularity assumptions for $\mathcal{T}(s, z)$ to implement optimal allocation

Under the optimal incentive-compatible allocation,

- $c(\theta)$, $s(\theta)$, $z(\theta)$ are smooth functions of θ,
- $c(\theta)$ is weakly increasing,
- Any type θ strictly prefers its allocation over any other.

Extended Spence-Mirrlees condition

$$\frac{s'(\theta)}{z'(\theta)} \frac{\partial}{\partial \theta} \left(\frac{U'_s}{U'_c} \right) + \frac{\partial}{\partial \theta} \left(\frac{U'_z}{U'_c} \right) > 0$$

[Back]