The Epistemic Spirit of Divinity

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Introduction

Epistemic approach to forward-induction refinements in signaling games: start from *explicit assumptions about interactive beliefs*. What are the possible outcomes of a signaling game when:

- the outcome is expected by both the sender and the receiver;
- the receiver interprets an unexpected message as follows (if possible):

"the sender is rational and her beliefs about my behavior are in line with the expected outcome distribution and not influenced by her type"

- the sender believes that the receiver interprets messages in this way;
- the receiver believes that the sender believes...

We capture the behavioral implications of these assumptions with a version of Strong Δ-Rationalizability (Battigalli 2003, Battigalli & Siniscalchi 2003)

The independence hypothesis is used to informally justify Divine Equilibrium (Banks & Sobel, 1987) and the weaker notion of Fixed-Equilibrium Rationalizable Outcome (Sobel, Stole & Zapater, 1990), but we obtain intermediate predictions.

Example

m^1	a^1	a^2	a^3
$\overline{\theta}$	**0**, *3*	**4**, *2*	**9**, *0*
$\underline{\theta}$	**0**, *3*	**4**, *2*	**9**, *0*

m^2	a^1	a^2	a^3
$\overline{\theta}$	**-2**, *3*	**2**, *5*	**7**, *3*
$\underline{\theta}$	**-3**, *3*	**1**, *2*	**6**, *0*

m^3	a^1	a^2	a^3
$\overline{\theta}$	**-5**, *3*	**-1**, *5*	**4**, *6*
$\underline{\theta}$	**-8**, *3*	**-4**, *2*	**1**, *0*

A potential employee can choose from 3 education levels: $M = \{m^1, m^2, m^3\}$.

There are two types of employee: $\Theta = \{\overline{\theta}, \underline{\theta}\}$.

Type $\overline{\theta}$ has a lower cost of education... and actually learns!
The employer can hire in three positions with increasing salary: $A = \{a^1, a^2, a^3\}$.

Ideally the employer would like to hire $\underline{\theta}$ in the lowest position and $\overline{\theta}$ in the position she is best qualified for.

Notation: for each $x = 1, 2, 3$, let $M^{\Theta}(m^x)$ denote the set of all $\sigma_1 \in M^{\Theta}$ such that $\sigma_1(\theta) = m^x$ for some $\theta \in \{\overline{\theta}, \underline{\theta}\}$. *[Comment on interpretation.]*

Example - Belief restrictions

Expected outcome distribution: $\overline{\theta}$ and $\underline{\theta}$ are equally likely, in both cases the sender chooses m^1, and the receiver replies with a^1.

Notation: Let p be the uniform distribution on types.

Δ_1: set of all $\mu^1 \in \Delta(A^M)$ such that

$$\mu^1 \left(\left\{ \sigma_2 \in A^M : \sigma_2(m^1) = a^1 \right\} \right) = 1.$$

Δ_2: set of belief systems $\mu_2 = (\mu_2(\cdot|\varnothing), (\mu_2(\cdot|m^x))_{x=1,2,3})$ over $\Theta \times M^\Theta$ s.t.:

- $\mu_2(\cdot|\varnothing) = p \times \delta_{m^1.m^1}$ ($\delta_{m^1.m^1}$ is the Dirac on $m^1.m^1$);
- there exists $\eta \in \Delta(M^\Theta) \setminus \{\delta_{m^1.m^1}\}$ such that, for each $x = 2, 3$,
  - if $\eta(M^\Theta(m^x)) > 0$, $\mu_2(\cdot|m^x)$ is derived by conditioning $p \times \eta$,
  - otherwise, $\mu_2(\cdot|m^x)$ is derived by conditioning $p \times \eta'$ for some other $\eta' \in \Delta(M^\Theta)$.

Example - Step 1: Sender's rationality

m^1	a^1	a^2	a^3
$\overline{\theta}$	0	4	9
$\underline{\theta}$	0	4	9

m^2	a^1	a^2	a^3
$\overline{\theta}$	-2	2	7
$\underline{\theta}$	-3	1	6

m^3	a^1	a^2	a^3
$\overline{\theta}$	-5	-1	4
$\underline{\theta}$	-8	-4	1

Consider the following beliefs in Δ_1: $\delta_{a^1.a^1.a^2}$, $\delta_{a^1.a^2.a^3}$, $\delta_{a^1.a^2.a^2}$.

(i) Under $\delta_{a^1.a^1.a^2}$, m^1 is optimal for both types.
(ii) Under $\delta_{a^1.a^2.a^3}$, m^3 is optimal for both types, and also m^2 is optimal for $\underline{\theta}$.
(iii) Under $\delta_{a^1.a^2.a^2}$, m^2 is optimal for both types.
Σ_1^1: "strategies" $\sigma_1 \in M^\Theta$ such that $\sigma_1(\overline{\theta})$ is optimal for $\overline{\theta}$ and $\sigma_1(\underline{\theta})$ is optimal for $\underline{\theta}$ *given the same* $\mu_1 \in \Delta_1$ (type-independent beliefs); (i)-(ii)-(iii) imply

$$m^1.m^1, m^3.m^2, m^3.m^3, m^2.m^2 \in \Sigma_1^1.$$

Example - Step 2: Receiver's reasoning, and 3: Sender

m^1	a^1	a^2	a^3
$\overline{\theta}$	3	2	0
$\underline{\theta}$	3	2	0

m^2	a^1	a^2	a^3
$\overline{\theta}$	3	5	3
$\underline{\theta}$	3	2	0

m^3	a^1	a^2	a^3
$\overline{\theta}$	3	5	6
$\underline{\theta}$	3	2	0

Receiver's beliefs $\mu_2 \in \Delta_2$ such that (i) $\mu_2(\Theta \times \Sigma_1^1 | \varnothing) = 1$ and (ii) $\mu_2(\Theta \times \Sigma_1^1 | m^x) = 1$ for every $x = 1, 2, 3$ such that $\Sigma_1^1 \cap M^\Theta(m^x) \neq \varnothing$.
(Emptiness if (i) cannot be satisfied! Here it can: $m^1.m^1$ is in Σ_1^1.)
Consider $\mu_2, \mu_2', \mu_2'' \in \Delta_2$ such that:

- $\mu_2(\cdot|m^2)$, $\mu_2(\cdot|m^3)$ are derived from $p \times (\frac{1}{2}\delta_{m^3.m^2} + \frac{1}{2}\delta_{m^3.m^3})$;
- $\mu_2'(\cdot|m^2) = p \times \delta_{m^2.m^2}$ and $\mu_2'(\cdot|m^3)$ is derived from $p \times \delta_{m^2.m^3}$;
- $\mu_2''(\cdot|m^2) = p \times \delta_{m^2.m^2}$ and $\mu_2'(\cdot|m^3) = p \times \delta_{m^3.m^3}$.

They justify, respectively, $a^1.a^1.a^2$, $a^1.a^2.a^3$, $a^1.a^2.a^2 \in \Sigma_2^2$ for the **receiver**.

With this, we can justify at **step 3** all the strategies of the **sender** that survived step 1. The procedure converged: $m^1.m^1, m^3.m^2, m^3.m^3, m^2.m^2 \in \Sigma_1^3 = \Sigma_1^1$.
Conclusion: *the expected behavior is consistent with the hypotheses.*

Comparison with Divine Equilibrium

$(m^1.m^1, a^1)$ is not consistent with Divine Equilibrium.

According to Divine Equilibrium, after m^2, the receiver cannot raise the probability of $\underline{\theta}$ and play a^1, because $\underline{\theta}$ prefers m^2 to m^1 for a smaller set of beliefs than $\overline{\theta}$.

Divine equilibrium analyzes each deviation as if there were no other available deviations.

Moreover, the sender's belief about the reaction of the receiver must coincide with an optimal mixed action of the receiver.

As if the sender were certain of the off-path belief of the receiver
($\rightarrow$ sequential equilibrium)

Our solution concept is weaker than Divine Equilibrium in these two ways.

FERO and complete consistency

The Fixed Equilibrium Rationalizable Outcome (FERO) concept is equivalent to our solution concept in the example, but weaker in general.

The reason is that FERO does not require *complete consistency* (Battigalli, Catonini & Manili, 2021, similar to full-support lexicographic beliefs), while we do (see definition. of Δ_2).

Complete consistency imposes discipline among the theories used to rationalize different messages.

Possible interpretations: deep introspection or a wired-in property of belief formation.

References

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