# Zero-hours Contracts in a Frictional Labor Market 

Juan J. Dolado ${ }^{1}$ Etienne Lalé ${ }^{2}$ Hélène Turon ${ }^{3}$

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[^0]
## Introduction

- The rise of alternative work arrangements, largely sparked by the digitization of the economy, has raised growing concerns about their employment and welfare effects
- Zero-hours contracts (ZHCs) have been under the spotlight, particularly in the U.K., where they have become the focus of a heated debate in the media and political arena
- ZHCs: contracts where employers are not obliged to provide any minimum working hours, and workers are not obliged to accept any work offered
- In this paper, we develop a structural model of ZHCs to assess the impact of these labor contracts on equilibrium allocation and welfare


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- Substitution: some jobs that would be otherwise viable under regular contracts become advertised as ZHCs
- Participation: individuals who prefer flexible work schedules join the labor market to take advantage of ZHCs


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- Participation: individuals who prefer flexible work schedules join the labor market to take advantage of ZHCs
- Calibrate / estimate the structural parameters of the model, and make inference about the heterogeneous types of workers and firms in the U.K. low-wage labor market
- Analyze and quantify the effects of a min. wage raise [not today] and of a ban on ZHCs


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(3) Welfare effects are not obvious, as both unemployment and regular employment increase $\Rightarrow$ Full impact of a ban on ZHCs is a welfare loss by about $0.9-1.1 \%$ in CEV
$\Rightarrow$ Eliminating the substitution effects of ZHCs would increase welfare by $0.2-0.5 \%$ in CEV

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$\Rightarrow$ Eliminating the substitution effects of ZHCs would increase welfare by $0.2-0.5 \%$ in CEV
(4) [Not today] Structural estimates of the willingness to pay (MWP) for shorter working shifts $\Rightarrow$ Wide dispersion in MWPs. The least attached workers on ZHCs would require at least $£ 10.9$ (= 1.45 times the hourly minimum wage) to accept working one extra hour

## Roadmap of presentation

The model

Calibration and inference on workers' and firms' types [Not today]

Policy experiments

Conclusion

The model

## The model

- Time is discrete. Think of a model time period as 2 weeks
$\Rightarrow$ Rapidly changing business conditions, independent across periods
- Economy is populated by heterogeneous workers $i$ and heterogeneous firms $j$
- All agents discount the future at rate $\rho$
- Focus on minimum-wage labor market. No wage dispersion. All jobs pay $w$ $\Rightarrow$ No bargaining. Abstract from compensating differential in wages


## Workers' types

- $N$ : a worker's asset value of being not employed
$-W_{Z}$ and $W_{R}$ : worker's asset values of being employed under resp. $Z$ and $R$ contracts

$$
\begin{cases}N<W_{Z}<W_{R} & \text { for type 1 }  \tag{1}\\ W_{R}<N<W_{Z} & \text { for type 2 } \\ W_{Z}<N<W_{R} & \text { for type 3 } \\ N<W_{R}<W_{Z} & \text { for type 4 }\end{cases}
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Notice that:

- Type-1 and type-4 workers have reasons to search on the job


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- If $Z$ jobs were to be banned, type-2 workers would remain inactive $\Rightarrow$ Participation effect


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- Type-1 and type-4 workers have reasons to search on the job
- If $Z$ jobs were to be banned, type-2 workers would remain inactive $\Rightarrow$ Participation effect
- Rankings must hold in equilibrium: $N, W_{Z}, W_{R}$ are equilibrium objects!


## Firms' types

$\rightarrow V_{Z}$ and $V_{R}$ : Firms' asset values of advertising a vacant position as either a $Z$ or $R$ contract

- Under free entry, value of a firm being inactive is 0 . Firms can be of one of three types:

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\begin{cases}V_{R}<0<V_{Z} & \text { for type } c  \tag{2}\\ 0<V_{R}<V_{Z} & \text { for type } s \\ 0<V_{Z}<V_{R} & \text { for type } r\end{cases}
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Notice that:

- Without ZHCs, type- $c$ firms would abstain from creating any jobs $\Rightarrow$ Job creation effect
- Type-s firms advertise $Z$ contracts but would be viable as $R$ contracts $\Rightarrow$ Substitution effect
- Job creation is somewhat of a misnomer, given that there is free entry of firms


## Workers' preferences

- No saving/borrowing, workers consume all their income
- When not employed, workers receive unemployment benefits $b$
- When working $h$ hours and earning labor income $w h$, workers lose their unemployment benefit at a taper rate $\tau$ :

$$
\begin{equation*}
\operatorname{inc}(h)=\max \{w h, b+(1-\tau) w h\} \tag{3}
\end{equation*}
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- The intra-period utility function is given by:

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u^{i}(h, a)=\frac{\operatorname{inc}(h)^{1-\eta}-1}{1-\eta}-\alpha_{i} \max \{h-a, 0\} \tag{4}
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$\alpha_{i}$ is heterogeneous, even potentially across workers of same type $i$

## Production technology

- $\widetilde{h}$ denotes the number of working hours that would meet the demand that a firm faces at a given point in time
- Deviations between actual hours $h$ and $\widetilde{h}$ are costly (reputation costs, marketing expenses)
- Firms' instantaneous profit function is:

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- Think of ZHCs as contracts that enable firms to set $h=\widetilde{h}$. In $R$ contracts, $h=\bar{h}$


## Search frictions

- Search is random, labor market tightness $\theta$ pins down the contact rate between job seekers and vacancies
- When employed workers choose to search on the job. Their relative search intensity is $x_{i}$
- Exogenous job destruction shock hits firms with probability $\delta$, making firm leave the market
- When a worker quits into another job, the firm remains and re-advertises its job
- Firms choose the contract type at the point of advertising, under full commitment


## Policy experiments

## Ban on ZHCs: Equilibrium allocation

Table: Equilibrium (re)allocation effects of a ban on $Z$ contracts

|  | Share of type- $c$ among $Z$ jobs |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathbf{0 . 0 0}$ | $\mathbf{0 . 2 5}$ | $\mathbf{0 . 5 0}$ | $\mathbf{0 . 7 5}$ | $\mathbf{1 . 0 0}$ |
| Employment rate (in \%) |  | 86.0 | 85.9 | 85.7 | 85.6 | 85.4 |
|  |  | -4.79 | -4.93 | -5.08 | -5.23 | -5.40 |
| Unemployment rate (in \%) |  | 11.2 | 11.4 | 11.5 | 11.7 | 11.8 |
|  |  | 2.03 | 2.17 | 2.33 | 2.48 | 2.65 |
| Duration of $R$ vacancies (in weeks) | 10.5 | 8.0 | 8.0 | 7.9 | 7.8 | 7.8 |
| Net output (1 = baseline) |  | -2.47 | -2.53 | -2.59 | -2.66 | -2.73 |
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- Important role for job creation even when substitution effects are largest (i.e., when $\gamma_{c}=0$ )


## Ban on ZHCs: Accession to regular employment

Change in regular employment, $\widetilde{e}_{R} / e_{R}$, depends on three channels:

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\begin{equation*}
\frac{\tilde{e}_{R}}{e_{R}}=\underbrace{\frac{\lambda(\tilde{\theta})}{\lambda(\theta)}}_{\text {job creation }} \times \underbrace{\frac{1}{v_{R} / v}}_{\text {vacancy competition }} \times \underbrace{\frac{\tilde{n}_{1}}{(1-\delta) x e_{1, Z}+n_{1}}}_{\text {search efficiency }} . \tag{6}
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- $\lambda(\widetilde{\theta}) / \lambda(\theta)=71 \%$, meaning that lower job creation would reduce regular employment by almost $30 \%$ ceteris paribus

Reduction in vacancy competition in isolation from the other effects would increase regular employment by $24 \%$

- Additional search efficiency units for employment following a ban on ZHCs would increase regular employment by $15 \%$ ceteris paribus


## Ban on ZHCs: Welfare impact

- What is the overall impact of the ban on time spent out of regular employment?
- After the policy reform, this duration is given by the duration of unemployment spells
- Prior to the ban, this is the duration that type-1 workers spent in unemployment as well as in $Z$ waiting to eventually transit to $R$ employment

Difference is readily measured in our model, as

$$
\begin{equation*}
\triangle=\frac{\omega_{1}}{\lambda(\widetilde{\theta}) \widetilde{n}_{1}}-\frac{\omega_{1}}{\lambda(\theta) \frac{v_{R}}{v}\left(x(1-\delta) e_{1, Z}+n_{1}\right)} \approx-7 \text { weeks } \tag{7}
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$\rightarrow$ Even though the unemployment rate increases after a ban on ZHCs, type-1 workers spend on average more time in regular employment

## Ban on ZHCs: Welfare impact

Table: Welfare (in \% of CEV) effects of a ban on $Z$ contracts

|  | Share of type- $c$ among $Z$ jobs |  |  |  |  |  |
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|  |  |  |  |  |  |  |
|  |  | $\mathbf{0 . 2 5}$ | $\mathbf{0 . 5 0}$ | $\mathbf{0 . 7 5}$ | $\mathbf{1 . 0 0}$ |  |
| At 1st percentile of $\alpha$ |  | -0.93 | -0.99 | -1.06 | -1.12 | -1.15 |
| At 25th percentile of $\alpha$ |  | -0.96 | -1.02 | -1.08 | -1.13 | -1.14 |
| At 50th percentile of $\alpha$ |  | -0.99 | -1.04 | -1.09 | -1.15 | -1.13 |
| At 75th percentile of $\alpha$ | 0.00 | -1.02 | -1.07 | -1.11 | -1.16 | -1.13 |
| At 99th percentile of $\alpha$ | 0.00 | -1.05 | -1.09 | -1.13 | -1.17 | -1.12 |
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- Still, the consequences of more unemployment is that welfare decreases by 0.9-1.1\%


## Ban on ZHCs: Welfare impact

Table: Welfare (in \% of CEV) effects of a ban on $Z$ contracts: The substitution channel

|  | Bhaseline of type- $c$ among $Z$ jobs |  |  |  |  |  |
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|  |  | $\mathbf{0 . 0 0}$ | $\mathbf{0 . 2 5}$ | $\mathbf{0 . 5 0}$ | $\mathbf{0 . 7 5}$ | $\mathbf{1 . 0 0}$ |
| At 1st percentile of $\alpha$ |  | 0.54 | 0.54 | 0.53 | 0.53 | 0.52 |
| At 25th percentile of $\alpha$ |  | 0.45 | 0.45 | 0.45 | 0.44 | 0.44 |
| At 50th percentile of $\alpha$ |  | 0.37 | 0.36 | 0.36 | 0.36 | 0.35 |
| At 75th percentile of $\alpha$ |  | 0.28 | 0.28 | 0.27 | 0.27 | 0.27 |
| At 99th percentile of $\alpha$ | 0.00 | 0.20 | 0.19 | 0.19 | 0.19 | 0.18 |
|  |  |  |  |  |  |  |

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|  | Baseline | $\mathbf{0 . 0 0}$ | $\mathbf{0 . 2 5}$ | $\mathbf{0 . 5 0}$ | $\mathbf{0 . 7 5}$ | $\mathbf{1 . 0 0}$ |
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- Not negligible. May explain the seemingly paradoxical responses to the spread of ZHCs
- Importance of a general equilibrium analysis that accounts for the other forces that come into play (Job creation, Labor force participation)


## Conclusion

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Our findings suggest that:

- Most workers employed under ZHCs would prefer having a regular contract
- At the same time, and in line with related studies, there is a substantial willingness to pay for flexible work schedules in some segments of the (potential) labor force
- There are productive opportunities in sectors facing highly volatile demand which may not be viable without the ability to adjust working hours at no cost
- Identifying such segments of economic activity requires the availability of richer data on firms' profitability and workers' time use and preferences

Appendix

## Related literature

- Growing literature (mostly empirical) on understanding alternative work arrangements
- Measuring the trends and characteristics of workers: Abraham et al. [2019, 2021], Katz and Krueger [2019], Collins et al. [2019], Boeri et al. [2020], etc.
- Mas and Pallais [2017]: Experiment with workers in a U.S. national call center to measure different MWPs:
- Workers willing to give up $8 \%$ of wages for the option to work from home
- Willing to pay $20 \%$ to avoid a schedule set by an employer on short notice
- Datta et al. [2019] document how the 2016 rise in the U.K. minimum wage has resulted in increased usage of ZHCs in the social care and related low-wage sectors
- Only few attempts to analyze flexible hours contracts in a structural model
- Scarfe [2020]: search-matching model of "casual work" in Australia (where casual work accounts for $10 \%$ of the labor force)
- Frazier [2017]: directed-search model with two types of jobs, one with fixed hours, the other with variable hours. Estimated with data on the U.S. retail sector


## Bellman equations

- Workers' asset values depend on their own type $i$ and contract $k$
- These asset values solve:

$$
\begin{equation*}
N^{i}=u_{N}+\frac{1}{1+\rho}\left[(1-\lambda(\theta)) N^{i}+\lambda(\theta) \sum_{k^{\prime}} \frac{v_{k^{\prime}}}{v} \max \left\{N^{i}, W_{k^{\prime}}^{i}\right\}\right], \tag{8}
\end{equation*}
$$

and

$$
\begin{equation*}
W_{k}^{i}=u_{k}^{i}+\frac{1}{1+\rho}\left[\delta N^{i}+(1-\delta)\left(\left(1-x_{i} \lambda(\theta)\right) W_{k}^{i}+x_{i} \lambda(\theta) \sum_{k^{\prime}} \frac{v_{k^{\prime}}}{v} \max \left\{W_{k}^{i}, W_{k^{\prime}}^{i}\right\}\right)\right] \tag{9}
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- Flow values of employment $u_{k}^{i}$ in (9) depend on the equilibrium mix of firm types:

$$
\begin{equation*}
u_{Z}^{i}=\frac{e_{c}}{e_{Z}} \int u^{i}(\widetilde{h}, a) d G_{c}(\widetilde{h})+\frac{e_{s}}{e_{Z}} \int u^{i}(\widetilde{h}, a) d G_{s}(\widetilde{h}) \text { and } u_{R}^{i}=u^{i}(\widetilde{h}, a) \tag{10}
\end{equation*}
$$

## Bellman equations

- Firms of a given type $j$ compare the asset value $V_{k}^{j}$ of holding a vacant position advertised as a contract $k=Z, R$
- These depend on $J_{i, k}^{j}$, which is the asset value of filling the position with a worker of type $i$
- These asset values solve:

$$
\begin{equation*}
V_{k}^{j}=-\kappa+\frac{1}{1+\rho}\left[V_{k}^{j}+\frac{\lambda(\theta)}{\theta} \sum_{i} \frac{\left.n_{i} \mathbb{1}_{\left\{W_{k}^{i}>N^{i}\right\}}+\sum_{j^{\prime}} x_{i} e_{i, j^{\prime}} \mathbb{1}_{\left\{W_{k}^{i}>W_{k\left(j^{\prime}\right)}^{i}\right\}}\right\}}{n+\sum_{i^{\prime}} x_{i^{\prime}} e_{i^{\prime}}}\left(J_{i, k}^{j}-V_{k}^{j}\right)\right] \tag{11}
\end{equation*}
$$

## Bellman equations

- Firms of a given type $j$ compare the asset value $V_{k}^{j}$ of holding a vacant position advertised as a contract $k=Z, R$
- These depend on $J_{i, k}^{j}$, which is the asset value of filling the position with a worker of type $i$
- These asset values solve:

$$
\begin{equation*}
V_{k}^{j}=-\kappa+\frac{1}{1+\rho}\left[V_{k}^{j}+\frac{\lambda(\theta)}{\theta} \sum_{i} \frac{\left.n_{i} \mathbb{1}_{\left\{W_{k}^{i}>N^{i}\right\}}+\sum_{j^{\prime}} x_{i} e_{i, j^{\prime}} \mathbb{1}_{\left\{W_{k}^{i}>W_{k\left(j^{\prime}\right)}^{i}\right\}}\right\}}{n+\sum_{i^{\prime}} x_{i^{\prime}} e_{i^{\prime}}}\left(J_{i, k}^{j}-V_{k}^{j}\right)\right] \tag{11}
\end{equation*}
$$

$>$ Notice the role $\mathbb{1}_{\{.\}}$: The worker's decision to accept the job depends on:

- The contract $k$ offered by the firm
- The worker's own current labor market status and preferred employment contract


## Bellman equations

- For filled jobs, the asset values solve:

$$
\begin{equation*}
J_{i, k}^{j}=\pi_{k}^{j}+\frac{1-\delta}{1+\rho}\left[V_{k}^{j}+\left(1-x_{i} \lambda(\theta) \sum_{k^{\prime}} \frac{v_{k^{\prime}}}{v} \mathbb{1}_{\left\{W_{k^{\prime}}^{i}>W_{k}^{i}\right\}}\right)\left(J_{i, k}^{j}-V_{k}^{j}\right)\right] \tag{12}
\end{equation*}
$$

- Probability of job continuation depends on the equilibrium offers from other firms (through $v_{k^{\prime}} / v$ ) and on workers' preferences over those offers
- The flow values of employing a worker under contract $k$ are:

$$
\begin{equation*}
\pi_{Z}^{j}=\int \pi(\widetilde{h}, \widetilde{h}) d G_{j}(\widetilde{h}) \text { and } \pi_{R}^{j}=\int \pi(\bar{h}, \widetilde{h}) d G_{j}(\widetilde{h}) \tag{13}
\end{equation*}
$$

## Job creation

- Firms pay a business creation cost $K$ to enter the market, and then draw their type $j$ from a distribution $\left(\gamma_{j}\right)_{j=c, s, r}$
- In equilibrium, these types must be consistent with firms' ranking of posted contracts
- As a result, under free entry, we have

$$
\begin{equation*}
K=\gamma_{c} \cdot V_{Z}^{c}+\gamma_{s} \cdot V_{Z}^{s}+\gamma_{r} \cdot V_{R}^{r} \tag{14}
\end{equation*}
$$

- Market tightness $\theta$, which is the ratio between $v$ and $n+\sum_{i} x_{i} e_{i}$, adjusts to satisfy Eq. (14)


## Steady-state equilibrium

A steady-state equilibrium is a list of asset values $N^{i}, W_{k}^{i}, V_{k}^{j}, J_{i, k}^{j}$; a stationary distribution of job matches $e_{i, j}$, non-employed workers $n_{i}$ and vacancies $v_{j}$; and labor market tightness $\theta$ such that:

1. Given the measures $e_{i, j}, n_{i}, v_{j}$, and market tightness $\theta$, the asset values $N^{i}, W_{k}^{i}, V_{k}^{j}, J_{i, k}^{j}$ solve the Bellman equations (8), (9), (11), (12)
2. Given $N^{i}, W_{k}^{i}$, worker types satisfy the rankings presented in (1); given $V_{k}^{j}, J_{i, k}^{j}$ firm types satisfy the rankings presented in (2)
3. Given $V_{k}^{j}$, where $k=Z, R$ is the contract offered by type- $j$ firms, market tightness $\theta$ solves the free entry condition in Equation (14)
4. Given market tightness $\theta$, the measures $e_{i, j}, n_{i}, v_{j}$ are time-invariant with respect to the law of motion of the economy

## Vacancy elasticity of the matching function

- Newly-formed matches $\left(M_{o, t}\right)$, unemployment claims $\left(U_{o, t}\right)$ and job vacancies $\left(V_{o, t}\right)$

Occupations $o$ : ('Administrative', 'Secretarial and related', 'Caring personal service', 'Leisure and other personal service',
'Process, plant and machine', 'Elementary trades, plant and storage related', 'Elementary administration and service')

- Run a linear regression:

$$
\begin{equation*}
\log \left(\frac{M_{o, t}}{U_{o, t}}\right)=\alpha_{o}+\sigma^{\prime} g(t)+\psi \log \left(\frac{V_{o, t}}{U_{o, t}}\right)+\varepsilon_{o, t} \tag{15}
\end{equation*}
$$

Table: Vacancy elasticity of the matching function

|  | Log- job finding $\left(\log \left(M_{o, t} / U_{o, t}\right)\right)$ |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Log- market tightness $\left(\log \left(V_{o, t} / U_{o, t}\right)\right)$ | $0.643^{* * *}$ | $0.701^{* * *}$ | $0.586^{* * *}$ | $0.703^{* * *}$ |
|  | $(0.027)$ | $(0.041)$ | $(0.025)$ | $(0.034)$ |
| R-squared | 0.859 | 0.896 | 0.802 | 0.871 |
| Time trend $(g(t))$ |  | $\boldsymbol{V}$ |  | $\boldsymbol{V}$ |
| Occupation fixed effect $\left(\alpha_{o}\right)$ |  |  | $\boldsymbol{V}$ | $\boldsymbol{\checkmark}$ |

## Parameters set externally

- Model period is 2 weeks. $\rho=0.0015$ to yield an annual discount rate of 4 percent [To ease interpretation, several parameter values are expressed in weekly values]
- $w=£ 7.50$ per hour (U.K. national minimum wage for workers aged 25 and over)
- Cobb-Douglas matching function:

$$
\begin{equation*}
m(s, v)=M v^{\psi} s^{1-\psi} \tag{16}
\end{equation*}
$$

where $v=$ vacancies and $s=$ job seekers weighted by their search intensity

- $\psi$ can be an important parameter for the job creation effects predicted by the model. Using U.K. data for the low-pay segment of the labor market, we estimate $\psi=0.65$


## First-step calibration parameters

- Set $M, \theta, \delta, x$, the $\omega_{i}$ 's and $\gamma_{r}$ to match data moments on job and worker turnover

Identification (somewhat heuristic):

- Either $M$ or $\theta \rightarrow$ transition out of $N, \gamma_{r} \rightarrow$ whether $N$ to $R$ as opposed to transition $N$ to $Z$
- $\delta, x$, the $\omega_{i}$ 's $\rightarrow$ transitions out of $Z$ and $R$, with $\omega_{i}$ 's identified by the distribution of job tenure within each contract type
- No type-4 workers (virtually no $R$ to $Z$ transitions). Rule out type- 3 workers as $\omega_{3} \rightarrow 0$
- We obtain the sum $\gamma_{c}+\gamma_{s}$, but not $\gamma_{c}$ and $\gamma_{s}$ separately from each other


## First-step calibration parameters

Table: Description of baseline equilibrium

|  |  | Model | Data |
| :---: | :--- | :---: | :---: |
| $n$ | Unemployment rate | 9.2 | 10.1 |
| $e_{Z} /\left(e_{Z}+e_{R}\right)$ | Employment share of ZHCs | 6.5 | 7.2 |
| $v_{Z} /\left(v_{Z}+v_{R}\right)$ | Vacancy share of ZHCs | 19.4 | - |
| $e_{1, Z} / e_{1}$ | Share of employed type-1 workers in ZHCs | 4.8 | - |
| $e_{1, Z} / e_{Z}$ | Share of filled ZHCs employing type-1 workers | 66.8 | - |

- Type-1 workers are key to sustain an equilibrium with ZHC jobs
- ZHCs exert a negative effect on $R$ vacancies by making it more difficult for these vacancies to contact type-1 workers (given that search is random)


## First-step calibration parameters



Figure: Model fit: Job tenure by labor contract

## Second-step calibration parameters

In the second step, we calibrate $p, \bar{h}, \phi, \kappa, K$, and the stochastic distributions $H_{j}($.$) for each type j$
> $p=8.25$, assuming that the marginal productivity of low-pay workers is $10 \%$ higher than $w$

- No direct empirical counterpart for $\phi$, but $(\phi, \kappa, K)$ pinned down by free-entry condition:
- Expected cost of vacancy posting (i.e. $\kappa \theta / \lambda(\theta)$ ) of $14 \%$ of average quarterly labor earnings (Elsby and Michaels [2013])
- Startup costs of creating a business, $K$, at around $£ 4,500$
- For job creation condition to hold, we find $\phi=0.16$
$\Rightarrow$ Deviating from $\tilde{h}$ by 5 hours $\downarrow$ firms' weekly accounting profits $((p-w) h)$ by $10 \%$


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- For job creation condition to hold, we find $\phi=0.16$ $\Rightarrow$ Deviating from $\tilde{h}$ by 5 hours $\downarrow$ firms' weekly accounting profits $((p-w) h)$ by $10 \%$
- $H_{j}($.$) 's are Beta distributions over [0,50]$, where $50=$ maximum weekly hours worked
- Choose $\mu_{j}$ and $\sigma_{j}$ within regions of the parameter space that are consistent with firm types, and given the $\neq$ between $R$ and $Z$ jobs documented in Section 3


## Parameter values

Table: Parameter values

| (a) Parameters set externally |  |  |
| :---: | :---: | :---: |
| $\rho$ | Discount rate of 4 percent per annum | 0.0015 |
| $\psi$ | Elasticity of job-filling rate w.r.t. tightness | 0.65 |
| $w$ | Minimum hourly wage in $£$ (U.K. policies) | 7.50 |
| (b) First stage calibration parameters |  |  |
| M | Matching function elasticity | 0.1278 |
| $\theta$ | Labor market tightness | 0.2418 |
| $\delta$ | Job destruction probability | 0.0047 |
| $x$ | On-the-job search efficiency | 0.3524 |
| $\omega_{1}$ | Share of type-1 workers | 0.9689 |
| $\gamma_{r}$ | Probability of type-r firms upon entry | 0.9498 |
| (c) Second stage calibration parameters |  |  |
| $p$ | Productivity of hours worked | 8.25 |
| $\left(\mu_{c}, \mu_{s}, \mu_{r}\right)$ | Average of weekly hours by firm type | $(18,18,28)$ |
| $\left(\sigma_{c}, \sigma_{s}, \sigma_{r}\right)$ | St. dev. of weekly hours by firm type | $(6,3,2)$ |
| $\phi$ | Marginal cost of deviating from targeted hours | 0.16 |
| $\kappa$ | Flow cost of vacancy posting, in $£$ per week | 38.0 |
| K | Startup cost of new businesses, in $£ 1,000$ | 4.38 |

## Heterogeneous firms' types



Figure: Firms' types across (some) regions of the parameter space

## Heterogeneous workers' types



Figure: Workers' types across (some) regions of the parameter space

## Heterogeneous workers' types

- Type-1 workers are individuals who do not value short hours much: at $a=22$ hours would give up $£ 0.5$ to $£ 4.6$ of consumption (per week) to avoid working one hour beyond $a$
- Type-2 workers have a higher valuation of short hours. They would be willing to give up at least $£ 10.9$, or 1.45 times the minimum wage, to avoid working one hour beyond $a$

Table: Parameter values for welfare assessment

| $\eta$ | Relative risk aversion coefficient | 2.0 |
| :---: | :--- | :---: |
| $b$ | Unemployment benefits in $£$ per week (U.K. policies) | 148.5 |
| $\tau$ | Taper rate (U.K. policies) | 0.63 |
| $a$ | Available hours per week | 22.0 |


[^0]:    ${ }^{1}$ Universidad Carlos III de Madrid, CEPR and IZA
    ${ }^{2}$ Université du Québec à Montréal, CIRANO and IZA
    ${ }^{3}$ University of Bristol and IZA

